Defending the Structural Concept of Representation

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ABSTRACT: The aim of this paper is to defend the structural concept of representation, as defined by homomorphisms, against its main objections, namely: logical objections, the objection from misrepresentation, the objection from failing necessity, and the copy theory objection. The logical objections can be met by reserving the relation 'to be homomorphic to' for the explication of *potential* representation (or, of the representational content). *Actual* reference objects ('targets') of representations are determined by (intentional or causal) representational mechanisms. Appealing to the independence of the dimensions of 'content' and 'target' also helps to see how the structural concept can cope with misrepresentation. Finally, I argue that homomorphic representations are not necessarily 'copies' of their representanda, and thus can convey scientific insight.

Key words: (structural concept of) representation, homomorphism, content.

1. Introduction

What is the essence of representation? This question has motivated a lively debate within both philosophy of science and the cognitive sciences. Discussions of this issue center on two questions. First, is the concept of representation appropriate and useful for the study of cognitive processes? Second, can representation, in general, be understood as a transfer of structure, from some original domain to some representing domain? The focus of this paper is on the second question. The structural concept of representation, as I call it in this paper, has been advocated by Mundy (1986), Watson (1995), Swoyer (1991) and French (2003), but resolutely rejected by Goodman (1976), Scholz (1991), Grush (1995), Hughes (1997), Suárez (2003, 2004), Bailer-Jones (2003) and others. Generally, those who reject the structural concept of representation do so for conceptual, not empirical, reasons (an exception is Grush (1995)). In what follows, I shall defend the structural concept of representation by demonstrating that the conceptual objections can be refuted. After introducing the structural concept of representation (section 2), I shall challenge the main objections against the structural concept of representation: logical objections (section 3), the objection from misrepresentation (section 4), the objection from failing necessity (section 5), and the copy theory objection (section 6).

2. The Structural Concept of Representation

The structural concept of representation claims that something, *B*, can *represent* something, *A*, only if some structure of the represented domain *A* is transferred to its image *B*. To make this idea more precise, *A* (the domain to be represented) and *B* (the domain representing A) are described by *similar*[1] *relational structures*. A relational structure is given by a set, on which one- up to n-place relations $R_1^A \dots R_m^A$ (respectively $R_1^B \dots R_m^B$) are defined. Now a mapping $f : A \rightarrow B$ can be defined which maps A *onto* B. The mapping f is not necessarily one-to-one and satisfies two following conditions:

(i) For all j and all elements a_i of A: if $R_j^B(f(a_1), \dots, f(a_n))$, then $R_j^A(a_1, \dots, a_n)$

Condition (i) requires that for all relations R_j^B, if some images $f(a_1) \dots f(a_n)$ of the arguments $a_1, \dots, a_n$ under f satisfy the relation, then the arguments also satisfy the corresponding relation R_j^A on A. If that is the case, then f is called a *faithful* mapping of A onto B. Representations should be modeled by means of faithful mappings. Otherwise there may be facts in the representing domain to which there are no corresponding facts in the represented domain.

The second condition is that the facts in B give *complete* information about facts in A, that is, for every fact in A there must be a corresponding (representing) fact in B.

(ii) For all j and all elements a_i of A: if $R_j^A(a_1, \dots, a_n)$, then $R_j^B(f(a_1), \dots, f(a_n))$

If (i) and (ii) are fulfilled, f is a homomorphism from A onto B, and B, by virtue of the existence of f, can be said to be an *homomorphic image* of A (Dunn and Hardegree 2001, 15). The structural concept of representation claims that B represents A only if B is a homomorphic image of A. (In the following, if B is an homomorphic image of A, I will say that 'A is homomorphic to B').

In section 3, I shall discuss a differentiation concerning 'B *represents* A' that turns out to be a necessary reaction to the logical objections against the structural concept of representation. I shall then introduce two independent components of the relation of representation: the representational *content* and the *target* of the representation. Accordingly, 'B represents A' can either mean 'A is a part of the representational *content* of B' or 'A is the target (reference object) of B'. Understood in the first sense, 'B represents A' is to be explained by the relation of A being homomorphic to B. However, the component of the relation of representation in the second sense cannot be understood by means of homomorphisms; I shall explain this in more detail in section 3.

In section 6, I shall discuss the reasons for employing homomorphism rather than isomorphism for modelling the representation relation. (Indeed, the failure to distinguish between homomorphism and isomorphism may be the main source of the copy-theory misunderstanding with respect to homomorphisms). For now, I shall merely cite the example given by Dunn and Hardegree (2001) for the case of homomorphism. They note that a photographic image (a clear case of representation) 'is not isomorphic to its subject, even in the ideal, for at least the following reasons: (1) the image is two-dimensional, whereas the subject is three-dimensional; (2) the image depicts

[1] Two relational structures A and B are similar, if they are of the same type, that is, if all corresponding relations on A and B have the same number of arguments (see Dunn and Hardegree 2001, 10).

only the surface of the subject, whereas the subject presumably has inner detail, not conveyed by the image; (3) the image may be in black and white, whereas the subject is presumably 'in color' (Dunn and Hardegree 2001, 15). As the example shows, to take isomorphisms as the core of representation would fail even with regard to some very common instances of representation.

On the other hand, it has to be admitted that 'homomorphism' is a very general notion, which has to be filled out by some specific types of mappings to model concrete cases of representation. The objects of homomorphic representations can, for instance, be perceptual objects. A specific type of relations, defined on the representing domain, are geometrical structures (distance structures on vector spaces) as they are used by Peter Gärdenfors to model conceptual representations (see Gärdenfors 2000), or mereological structures, as they may be useful to model non-conceptual representations. In all these cases, different concrete types of relations specify the relations defining the relational structures, but the claim in all the cases is that structures are transferred from the represented to the representing domain.

Homomorphisms, as defined above, describe an idealized case. The conditions that hold for homomorphisms can be weakened so as to fit the cases in which representations do not work perfectly. This can happen with respect to two criteria: faithfulness and completeness.

The faithfulness that is required in the definition of homomorphisms is in a sense 'absolute', as the satisfaction of the corresponding relation in the represented domain A is required with respect to *all* ur-images of $f(a_1), \dots, f(a_n)$. (Since f is not necessarily one-to-one, there can be more than one ur-image.). The absolute condition of faithfulness for the information the representation provides about the represented domain may be weakened to the restricted notion of '*minimal fidelity*'. This weaker notion only requires that *there are* ur-images which satisfy the corresponding relation defined on A.[2] Compared to absolute faithfulness, this notion ensures that for every fact in B there is a corresponding fact in A. If a representational mechanism fulfils only minimal fidelity, the representation may lead to false expectations concerning facts in the represented domain A. For instance, the visual system of an organism indicates directions of stimuli in the visual field only up to a range of fuzzyness; the representational mechanism then produces *non-exact* representations (as it is to be expected for representational mechanisms in the biological world).

Non-exact representations blur some of the fine grained differences existing in the represented domain. In other words, their representational content does not reflect those differences. In order to be able to describe the representational content even in those cases by means of the transduced homomorphic structure, we have to 'adapt' the represented domain A by identifying all arguments in A which are mapped to the same element of B by the function f; thereby the old arguments of f are replaced by new arguments which are equivalence classes of old arguments (the equivalence relation being the relation of 'being mapped to the same element of B'). By this identifica-

[2] This condition is called 'minimal fidelity' by Dunn and Hardegree 2001, 17.

tion procedure the description of representational content by means of transduced homomorphic structure can be restored. In the extreme, the representation blurs all differences existing in A. Then the representation has degenerated into a detector of A-like events.

The conditions that hold for homomorphisms can also be weakened with regard to *completeness*. In the ideal case, facts about A are preserved by facts about B 'for all j', i.e. for all relations that are defined on A. In less ideal cases, there may only exist *some relations* for which this preservation holds. In one type of weakening the homomorphism can be trivially eliminated by cutting off from the relational structure A all relations for which the homomorphism-condition is not satisfied.[3] A second type of weakening allows a representation to represent a certain property or relation only for a *limited range of arguments*.[4] For example, the representational capacity can be limited to a certain region of stimuli from the environment. The visual system of an organism, for instance, is sensitive only for a certain range of wave lengths.

The fewer relations for which the transfer of structure holds, and the fewer the number of elements of A to which the transfer is restricted, the poorer the representation will be with respect to content. In an extreme case, no content will be left.

3. Logical Objections

Many critics of the structural concept of representation think that the concept, first of all, fails to meet the most obvious logical conditions for defining representation. One difficulty evaluating those objections is that the critics' attacks are directed against what they call the 'similarity theory of representation' (Goodman 1976) or the 'isomorphism conception of representation' (cf. Scholz 1991,[5] Suárez 2003, 2004). Simi-

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[3] In the same vein, Swoyer (1991, 470f.) discusses cases of representations in which the mapping between the represented and the representing domain 'does not respect all of the relations in the original system, but only some'. One of his examples is the two-dimensional projection of a sphere that 'cannot depict all of its features without distortion, so when we use flat maps to represent the Earth, something has to give'. The structural theory of representation can be accommodated to those cases, according to Swoyer, by restricting the operation of the representing function to the respected relations (Swoyer 1991, 472). French and da Costa (2003) have coined the notion of a 'partial isomorphism' to describe representation relations, which are restricted in their scope to a certain substructure of a given structured domain. For instance, the representation provided by the billiard ball model for gases, as described by the kinetic theory of gases, is a representation "in certain respects and to certain degrees" (French and da Costa 2003, 49), and therefore has to be described by partial isomorphisms operating on partial structures.

[4] This sort of weakening of the homomorhism conditions is discussed by Swoyer (1991, 470/71): 'In some cases of representation, relations are respected only under certain conditions (e.g., boundary conditions). For example, a mercury thermometer may reliably represent the temperature if it is neither too hot nor too cold, but it would fare poorly in liquid helium or near the surface of the sun.'

[5] According to Scholz, isomorphisms not only lack adequate logical properties for explaining the representation relation, but also is in danger of trivialization (see Scholz 1991, 59). According to this argument, it is possible to define an isomorphism between arbitrary complexes. Now, if my television set, by means of its being isomorphic to my body, were a representation of my body, the whole structural

larity, or isomorphism, it is argued, lack adequate logical properties for explaining representation relations, because (i) representation relations are *non-reflexive*, whereas similarity (and isomorphism) are reflexive, and (ii) representation relations are non-*symmetric*, whereas similarity (and isomorphism) are symmetric. Since 'to be homomorphic to' is non-symmetric, only objection (i) can also be raised automatically against the homomorphism theory of representation. Nevertheless, objection (i) suffices to show that in general, for *A* to be represented by *B*, it would not be *sufficient* that *A* is homomorphic to *B*. There has to be some additional component of the representation relation, which prevents that each object, endowed with a relational structure, represents *itself*.

The *locus classicus* for logical objections against the structural concept of representation is Goodman's *Languages of Art*.

> The most naive view of representation might perhaps be put somewhat like this: "A represents B if and only if A appreciably resembles B", or "A represents B to the extent that A resembles B". Vestiges of this view, with assorted refinements, persist in most writing on representation. Yet more error could hardly be compressed into so short a formula. Some of the faults are obvious enough. An object resembles itself to the maximum degree but rarely represents itself; resemblance, unlike representation, is reflexive. Again, unlike representation, resemblance is symmetric: B is as much like A as A is like B, but while a painting may represent the Duke of Wellington, the Duke doesn't represent the painting. Furthermore, in many cases neither one of a pair of very like objects represents the other: none of the automobiles off an assembly line is a picture of any of the rest; and a man is not normally a representation of another man, even his twin brother. Plainly, resemblance in any degree is no sufficient condition for representation. (Goodman 1976, 3-4)

Goodman makes three points: the reflexivity and symmetry objections, and he notes that objects resembling each other do not necessarily also *represent* each other. Goodman's criticism is, with the exception of the symmetry objection (ii), also appropriate with respect to the homomorphism theory of representation. The proponent of the homomorphism theory has to admit that the extension of 'to represent' is at most a *proper subset* of the extension of the relation 'to be homomorphic to', and he is obliged to explain *why* the extensions do not coincide.

In order to explain why the extensions do not coincide, I shall introduce the distinction between *potential* representations and *actual* representations. *B* is a potential representation of *A*, if *B* can be used to correctly represent *A*, given the existence of some representational *mechanism* connecting *A* with *B*. I will say, then, that *A* is part of the representational content of *B*. For example, one can use a road map to correctly

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similarity theory would be trivialized. The force of this argument is not very strong. Of course, it will be possible to identify the same number of parts in both of the complexes, the television set and my body, and it will also be possible to impose relations on these complexes such that there is a one-to-one mapping between them, preserving the relations. For instance, simply take the relation of 'being identical with' for both complexes. Now the claim is that my television set would have to be accepted as a representation of my body with respect to this relation. I have no problem with admitting this, since this merely means that there exist rather uninteresting forms of representation. What makes a representation interesting is that the preserved relations tell us something important about the object to be represented with respect to the representational context.

represent one's way home, if one intentionally takes a certain red curve on the map to stand for the highway which one has to pass etc. Since the road map is endowed with the relevant structure, it entails a potential representation of his or her way home that can be exploited by means of an intentional representational mechanism. Thus, I shall claim that *A* being homomorphic to *B* is sufficient for *A* to be potentially represented by *B*, i.e. that the extensions of the relations 'to be homomorphic to' and 'to represent potentially' coincide. In order for *B* to be also a correct *actual* representation of *A*, *A* has to be selected as the *target* of the representation from the set of objects potentially represented by *B* (i.e., from the content of *B*) by some representational mechanism connecting *A* with *B*. If there is a representational mechanism connecting *A* with *B*, but *B* is not a potential representation of *A*, then *B misrepresents A*.

The most important sorts of representational mechanisms are *representational intentions* and *causal relations*. The existence (or non-existence) of representational mechanisms of these sorts explains how we get (or fail to get) actual representations out of potential ones. For example, a chair, considered as a structured object formed by its parts, is homomorphic to itself, and therefore potentially represents itself. But in this case a representational intention has to occur, in order to turn this potential representation into an actual representation. The chair usually does not represent anything, but serves a certain purpose. On the other hand, if you find the chair being an exhibit in an art exhibition, then the idea is not too far fetched that the chair might be an (actual) representation of something. Now if you ask what kind of thing the chair represents, it may turn out that the artist intends, by placing the chair in a certain place or way, that the chair (reflexively) represents *itself*, in order to suggest to the visitor of the exhibition that a thing is not necessarily what it is most of the time, namely something made for a certain purpose, but that it can also be seen as 'standing for itself'. The intention of the artist figures as a representational mechanism turning a potential representation into an actual representation. Thus, Goodman was right to insist that, as a matter of fact, 'an object rarely represents itself'. But we now see that this does not count against a structural concept of representation. Instead, it brings to our attention the requirement that to make something an *actual* representation of itself representational mechanisms are needed, and that this requirement is rarely satisfied.

Causal representational mechanisms are exemplified by photography. A photograph may be a potential representation of my son, and *vice versa*. But it is an actual representation only in one direction, because there is a one-way causal process connecting the light rays emanating from my son's body and resulting in the photography, but not the other way round. Nevertheless, there may be 'irregular' contexts, in which representational intentions do not follow the regular causal direction. Again, Goodman is right in insisting 'the Duke doesn't represent the painting.' But in some odd contexts, the Duke could nevertheless be seen as representing the painting of the Duke. Imagine, for example, an impoverished Duke who is now forced to imitate the painting on fairs. Perhaps the painting has become famous during the Duke's personal decline. Whereas in the common context of portrait painting only pairs of the relation are used to instantiate actual representations in which the first element is a person, and the second a picture, a pair with the reversed order fulfils the actual representation relation in this strange example.

Causal or intentional representational mechanisms also determine *how the relational structures* that are related by the homomorphism are defined: this includes the identification of the elements of the base set and the relations that are seen as relevant for being represented with respect to the context in question. I refer to all this as the *pragmatic* conditions of actual representations (see Bailer-Jones 2003). My concern, in this paper however, is potential representation as the necessary condition of correct actual representation. It was my aim in this section to show that the standard logical objections against the structural concept of representation don't have the force to exclude a homomorphism conception of potential representation. The structural properties of an object determine what the object potentially represents. If these representational resources are exploited by intentional users or by causal processes, then actual representations emerge.

4. The Objection from Misrepresentation

A second general objection against the homomorphism theory of representation is that homomorphisms do not allow for misrepresentations. Misrepresentation is a common empirical phenomenon, thus no concept of representation will be empirically adequate without being able to explain it. What is more, permitting misrepresentation is a condition of *conceptual adequacy*, since the very concept of representation presupposes the possibility of a distinction between the case in which some X *misrepresents* some Y and the case, in which X does *not* represent Y at all.

Why are homomorphisms perceived to be unable to fulfill this condition? The reason is that a homomorphism between relational structures A and B either exists or does not exist; in the first case, B represents A, whereas in the second case B does *not* represent A. What would it mean for B to represent A, but *incorrectly*?

Contrary to first appearances, the homomorphism theory does not have problems to allow for misrepresentation. If B represents A, then B *refers* to A. There is also a *content* of that representation which is not necessarily identical with its reference. B misrepresents A just in case B refers to A but the representational content does not entail A. Intuitively this means that B is about A, but does not match A in what it says about A. Problems with misrepresentation arise because some theories of representation do not have the resources to identify reference and content independently. If and only if reference and content are conceptually identified do we lack resources to explain misrepresentation. The following demonstrates that this is not the case for the homomorphism theory:

(a) According to the homomorphism theory the reference of B is A iff A is the target of B, which is determined by a representational mechanism.[6]

[6] The notion of a 'target' follows Cummins (1996). But whereas in Cummins' sense, the target is determined by the intended use a certain representational state is supposed to serve, I assume that the ref-

(b) The content of B entails A iff A is homomorphic to B.

Hence reference and content do not coincide from which it follows that misrepresentation is possible

This shows that the impression of the theories' inability to cope with misrepresentation arises from the wrong assumption that by the existence of a homomorphism between A and B the *reference* of B would be determined. This assumption cannot be right, simply because no unique reference object for B can be determined on the basis of the property of being homomorphic to B. Therefore, the determination of a reference object for B has to be explained independently of the homomorphism theory. The reference of B is fixed by a representational mechanism, i.e. either by an intentional or a causal process (cf. section 3). In contrast, the representational *content* of B is determined by B's structural properties, i.e. by the relational structure B is endowed with. This relational structure of B determines what objects (which are themselves relational structures) are homomorphic to B, i.e. it determines the set of objects that are *potentially represented* by B (cf. section 3). Thus, the representational content of B is identical with the set of objects that are potentially represented by B. Reference objects for B will be determined independently of the content of B. If a reference object for B is chosen by a representational mechanism out of the set of objects potentially represented by B, then B will correctly represent this object. If a reference object for B is chosen which does not belong to this set, then this reference object will be *misrepresented* by B. Thus, the case in which something A is misrepresented by B and the case in which A is *not* represented by B (i.e. A is not a reference object of B) are clearly distinct. This means that the homomorphism theory of representation has the resources to explain misrepresentation.

A nice example of the ability of the homomorphism theory to explain misrepresentation is Cummins' (1996) example of a chess computer. The calculations of the chess computer are intended (by the constructor) to result in a certain position, in response to the moves of the computer's opponent (this position is the *target*, or the *reference object*), but —by some failure in the computer's architecture— the computer performs by indicating a different position. The relevant structure of this different position is the *representational content* and the appearance of a difference between the two positions means that a misrepresentation has occurred (see Cummins 1996, 5f.).

5. The Objection that Homomorphism is not Necessary

Even if it is accepted that the structural concept is able to overcome both the logical and the misrepresentation objections, this may only mean that *some* representations may exhibit homomorphic structure. But perhaps homomorphisms are not necessary for representations? Indeed, most critics (e.g. Scholz 1991, Suárez 2004) argue that homomorphism is neither sufficient nor necessary for representation. Instead of going

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erence could also be fixed causally. I do not intend in this paper to give any reasons that could favour one theory of reference against some other theory.

into the details of their argumentation, with regard to this objection, I prefer to consider two types of phenomena that are, as far as I can see, the most obvious paradigms of representation that allegedly work without homomorphism. In the first case, the result will be that representations are involved, but (contrary to first appearances) invoke homomorphisms, whereas in the second case, it will turn out that no representations are involved. The first paradigm type is detectors that have no internal structure.

There are both natural and artificial systems using some detector systems in order to represent certain conditions occurring in their environment, but the detectors cannot be interpreted as being homomorphically related to the represented condition. Examples are the detection of the direction of the magnetic field by the magnetosomes of sea bacteria, or neuronal systems in the human retina that are able to detect certain directions of moved stimuli occuring in the visual field (see Goldstein 1996, 274). The representation,[7] in the latter example, is performed by 'yes'- or 'no'-answers which are produced by the neuronal system corresponding to whether the stimulus occurs within a given range of spatial directions or not.

The answer given by the detector system depends on whether the direction of the stimulus occurring in the visual field is such as to generate neuronal inputs which are transduced through the system up to the central neuron. The firing of the central neuron then means a 'yes'-answer of the system with regard to the stimulus. In that case, the system represents the corresponding type of stimuli. If the inputs inhibit each other, the signal is cancelled out with the result that no firing of the central neuron occurs. This means a 'no'-answer of the system. It is crucial for the answer of the system, in which direction the stimulus moves through the visual field. The direction determines, in what temporal succession the neuronal inputs enter the detector system, and the succession determines whether the inhibiting neuronal connections are able to cancel out the signal or not. For each direction, there is an optimal detector, such that to stimuli in that direction it is maximally responsive, whereas stimuli moving in the opposite direction are completely ignored by the detector.

Whether a certain type of stimuli is represented by the neuronal system, depends on whether its direction fits the *internal structure* of the system. The detector system is like a lock in that a key (the direction in which the stimulus moves) fits or not. Since the internal neuronal structure of the detector explains its representational performance, the internal structure of some detector may be seen as determining its representational *content*. But, there is no internal structure of some single stimulus. Thus, a 'yes'-answer of the system corresponding to that stimulus cannot be explained by the existence of a homomorphism between the stimulus (described as a relational structure) and the neuronal system. This means that a single neuronal detector *does not represent* a single stimulus by means of the stimulus being *homomorphic* to the detector. The

[7] Neuronal detectors of direction stimuli are typical examples of representations described in studies of human and animal visual systems. I will presuppose that such examples have to be understood as representations by any serious theory of representation.

single neuronal detector has no representational content in the sense of the homomorphism theory.

Representation occurs in this case in the first instance not between the single concrete entities but between the stimulus space S and the detector-space D, by means of a homomorphism d operating between the space of all types of stimuli (each defined by a certain direction a) and the corresponding detectors $d(a)$. The stimulus-types are related by rotations with respect to the horizontal plane through the retina, and the same applies to the corresponding detector systems. Thus, the homomorphism d preserves the connection structure of mappings (rotations) o on S, since every mapping on S corresponds to a mapping o^* on D (which is a rotation of detectors):[8]

For rotations r_1 and r_2 on S: $d(r_1 \, o \, r_2) = d(r_1) \, o^* \, d(r_2)$.

The representation that holds in the first instance between the spaces induces a representation in a derivative sense also between single stimuli and single detectors by means of their occupying corresponding places in the homomorphic mapping structures.

The second paradigm case of alleged representations that work without homomorphisms are intentional denotations by arbitrary signs, be they pencils representing kings in a child's play, coins standing for soccer players used to demonstrate a possible move, or what ever. Arbitrary signs *denoting* objects, phenomena, situations, or types of behavior, are often held to be *the* paradigmatic instances of representation. Indeed, an arbitrary sign has something of a representation, insofar as some intentional representational mechanism (a decision about its denotation) has assigned it to a reference object. But an arbitrary sign cannot *misrepresent*. The reference object to which the sign has been assigned by an intentional act cannot *fail* to belong to the set of objects potentially represented by the sign, that is, it cannot fail to belong to the representational content of the sign. The reason is that the sign simply *has* no representational content. In order to have representational content conceptually independent of its reference object, there would need to be properties of the sign delimiting the set of objects the sign could be correctly used to represent. Since, by definition, no property of an arbitrary sign has any representational relevance (beside the denotation act applied to it), arbitrary signs cannot misrepresent. Since, for any conception of representation, permitting misrepresentation is a condition of *conceptual adequacy* (cf. section 4), arbitrary signs, contrary to first appearances, are not representations.

Even Nelson Goodman, supposedly one of the main advocates of the denotation view, has actually been very sceptical about denotation as a means of representation.

[8] Representations that work by transducing the connection structure of the represented domain to the representing domain are discussed by Ibarra and Mormann (2000) under the notion of *homology*. Contrary to what is claimed by Suárez (2004, 769), the homology theory explains representation as structural relation between the represented and the representing domain; both, the represented and the representing domain are conceived of as algebraic structures, namely as group structures of morphisms, and the relation between them is 'structural', since it is a homomorphism operating on the group structures.

True, Goodman argues that '[d]enotation is the core of representation and is independent of resemblance' (Goodman 1976, 5). This means that the *reference* of a representation cannot be determined by similarity, but has to be conceived as a primitive relation. On the other hand, he notes that representations, for instance paintings, do have a *content* that is independent of the denotation (reference). Now, the fact that paintings can have content but not necessarily denotation (for example, a painting of a unicorn does not refer to an actual object) makes things difficult for a pure denotation view: 'a picture must denote a man to represent him, but need not denote anything to be a man representation' (Goodman 1976, 25).

From considerations like these it follows that denotation is not even a *necessary* component in the explanation of representational content. Thus, the ability of something to represent, in the sense of having representational content, has nothing to do with its denotation. It is simply contingent, whether a painting has a denotation or not, although the painting has representational content in any case. Goodman is very explicit in some places in *Languages of Art* that (in cases, where denotation is not null) denotation alone does not suffice to make something a representation of something else. After all, an officer may use the paintings of a museum which has been occupied by his unit to denote the positions held by the enemy. In cases like that, no representation appears. In order to represent, the *content* of a painting has to relate to its denotation in some adequate way, such that 'what is denoted depends solely upon the pictorial properties of the symbol' (Goodman 1976, 42).

If representation could be constituted merely by denotation, we would get a concept of representation so weak that it would not be possible, for example, to explain interesting abilities of organisms like the successful homing behavior of desert ants by means of their ability to represent their own movements in their environment (see Gallistel 1990, 59f.). True, representations denote what they represent, but to understand how an organism performs well using a certain representational system we have to consider the specific *contents* of the representation and how they relate to its reference objects. Content is a necessary component of representation, and homomorphisms are necessary to explain this necessary component.

6. *The Copy Theory Objection*

Finally, I want to reject the common, but misguided view that the homomorphism theory is nothing more than a (more precise) version of the similarity theory of representation. My impression is that this misconception builds the core of the bundle of arguments against the theory that I have discussed before.

In section 3 it was noted that 'to be homomorphic to' is not a symmetric relation. Therefore, this relation is not a 'similarity'. Similarities are reflexive and symmetric. Since the notion of a 'similarity theory of representation' has never been made very clear, many (and often misplaced) connotations can invade that notion. One of the most popular connotations is that of a *copy theory*. The following short remarks are intended to show how the homomorphism theory is distinguished from a copy theory of representation:

It is a very common phenomenon that classes of objects be used to represent a given domain, for which no *analogous intrinsic relations* exist. For example, the natural numbers are represented by decimal symbols, although finite sequences of decimal symbols do not possess any intrinsic structure similar to the relation of addition defined on the natural numbers. In this case, we *extrinsically* endow the system of decimal symbols with the desired structure by imposing rules of calculation, in order to enable them to represent the addition structure of the natural numbers (such that the decimal representation of the sum of two natural numbers equals the sum of the decimal representations).

The representing objects very often are not simply there, but have to be constructed together with their relational structure for a certain representational use. In such cases, only when those objects have been constructed, the representandum can be identified with some part of the newly built object class (it is then said that the representandum has been *embedded* into the new domain). The representing domain is then not a 'copy' of the original domain in a twofold sense: firstly, because the representing domain is the result of a construction, and secondly, because the representing domain includes the original domain as a subdomain. For an example see Carnap's construction of 'quality classes' as representations (logical reconstructions) of intuitive perceptual qualities (Carnap 1998, 98). In this case, neither the representandum, namely the qualities, nor the entities which represent them, are simply 'given'. The qualities exist only in the sense that it is a common way of speaking to refer to 'qualities' as 'parts' of elementary experiences. This common way of speaking is, from the phenomenal perspective, incorrect. From such a perspective, elementary experiences do not have any parts. This incorrect mode of speech, according to Carnap, has to be replaced by a correct explication of 'qualities' by means of a logical reconstruction of quality classes. Thus, the quality classes themselves are clearly not 'given' before the representation procedure starts.

Carnap's guideline for the construction of quality classes is that the classes have to fulfill a certain structural characterization: a quality class counts as an adequate reconstruction of a certain quality if the elementary experiences belonging to this quality class relate to each other in the same way as elementary experiences relate to each other when they 'contain' that quality. Quality classes exemplify qualities, that is, they are taken as models of the common expression 'some two elementary experiences contain the same quality'. Whereas 'copying' of something does not lead to any new knowledge about that something, the construction of new objects —given the constraint of structure preservation, as explained above— may improve our knowledge with respect to both, its precision and its scope.

These cases demonstrate how representations can be generated by homomorphisms, although they are not 'copies'. The decimal representation does not copy the additive structure of the natural numbers, and Carnap's quality classes do not copy the qualities of our phenomenal experience. As I hope, the reader will be convinced by now that the most common conceptual objections against the structural concept of representation are misplaced. They seem to originate from a common source, the copy theory misunderstanding. As this misunderstanding is swept away in the light of the notion of homomorphism, a more promising pursuit with regard to the structural theory is waiting: the pursuit of its empirical prospects.

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