# Hudson on Receptacles

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In a recent paper, Hud Hudson argues for what he calls the 'liberal view of receptacles' [Hudson 2002]. Here, a 'receptacle' is a set of spatial points which could be exactly occupied by a material object, and the liberal view is the view that *any* set of points constitutes a receptacle. Hudson's argument for this conclusion is complicated. For one thing, it's structured indirectly. Hudson argues *against* three rival views of receptacles (the view that material objects must occupy open regions, the view that they must occupy closed regions, and the view that they must be three-dimensional); he then suggests that the liberal view 'appears considerably more attractive' in the aftermath of these arguments. For another thing, the nature of the views Hudson is arguing against leads him to engage in some complicated *topological* constructions.

Now, I have no desire to challenge Hudson's arguments here. (Not being a metaphysician myself, I find that I have *no* independent views on the nature of receptacles.) But I think it's worth noting that the mathematics in Hudson's paper can be dramatically simplified—in particular, the topology can be almost completely eliminated. Doing so will, I think, help to clarify the significance of Hudson's arguments.

## 1   The Philosophy

Hudson's arguments rest on three principles. The first two are highlighted near the beginning of his paper [434–435]; the third comes out later in his argument [438].

**(U)** Necessarily, for any collection of material objects, *the xs,* there exists a material object, y, such that *the xs* compose y.

**(E)** Necessarily, there are no spatially-extended material simples.[1]

**(RP)** Necessarily, for any material objects x and y, if x has y as a part, and y does not have x as a part, then there exists a material object z such that x has z as a part and z does not overlap y.

Along with these principles, Hudson assumes that space consists of a continuum of spatial points (so, it's neither granular nor gunky). He also alludes in passing to a 'recombination principle' which allows us to argue directly from the existence of a one-point receptacle to the truth of the liberal view [436 and 438]. Without

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[1]It's not clear just what Hudson means here. He could mean simply that any extended material object *has* a proper part. Or, he could mean that any extended material object is *decomposable into* proper parts—i.e., that there exist disjoint parts $\langle A_i \mid i < \alpha \rangle$ such that the fusion of these parts gives us the original object. Judging from arguments on 435, 436 and 437, I would guess that Hudson intends the latter principle. For our purposes, however, the earlier (weaker) principle will suffice.

Also, it's not quite clear what Hudson means by 'spatially-extended.' In section 2, I assume that an object is 'extended' if its receptacle contains more than one point. In section 3, I discuss the impact of other conceptions of 'extension' on my argument.

worrying about the exact nature of this principle,[2] I'll focus on a related principle which is supposedly a consequence of Hudson's principle—call it the 'liberals win on points' principle:

**(LWP)** If any point is a receptacle, then the liberal view is correct.

Together, these principles and assumptions provide the background for Hudson's discussion.

In the next section, I will present a simple, non-topological, proof of the following result: *(U), (E), and (RP) entail that there are one-point receptacles.* This result has three immediate consequences. First, it provides a simple argument against two of Hudson's rivals—the 'open' theory and the 'three-dimensional' theory—as neither of these two theories allows one-point receptacles. Second, it provides an argument against the 'closed' theory, provided that this theory admits receptacles which have non-closed subsets.[3] Finally, if we go ahead and accept (LWP), then it gives a *direct* argument for Hudson's liberal view. At the end of the day, then, we get all of Hudson's conclusions (and more!) with almost none of his work.

## 2 The Mathematics

Let $A$ and $B$ be material objects. For notational purposes, I will use $\bar{A}$ and $\bar{B}$ to denote the receptacles occupied by $A$ and $B$, I will use $A < B$ to abbreviate '$A$ is a proper part of $B$,' and I will use $A \cup B$ to denote the fusion of $A$ and $B$. I begin with a lemma:[4]

**Lemma 1.** *Let $A < B$. Then $\bar{A} \subsetneq \bar{B}$.*

*Proof.* Let $A < B$. By (RP), there exists $A' < B$ such that $A$ and $A'$ do not overlap. This is incompatible with $\bar{A} = \bar{B}$. Q.E.D.

**Theorem 2.** *Let $p$ be a point in $\bar{A}$. Then $\{p\}$ is the receptacle for a part of $A$.*

*Proof.* If $\{p\} = \bar{A}$, then we are done. Otherwise, we construct an increasing sequence, $A_0 < A_1 < A_2 \ldots$, of proper parts of $A$ such that for each $\alpha$, $p \notin \bar{A}_\alpha$. We begin by applying (E) to obtain $B < A$ and then (RP) to obtain $B' < A$ such that $B$ and $B'$ do not overlap. If $p \in \bar{B}$, we let $A_0 = B'$; otherwise we let $A_0 = B$.

Given $A_\alpha$, we apply (RP) to generate $B_\alpha < A$ such that $A_\alpha$ and $B_\alpha$ do not overlap. There are three cases to consider:

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[2] Presumably, Hudson's recombination principle is supposed to do two things. First, it lets us 'move receptacles around' so that, if one point is a receptacle, then any point is a receptacle. It also has some mechanism for ensuring that it's possible for any *collection* of receptacles to be jointly occupied (perhaps the 'patchwork principle'?).

[3] Two remarks are in order. First, the result in the next section actually shows that *any* point within a receptacle is also a receptacle. This, together with Hudson's (U), is enough to generate the argument mentioned above. Second, the qualification on 'admitting receptacles with non-closed subsets' is important. In principle someone could think that matter is only possible at points with rational coordinates of the form $(\frac{m_1}{2^{20}}, \frac{m_2}{2^{20}}, \frac{m_3}{2^{20}})$ where $m_1 \ldots m_3$ are integers. This view is perfectly consistent with (U), (E), and (RP), and it easily entails that all receptacles are closed.

[4] In the mathematics which follows, I make use of the Axiom of Choice (to justify transfinite induction up to $(2^{\aleph_0})^+$). While few mathematicians today have scruples about this axiom, it's use in a metaphysical context should probably be noted. As far as I can tell, Hudson's own arguments do not invoke Choice.

**Case 1.** Suppose $p \notin \bar{B}_\alpha$. Then we let $A_{\alpha+1} = A_\alpha \cup B_\alpha$, noting that $A_{\alpha+1} < A$ by (U).[5]

**Case 2.** Suppose $p \in \bar{B}_\alpha$ but $\{p\} \neq \bar{B}_\alpha$. Then we apply (E) to obtain $C_\alpha < B_\alpha$ and then (RP) to obtain $C'_\alpha < B_\alpha$ such that $C_\alpha$ and $C'_\alpha$ do not overlap. If $p \in \bar{C}_\alpha$, we let $A_{\alpha+1} = A_\alpha \cup C'_\alpha$; otherwise we let $A_{\alpha+1} = A_\alpha \cup C_\alpha$.

**Case 3.** Suppose $\{p\} = \bar{B}_\alpha$. Then $\{p\}$ is the receptacle for $B_\alpha$ and our theorem is done.

At limits, we let $A_\lambda = \bigcup_{\alpha < \lambda} A_\alpha$ and note that $A_\lambda < A$ by (U). By Lemma 1, the sequence of receptacles, $\bar{A}_0 \subsetneq \bar{A}_1 \subsetneq \bar{A}_2 \ldots$, is constantly increasing. For cardinality reasons, therefore, our construction must terminate in less than $(2^{\aleph_0})^+$ steps at some 'Case 3' situation. Eventually, therefore, we obtain a $B_\alpha < A$ which has $\{p\}$ as its receptacle.                                           Q.E.D.

## 3   A Caveat

Before concluding, I want to briefly mention a minor worry concerning the above argument. In applying (E) in the proof of Theorem 2, I assumed that a region is 'extended' if it contains more than one point. It is possible, however, that Hudson has a more restrictive conception of extension (perhaps, that a region is extended iff it contains an open ball or iff it has non-zero topological dimension).

There are two things to say here. First, I think Hudson *is* committed to the 'one-point' version of (E). Some of his arguments seem to presuppose that discontinuous objects have proper parts [435, 436 and 437]; others presuppose that continuous, non-point sized, objects have proper parts [435, 436 and 437]. Together, these assumptions entail the 'one-point' version of (E). Second, and more importantly, a trivial modification of the proof of Theorem 2 gives the following (for *any* conception of extension).

**Theorem 2′.** *Let $p$ be a point in $\bar{A}$. Then there exists $X \subset \bar{A}$ such that $p \in X$ and $X$ is the receptacle for a simple (hence, non-extended) material object.*

On any reasonable conception of extension, this theorem is still enough to rule out the 'open' view and the 'three-dimensional' view. Further, Hudson clearly assumes that the existence of non-extended closed receptacles is incompatible with the 'closed' view [436]. If so, then the theorem rules out that view as well. (And if not, then Hudson's *own* arguments against the 'closed' view fail too.) At the end of the day, therefore, we still get all of Hudson's conclusions with almost none of his work (and, in particular, with virtually *no* topology).

## References

Hudson, Hud 2002. The Liberal View of Receptacles, *Australasian Journal of Philosophy* 80: 432–439.

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[5]Throughout this paper, I interpret (U) as entailing that a fusion of parts of $A$ is also a part of $A$ (and not just a material object whose receptacle is a subset of $\bar{A}$). It is clear from, e.g., 438 that this is Hudson's own understanding of (U).