

HOW TO PRECISIFY QUANTIFIERS

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Most people would deny that there is something entirely composed of Alpha Centauri and my left thumb. We are more tolerant of composites whose parts are more “connected”, a prime example being molecules. And then there are the various “intermediate” candidates, perhaps the mereological sum of a pregnant woman and the foetus, or schools of fish—cases in which we are not sure whether “there are” such putative things. As we move along the succession of mereological sums from the very connected toward the very scattered, we are decreasingly prone to agree with the claim that there is a thing composed of the objects in question. Since this will be so even as we have eliminated all vagueness in “composed of” and in terms referring to the putative parts, we have a *prima facie* reason for claiming that the unrestricted existential quantifier is vague.

As part of his case for four-dimensionalism, however, and against the neo-Carnapian metaontology of philosophers like Eli Hirsch (e.g., (2009)), Theodore Sider has argued, following David Lewis (1986: 213), that quantifiers and other logical constants cannot be vague (Sider (2001: 9.1), (2003), (2007), (2009)). The first of his two arguments, the one I will here try to refute, is what Liebesman and Eklund (2007) call the “indeterminacy argument”. Its major premise is that if an expression is vague, it must have *precisifications*, where a precisification is a meaning of a precise expression (2003:

137f.). More precisely, a precisification of a vague expression e is the meaning an expression e' , which is like e , except that it has been stipulated to have a sharp cut-off point on the scale relative to which e is indeterminate (in the case where there is only one such scale). Further, a precisification of an expression e should yield determinate readings to sentences that are indeterminate due to vagueness in e . For the case of quantifiers, this requirement comes down to the following. If “ \exists ” is vague in the way suggested above, there will be sentences of the form,

(E) $\exists x$ (x is composed of the F and the G),

that are indeterminate in truth value due to vagueness in “ \exists ”. Thus, if “ \exists ” is vague, then a sentence of this form could be indeterminate even if “is composed of” is precise and it is definitely true that the F and the G exist. So, a precisification of “ \exists ” must be such that on a reading of (E) on which “ \exists ” is interpreted as expressing the precisification, (E) comes out as definitely true or definitely false.

A second, crucial premise in Sider’s argument is that there *are no* precisifications of “ \exists ”, because there is no coherent way of *describing them*. Thus, by the major premise and *modus tollens*, “ \exists ” is not vague. Sider argues against the coherence of describing a precisification of “ \exists ” by considering two candidate description of precisifications of “ \exists ”, and how they fare with the indeterminate (E) above. The first description Sider considers, which I will call (D1), reads:

‘ \exists ’ has at least two precisifications, call them \exists_1 and \exists_2 . There is an object, x , that is in \exists_1 ’s domain but not in \exists_2 ’s domain, and which is composed of the F and the G . Thus, (E) is neither definitely true nor definitely false (2003: 139).

If this is how the candidate meanings are to be described, then, Sider claims, the idea that “ \exists ” is vague is incoherent. For it is here stated that *there is something* which is not in the domain of \exists_2 . In his (2009), Sider is clear on what is wrong with this: “If [(D1)] is assertible, it must be determinately true” (2009: 1). Thus, (D1) presupposes that it is definitely true that *there is something* not in the domain of \exists_2 . But this entails that (E) is not indeterminate, contrary to our assumption. Thus, (D1) is not a coherent description of the alleged precisifications of the quantifier.ⁱ

The second candidate description Sider considers, a proposal originally made by Hirsch (2002), goes as follows: “ \exists_1 is to be a precisification according to which *any* two objects have a mereological sum. The truth condition it assigns to (E) is the following: (E₁) $\exists x(x=\text{the } F) \ \& \ \exists x(x=\text{the } G)$. [...] Since by hypothesis the F and the G exist, (E₁) is definitely true. [...] \exists_2 , on the other hand, is to be a precisification on which only suitably related objects have a mereological sum.” (2003: 141). Sider’s main complaint with this proposal is that (E₁) cannot reasonably be taken as a candidate meaning of (E), since (E₁) does not have a quantifier as its main constant.ⁱⁱ He writes, “‘The F and the G exist’ seems clearly not to be a way of making precise the idea that ‘The F and the G exist, *and in addition there exists something made up of the F and the G* , which is what (E) says” (2003: 141).

These complaints about two specific attempts to precisify quantifiers of course do not establish that precisifications of quantifiers cannot be coherently described. What is needed, and what Sider adds (2003: 141), is the further premise that *any* candidate description of a precisification of the quantifier will be subject to either of these complaints. It is this claim I will here try to refute, by devising a different way of describing precisifications of quantifiers that is not subject to Sider's complaints (whether the latter are cogent or not).

Before presenting my proposal, I will make some general claims about the proper demands on precisifications of an expression. But first, I should clarify the dialectical situation. Since I will provide an argument against an argument against a claim (a counter-counter argument), I can legitimately assume the truth of that claim in doing so. Thus, we may legitimately assume that quantifiers are vague and that for sufficiently scattered objects, it is not definitely true that there is something composed of them. It is not that I need these assumptions to state the proposal. It is rather that, in general, making them would not illegitimately presuppose what is to be demonstrated. For, strictly speaking, what is to be demonstrated is merely that Sider's case against vagueness in quantifiers fails. By the same token, an argument that quantifiers are not vague is not an argument against the present paper.

I will follow Sider in taking precisifications to be *meanings of precise expressions*. To say that a precisification is a candidate meaning of a vague expression *e* is then not, of course, to say that it is the meaning of *e*. This relation, rather, will be specified below. Now, a meaning can be referred to as "the meaning of (expression) *e*", if *e* has been appropriately defined. For instance, if *e* is a predicate, the definition (a

sentence) may be a universally quantified biconditional that is stipulated to be true, but a definition may of course take other forms. For a definition of a term t to yield a proper precisification of a vague term v , it is reasonable to demand the following two things: firstly, for every circumstance C , and every sentence containing v that is a borderline case in C , and which owes its borderline status only to vagueness in v , the result of replacing v by t should yield a sentence with a definite truth value in C . Secondly, for every circumstance C , and every sentence containing v that is definitely true (false) in C , the result of replacing v by t should be definitely true (false) in C . (I will henceforth omit the relativisation to circumstances, however, since nothing below turns on it.)

The first demand arguably comes with the very notion of a *precisification*. The second is to ensure that the definition will yield a precisification of v , rather than some other term. I take these two necessary conditions for a definition to yield a proper precisification of a vague term to also be jointly sufficient. Thus, we may now define: m is a precisification of v just in case there is a term t , which has been defined so as to conform to these two demands, and m is the meaning of t .

With these demands in mind, we may now try to describe precisifications of the existential quantifier, corresponding to various degrees of connectedness. It is reasonable that there will be one aspect which is common to all of them, and which presumably makes them meanings of a *quantifier*. Let us call this element the *auxiliary logic* of the quantifier. This could be determined, e.g., by inference rules, axioms or axiom schemata, or a truth-theoretic characterisation. This auxiliary logic would, *inter alia*, validate such definitely true sentences as “If $F(a)$, then $\exists xF(x)$ ”, and so play a role in satisfying the second demand.

Second, a further element is needed in order to distinguish various precisifications of “ \exists ” from one another. Let us, to this end, stipulate that R_d be the multigrade relation of being connected to the precise degree d or higher, where d is any real number. Whether such a precise notion of connectedness can be defined is debatable, of course, but the problems involved here need not concern us, since this question is irrelevant to Sider’s argument. Now, we can simply take a given precisification of “ \exists ” to be *the* meaning of “ \exists_1 ”, and claim that (R) is a partial definition of “ \exists_1 ”:

(R) Not $\exists_1 x_1, \dots, x_n$ not: $(\exists_1 y (y \text{ is composed of } x_1, \dots, x_n) \equiv (x_1, \dots, x_n \text{ stand in } R_d))$.

(R) is not kind to the eye, but note that it is equivalent to the claim that all₁ objects are such that something₁ is composed of them iff they stand in R_d to one another (I am assuming that a corresponding universal quantifier can be introduced together with “ \exists_1 ”, e.g., by being defined in terms of the latter in the usual way). Note here that the smaller d is, the more “tolerant” the quantifier.

I said that (R) is a *definition* of “ \exists_1 ”. This merely means that it is a sentence stipulated to be true. However, it is only a *partial* definition, since it is not intended to imbue “ \exists_1 ” with a meaning by itself—the auxiliary logic is required as well. Further, it is an *implicit* definition, i.e., it is not of the form “ $\exists_1 x \phi$ iff ...”, where the right-hand side does not contain “ \exists_1 ”. But this, I submit, does not affect its adequacy. What is important is that it is stipulated as true, and makes the defined term accord with the two demands in virtue of this stipulation. As we will see, (R) exactly parallels explicit definitions of vague predicates in this respect, except that the latter definitions are ordinary universally

quantified biconditionals. Of course, (R) is equivalent to a universally quantified biconditional. But in ordinary explicit definitions, no quantifier is being defined, but rather a predicate. In any case, I submit, and will argue further below, that these syntactic differences are irrelevant.

Let us now compare this idea to a paradigmatic precisification of a vague term, say, “red”. The obvious way of describing the admissible precisifications of “red” goes by saying that they are the respective meanings of the terms “red₁”, “red₂”, ..., where each of these are defined by a biconditional of the form “For all x , x is red _{n} iff x has light reflectance property P_n ”, subject to the two demands stated above. The first of these will in this case require that if “ a is red” is a borderline sentence, owing its borderline status only to vagueness in “red”, then replacing “red” with any of the precisely defined terms “red₁”, “red₂”, ... will result in a sentence with a definite truth-value. As long as “ P_n ” is appropriately defined, this demand will be met since, for any object, it will then be definite whether it has wavelength property P_n , and for all n , it is definite (because definitional) that something is red _{n} iff it has property P_n . Clearly, the demand is met in virtue of the logical relationships between the definition and other sentences.

I claimed that (R) provides precise readings of borderline sentences in the same way. So let us look at the borderline sentence (E). If the vague quantifier in (E) is replaced by “ \exists_1 ”, the result should be a sentence with a definite truth-value. To show that this holds, we can argue as follows. First, we assume that (R) is not a *failed* definition (for instance, it does not make the underlying theory inconsistent). Then, since it is a definition, it is definitely true. Second, we are assuming that the F and the G definitely exist. But then,

$$(1) \quad \exists_1 x(x=\text{the } F) \ \& \ \exists_1 x(x=\text{the } G)$$

is definitely true, if the definition meets the second demand on precisifications (and it will be met provided d is small enough). Further, since “ R_d ” is assumed to be precise,

$$(2) \quad \text{The } F \text{ and the } G \text{ stand in } R_d$$

will come out as definitely true or definitely false. If (2) is definitely true, then

$$(3) \quad \exists_1 x (x \text{ is composed of the } F \text{ and the } G),$$

i.e., the result of replacing “ \exists ” by “ \exists_1 ” in (E), comes out as definitely true, too, since it follows logically from (R) and (2), which are definitely true.ⁱⁱⁱ If (2) is definitely *false*, by contrast, then its negation is definitely true. But from (R) and the negation of (2), we can derive the negation of (3). By essentially the same reasoning as above, then, (3) is definitely false. Thus, in either case, the sentence resulting from replacing “ \exists ” with “ \exists_1 ” in (E) comes out as definitely true or definitely false (depending on the truth value of (2)), whence the first demand is met. To meet the second demand, we must ensure, firstly, that d is not too great (if it is, then sentences “ $\exists_1 x\phi$ ” will come out as false when “ $\exists x\phi$ ” is definitely true). Secondly, if, for sufficiently scattered objects, it is definitely false that there is something composing them (and not just not definitely true), we must also require that d not be too small.

The observant reader will have noticed that (1) played no role in the derivation above. What was shown there was that (3) or its negation follows *classically* from definitely true claims. But, for reasons that will emerge below, it would be nice if we could do with the weaker *free logic* (see, e.g., Nolt (2006) for an introduction), in which universal instantiation is invalid, and which instead condones the more restricted rule $\forall\phi$, $\exists x(x=t) \Rightarrow \phi(x/t)$. Now, since we assumed that (1)—the relevant required existential premise—is definitely true, we can show that (3) is definitely true or definitely false assuming merely free logic.

Let us now consider how this proposal fares with Sider’s complaints about the two descriptions of precisifications considered above. We should first note that Sider’s objection from difference in logical form cannot plausibly be levelled at this proposal. His idea was that the meaning of “ $\exists x(x=\text{the } F) \ \& \ \exists x(x=\text{the } G)$ ” could not be an acceptable candidate meaning of (E), since the two sentences differ in logical form. But on my proposal, the relevant candidate meaning of (E) (i.e., the one had by reading “ \exists ” in (E) as having the meaning of “ \exists_1 ”) is that of (3), and I see no reason to regard (E) and (3) as having different logical forms (although, since Sider does not set any criteria for this, it is hard to be sure how exactly to interpret his argument). Of course, (R) itself has a peculiar logical form, but that should not matter. The logical form of (R) is of course motivated by its role in differentiating quantifiers with respect to degree of connectedness. Stipulating that (R) is true only affects the semantic content of the quantifier so defined. It helps fix the inferential role of “ \exists_1 ” just like ordinary introduction and elimination rules fix the inferential role of “ \exists ”. I see no reason to think that (R) should have the effect that “ \exists_1 ” and “ \exists ” come out as belonging to different syntactic types. Also, it is plausible to assume

that, in the case of predicate calculus quantifiers, the syntactic (surface) form *is* logical form—there is no hidden deep-structure. So Sider’s objection against the second candidate description does not seem apt here.

What about his complaints against (D1)? We saw that (D1) is inadequate as a description of precisifications because it presupposes something that contradicts the claim that (E) is not definitely true. This came about because of the way the ordinary quantifier was used in (D1). But (R) does not contain the ordinary quantifier at all, so there seems to be no reason to suspect that (R) should fall prey to his objection against (D1). All that follows from the present account is that, for various m and n , there is $_m$ something not in the domain of \exists_n . But this is nothing like the disastrous consequence of (D1).

Might there be other problems with this consequence? One possible objection is that if, for some m and n , there is $_m$ an object which is not in the domain of \exists_n , then the latter cannot be a proper precisification, since it is then not a candidate meaning of an *unrestricted quantifier*. To assess this objection, we must first ask what is meant by an “unrestricted quantifier”. It is perhaps most naturally understood as one whose domain contains *everything*. But one can also read “unrestricted” as relative to various precisifications. Thus, we can say that if there is $_m$ something not in the domain of \exists_n , then \exists_n is not “ m -unrestricted”. But it is hard to see why we should demand, for all i and j , that \exists_i come out as j -unrestricted. On the face of it, this is precisely what should *not* follow, given our desideratum of a series of quantifiers of increasing tolerance.

Perhaps, it may be thought, the claim that \exists_n is not m -unrestricted entails that \exists_n is not unrestricted in the natural sense, i.e., that not *everything* is in the domain of \exists_n . And this may lead one to suspect that \exists_n is not a proper precisification of the ordinary,

unrestricted quantifier. But, assuming that \exists_n meets the second demand for precisifications, we can argue that this entailment fails. Note first that the objector’s suspicion, the alleged entailment, is equivalent to the claim that “ $\exists_m x \neg(x$ is in the domain of $\exists_n)$ ” entails “ $\exists x \neg(x$ is in the domain of $\exists_n)$ ”. The former, if true, is definitely true. If it entails “ $\exists x \neg(x$ is in the domain of $\exists_n)$ ”, then the latter must be definitely true too, since definite truth is closed under entailment. But if “ \exists_n ” meets the second demand, then, for any sentence containing “ \exists ” that is definitely true, the result of replacing “ \exists ” by “ \exists_n ” must be true. But “ $\exists_n x \neg(x$ is in the domain of $\exists_n)$ ” is clearly false. Hence, “ $\exists x \neg(x$ is in the domain of $\exists_n)$ ” is not definitely true, so the entailment fails. (Could the sentence “ $\exists x \neg(x$ is in the domain of $\exists_n)$ ” be definitely *false*? That is, could it be definitely true that everything is in the domain of \exists_n ? Not if there is a precisification \exists_m which meets the second demand and which is such that “ $\exists_m x \neg(x$ is in the domain of $\exists_n)$ ” is true. Hence, in such a case, it must be indeterminate whether “ \exists_n ” is unrestricted in the natural sense.)

There is a different way to take the objection that if there is_m something not in the domain of \exists_n , then \exists_n is not properly unrestricted. That is to read it as *demanding* that, for every precisification of “ \exists ”, it comes out as definitely true that everything is in its domain. But adherents of the vagueness view should reject this demand as both unmotivated and question-begging, for to meet this demand is simply to show that the ordinary quantifier is precise. We have now looked at two interpretations of the argument that if \exists_n is not *m*-unrestricted, then \exists_n is not an admissible precisification, and found it wanting on both. Before moving on, I would like to note an interesting corollary of the above demonstration that “ $\exists_m x \neg(x$ is in the domain of $\exists_n)$ ” does not entail “ $\exists x \neg(x$ is in

the domain of \exists_n ”, which concerns the inadequacy of (D1). To wit, we can now give an explanation of why (D1) fails that is more direct and does not mention an allegedly indeterminate sentence like (E). (D1), recall, presupposes that it is determinately true that there is an object, which is in the domain of \exists_1 but not in that of \exists_2 . We have just seen that if distinct precisifications \exists_m and \exists_n satisfy the two demands, then it will be indeterminate whether something is in one domain but not in the other. What (D1) presupposes (or what must be the case if it is assertible) is thus inconsistent with the assumption that \exists_1 and \exists_2 are proper precisifications.

Let us now look at some other possible problems with our method of devising precisifications of the quantifier. Recall that (R)-like definitions are to be taken as merely partial definitions, to be supplemented with logical principles common to all quantifiers, such as basic inference rules. But suppose we use an ordinary introduction rule,

$$(\exists_1 I) \quad F(t) \Rightarrow \exists_1 x F(x)$$

to partially define “ \exists_1 ”. Now, we might worry that one could infer “ $\exists_1 x F(x)$ ” from a sentence “ $F(\text{the object composed of } a \text{ and } b)$ ” even if a and b are connected to a degree lower than d (so that there is₁ no object composed of a and b). After all, “ t ” in $(\exists_1 I)$ is a schematic *term* letter and “the object composed of a and b ” is a term. I think there are many possible solutions to this problem. However, I will not recommend any particular one here, but merely give a rough map of the territory. The solutions can be divided, firstly, into those on which definite descriptions are terms (in the sense of $(\exists_1 I)$) and those on which they are not. On the first option, we can go two ways. We can stick with $(\exists_1 I)$

and argue as follows. Since there are no true sentences containing the problematic kind of terms, the rule could not take us from true sentences to false conclusions, wherefore it would be harmless. Alternatively, we could reject $(\exists_1 I)$ —perhaps because we think there are true sentences containing such terms. On this proposal, a reasonable alternative to $(\exists_1 I)$ is its analogue in free logic, i.e., $F(t), \exists_1 x(x=t) \Rightarrow \exists_1 xF(x)$. And we saw above that a major desideratum of this theory can be satisfied also on free logic.

On the second type of solution, we deny that definite descriptions are terms (in the relevant sense of “term”). The most obvious account of “the”, on this option, is to adopt a broadly Russellian analysis of definite descriptions as quantificational. In that case “the” will be vague along the same dimension as quantifiers. Then, it has to be precisified in tandem with quantifiers, in the sense that whatever relations we take to hold between “the” and “ \exists ” should also hold between precisifications thereof, the latter being the meanings of precisely defined expressions “ \exists_n ” and “the_n”. We can then ensure that there be no case in which “ $F(\text{The}_1 \text{ object composed of } a \text{ and } b)$ ” comes out as true and “ $\exists_1 x F(x)$ ” as false. So far so good. However, this solution does not necessarily save $(\exists_1 I)$ from an analogous objection involving *proper names*. For suppose we could introduce a name by way of an identity sentence linking the name with a definite description. Then, since names must surely count as terms, $(\exists_1 I)$ must be reconsidered again.

We have seen that there are several takes on this problem, and there is certainly much more to say about it. Still, I will leave the matter here, not so much because it is a trivial problem, but because, on the contrary, I think it is an instance of the more general problem of empty terms and existential generalisation. If that is true, then, fortunately for the present account of precisifications, the problem is not produced by the account, and

thus does not tell against it. Further, since the general problem presumably has a solution, that solution can reasonably be taken as solving the problem with $(\exists_1 I)$.

Finally, I will try to make good the promise made early on in the paper, and show how precisifications of “ \exists ” might be given a truth-theoretic semantic description to the effect that a sentence of the form “ $\exists_1 x \phi$ ” is true iff p . This is of course possible if “ \exists_1 ” itself is allowed into the metalanguage. But then, the interpretation of “ \exists_1 ”, as used in the metalanguage would still rely on an implicit definition like (R). But contrary to what might be expected given Sider’s criticism of (D1), the truth-conditions of “ $\exists_1 x \phi$ ” can also be given without using “ \exists_1 ”, indeed, in a way that provides a uniform treatment of all precisifications of “ \exists ”. So we need not say that the only way of defining “ \exists_1 ” is by saying that (R) implicitly defines it (although, I should say, I see no reason to regard that as an unacceptable consequence). For we can give the truth-conditions of quantified sentences in terms of a *maximally tolerant quantifier*, “ \exists_M ”, partially defined by

$$(R_M) \quad \text{Not-}\exists_M x_1, \dots, x_n \text{ not: } \exists_M y \text{ (} y \text{ is composed of } x_1, \dots, x_n \text{)}.$$

Now, “ \exists_1 ” can be truth-theoretically specified by the clause,

$$\begin{aligned} \text{“}\exists_1 x \phi\text{” is true iff } & \exists_M x_1, \dots, x_n \text{ (} x_1, \dots, x_n \text{ are connected to degree } d \text{ or higher, and} \\ & \exists_M y \text{ (} x_1, \dots, x_n \text{ are parts of } y \text{ and } y \text{ satisfies } \phi \text{))}. \end{aligned}$$

Or we could say, which comes to the same thing, that the domain of “ \exists_1 ” is the set of every_M object all_M of whose parts are connected to degree d or higher. This avoids

Sider's objection against the first candidate precisifications simply by taking the quantifier used to characterise " \exists_1 " to be maximally tolerant.

Of course, if it is definitely false that there are things composed of very scattered things, then \exists_M is not an admissible precisification of the ordinary quantifier, however natural (in David Lewis's sense of "natural"). But this does not mean that adherents of the vagueness view cannot define, use, or understand \exists_M , for that is surely independent of the actual meaning of the ordinary quantifier.

In view of Sider's consideration about (D1), however, it is hard to see how " \exists_M " could itself be defined without recourse to a (partial) implicit definition. For if the vagueness view is true and, for sufficiently scattered things, it is not definitely true that something is composed of them, then the ordinary quantifier cannot be used to describe the domain of " \exists_M " or state truth-conditions of sentences containing it. So, probably, precisifications of quantifiers must fundamentally rely on the idea of (partial) implicit definitions.

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ⁱ One might think that this consideration is itself an objection against vagueness in quantifiers. That is, it may be thought that the claim that (1) " \exists " is vague *entails* that (2) there are admissible readings of " \exists ", \exists_1 and \exists_2 , such that *there is* an object in the domain of \exists_1 , which is not in the domain of \exists_2 . But, the objection goes, since (2) entails that (E) is not indeterminate, and since parallel reasoning applies to any such allegedly

indeterminate sentence, “ \exists ” is not vague after all. It is clear from Sider’s words that this is not the argument he intends to give. If it were, however, we should point out that we have not been given any reason to believe that (1) entails (2). (1) will presumably entail a claim which is like (2), except containing a suitably defined precise quantifier in place of the ordinary one (which will be illustrated toward the end of the paper). But that is irrelevant to (1) and (2).

ⁱⁱ He also thinks the proposal as stated here cannot be properly generalised, but in his (2007) and (2009), he devises a version of the proposal that avoids this problem. But the objection by Sider mentioned here stands, why we may stick with this simpler proposal.

ⁱⁱⁱ To see how, note first that the relevant instance of (R), i.e., “Not $\exists_1 x, y$ not: ($\exists_1 z$ (z is composed of x, y) \equiv (x, y stand in R_d))”, is equivalent (both classically and on free logic) to “ $\forall_1 x, y$ ($\exists_1 z$ (z is composed of x, y) \equiv (x, y stand in R_d))”. By universal instantiation, we infer, “ $\exists_1 z$ (z is composed of the F , the G) \equiv (the F , the G stand in R_d)”. By equivalence elimination, (2), and *modus ponens*, we derive “ $\exists_1 z$ (z is composed of the F , the G)”.

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