

“Etchemendy, Tarski, and Logical Consequence”¹

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Abstract: John Etchemendy (1990) has argued that Tarski's definition of logical consequence fails as an adequate philosophical analysis. Since then, Greg Ray (1996) has defended Tarski's analysis against Etchemendy's criticisms. Here, I'll argue that—even given Ray's defense of Tarski's definition—we may nevertheless lay claim to the conditional conclusion that 'if' Tarski intended a conceptual analysis of logical consequence, 'then' it fails as such. Secondly, I'll give some reasons to think that Tarski 'did' intend a conceptual analysis of logical consequence.

1. Introduction

Tarski's definition of logical consequence has clearly stood the test of time, as far as definitions in philosophy go.² But recently John Etchemendy has put forth a meticulous evaluation and thorough criticism of Tarski's definition.³ Among the numerous shortcomings Etchemendy reveals, he argues forcefully that either (i) Tarski's definition gets wrong the extension of our concept of consequence⁴ or (ii) the definition gets the extension right, but for the wrong reasons. Now, if Etchemendy is right, then Tarski's definition fails as an adequate conceptual analysis of logical consequence. This is because a good conceptual analysis has not only to get the extension of the concept right, but it must further provide the basis for an intuitive explanation of why it has just the extension it does have. However, not everyone this is a fair criticism of Tarski's definition. Namely, Greg Ray attempts a vindication of Tarski's definition to the effect that (i) *in fact* (if not necessarily) it gets the extension right, and (ii) that it gets the extension right for the “wrong reasons” is no criticism of Tarski's definition because it simply is not a piece of conceptual analysis.⁵

Here, I intend to side with Etchemendy. That is to say, I will show that, despite Ray's considerations, we may still lay claim to the result that if Tarski's definition gets the extension right, then it gets it for the wrong reasons. So, *if* Tarski intended to give a conceptual analysis of logical consequence, he was unsuccessful. Moreover, I will attempt to undermine Ray's motivation for thinking that Tarski did not intend a conceptual analysis of consequence. To do so is, in effect, to reinstate Etchemendy's conclusion that Tarski's definition is unsatisfactory. I turn now to a concise recapitulation of the debate up to now.

2. Tarski's Definition

Tarski, in his 1963 essay, put forth the first model-theoretic account of the concept of logical consequence, having found various other definitions inadequate.⁶ We can state Tarski's definition of logical consequence as follows:

For any sentence S , and set of sentences, K , S is a logical consequence of K in case every sequence that satisfies K also satisfies S .⁷

This is to say that a sentence, S , is a logical set of sentences, K , if, and only if, every sequence that satisfies also the sentential function, S' , of the sentence, S .⁸

Now, as it happens, Etchemendy's criticisms of Tarski's definition are not criticisms of the definition of logical consequence *as such*, but are rather criticisms of Tarski's definition of logical truth—which Tarski does not give in his 1936 essay on consequence. But, surely Tarski's model-theoretic approach to logical consequences has also to work for the other logical properties. And, since we can (and do) think of logical truths as following logically from sets of sentences that are empty, the following definition of logical truth suggests itself naturally:

For any sentence, S , S is logically true just in case every sequence satisfies S .

3. Etchemendy's objection

Etchemendy's main problem with Tarski's definition of logical truth is that it relies on a reduction of logical truth to truth *simpliciter*. To see this more perspicuously, notice that we can replace our talk of *satisfaction by a sequence* with talk of *truth in an interpretation*. Thus, a sentence, S , is logically true, just in case it is true in all interpretations. According to Etchemendy, this is just to say that a sentence, S , is logically true just in case the *universal closure* of S' is true—where the *universal closure* of S' is the result of universally quantifying over the variables in the sentential function, S' , of S (96).⁹ Thus, Tarski must be implicitly relying on the following principle, which Etchemendy terms “the reduction principle”:

(R) If the universal closure of a sentential function (in which only logical terms are constant) is true, then all of its instances are logically true. (cf. Etchemendy 110)

Etchemendy argues that if Tarski's account explicates logical truth according to (R), then the account fails for two reasons: First, the account gets wrong the extension of

our ordinary concept of logical truth; second, even assuming that it gets the extension right, the definition gives the wrong account of why that is. Etchemendy finds that sentences concerning the size of the universe can be seen to derail Tarski's definition of logical truth for these reasons. Consider the following sequences of sentences:

- α_2 : There are fewer than 2 things in the universe $\neg\exists x\exists yx\neq y$
- α_3 : There are fewer than 3 things in the universe $\neg\exists x\exists y\exists z(x\neq y \wedge y\neq z \wedge x\neq z)$
- .
- .
- .

Now, none of these is a logical truth, intuitively. However, according to the reduction principle, if any of these is such that the universal closure of its sentential function is true, then it *is* a logical truth.¹⁰ But, where we treat the existential quantifier as ranging over all subcollections of the universe, the universal closure of α_2 ' says that every subset of the universe contains fewer than two individuals. Now, obviously, this ununiversal generalization is false; that is, some subcollections of the universe contain at least two members. But, if the universe is finite, then somewhere in the sequence of sentences above we'll find a logical truth. That is, suppose the universe has exactly n individuals in it. In that case, the universal closures of α_{n+1} , α_{n+2} , etc. will all be true. This means that α_{n+1} , α_{n+2} , etc. are all logically true, according to the reduction principle. However, recall, intuitively, *none* of these is a logical truth. Thus, if the universe is finite, then (R) gets wrong the extension of our concept of logical truth.

However, as it happens, the universe is finite. And, in that case, all of the universal closures we would get from the sentences above are all false. Thus, [®] does not place any of the α -sentences in the extension of 'logical truth.' This, as we said earlier, is the *right* result. But, surely these sentences do not fail to be logically true *because of the size of the universe*—as Tarski's account seems to tell us. Rather, as Etchemendy suggests, none of these is a logical truth because none of them is true by virtue of the meanings of the logical constants. To drive this point home, Etchemendy claims that *had* the universe been finite (i.e., *had* some other substantive feature of the world been different), then these sentences *would have been* classified as logical truths, on Tarski's definition. The problem is this: Instead of giving us the right account of why these sentences are not logical truths, Tarski's definition adverts to *logically* contingent (though perhaps *metaphysically* necessary) facts about the size of the world to explain their status.¹¹

4. Ray's defense

Ray's defense of Tarski's definition, like Etchemendy's objection to it, is two-pronged. On the one hand, Ray maintains that Tarski's definition correctly classifies the α -sentences as other than logical truths. This is because, as a matter of fact, the universe is infinite. So, as it pertains to the sentences we have been considering, Tarski's definition gets the extension of 'logical truth' right. On the other hand, Ray argues that, given its extensional adequacy, no further criticism of Tarski's definition is warranted. This is the most complicated defense, and so it is the one on which I will continue most of my efforts. (The first defense—namely, that Tarski's definition gets right the extension of 'logical truth'—is one that can easily be granted, for convenience, without at all giving up Etchemendy's conclusion.)

Recall that Etchemendy thinks that Tarski's definition attains to extensional adequacy on pain of getting wrong something else crucial to analysis. Namely, Etchemendy thinks that where Tarski's definition gets the extension right, it does so for the wrong reasons. This is so because the verdicts it yields (especially as they pertain to the α -sentences) are influenced by non-logical, substantive facts about the world. To illustrate this point further, Etchemendy says the following.

If the universe is infinite, none of these α -sentences will be declared logically true. But is that because the account has captured our ordinary notion of logical truth? After all, these sentences are not *in fact* logically true, but neither would they *be* logically true if the universe were finite. Yet according to [Tarski's] account, the sentence [α_n is not logically true]...only because there are more than n objects in the universe (113-4).

This, Etchemendy maintains, clashes with our intuitions about what makes a sentence fail to be a logical truth. If Tarski's analysis gets *that* wrong, then it should still be rejected.

Now, Ray does not agree. He holds that Etchemendy needs two conditions in order for this second criticism to go through—neither of which will Ray allow him. First, Etchemendy needs that if a sentence *had* satisfied Tarski's definition of logical truth, then that sentence *would* have been a logical truth (Ray 640-2). Secondly, and not unrelatedly, Etchemendy needs that such counterfactual considerations are of concern for Tarski's definition in the first place. That is, Etchemendy needs that Tarski was trying to give a *conceptual analysis* of logical truth—an analysis that held not only in the actual world, but across all possible worlds. As Ray sees it, this latter condition is necessary for the former. That is to say, Tarski's definition can be held accountable for sentences that

are “counterfactually” logically true only if it is an account of logical truth in all possible worlds.

To the contrary, Ray maintains that Tarski's definition has no such scope. In other words, Ray argues that Tarski never intended to give a *conceptual analysis* of logical truth. To this end, Ray has basically two lines of underlying support: (1) The textual evidence in Tarski's 1936 essay is at best ambiguous, and a charitable reading would attribute mere material adequacy to Tarski's definition, i.e., his intention was not to give a conceptual analysis. (2) Tarski grounds his definition of logical truth in his definition of truth. His definition of truth is notoriously only extensionally adequate, so his definition of logical truth can only hope to be extensionally adequate as well (and so, not a conceptual analysis). Tarski's definition, then, is not a piece of conceptual analysis. Therefore, Etchemendy is not warranted in making the needed assumption that Tarski's definition is accountable for logical truths across possible worlds.

5. The Inadequacy of Tarski's Definition Reiterated

Does Etchemendy's objection need that if a sentence were to qualify on Tarski's definition as a logical truth, then it would have been a logical truth? It might appear that it does. After all, in the passage quoted above, as well as in others, Etchemendy's argument suggests that, “[a]fter all, these [α]-sentences are not *in fact* logically true, but neither would they *be* logically true if the universe were finite” (113-4). If the universe were finite, for sure, some of these sentences would have qualified as logical truths. But, while Etchemendy illustrates his point thus, his criticism runs much deeper. In his own words,

The problem these sentences bring out remains...even if we take the axiom of infinity to be a necessary truth. All we need to recognize is that the axiom of infinity, and its various consequences, are *not* logical truths. This is all that is required to see that the output of Tarski's account is still influenced by extralogical facts—in this case, by the set-theoretic fact expressed by the axiom of infinity. (116)

So, it is not that Etchemendy is assuming that some of our α -sentences qualify as logically true in some possible world, and that, therefore, Tarski's account is untenable. Rather, Etchemendy's point holds that even if *none* of our α -sentences qualify as logical truths in any possible world. Why is this? Because, in so classifying them, *in whichever possible world*, Tarski's account adverts to extralogical facts about the size of the universe. Thus, Etchemendy may at least lay claim to the *conditional* conclusion that *if* Tarski's definition is a piece of conceptual analysis, *then* it is unsatisfactory as such.

But, *is* Tarski's definition supposed to be a conceptual analysis? Ray surely thinks not. Let's turn to Tarski's own words:

In order to obtain the proper concept of consequence, which is close in essentials to the common concept, we must resort to quite different methods and apply quite different conceptual apparatus in defining it. (413)

I should like to sketch here a general method which, it seems to me, enables us to construct an adequate definition of the concept of consequence ... Certain considerations of an intuitive nature will form our starting-point. Consider any class *K* of sentences and a sentence *X* which follows from the sentence of this class. From an intuitive standpoint it can never happen that both the class *K* consists of only true sentences and the sentence *X* false. Moreover, since we are concerned here with the concept of logical, i.e. *formal* consequence, and thus with a relation which is to be uniquely determined by the form of the sentences between which it holds, this relation cannot be influenced in any way by empirical knowledge, and in particular by knowledge of the objects to which the sentence *X* or the sentences of the class *K* refer ... The two circumstances just indicated, which seem to be very characteristic and essential for the proper concept of consequence, may be jointly expressed in the following statement ... (414-5)

It seems to me that everyone who understand the content of the above definition must admit that it agrees quite well with common usage. (417)

Now, in light of these comments, it sure looks like Tarski is trying to capture the content of our common concept of logical consequences. That is, given the above statements (especially those in the second passage), we have strong *prima facie* evidence that Tarski is going about a conceptual analysis of logical consequence. Ray asserts that the most charitable reading of the text would understand Tarski's project as developing a materially adequate definition of logical consequence (643-4); but, then, Ray does not consider the remarks made in the second quoted passage. It is hardly ambiguously asserted there that Tarski is attempting to capture our ordinary concept of logical consequence. As he states, "[t]he two circumstances just indicated, *which seem to be*

very characteristic and essential for the proper concept of consequence, may be jointly expressed in the following statement” (415). Immediately following this claim, Tarski gives a first approximation of his ultimate definition. Thus, to attribute to Tarski the intention of capturing our ordinary concept of consequence is, indeed, well-founded, given Tarski's statements in his 1936 essay.

Ray, however, adduces another passage from Tarski's essay:

I am not of the opinion that in the result of the above discussion the problem of a materially adequate definition of the concept of consequence has been completely solved. (418)

Ray infers from these remarks that *the* problem with which Tarski is concerned is the problem of developing a materially adequate definition of logical consequence (644). But, there are other explanations, of at least equal plausibility, for this remark. One such explanation is that Tarski thinks his definition unsatisfactory because, for starters, *it is not even materially adequate*.¹² Any good conceptual analysis will be at least materially adequate. In other words, solving the problem of a conceptually (or, metaphysically) adequate definition *requires* solving the problem of a materially adequate definition, and more. Thus, we have only the other passages cited above to adjudicate between the two possible interpretations on the table. For the reasons given above, then, I think we may permissibly attribute to Tarski the intention of providing a conceptual analysis of logical consequence.

But, perhaps Tarski's intentions—whatever they may have been—are not what determines the importance of Etchemendy's considerations. We may wonder if we have a good conceptual analysis of logical truth. And, of all the possible candidates we have on hand, Tarski's definition emerges head and shoulders above the rest as the best possible candidate. Why is Tarski's the best hopeful? Because, all of the others have been found to be less than materially adequate.¹³ Having settled that Tarski's definition is the one most likely to play the role of a conceptual analysis of logical truth, we may ask whether it succeeds. For the reasons already iterated, we know that it does not. That is, Tarski's definition, whether it was intended to be a conceptual analysis or not, fails to be a good definition of logical truth. The ultimate result is the same, we have no satisfactory, precise definition of logical truth.

I may underscore this result by illustrating that this latter line of response highlights the *importance* of Etchemendy's critique. According to Richard Jeffrey, for example, formal logic is the science of systematically determining whether and when the relationship between premises and conclusions expressed by Tarski's definition actually holds.¹⁴ Thus, contemporary logicians, for right or wrong, do appear to take Tarski's

definition to be *the* definition of logical consequence. So, any demonstration that Tarski's definition fails to succeed in capturing the *concept* of logical consequence is at least important to the philosophical community—even if it was never Tarski's aim to proffer a conceptual analysis to begin with. Thus, Ray's urgings that Tarski's definition was not intended to be a conceptual analysis does not undercut the *importance* of Etchemendy's objection: If the larger philosophical community takes Tarski to have captured our concept of logical consequence, then it is important to see that he has not.

Notes

¹ I would like to thank Paul Weirich for valuable comments on an earlier draft of this paper.

² Alfred Tarski, "On the concept of logical consequence" in *Logic, semantics, metamathematics* (Oxford: 1956). The essay was originally published in Polish in 1936. Hereafter, I will refer to this as Tarski's 1936 essay, though page numbers will refer to the 1956 edition.

³ John Etchemendy, *The concept of logical consequence* (Harvard: 1990).

⁴ Actually, Etchemendy is arguing against the definition of *logical truth*, rather than *logical consequence*. We'll see later how this difference doesn't in the end matter.

⁵ Greg Ray, "Logical consequence: a defense of Tarski," *Journal of Philosophical Logic* 25 (1996): 617-677.

⁶ Tarski rejects the proof-theoretic account (410-2) and Carnap's account (413).

⁷ Cf. Tarski (417). To see that Tarski's statement of his definition amounts to my statement, see Etchemendy (53-5).

⁸ The sentential function of a sentence is, for our purposes, the result of replacing all nonlogical terms in the sentence with a variable that ranges over the semantic category of the term replaced. For example, in 'John is at home or John is not at home,' 'John' is replaced by a variable, x , ranging over individuals, and 'is at home' is replaced by a variable, P , ranging over properties, and all logical terms (*or*, *not*) remain constant. Thus,

the sentential function of our sentence is 'xP or not-xP' – or, with more symbols, 'xP \vee \neg xP.'

⁹ For example, the universal closure of the sentential function above in note 8: $\forall x \forall P (xP \vee \neg xP)$.

¹⁰ For clarity, the universal closure of, say, α_2 ': $\forall E (\neg ExEyy \neq x)$, where 'E' ranges over subcollections of the universe.

¹¹ Etchemendy's own argument, just sketched here, can be found on pp.113-17.

¹² It is important to take this comment in context. Tarski hasn't just rejected his own definition, here. Rather, he is grappling with the problem of distinguishing purely logical from nonlogical signs – which is critical to the success of his definition. Thus, *pending an acceptable distinction between logical and nonlogical signs*, Tarski's definition regains its adequacy. See Tarski (418-19).

¹³ Indeed, we can lay the responsibility in Tarski's hands for finding *some* of the alternatives inadequate – e.g., the proof-theoretic approach and Carnap's approach. See Tarski (409-14) to see Tarski's criticisms of these theories.

¹⁴ See Jeffrey's *Formal Logic: its Scope and Limits* (McGraw-Hill, 1991). The definition provided by Jeffrey is not identical to Tarski's, but it is evidently equivalent.