The Need for Adaptive Logics in Epistemology*

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Abstract

After it is argued that philosophers of science have lost their interest in logic because they applied the wrong type of logics, examples are given of the forms of dynamic reasoning that are central for philosophy of science and epistemology. Adaptive logics are presented as a means to understand and explicate those forms of reasoning. All members of a specific (large) set of adaptive logics are proved to have a number of properties that warrant their formal decency and their suitability with respect to understanding and explicating dynamic forms of reasoning. Most of the properties extend to other adaptive logics.

1 Aim of this Paper

In the first half of the twentieth century, epistemology largely reduced to the philosophy of science and logic played a central role in it. We are here interested in the last half of the previous sentence. This raises at once the question why logic lost its central role in epistemology, including the philosophy of science. We all know when this happened—the Vienna Circle was succeeded by people like Hanson, Kuhn, Lakatos, Feyerabend, and Laudan, just to name a few central ones. There is no logic in their writings, they hardly ever mention logic. But *why* did it happen?

Of these philosophers of science, only Feyerabend made some explicit claims on the topic. While arguing, in [30], that often inconsistencies occur in episodes of the history of some sciences, he remarks that 'logic' cannot handle inconsistencies, and comments that this is a problem for logic, not for the history and philosophy of science. I do not think that this diagnosis is correct. For one thing, logics that can handle inconsistencies had been around for a while—see, for example, [28] and many other papers on paraconsistent[1] logics (by da Costa and associates); even some of Jaśkowski's work had been translated into English—see [32].

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[1]A logic is paraconsistent iff it does not validate ex falso quodlibet ($A, \sim A \vdash B$).

The correct diagnosis, it seems to me, is that philosophers of science had been applying the wrong *type* of logic. They had mainly been applying **CL** (Classical Logic). Even the few that recurred to 'non-standard logics' applied modal logic or intuitionistic logic—systems that share their central properties with **CL**. I do not claim that there is anything wrong with those logics. I only claim that that they were used for purposes for which they are unfit.

That the wrong type of logics was chosen was not merely an accident. The mainstream in Western epistemology has always been foundational. Notwithstanding some occasional remarks (as Neurath's idea that we have to rebuild our ship in the open sea) concerning the revision of (mainly) theories, the Vienna Circle fitted perfectly within this tradition. The 'Protokollsätze' provided the absolute basis. The only epistemological role for logic was to relate Protokollsätze to theories, mainly by deriving consequences from theories together with Protokollsätze. Needless to say, **CL** performs that role in an excellent way, especially as the Vienna Circle's view on the relation between observation and theory was simple and one-dimensional: saving the phenomena.[2]

When Hanson, Kuhn, and the others came around, it soon became clear that the most interesting aspects of scientific reasoning do not concern the relation between observations and theories, *and* that the Vienna Circle's view on the relation was utterly simplistic and mistaken—see especially [34]. Soon, there was again attention for discovery processes—the definite breakthrough were [40] and [41]. Not long thereafter, the study of explanation as a logical relation was replaced by a study of the *process* of explanation: how does one, given an *explanandum* E and a theory, arrive at 'initial conditions' to explain E?[3] And these are just a few examples.

In Section 2, I shall consider several reasoning processes that are essential to epistemology and philosophy of science, and that clearly cannot be handled by logics of the same type as **CL**. In Section 3, I shall introduce adaptive logics, the type of logics that is able to handle such reasoning processes. Characterizing such logics semantically will enable me, in Section 4, to prove that the consequence relations of these logics have the required properties. Much more important, however, is the dynamic proof theory of adaptive logics, which I shall discuss in Section 5. Unlike the semantics, which characterizes the consequence relations by means of abstract definitions, the dynamic proof theory enables one to explicate the actual reasoning processes. Moreover, the metatheory enables one to show that, notwithstanding their dynamic character, these proofs (i) lead to the correct conclusions in the long run, and (ii) lead to conclusions that provide a basis for decision and action even in the short run.

2 The Problem

The most common mechanism that, in specific situations, leads to general knowledge is inductive generalization. To be more precise, I mean the 'derivation' from data of formulas of the form $\forall(A \supset B)$ (the universal closure of $A \supset B$) and of

[2] Logic also had a curative function: to prevent people from talking nonsense. I shall not discuss this here, as the topic has long been settled by now.

[3] See, for example, the work of Hintikka and associates, who were among the few philosophers of science that tried to keep applying logic.

CL-consequences of the data and the generalizations.[4] It often has been argued that there is no logic of induction. The only argument that has ever been adduced for this claim, is that the resulting consequence relation is not monotonic: a generalization that is derivable from a set of data, need not be derivable after the set of data is extended. Today, however, many non-monotonic consequence relations have been decently defined and well-studied.

Non-monotonic consequence relations display an *external dynamics*. Suppose that $\Gamma \vdash_{\mathbf{LI}} A$ states that A is an inductive consequence (defined by the logic of induction **LI**) of Γ.[5] If Γ is the set of data available at some point in time, then $\Gamma \vdash_{\mathbf{LI}} A$ enables one to accept A. At a later point in time, the set of available data might be $\Gamma \cup \Delta$ and if $\Gamma \cup \Delta \nvdash_{\mathbf{LI}} A$, one will have to give up the conclusion A. This dynamics is external because it does not occur within the reasoning process. Both $\Gamma \vdash_{\mathbf{LI}} A$ and $\Gamma \cup \Delta \nvdash_{\mathbf{LI}} A$ always were true and always will be true. If one knows them to be true, then one justly accepted A at the point in time where Γ was the set of available data, and one justly rejected A after Γ was extended with Δ.

Suppose that one is only interested in inductive generalizations $\forall(A \supset B)$ in which both A and B do not contain any quantifiers or individual constants. The basic mechanism behind (thus restricted) inductive generalization is that the derived generalizations should together be compatible with the data. Given that the data are singular formulas and given the form of the generalizations, it is (effectively) decidable whether the data together with any finite set of generalizations is consistent. However, let us make the picture slightly more realistic and suppose that some background theories are available. Suppose moreover, to keep things simple, that the data do not contradict the background theories. The available knowledge now consists of the data, the background theories, and their (**CL**-)consequences. Which set of inductive generalizations is compatible with such a knowledge set is not in general a decidable matter. Worse, there is no positive test for it.[6] So, even if the set is consistent, there is in general no finite proof of this.

Given the absence of a positive test, how is is possible that people ever arrive at inductive generalizations? The answer is quite obvious: by reasoning. This reasoning cannot lead to a final judgement, even if the set of data remains stable during the reasoning process. It can, however, lead to a good estimate. Some people will arrive at a better estimate than others, and the efficiency of such reasoning processes may be studied. In specific cases, the reasoning may enable one to arrive at a final judgement. Even if it does not, one may consider the obtained estimate as sufficient for taking a decision—one may know that a final judgement is impossible, one may consider it too expensive or time consuming to obtain a better judgement, etc.

The absence of a positive test makes the reasoning process necessarily dynamic. Even when reasoning from a stable set of premises, one will have to consider certain formulas as derived provisionally. In other words, it cannot be

[4]Nearly always, background knowledge plays a role. I return to that in this very section.

[5]Actually, the adaptive logic **LI** is described in [16] and several further results will appear in papers by Lieven Haesaert and myself. However, one needs not to know those systems in order to follow the argument in the text.

[6]A positive test for a property is a systematic procedure that leads, after finitely many steps, to a "yes" if the property applies, but may go on forever if it does not. See [26] for such matters.

avoided that, at some point in the reasoning process, one considers as derived certain formulas that later have to be considered as not derived. This I shall call the *internal dynamics*. It is not caused by the introduction of new premises, but is a property of the reasoning process itself, even if the premise set is stable. For example, if background knowledge is present, one cannot avoid deriving certain inductive generalizations that later turn out incompatible with the (stable) available knowledge.

In the subsequent paragraphs, I give some more examples of reasoning processes that display an internal dynamics. However, let me stress at once that the discussed logic of inductive generalization is only a special case of a broader phenomenon. Whenever a new theory is adduced, it is supposed to be compatible[7] with available knowledge (the data and formerly accepted theories, or at least part of them). As there is no positive test for compatibility, any reasoning that leads to accepting the new theory necessarily displays the internal dynamics and, except in the specific cases in which a final judgement can be reached, the decision taken as the result of this reasoning is necessarily defeasible and hence provisional.

In [31], Ilpo Halonen and Jaakko Hintikka present a recent version of the theory of the process of explanation. In Section 6, they discuss the conditions on (nonstatistical) explanations (with a number of restrictions). The conditions (I slightly change their notation) concern an explanandum Pb, a background theory T (in which the predicate P occurs) and an initial condition (antecedent condition) I (in which b occurs). Among the six conditions are the following:

(iii) I is not inconsistent ($\nvdash_{\mathbf{CL}} \sim I$).
(iv) The explanandum is not implied by T alone ($T \nvdash_{\mathbf{CL}} Pb$).
(vi) I is compatible with T, i.e. the contextual evidence does not falsify the background theory ($T \nvdash_{\mathbf{CL}} \sim I$).

Obviously, there is no positive test for any of these three conditions. In other words, no finite reasoning process can (in general) lead to the conclusion that Pb is explained by I and T.

So, although the 'logic' appears to be **CL** (see the formal conditions above), it is quite obvious that the reasoning process that leads to the conclusion that I and T together explain Pb cannot possibly be explicated in terms of **CL** (at the object level). The reasoning is *about* **CL**-derivability, and necessarily displays the internal dynamics. This is why it cannot be explicated by a **CL**-proof but only by a dynamic proof.[8]

The logic of questions forms a further example. According to [52] and [51], where this problem is studied and solved, a question Q is evoked by a set of declarative statements Γ iff the presupposition of Q is derivable from Γ but no direct answer to Q is derivable from Γ. However, there is no positive test for non-derivability from Γ. Hence, although the definition is itself unobjectionable, only a dynamic proof may (in general) lead to the conclusion that Q is evoked by Γ.

Another example concerns handling inconsistency. Consider the case in which a scientific (empirical or mathematical) theory T was meant to be consistent and was formulated with **CL** as its underlying logic, but turned out to

[7]See [18] for the adaptive logic of compatibility in the framework of **CL**.
[8]See [20] for adaptive logics that explicate several forms of reasoning that underly the search for explanations.

be inconsistent. As we know from the literature,[9] scientists do not just throw away such a theory. They *reason* from T in search for a consistent replacement. Of course, they do not reason in terms of **CL**, because this is known to lead to triviality. However, they also do not reason in terms of some monotonic paraconsistent logic **PL**. In their reasoning, they want to interpret T *as consistently as possible*. After all, T was meant as a consistent theory.

Let us consider an utterly simplistic but instructive example. Let the 'theory' consist of $\sim p$, $p \vee r$, $\sim q$, $q \vee s$, and p. One obviously should not derive r from $\sim p$ and $p \vee r$ by Disjunctive Syllogism. To do so would lead to triviality: $p \vee r$ is itself a consequence of p, and so is any formula of the form $p \vee A$. However, as the theory was meant to be consistent, one will apply Disjunctive Syllogism to derive s from $\sim q$ and $q \vee s$. Indeed, it is quite obvious that q behaves consistently on this theory. To be more precise, q is consistently false on the theory, for $\sim q$ is obviously derivable whereas q is not, except of course by explicitly or implicitly applying Ex Falso Quodlibet to p and $\sim p$.[10]

So, the reasoning from T should proceed in such a way that one obtains T in its full richness, except for the pernicious consequence of its inconsistency. Precisely for this reason, the reasoning cannot proceed in terms of some monotonic paraconsistent logic **PL**. Indeed, **PL** will invalidate certain **PL**-rules, for example Disjunctive Syllogism.[11] However, as we saw from the previous example, the requested reasoning should not invalidate certain rules of inference of **CL**, but only certain *applications* of these rules. Let me express this more precisely. For certain rules, an application should be valid if specific involved formulas behave consistently on the theory, and invalid otherwise. Precisely this proviso causes the reasoning to be internally dynamic: there is no positive test for the consistent behaviour of some formula on a set of premises.[12]

Up to now, we have considered forms of reasoning that display an internal dynamics. All of them concerned a single unstructured set of premises. In many cases, however, the premises are structured, usually as an n-tuple of sets. I shall now consider some examples of this type.

Let us return for a moment to inductive generalization. Apart from the data, we usually have background knowledge. Let us restrict the discussion to the case where the background knowledge consists of generalizations in the sense meant before. An obvious complication is that the data may falsify some of the background generalizations. So, two forms of dynamics have to be combined. First, we retain the background generalizations in as far as they are not falsified by the data. Next, from the data and the retained background generalizations, we obtain new generalizations by the aforementioned logic **LI**. Remark, however, that it is in general impossible to perform the first selection (of background generalizations) before proceeding to the second selection (of new inductive generalizations). This means that both forms of dynamics are necessarily combined in the reasoning process. After deriving some new inductive generalizations, one may be forced to change one's judgement on the compatibility of some back-

[9] See for example [42], [43], [47], [27], [39], [36] and [38]. Not all of these authors side with me on the required approach.

[10] One way to implicitly apply Ex Falso Quodlibet proceeds by first applying Addition to obtain $p \vee q$ from p and next applying Disjunctive Syllogism to obtain q from $\sim p$ and $p \vee q$.

[11] See [37] for an exception: the paraconsistent logic **AN** validates Disjunctive Syllogism (and all 'analysing' rules of **CL**) but invalidates Addition (and Irrelevance and similar rules).

[12] Explicating this type of kind reasoning was at the origin of the adaptive logic programme—see [5], [8], and many other papers.

ground generalization with the data, and this will affect the derivability of new inductive generalizations.[13]

Often not all background generalizations will be considered equally trustworthy. So, instead of a set of background generalizations, one confronts a sequence of such sets, each having a different priority. In this case one has to combine a multiplicity of dynamics concerning the background generalizations with the dynamics that pertains to the new generalizations. Moreover, even falsified background generalizations may be considered as applying 'normally'. This means that an *instance* of the generalization is considered to hold unless and until proven incompatible with the data. Such 'pragmatic generalizations' may also be ordered by some priority relation. All this leads to more forms of dynamics (which, however, are all of three kinds).

Let us now consider a very different example. A participant in a discussion may change his or her position in view of arguments adduced by other participants. As a result, the interventions of the participant will be mutually incompatible, even if the participant's position is consistent during each intervention. However, the participant needs not state his or her full new position whenever there is a change. So, after an intervention, the participant's position has to be reconstructed from all his or her interventions. In order to do so, one starts with (the consistent part of) the last intervention, to this one adds that part of the previous intervention that is compatible with it, and so on. Remark that, while doing so, one does not select statements that are made during an intervention, but rather their consequences.[14]

Diagnostic reasoning forms a further example in which the premises are prioritized and hence require a multiplicity of dynamics. One reasons from on the one hand data and on the other hand expectancies (that may have varying degrees of trustworthiness). The expectancies, or rather their consequences, are retained (in their order of priority) until and unless proven inconsistent with the data. (See [50], [50] and [22] for the adaptive logics.)

In all examples mentioned before, the flat ones as well as the prioritized ones, the reasoning displays both the internal dynamics and the external dynamics. It is worth mentioning that, whenever the external dynamics (non-monotonicity) is present, the reasoning necessarily displays the internal dynamics (defeasible conclusions even if the premises are stable). The converse, however, does not hold. The *Weak* consequence relation, of Rescher and Manor—see, for example, [46] and [24]—is monotonic. Nevertheless, it may be shown that the reasoning from premises to weak consequences requires an internal dynamics.[15] Some consequence relations that are monotonic as well as decidable may even be characterized (in an enlightening and attractive way) by a form of reasoning that displays an internal dynamics—see [11] for an example.

The preceding paragraphs do by no means contain an exhaustive list of all reasoning mechanisms (or even of all types of reasoning mechanisms) that display an internal dynamics. Nevertheless, the problem should be clear by now. Essential forms of human reasoning, that are common and that are important

[13] The complication of falsifiable background generalizations is dealt with in [16]. The complications discussed in the subsequent paragraph of the text will be handled in forthcoming papers.

[14] Adaptive logics for this reconstruction are spelled out in [48] and [12].

[15] A is a Weak consequence of Γ iff it is a **CL**-consequence of some consistent subset of Γ—remember that there is no positive test for consistency.

for understanding the way in which humans arrive at knowledge and revise it, display an internal dynamics.

In order to arrive at a precise theory of knowledge, one needs to explicate such forms of reasoning. In order to do this, one needs specific logics: adaptive logics. These logics unavoidably have some non-standard properties. More important, however, is that they are characterized in a formally stringent way, and that their properties are studied in agreement to the professional standards.

3 What are Adaptive Logics?

In the loose sense of the term, a logic is adaptive iff it adapts itself to the specific premises to which it is applied. This should not be misunderstood. First, I do not mean to say that the consequence set, determined by the logic, depends on the set of premises. This obviously holds for nearly all[16] logics. I mean that the logic itself adapts to the premises in that it depends on properties of the premise set whether some formula is derivable from some of the premises. Next, I really mean that the logic adapts *itself* to the premises. The reasoner does not interfere in this. The logic is defined by a set of rules as well as by a semantics, and these lead to the adaptive effect, independently of any decision of the human or machine that applies the rules.

The previous paragraph describes the underlying idea. I shall also present a more technical characterization. This should not be understood as a definition, but rather as a hypothesis on the properties of all adaptive logics. It relies on present best insights. These may change as more logics are studied or new insights in them are gained. I have a good reason to insert this remark: during the last twenty years the dynamics of the adaptive logics programme forced the Ghent logic group several times to revise the technical characterization.

Flat (non-prioritized) adaptive logics. Let us start with these, as the prioritized ones require a somewhat more complex treatment. An adaptive logic **AL** may be characterized by a triple: the lower limit logic, the set of abnormalities, and the strategy. The *lower limit logic* **LLL** is a monotonic logic that is characterized by $Cn_{\mathbf{LLL}}(\Gamma) = \bigcap\{A \mid \Gamma \cup \Delta \vdash_{\mathbf{AL}} A; \emptyset \subseteq \Delta \subseteq \mathcal{W}\}$ in which $\mathcal{W}$ is the set of all closed formulas of the language. Intuitively, the lower limit logic is the stable part of the adaptive logic, the part that is not subject to any adaptation. From a proof theoretic point of view, the lower limit logic delineates the rules of inference that hold unexceptionally. From a semantic point of view, all adaptive models of Γ are lower limit models of Γ (but not conversely). It follows that $Cn_{\mathbf{LLL}}(\Gamma) \subseteq Cn_{\mathbf{AL}}(\Gamma)$.

Suppose that we are dealing with a context in which **CL** is taken as the standard of deduction. If the lower limit logic of **AL** is **CL** (or, for example, a modal extension of **CL**), it is said that **AL** is *ampliative*. This is the case

[16]The two obvious exceptions are zero logic, according to which nothing is derivable from any premise set (not even the premises themselves) and the logic according to which everything is derivable from any premise set. These logics may seem completely uninteresting, but actually zero logic is not. From it, an adaptive logic may be defined, thus making all logical reasoning contingent on specific properties of the premises (put differently: on 'the world'). For example, adaptive zero logic assigns to consistent sets of premises exactly the same consequence set as **CL**. See [9] for a study of zero logic and the (most straightforward) adaptive logic definable from it.

for inductive generalization (without background knowledge), for compatibility, etc. If the lower limit logic is weaker than **CL**, as in the case of inconsistency-adaptive logics, the adaptive logic is called *corrective*—the theory was intended to be interpreted in terms of **CL**, but turned out to be inconsistent and hence is interpreted as consistently as possible.[17]

The second (and crucial) component of an adaptive logic is the set of *abnormalities* Ω. These are the formulas that are presupposed to be false, unless and until proven otherwise. Thus, in an inductive logic, the set of abnormalities may consist of all formulas of the form $\exists(A \wedge B) \wedge \exists(A \wedge {\sim}B)$ in which no individual constants or quantifiers occur in A and B, and $\exists A$ abbreviates the existential closure of A. In handling inconsistency, the set of abnormalities will be some set of formulas of the form $\exists(A \wedge {\sim}A)$.

There seems to be a natural restriction on the set of abnormalities Ω: extending the lower limit logic with the requirement that no abnormality is logically possible, should result in a monotonic logic, which is called the *upper limit logic*. Presumably the effect is most easily seen by considering the semantics. If from the lower limit logic models one eliminates those that verify an abnormality, the resulting models should characterize the upper limit logic. If the adaptive logic is corrective, the lower limit logic is weaker than **CL**, and the upper limit logic will be (and is in all cases studied up to now) **CL**. If the adaptive logic is ampliative, the lower limit is (in all cases studied so far) **CL** or a modal extension of **CL**, and the upper limit logic is an extension of this.

Some examples are useful to clarify the matter. If the lower limit logic is **CL** and the set of abnormalities comprises all formulas of the form $\exists(A \wedge B) \wedge \exists(A \wedge {\sim}B)$ (see the previous paragraph), then the upper limit logic is obtained by adding to **CL** the axiom $\exists A \supset \forall A$.[18] If, as in the case of an inconsistency-adaptive logic, the lower limit logic is a paraconsistent logic **PL** that is a fragment of **CL**, and the set of abnormalities comprises all formulas of the form $\exists(A \wedge {\sim}A)$, then the upper limit logic is **CL**. The importance of the set of abnormalities is obvious. If the premise set does not require any abnormality to obtain, the adaptive logic will deliver the same consequences as the upper limit logic. If the premise set requires some abnormalities to obtain, the adaptive logic will still deliver more consequences than the lower limit logic, viz. all upper limit consequences that are not 'blocked' by those abnormalities.

It became only clear during the last years that, given a lower limit logic, different sets of abnormalities may result in the same upper limit logic but in a different adaptive logic. This may be easily exemplified in terms of inductive logics. To avoid technical clutter, I phrase the matter informally. Suppose that one replaces the aforementioned set of abnormalities (all formulas of the form $\exists(A \wedge B) \wedge \exists(A \wedge {\sim}B)$) by the set of couples consisting of, first, an instance of $\exists(A \wedge B)$ and, second, an instance of $\exists(A \wedge {\sim}B)$.[19] The upper limit logic is still obtained by adding to **CL** the axiom $\exists A \supset \forall A$. The resulting adaptive logic, however, is drastically different from the original. Consider the premises set $\Gamma = \{Pa, Qa, Pb, Qb, Pc, {\sim}Qc, Pd\}$. According to the original adaptive logic

[17]Remark that the lower limit logic may be zero logic—see footnote 16.

[18]Semantically, this logic is characterized by those **CL**-models in which, for each predicate π of adicity i, $v(\pi) \in \{D^i, \emptyset\}$ in which D^i is the i-th Cartesian product of the domain.

[19]To avoid complications, I disregard models that are not ω-complete. Such models require that one measures the abnormalities with respect to the model rather than with respect to the formulas it verifies.

of induction, neither Qd nor $\sim Qd$ is a consequence of this premise set. According to the present adaptive logic, Qd is derivable because it causes only one *new* abnormality, viz. $(Pd \wedge Qd) \wedge (Pc \wedge \sim Qc)$ whereas $\sim Qd$ causes two new abnormalities. The original set of abnormalities leads to an adaptive logic of inductive generalization; the set of abnormalities introduced in this paragraph leads to an adaptive logic of (qualitative) inductive prediction. However, the upper limit logic is the same in both cases.

A very important matter has to be brought up at this point. For all that was said before, an adaptive logic is obtained by presupposing that all formulas behave normally, except for those that need to behave abnormally in view of the premises. This formulation suggests that there is a well-defined set of formulas that need to behave abnormally in view of the premises, but this is not the case. The complication derives from the fact that, except for some specific lower limit logics—I shall discuss this when introducing the Simple strategy—a set of premises may entail a disjunction of abnormalities (members of Ω) without entailing any of its disjuncts. Let us again consider the original adaptive logic of induction and let the premise set be $\{Pa, Qa, Rb, \sim Qb\}$. Even with so small a data set, $(\forall x)(Px \supset Qx)$ and $(\forall x)(Rx \supset \sim Qx)$ are derivable. Suppose next that the premise set is extended to $\{Pa, Qa, Rb, \sim Qb, Pc, Rc\}$. Remark that neither $(\exists x)(Px \wedge Qx) \wedge (\exists x)(Px \wedge \sim Qx)$ nor $(\exists x)(Rx \wedge Qx) \wedge (\exists x)(Rx \wedge \sim Qx)$ is **CL**-derivable from these premises. However, their disjunction *is* **CL**-derivable from the premises.

Disjunctions of abnormalities will be called *Dab-formulas* and will be written as $Dab(\Delta)$, in which Δ is a finite set of formulas.[20] The *Dab*-formulas that are derivable by the lower limit logic from the premise set Γ will be called *Dab-consequences* of Γ. If $Dab(\Delta)$ (the disjunction of the members of $\Delta \subseteq \Omega$) is a *Dab*-consequence of Γ, then so is $Dab(\Delta \cup \Theta)$ for any (finite) Θ.[21] For this reason, the following definition is important. Let **LLL** be the lower limit logic as before.

Definition 1 *$Dab(\Delta)$ is a* minimal *Dab-consequence of Γ iff $\Gamma \vdash_{\mathbf{LLL}} Dab(\Delta)$ and there is no $\Theta \supset \Delta$ such that $\Gamma \vdash_{\mathbf{LLL}} Dab(\Theta)$.*

If $Dab(\Delta)$ is a minimal *Dab*-consequence of Γ, then it is derivable (by the lower limit logic) from Γ that some member of Δ behaves abnormally, but it is not derivable which member of Δ behaves abnormally. Adaptive logics are obtained by interpreting a set of premises 'as normally as possible'. But clearly, this phrase is not unambiguous. This is why we need to disambiguate it by choosing a specific adaptive strategy.

The oldest known *strategy* is *Reliability* from [5], where it is discussed at the propositional level. Let $U(\Gamma) = \{A \mid A \in \Delta$ for some minimal *Dab*-consequence $Dab(\Delta)$ of $\Gamma\}$ (the set of formulas that are unreliable on Γ). The Reliability strategy considers a formula as behaving abnormally iff it is a member of $U(\Gamma)$. As for the other strategies, the effect of this on the semantics and proof theory will be discussed in subsequent sections.

[20]Remark that $Dab(\Delta)$ is the disjunction of the members of Δ. In many previous papers on specific adaptive logics, $Dab(\Delta)$ has a slightly different function.

[21]I suppose that the lower limit logical validates Addition, and justly so. Whenever we are dealing with an adaptive logic in which disjunction is non-standard, the matter is handled by extending the language with classical disjunction—see [9] for an example.

The *Minimal Abnormality* strategy (first presented in [4] for the discussion in semantic terms of the propositional level) delivers some more consequences than the Reliability strategy. If, for example, $Dab(\Delta_1)$, ..., $Dab(\Delta_n)$ are the minimal *Dab*-consequences of Γ, the Minimal Abnormality strategy takes one member of each Δ_i to behave abnormally, while all other formulas behave normally.[22] Obviously, the Minimal Abnormality strategy does not pick out a specific such combination, but considers all of them. Consider a simple propositional example for an inconsistency-adaptive logic: $\Gamma = \{\sim p, \sim q, p \vee q, p \vee r, q \vee r\}$. If the lower limit logic validates all of full positive logic, $(p \wedge \sim p) \vee (q \wedge \sim q)$ is a minimal *Dab*-consequence of Γ. On the Reliability strategy, both p and q are unreliable with respect to Γ, and hence r is not an adaptive consequence of Γ. However, if the Minimal Abnormality strategy is chosen, then r is an adaptive consequence of Γ. Indeed, if p behaves abnormally, then q behaves normally and hence r is true in view of $\sim q$ and $q \vee r$; if q behaves abnormally, then p behaves normally and hence r is true in view of $\sim p$ and $p \vee r$. In subsequent sections, we shall see that both strategies are simple and perspicuous from a semantic point of view, and that the Reliability strategy leads to simple dynamic proofs, but that the dynamic proofs determined by the Minimal Abnormality strategy are rather complicated. Which strategy is adequate in a specific context of application is obviously a very different matter.

For some specific lower limit logics and sets of abnormalities, any minimal *Dab*-consequence $Dab(\Delta)$ of any premise set is such that Δ is a singleton. In such cases, the Reliability and Minimal Abnormality strategies lead to the same result and coincide with what is called the *Simple* strategy: a formula behaves abnormally just in case the abnormality is derivable from the premise set. Examples may be found in [37] and [18].

Several other strategies have been studied, but seem to have a less general import. Most of them were the result of characterizing an existing consequence relation by an adaptive logic. Examples may be found in [14], [17], [29] and [49].

A different way to characterize most flat adaptive logics is by seeing them as formula-preferential systems. The idea was first presented in [35] (see also [2]). I am not sure, however, that this will work for any adaptive logic. As was remarked in footnote 19, the aforementioned adaptive logic of inductive prediction, if formulated at the predicative level and if one allows for ω-incomplete models, requires that we define the 'abnormal part' of a model with respect to the model itself rather than with respect to the formulas it verifies. Also, Graham Priest system $\mathbf{LP}^m$ from [45] defines the abnormal part of a model in a similar way—see [15] for some mistakes in this construction.

The idea to combine a lower limit logic with an arbitrary set of abnormalities was first presented in [35] and [2]. Of course, not any set of abnormalities will define a sensible upper limit logic, let alone a sensible adaptive logic. However, we shall see that the idea is sensible in general for so-called direct formulations of prioritized adaptive logics.

Prioritized adaptive logics and combining adaptive logics. Consider $\Sigma = \langle \Gamma_0, \ldots, \Gamma_n \rangle$, in which Γ_0 is a set of data (that are taken to be certain) and Γ_1, ..., Γ_n are sets of expectancies—formulas that are supposed to obtain

[22] For some sets of minimal *Dab*-consequences, it cannot be avoided that at least two members of some Δ_i behave abnormally—see [8, p. 468] for an example.

but may be overruled. The members of Γ_i ($1 \leq i \leq n$) carry a higher degree of certainty as i is smaller.

One prioritized adaptive logic to handle such n-tuples is obtained as follows. Where $\Diamond^i$ abbreviates a sequence of i diamonds, $\Sigma^\Diamond = \{\Diamond^i A \mid A \in \Gamma_i\}$. The lower limit logic is the modal logic[23] **T** and the set of abnormalities are all formulas of the form $\Diamond^i A \wedge {\sim} A$ in which A is either a primitive formula (sometimes called an atom) or its negation and $1 \leq i \leq n$. Remark that the upper limit logic is **Triv**, which is obtained by extending **T** with (for example) the axiom $\Diamond A \supset A$. Finally, one combines the above with either the Reliability or Minimal Abnormality strategy with the following proviso: an abnormality $\Diamond^i A \wedge {\sim} A$ is considered as worse according as i is smaller. This means that, if either an abnormality of level i or an abnormality of level j is unavoidable in view of the premises, then the abnormality of level i is avoided if $i < j$. The results are the nice adaptive logics $\mathbf{T}^{sr}$ and $\mathbf{T}^{sm}$ from [22]. For more examples see [48] and [49], the latter containing adaptive logics that characterize all the prioritized Rescher–Manor consequence relations from [25].

A different way to characterize prioritized adaptive logics is by seeing them as the result of applying a sequence of flat adaptive logics, each of these logics having a (nearly) similar structure. This characterization of prioritized adaptive logics is a special case of a more general mechanism, viz. that adaptive logics, which may have a very different structure, are combined with each other. Consider the search for an explanation of some singular fact, given a theory. If one relies on Hintikka's conditions, one needs an adaptive logic to deal with the conditions for which there is no positive test—see Section 2. Suppose that moreover the involved theory is inconsistent (or that the data are inconsistent)—see [13] for the discussions of such inconsistencies. In such a case one needs to combine the adaptive logic for the process of explanation with an inconsistency-adaptive logic that interprets the theory as consistently as possible. The result is a sequence of (two) adaptive logics.

The term 'sequence' deserves a comment. The definition may refer to such a sequence, and intuitively one may understand the logic as resulting from applying one adaptive mechanism after the other. However, as there is no positive test for any of the two, it is essential that the dynamic proof theory is able to handle all the adaptive steps in random order.

Applying the above logics $\mathbf{T}^{sr}$ and $\mathbf{T}^{sm}$ requires the transition from the n-tuple Σ to the set $\Sigma^\Diamond$.[24] Given such transition, the characterization of the prioritized adaptive logic comes to a combination of (similar) flat adaptive logics. However, it might be objected that the prioritized adaptive logic does not define the consequence relation itself, but defines it only under some translation.[25]

In all cases studied up to now, we were able to also articulate direct formulations. These are formulations in terms of the original language. The same

[23]Given the variety of predicative extensions of propositional modal logics, one needs to pick a specific predicative version of **T**. I shall only discuss the propositional case here and refer to [22] for a predicative version that is adequate for diagnosis logic.

[24]Actually, the situation is similar for several flat adaptive logics that characterize formerly known consequence relations.

[25]Still, the matter is not as simple as it looks. If A is not a premise but only an expectancy, it seems desirable that this is expressed in the object language. Typically, in [50], which started the work on adaptive logics for diagnosis, that A is an expectancy is rendered as $E(A)$. So, one might just as well reinterpret $\Diamond A$ as "it may be expected that A" and $\Box A$ as "it cannot be expected that ${\sim}A$". In this case, there is no translation.

applies for the dynamic proofs. It not completely clear whether these formulations fit within the technical characterization of an adaptive logic. Apparently, more work will be required before the matter can be settled. However, it is quite obvious that the direct formulations may be seen as adaptive logics in the sense of formula-preferential logics: the logic selects the models that verify 'as much as possible' the members of each $\Gamma_i \in \Sigma$ (in its order of priority). If adaptive logics are seen as formula preferential, and the set of abnormalities is arbitrary (and not defined by some logical form), the lower limit logic and the requirement that no abnormalities obtain might not together define an upper limit logic. Formulations that recur to a translation are very different in this respect. There the members of the different Γ_i are translated in a different way (in the above example as $\Diamond^i A$) and abnormalities have a specific logical form (in the example $\Diamond^i A \wedge {\sim} A$ in which A is either a primitive formula or its negation). This matter too requires more study.

It is not always clear whether a prioritized adaptive logic (in its direct formulation) is corrective or ampliative. Consider again some $\Sigma = \langle \Gamma_0, \ldots, \Gamma_n \rangle$. If only the members of Γ_0 are considered as the premises, then logics such as $\mathbf{T}^{sr}$ are ampliative: some adaptive consequences may be **CL**-consequences of one or more Γ_i $(1 \leq i \leq n)$ that are not **CL**-consequences of Γ_0. If $\Gamma_0 \cup \ldots \cup \Gamma_n$ is considered as the premise set, then the adaptive logic is corrective in that not all **CL**-consequences of this set need to be adaptive consequences. This mainly shows that the distinction between corrective and ampliative adaptive logics, which is a merely pragmatic distinction that is useful for flat adaptive logics, does not apply in a clearcut way to prioritized adaptive logics.

Other adaptive logics. The distinction between flat and prioritized adaptive logics does not apply to all adaptive logics; some of them do not fit in either category (as characterized above). As this concerns forthcoming work, I shall only mention two examples.

Consider a flat adaptive logic, and suppose that $Dab\{A_1, \ldots, A_n\}$ is a minimal *Dab*-consequence of some premise set Γ. If the adaptive logic is inductive, such reasons apparently always derive from the set of data, and hence are adequately taken care of by the fact that all data are premises. For other adaptive logics, for example for all inconsistency-adaptive ones, there may be reasons to believe that specific A_i do not behave abnormally and these reasons need not be expressed by the premises. Indeed, if it follows from the premises that some disjunction of inconsistencies obtains, one may have a good reason to believe that some of the disjuncts are false—for example, they may pertain to well-entrenched theories, or to observational criteria that are considered as unproblematic. In such cases, one may want to posit that some of the A_i are not abnormal. If classical negation (written as $\neg$) is available, one may want to introduce $\neg A_i$—remember that A_i is a formula of the form $\exists(B \wedge {\sim} B)$). However, such 'new premises' should be defeasible. Indeed, there is no positive test for "is consistent" and hence one cannot in general be sure whether some *Dab*-consequence of Γ is minimal—see also the proof theory in Section 5. In summary, this form of reasoning requires for the possibility, within the framework of an inconsistency-adaptive logic, to introduce revokable premises of a specific form next to the standard premises, thus introducing a separate adaptive mechanism.[26]

A different type of adaptive logic is also most easily illustrated in terms of inconsistency. Suppose that one confronts an inconsistent theory, that the inconsistency is taken to render the theory inadequate, and that one is interested in the 'consistent part' of the theory, which one deems unproblematic. To locate this consistent part (in other words, to obtain consistency by brute force) is a task for logic. Given the absence of a positive test for consistency, it is a task for adaptive logic. The task is a difficult one; so far this adaptive logic has not been adequately characterized.

After first reading or hearing about adaptive logics, some people think that adaptive logics are (what I like to call) flip-flop logics. Thus some people think that inconsistency-adaptive logics (i) deliver the classical consequences of a premise set if it is consistent, and (ii) deliver the paraconsistent consequences (defined by the lower limit logic) of the premise set if it is inconsistent. While (i) is correct, (ii) is false for most inconsistency-adaptive logics. Even if the premise set is abnormal, most adaptive logics still interpret the set as normally as possible, and hence deliver more consequences than the lower limit logic. It is amusing, however, to note that it is very easy to define adaptive logics that are flip-flop logics. They indeed adapt themselves to the premise set, even if only in a very crude way.

4 Semantics

The dynamic proof theory of adaptive logics is certainly their most fascinating feature. It was this proof theory that led to the discovery of adaptive logics—see [5]. I shall nevertheless start by discussing the semantics because this will sound more familiar to most people.

I shall at first concentrate on adaptive logics **AL** (defined from a lower limit logic **LLL**, a set of abnormalities Ω and some strategy) that fulfil the following conditions:

C1 **AL** is a flat adaptive logic.

C2 **LLL** is monotonic and compact.[27]

C3 The strategy is either Reliability or Minimal Abnormality.[28]

C4 **LLL** and Ω define a monotonic upper limit logic **ULL**. This means that the set of **LLL**-models that verify no member of Ω form an adequate semantics for the monotonic **ULL**.

C5 For any $A \in \Omega$ there is a (finite or infinite) $\Pi_A \subset \mathcal{W}$ such that
  (i) If $B \in \Pi_A$, then any **ULL**-model verifies B ($\vDash_{\mathbf{ULL}} B$).
  (ii) Any **LLL**-model either verifies A or verifies all members of Π_A.

[26] That standard premises may be revoked is not a problem, not even in the context of **CL**: consequences of Γ need not be consequences of some $\Delta \not\subseteq \Gamma$. The new premises, however, are conjectures and, given that they express claims that transcend theories and observations, are revokable in the more fundamental sense that is captured by the internal dynamics of the reasoning.

[27] I shall especially need semantic compactness: Γ has a model iff every finite subset of Γ has a model.

[28] Whenever the Simple strategy applies, the theorems below extend to it immediately because both Reliability and Minimal Abnormality come to the Simple strategy in such cases.

(iii) No **LLL**-model[29] verifies A as well as all members of Π_A.

The set Π_A (or its member, if it is a singleton) 'expresses' that A is false in the sense of (iii). Convention C5 is easily illustrated in terms of an inconsistency-adaptive logic. Suppose that the lower limit logic is a paraconsistent logic **PL** and that $p \wedge \sim p \in \Omega$. Which sets fulfil the condition imposed upon $\Pi_{p\wedge\sim p}$? Depending on the presence of classical negation ($\neg$), material implication ($\supset$) and bottom (characterized by the theorem $\bot \supset A$), each of the following sets does: $\{\neg(p \wedge \sim p)\}$, $\{(p \wedge \sim p) \supset \bot\}$ and $\{(p \wedge \sim p) \supset A \mid A \in \mathcal{W}\}$.[30] From now on, I suppose that Π_A denotes some such set (or the union of all of them). Condition C5 holds for all adaptive logics studied so far by the Ghent logic group. As far as I can see, it holds for all sensible adaptive logics.

Where it matters, I refer to the strategy by the name of the adaptive logic thus: $\mathbf{AL}^r$ and $\mathbf{AL}^m$. I shall suppose that an adequate semantics is available for **LLL** as well as for **ULL**, and hence that I can pass freely from the proof theory to the semantics.

The **AL**-models of a premise set Γ are a subset of the **LLL**-models of Γ. How the selection is made depends on the strategy. For both strategies, we need $Ab(M)$ (the abnormalities verified by the model M). For the Reliability strategy, we moreover need the minimal *Dab*-consequences of Γ (defined by the semantic counterpart of Definition 1), and $U(\Gamma)$ is defined as in Section 3, viz. as $\{A \mid A \in \Delta$ for some minimal *Dab*-consequence $Dab(\Delta)$ of $\Gamma\}$.

Definition 2 *A* **LLL**-*model* M *of* Γ *is* reliable *iff* $Ab(M) \subseteq U(\Gamma)$.

Definition 3 $\Gamma \vDash_{\mathbf{AL}^r} A$ *iff* A *is verified by all reliable models of* Γ.

Definition 4 *A* **LLL**-*model* M *of* Γ *is* minimally abnormal *iff there is no* **LLL**-*model* M' *of* Γ *such that* $Ab(M') \subset Ab(M)$.

Definition 5 $\Gamma \vDash_{\mathbf{AL}^m} A$ *iff* A *is verified by all minimally abnormal models of* Γ.

I shall prove some theorems to hold for all adaptive logics under consideration. To simplify the notation, let $\mathcal{M}^{\mathbf{L}}_\Gamma$ denote the set of **L**-models of Γ. Where the logic is $\mathbf{AL}^r$ or $\mathbf{AL}^m$, I shall write $\mathcal{M}^r_\Gamma$ and $\mathcal{M}^m_\Gamma$ respectively.

Given that the adaptive logics were defined by a selection of lower limit models, it is important to prove that this selection is proper. A very strong property is that, for any lower limit model M that is not selected ($M \notin \mathcal{M}^{\mathbf{AL}}_\Gamma$), there is a selected model M' that is less abnormal than M ($Ab(M') \subset Ab(M)$). I call this property Strong Reassurance, Avron calls it Stopperedness, and it is closely related to what is called Smoothness in [33]. That its absence leads to undesired results is shown, for example, in [10]. I first prove the property for the Minimal Abnormality strategy. An unqualified "model" will always refer to a **LLL**-model.

[29] If one prefers to allow for the trivial model, the requirement should read: only the trivial **LLL**-model verifies A as well as all members of Π_A. In the sequel of the text, I suppose that there are no trivial models. The alternative requires nearly no change to the proofs below.

[30] Graham Priest's $\mathbf{LP}^m$ from [45] contains neither classical negation nor (detachable) material implication and indeed the convention does not apply to it. The upshot is that much of what follows is not provable for it—for more problems with $\mathbf{LP}^m$, see [15].

Theorem 1 *If* $M \in \mathcal{M}^{\mathbf{LLL}}_\Gamma - \mathcal{M}^m_\Gamma$*, then there is a* $M' \in \mathcal{M}^m_\Gamma$ *such that* $Ab(M') \subset Ab(M)$*. (Strong Reassurance for Minimal Abnormality.)*

Proof. The theorem holds (vacuously) if Γ has no **LLL**-models or if $\mathcal{M}^m_\Gamma = \mathcal{M}^{\mathbf{LLL}}_\Gamma$. Consider a $M \in \mathcal{M}^{\mathbf{LLL}}_\Gamma - \mathcal{M}^m_\Gamma$, let $D_1, D_2, \ldots$ be a list of all members of Ω, and define

$$\Delta_0 = \emptyset$$

$$\Delta_{i+1} = \Delta_i \cup \Pi_{D_{i+1}}$$

if there is a model M' of $\Gamma \cup \Delta_i \cup \Pi_{D_{i+1}}$ such that $Ab(M') \subseteq Ab(M)$, and

$$\Delta_{i+1} = \Delta_i$$

otherwise. Finally,

$$\Delta = \Delta_0 \cup \Delta_1 \cup \Delta_2 \cup \ldots$$

Given the compactness of **LLL**, $\Gamma \cup \Delta$ has models in view of the construction. Let $M' \in \mathcal{M}_{\Gamma \cup \Delta}$.

Step 1. I first show that, if M' is a model of $\Gamma \cup \Delta$, then $Ab(M') \subset Ab(M)$. Suppose that there is a $D_j \in \Omega$ such that $D_j \in Ab(M') - Ab(M)$. Let M'' be a model of $\Gamma \cup \ \Delta_{j-1}$ such that $Ab(M'') \subseteq Ab(M)$. As $D_j \notin Ab(M)$, $D_j \notin Ab(M'')$. Hence M'' is a model of $\Gamma \cup \Delta_{j-1} \cup \Pi_{D_j}$ and $Ab(M'') \subseteq Ab(M)$. So, $\Pi_{D_j} \subseteq \Delta_j \subseteq \Delta$. As M' is a model of $\Gamma \cup \Delta$, $D_j \notin Ab(M')$. But this contradicts the supposition.

Step 2. I now show that any model of $\Gamma \cup \Delta$ is a minimally abnormal model of Γ. Suppose that M' is a model of $\Gamma \cup \Delta$, but is not a minimally abnormal model of Γ. Hence, some model M'' of Γ is such that $Ab(M'') \subset Ab(M')$. It follows that M'' is a model of $\Gamma \cup \Delta$. If it were not, then, as M'' is a model of Γ, there is a $\Pi_{D_j} \subseteq \Delta$ such that M' verifies all members of Π_{D_j} and M'' falsifies some members of Π_{D_j}. By Condition C5, M' falsifies D_j and M'' verifies D_j, which is impossible in view of $Ab(M'') \subset Ab(M')$.

Consider any $D_j \in Ab(M') - Ab(M'') \neq \emptyset$. As M'' is a model of $\Gamma \cup \Delta_{j-1}$ that falsifies D_j, it is a model of $\Gamma \cup \Delta_{j-1} \cup \Pi_{D_j}$. As $Ab(M'') \subset Ab(M')$ and $Ab(M') \subseteq Ab(M)$, $Ab(M'') \subset Ab(M)$. It follows that $\Delta_j = \Delta_{j-1} \cup \Pi_{D_j}$ and hence that $\Pi_{D_j} \in \Delta$. But then $D_j \notin Ab(M')$. Hence, $Ab(M'') = Ab(M')$. So, the supposition leads to a contradiction. ■

As $\mathcal{M}^m_\Gamma \subseteq \mathcal{M}^r_\Gamma$ (by property 1 of Theorem 3 below), it follows that:

Theorem 2 *If* $M \in \mathcal{M}^{\mathbf{LLL}}_\Gamma - \mathcal{M}^r_\Gamma$*, then there is a* $M' \in \mathcal{M}^r_\Gamma$ *such that* $Ab(M') \subset Ab(M)$*. (Strong Reassurance for Reliability.)*

Corollary 1 *If* Γ *has lower limit models, then it has minimally abnormal as well as reliable models. (Reassurance.)*

At this point, I need some lemmas that require a specific characterization of the minimal abnormality strategy. In [8], such a characterization is offered in terms of a set Φ_Γ, which is a set of sets of abnormalities.[31] It is then shown that, where M is a minimally abnormal model of some Γ, $Ab(M)$ is characterized by some $\phi \in \Phi_\Gamma$—all members of $Ab(M)$ are **LLL**-consequences of some $\phi \in \Phi_\Gamma$.

[31] In [8] Φ_Γ is a set of sets of factors of abnormalities, but this modification is inconsequential and factors of abnormalities are a nuisance in the present setup.

Recently, I found a drastically simpler such characterization, which only has the disadvantage to be less 'finitistic'. I shall now redefine Φ_Γ. The proofs in [8] may be easily modified in view of this change and may be generalized to all adaptive logics considered, but I cannot, in the present paper, spell out the required modifications to the proofs. Let Φ°_Γ comprise all sets that contain a disjunct out of each minimal *Dab*-consequence of Γ and that are **LLL**-closed with respect to Ω.[32] Let Φ_Γ contain all members of Φ°_Γ that are not supersets of other members of Φ°_Γ. Suitably modifying and generalizing the proof of Lemmas 7.2 and 7.3 of [8] gives us:

Lemma 1 *M is a minimally abnormal model of Γ iff $M \in \mathcal{M}^{\mathbf{LLL}}_\Gamma$ and $Ab(M) \in \Phi_\Gamma$.*

The strength of this lemma may be seen from the fact that each of the following properties are immediate or nearly immediate consequences of it:

Theorem 3 *Each of the following holds:*

1. *Each minimally abnormal model of Γ is a reliable model of Γ ($\mathcal{M}^m_\Gamma \subseteq \mathcal{M}^r_\Gamma$). Hence $Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma)$.*
2. *If $A \in \Omega - U(\Gamma)$, then $\Pi_A \subseteq Cn_{\mathbf{AL}^r}(\Gamma)$.*
3. *If $Dab(\Delta)$ is a minimal Dab-consequence of Γ and $A \in \Delta$, then there is a minimally abnormal model M of Γ that verifies A and falsifies all members (if any) of $\Delta - \{A\}$.*
4. *All minimally abnormal models of Γ are minimally abnormal models of $Cn_{\mathbf{AL}^m}(\Gamma)$ and vice versa ($\mathcal{M}^m_\Gamma = \mathcal{M}^m_{Cn_{\mathbf{AL}^m}(\Gamma)}$) and hence $Cn_{\mathbf{AL}^m}(\Gamma) = Cn_{\mathbf{AL}^m}(Cn_{\mathbf{AL}^m}(\Gamma))$. (Fixed Point.[33])*
5. *All reliable models of Γ are reliable models of $Cn_{\mathbf{AL}^r}(\Gamma)$ and vice versa ($\mathcal{M}^r_\Gamma = \mathcal{M}^r_{Cn_{\mathbf{AL}^r}(\Gamma)}$) and hence $Cn_{\mathbf{AL}^r}(\Gamma) = Cn_{\mathbf{AL}^r}(Cn_{\mathbf{AL}^r}(\Gamma))$. (Fixed Point.)*
6. *For all $\Delta \subseteq \Omega$, $Dab(\Delta) \in Cn_{\mathbf{AL}}(\Gamma)$ iff $Dab(\Delta) \in Cn_{\mathbf{LLL}}(\Gamma)$. (Immunity.)*
7. *If $\Gamma \vDash_{\mathbf{AL}} A$ for every $A \in \Gamma'$, and $\Gamma \cup \Gamma' \vDash_{\mathbf{AL}} B$, then $\Gamma \vDash_{\mathbf{AL}} B$. (Cautious Cut.)*
8. *If $\Gamma \vDash_{\mathbf{AL}} A$ for every $A \in \Gamma'$, and $\Gamma \vDash_{\mathbf{AL}} B$, then $\Gamma \cup \Gamma' \vDash_{\mathbf{AL}} B$. (Cautious Monotonicity.)*

A premise set Γ will be called normal if $\mathcal{M}^{\mathbf{ULL}}_\Gamma \neq \emptyset$; it is called abnormal otherwise. Remark that Γ is normal iff $\Omega \cap Cn_{\mathbf{LLL}}(\Gamma) = \emptyset$.

Theorem 4 *Each of the following obtains:*

1. *$\mathcal{M}^{\mathbf{ULL}}_\Gamma \subseteq \mathcal{M}^m_\Gamma \subseteq \mathcal{M}^r_\Gamma \subseteq \mathcal{M}^{\mathbf{LLL}}_\Gamma$ and hence $Cn_{\mathbf{LLL}}(\Gamma) \subseteq Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma) \subseteq Cn_{\mathbf{ULL}}(\Gamma)$.*

[32] By the second half of the requirement I mean that $\varphi = Cn_{\mathbf{LLL}}(\varphi) \cap \Omega$.

[33] The label might suggest that recurrent applications of some closure operation ultimately lead to a fixed point. This, however, is not the case: a single application of the closure operation leads to a fixed point ($Cn_{\mathbf{AL}^m}(\Gamma)$ is a fixed point with respect to **AL**-closure).

2. *If* Γ *is normal, then* $\mathcal{M}^{\mathbf{ULL}}_{\Gamma} = \mathcal{M}^{m}_{\Gamma} = \mathcal{M}^{r}_{\Gamma}$ *and hence* $Cn_{\mathbf{AL}^r}(\Gamma) = Cn_{\mathbf{AL}^m}(\Gamma) = Cn_{\mathbf{ULL}}(\Gamma)$.

3. *If* Γ *is abnormal and* $\mathcal{M}^{\mathbf{LLL}}_{\Gamma} \neq \emptyset$*, then* $\mathcal{M}^{\mathbf{ULL}}_{\Gamma} \subset \mathcal{M}^{m}_{\Gamma}$ *and hence* $Cn_{\mathbf{AL}^m}(\Gamma) \subset Cn_{\mathbf{ULL}}(\Gamma)$.[34]

4. $\mathcal{M}^{r}_{\Gamma} \subset \mathcal{M}^{\mathbf{LLL}}_{\Gamma}$ *iff* $\Gamma \cup \{A\}$ *is* **LLL***-satisfiable for some* $A \in \Omega - U(\Gamma)$.

5. $Cn_{\mathbf{LLL}}(\Gamma) \subset Cn_{\mathbf{AL}^r}(\Gamma)$ *iff* $\mathcal{M}^{r}_{\Gamma} \subset \mathcal{M}^{\mathbf{LLL}}_{\Gamma}$.

6. $\mathcal{M}^{m}_{\Gamma} \subset \mathcal{M}^{\mathbf{LLL}}_{\Gamma}$ *iff there is a (possibly infinite)* Δ *such that* $\Gamma \cup \Delta$ *is* **LLL***-satisfiable and* $\Delta \nsubseteq \varphi$ *for every* $\varphi \in \Phi_{\Gamma}$.

7. *If there are* $A_1, \ldots, A_n$ *(*$n \geq 1$*) such that* $\Gamma \cup \{A_1, \ldots, A_n\}$ *is* **LLL***-satisfiable and* $\{A_1, \ldots, A_n\} \nsubseteq \varphi$ *for every* $\varphi \in \Phi_{\Gamma}$*, then* $Cn_{\mathbf{LLL}}(\Gamma) \subset Cn_{\mathbf{AL}^r}(\Gamma)$.

Proof. Ad 2. If Γ is normal, then $U(\Gamma) = \emptyset$ and only **ULL**-models of Γ are minimally abnormal.

Ad 3. If Γ is abnormal, then $\mathcal{M}^{\mathbf{ULL}}_{\Gamma} = \emptyset$.

Ad 1. $\mathcal{M}^{\mathbf{ULL}}_{\Gamma} \subseteq \mathcal{M}^{m}_{\Gamma}$ follows from 2 and 3. $\mathcal{M}^{r}_{\Gamma} \subseteq \mathcal{M}^{\mathbf{LLL}}_{\Gamma}$ is immediate in view of the definition of reliable model of Γ. $\mathcal{M}^{m}_{\Gamma} \subseteq \mathcal{M}^{r}_{\Gamma}$ is item 1 of Theorem 3.

Ad 4. Immediate in view of Definitions 2 and 3.

Ad 5. By 4, there is an $A \in \Omega - U(\Gamma)$ such that all $M \in \mathcal{M}^{r}_{\Gamma}$ verify Π_A whereas some $M \in \mathcal{M}^{\mathbf{LLL}}_{\Gamma} - \mathcal{M}^{r}_{\Gamma}$ does not.

Ad 6. Immediate in view of Definitions 4 and 5.

Ad 7. Suppose that the antecedent is true. For all $B_1 \in \Pi_{A_1}$, ..., and $B_n \in \Pi_{A_n}$, all $M \in \mathcal{M}^{m}_{\Gamma}$ verify $B_1 \vee \ldots \vee B_n$ whereas some $M \in \mathcal{M}^{\mathbf{LLL}}_{\Gamma}$ (viz. an $M \in \mathcal{M}^{\mathbf{LLL}}_{\Gamma \cup \{A_1, \ldots, A_n\}}$) does not in view of Condition C5. ■

Other known adaptive logics are obtained by combining adaptive logics of the type described above. Usually, it is easy to check that all aforementioned properties extend to them.[35]

5 Dynamic Proof Theory

I shall restrict the discussion to adaptive logics that fulfil the conditions C1–5 as well as two new conditions. These concern a function f that associates with any set of formulas Γ a set of formulas $f(\Gamma) \subseteq \Omega$.

C6 Every **LLL**-model M of Γ that falsifies all members of $f(\Gamma)$, verifies all formulas verified by some **ULL**-model of Γ.

C7 If Γ is finite, so is $f(\Gamma)$.

The conditions may look somewhat weird, but they are not. For example, for most inconsistency-adaptive logics $f(\Gamma)$ is the set of all formulas of the form $\sim(A \wedge \sim A)$ (sometimes formally restricted, for example to primitive formulas)

[34] If Γ is abnormal, it has no **ULL**-models and $Cn_{\mathbf{ULL}}(\Gamma)$ is trivial.

[35] More properties (see for example [1] and [2]) may be established for the adaptive logics under consideration. For example, Right Cautious Cut (and hence Plausibility) holds for $\mathbf{AL}^m$ (but not for $\mathbf{AL}^r$).

for which $\sim A$ is a subformula of some member of Γ. Remark that this set is finite whenever Γ is so. In general, the conditions are easily established if the set of abnormalities, Ω, is characterized by some logical form (or by finitely many logical forms)[36] and if the **LLL**-semantics is recursive.

As before, I shall suppose that $\vee$ denotes classical disjunction. If disjunction behaves abnormally in **LLL**, the language is extended with classical disjunction. This may be done by means of an explicit definition or, where this is impossible, by a straightforward extension of the language—see [9].

Theorem 5 *$\Gamma \vdash_{\mathbf{ULL}} A$ iff there is a finite $\Delta \in \Omega$ such that $\Gamma \vdash_{\mathbf{LLL}} A \vee Dab(\Delta)$. (Derivability Adjustment Theorem.)*

Proof. For the left–right direction, suppose that $\Gamma \vdash_{\mathbf{ULL}} A$. By the Compactness of **ULL**, there is a finite $\Gamma' \subseteq \Gamma$ such that $\Gamma' \vdash_{\mathbf{ULL}} A$. Let $\Delta = f(\Gamma')$, which is a finite set. If a **LLL**-model of Γ' verifies some member of $f(\Gamma')$, it verifies $Dab(\Delta)$. If a **LLL**-model of Γ' falsifies all members of $f(\Gamma')$, then, by C6, it verifies all formulas verified by some **ULL**-model of Γ, and hence it verifies A. It follows that all **LLL**-models of Γ' verify $A \vee Dab(\Delta)$.

For the right–left direction, suppose that $\Gamma \vdash_{\mathbf{LLL}} A \vee Dab(\Delta)$. By C5, no **ULL**-model verifies any member of Ω. It follows that all **ULL**-models of Γ (if any) verify A. ■

This theorem provides the motor for the dynamic proof theory. $Dab(\Delta)$ expresses that some formulas behave abnormally. Adaptive logics suppose that all formulas behave normally unless and until shown otherwise. So, if $\Gamma \vdash_{\mathbf{LLL}} A \vee Dab(\Delta)$ and the members of Γ have been derived from the premises,[37] one may derive A on the condition that certain formulas behave normally—what this means depends again on the strategy.

Just like any other proof, a dynamic proof consists of a sequence of formulas. Annotated proofs consist of a sequence of lines that have five elements: (i) a line number, (ii) the derived formula A, (iii) the line numbers of the formulas from which A is derived, (iv) the rule by which A is derived, and (v) a (possibly empty) 'condition'. The condition specifies which formulas have to behave normally in order for A to be so derivable.

Apart from the fifth element of the lines, the only unusual thing is that lines of a dynamic proof may be marked. The marks may change from one stage of the proof to the next (where adding a line brings the proof to its next stage). The formula (second element) of a line that is marked at stage s is considered as not derived at stage s. Marking is governed by a definition, which depends on the strategy.

I first list the rules for a proof from Γ. I list them in the form of generic rules.[38] Apart from a premise rule, there is an unconditional rule and a conditional rule.

PREM If $A \in \Gamma$, then one may add a line consisting of

(i) the appropriate line number,

[36] Remark that C4 will always be fulfilled in this case.
[37] Γ is an arbitrary set of formulas here, not the premise set.
[38] While this is unavoidable in the present general setup, it is even most convenient and transparent to characterize the proof theory of specific adaptive logics in terms of generic rules.

(ii) A,
(iii) "–",
(iv) "Prem", and
(v) $\emptyset$.

RU If $B_1, \ldots, B_m \vdash_{\mathbf{LLL}} A$ and $B_1, \ldots, B_m$ occur in the proof with the conditions $\Delta_1, \ldots, \Delta_m$ respectively, then one may add a line consisting of

(i) the appropriate line number,
(ii) A,
(iii) the line numbers of the B_i,
(iv) "RU", and
(v) $\Delta_1 \cup \ldots \cup \Delta_m$.

RC If $B_1, \ldots, B_m \vdash_{\mathbf{T}} A \vee Dab(\Theta)$ and $B_1, \ldots, B_m$ occur in the proof with the conditions $\Delta_1, \ldots, \Delta_m$ respectively, then one may add a line consisting of

(i) the appropriate line number,
(ii) A,
(iii) the line numbers of the B_i,
(iv) "RC", and
(v) $\Theta \cup \Delta_1 \cup \ldots \cup \Delta_m$.

At any stage of the proof, zero or more *Dab*-formulas will be derived. Some of them are minimal (at that stage). Let $U_s(\Gamma)$ be the union of all Δ for which $Dab(\Delta)$ is a minimal *Dab*-formula at stage s. Let $\Phi^\circ_s(\Gamma)$ be the set of all sets that contain one disjunct out of each minimal *Dab*-formula at stage s, and let $\Phi_s(\Gamma)$ contain those members of $\Phi^\circ_s(\Gamma)$ that are not proper supersets of other members of $\Phi^\circ_s(\Gamma)$.[39]

Definition 6 *Marking for* $\mathbf{AL}^r$*: Line i is marked at stage s iff, where* Δ *is its fifth element,* $\Delta \cap U_s(\Gamma) \neq \emptyset$.

Definition 7 *Marking for* $\mathbf{AL}^m$*: Line i is marked at stage s iff, where A is the second element and* Δ *the fifth element of line i, (i) there is no* $\varphi \in \Phi_s(\Gamma)$ *such that* $\varphi \cap \Delta = \emptyset$*, or (ii) for some* $\varphi \in \Phi_s(\Gamma)$*, there is no line k that has A as its second element and has as its fifth element some* Θ *such that* $\varphi \cap \Theta = \emptyset$.

At this point I can define **AL**-derivability:

Definition 8 *A is* derived at stage *s in an* **AL***-proof from* Γ *iff A is the second element of a line that is not marked in the proof (at stage s).*

Definition 9 *A is* finally derived *on line i of an* **AL***-proof (at a stage) from* Γ *iff (i) A is the second element of line i, (ii) line i is not marked at stage s, and (iii) any extension of the proof in which line i is marked may be further extended in such a way that line i is unmarked.*

[39]The proofs may made be somewhat more efficient by introducing some closing operations in the definitions of $U_s(\Gamma)$ and $\Phi_s(\Gamma)$. However, one should take computational matters into account: it should be decidable whether a line is marked or unmarked.

Definition 10 $\Gamma \vdash_{\mathbf{AL}} A$ *(A is* finally derivable *from* Γ*) iff A is finally derived on some line of an* **AL***-proof from* Γ*.*

For the specific logics that were studied, the Soundness and Completeness of the dynamic proof theory with respect to the semantics was proved. Apparently, these proofs may be generalized for all adaptive logics under consideration. For examples of dynamic proofs, I refer to the many papers on specific logics.[40]

While Definition 9 may be taken at face value for Reliability, some weird premise sets require that, in the case of Minimal Abnormality, infinite extensions of proofs are considered—see [8, p. 466] for an example.[41]

A central theorem for the dynamic proof theory is

Theorem 6 *If* $\Gamma \vdash_{\mathbf{AL}} A$*, then any proof from* Γ *can be extended into a proof in which A is finally derived from* Γ*. (Proof Invariance.)*

Proof. Consider any proof from Γ—call it P1. If $\Gamma \vdash_{\mathbf{AL}} A$, there is a proof from Γ—call it P2—in which A has been finally derived at some line i and that, if extending it with proof 1 results in line i being marked, may be further extended in such a way that line i is unmarked. Call the last extension E. Definitions 6 and 7 warrant that, if P2 is first extended with P2 and then with E, then the line that had number i in P1 is unmarked. ∎

What about decidability? The propositional fragments (and some other fragments) of most adaptive logics are decidable. This means that the dynamics of the proofs can in principle be avoided by deriving formulas in a suitable order and by not deriving any formulas that will be marked in view of other derived formulas. The full predicative versions of adaptive logics are obviously undecidable and have no positive test for final derivability.

Even when one is swimming in undecidable waters, there may be certain criteria that enable one to decide that a specific formula has been finally derived at some line of a dynamic proof from Γ. Some such criteria provide from work on the block approach (see for example [7]) and from work on tableau methods for adaptive logics (see [19] and [21]). Much more efficient criteria derive from goal directed dynamic proofs (work in progress, partly with Dagmar Provijn).

But what if no such criterion applies? It was shown in [7]—the result may be easily generalized to all considered adaptive logics—that as dynamic proofs proceed, the set of formulas derived at a stage offers an increasingly better estimate of the set of finally derivable formulas. This estimate is not merely a computational approximation, but there is an idea behind it: as the proof proceeds, it provides an increasingly better insight in the premises, and hence in the minimal *Dab*-formulas that are derivable from them. Moreover, the goal directed dynamic proofs provide means to speed up the gain of insight in the premises. The upshot is that dynamic proofs form a sensible basis for decision and action. In this sense, they not only enable one to explicate actual forms of dynamic reasoning, but also justify such forms of reasoning.

I shall be brief on prioritized and combined adaptive logics. The essential point was already mentioned: where different adaptive mechanisms are combined, one obtains dynamic proofs in which the dynamic mechanisms do not

[40] A list is available: `http://logica.rug.ac.be/adlog/albib.html`.

[41] Extensions of infinite proofs are obtained by inserting formulas in the proof.

act consecutively but at the same time. As a result, the dynamic proofs obtain their full explicatory and justificatory function.

6 In Conclusion

Several open problems have been mentioned in the previous sections and I shall not repeat them here. I shall rather add a final comment concerning the epistemological function of adaptive logics.

In my own work on epistemology (see for example [3] and [6]), I have stressed that the dynamics of human knowledge depends essentially on the fact that humans (as individuals or as groups) shift from one context to the other in solving problems. Adaptive logics do not offer an explication for this inter-contextual dynamics. They are meant to apply within contexts and to explicate the intra-contextual dynamics. Once we understand the latter, it may be hoped that one will be able to move on to understand the inter-contextual dynamics, to explicate it, and to find means to increase its computational as well as its problem-solving efficiency.[42]

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