PROPERTY THEORIES

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INTRODUCTION: WHAT IS A PROPERTY THEORY?

We begin with a truism.[1] A property theory is a theory that deals with properties. More precisely, it is a theory that formulates general, non-contingent laws that deal with properties. There are two salient ways of talking of properties. First, they can be talked about as *predicables* (i.e., as *instantiables*). Accordingly, one sort of property theory would be a

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theory that provides general, noncontingent laws for the behavior of the predication relation (instantiation relation). Nothing prevents the logical framework of such a theory from being extensional; that is, it could be formulated in a logical framework in which equivalent formulas are intersubstitutable *salva veritate*. For example, this sort of property theory could be formulated in a first-order extensional language with identity and a distinguished binary logical predicate for the predication relation. The major challenge facing this sort of property theory is to resolve various paradoxes that result from naive predication principles such as the following analogue of Russell's paradox: $(\exists x)(\forall y)$ (x is predicable of $y \leftrightarrow y$ is not predicable of itself). The second salient way of talking about properties is by means of *property abstracts* such as 'the property of being a man'. Property abstracts belong to a family of complex singular terms known as *intensional abstracts*. These include gerundive phrases, infinitive phrases, and 'that'-clauses. These singular terms are intensional in the sense that expressions occurring within them do not obey the substitutivity principles of extensional logic. Accordingly, another sort of property theory would be a theory that provides general, noncontingent laws for the behavior of intensional abstracts. The major challenge facing this sort of property theory is to systematize various subtle nonextensional substitutivity phenomena such as the nonsubstitutivity of necessarily equivalent formulas, the nonsubstitutivity of co-denoting names and indexicals, the paradox of analysis, and Mates' puzzle. These two types of property theory can be developed independently. Once this is done, one would then want to combine them to arrive at a single theory that treats both predication and intensional abstraction.

Although both types of property theory are important, the second type has an epistemological primacy, which we will now explain. Evidently, the best argument for the existence of properties – and for intensional entities, generally – is the following *argument from intensional logic*.[2] (Intensional logic is that part of logic in which the principle of the substitutivity of equivalent formulas fails.) The argument has three premises, which of course must be established. First, the best way to formulate intensional logic is to adjoin an intensional abstraction operation to standard extensional logic with identity and then to formulate laws governing the substitutivity conditions on expressions occurring within intensional abstracts. Second, on any acceptable interpretation of this intensional logic, intensional abstracts must be interpreted as being semantically correlated with real intensional entities, specifically, entities whose identity conditions match the substitutivity conditions of the intensional abstracts with which they are semantically correlated. From this premise it follows that intensional logic is committed to the existence of intensional entities. Third, intensional logic is an indispensable part of any acceptable comprehensive theory of the world. (It is understood here that any acceptable comprehensive theory of the world should include an account of its own acceptability.)[3] From these three premises it follows that intensional entities – and properties, in particular – are indispensable to any acceptable comprehensive theory of the world. Every acceptable comprehensive theory of the world is committed to the existence of intensional entities.[4]

However, suppose *per impossibile* that intensional logic could be omitted from an acceptable comprehensive theory. In that event, it is plausible that intensional entities – and properties, in particular – could be dispensed with. True enough, the sort of property theory that codifies laws for the predication relation might have nice theoretical payoffs; for example, it might provide an elegant construction of the foundations of mathematics or of the extensional semantics of extensional language. However, these payoffs on their own do not appear to justify an ontology of properties. The reason is that, for all we can tell, each of these theoretical payoffs could be provided by a theory of extensional entities (such as sets, or mathematical categories, or perhaps some new type of extensional theoretical posit). Thus, in the absence of the argument from intensional logic, intensional entities would, for all we can tell, be dispensable in favor of such extensional entities. It is for this reason that property theory construed as an intensional logic is epistemologically more primary than property theory construed as a theory of predication. It is needed to show that properties really exist.

Once one has established the need for the former sort of property theory, one would be justified in going on to develop the latter sort of property theory. There are two reasons. First, once one has established that properties exist, Ockham's razor directs one to attempt to dispense with the more complex ontology of both sets and properties in favor of the simpler ontology of just properties. However, one can accomplish this ontological simplification only if one has a property theory that has all the theoretical payoffs that set theory has. This is what a satisfactory theory of predication promises to do. Second, when a theory of intensional abstraction is combined with a satisfactory theory of predication, the resulting theory promises to yield several additional theoretical payoffs (for example, a definition of truth for propositions, a definition of necessity for propositions, a definition of logical validity and analyticity for propositions, and so on).[5]

Although there are good reasons to look forward to a unified theory of intensional abstraction and predication, there are nevertheless good methodological reasons for proceeding in separate stages. For example, the immediate prospects of finding a truly satisfactory (as opposed to merely workable) resolution of the paradoxes of predication are much less bright than those for a satisfactory intensional logic. This and other methodological reasons for keeping the two projects separate at this stage of research will be elaborated upon below.

Our plan in this paper is the following. In Part I, we will spell out the argument from intensional logic. In Part II, we will show in detail how to construct a property theory that is suited to serve as an intensional logic. In Part III, we will close with a discussion of a few somewhat more sophisticated issues in property theory, namely, the propositional-function thesis, type-free predication theories, and a proof of nonextensionality within predication theories with unrestricted abstraction principles.

The propositional-function thesis is the thesis that there is a strong correlation between properties and propositional functions. A theory that takes properties as primitive entities can capture the extremely fine-grained substitutivity conditions that hold in propositional-attitude contexts. The question we will address in connection with the propositional-function thesis is whether a propositional-functional theory can capture these extremely fine-grained substitutivity conditions as well.

When we speak of type-free predication theories, we have in mind theories in the style of Gilmore [1974], Feferman [1975], Scott [1975], Aczel [1980], Feferman [1984], Reinhardt [1985], Flagg and Myhill [1987], Turner [1987], and others. As we indicated above, we do not believe that any of the existing theories can be singled out as embodying a final resolution of the paradoxes. Nevertheless, these recent type-free predication theories have an attractive feature from our point of view: one can actually prove within them that the most general form of abstraction principle for the predication relation entails a principle of nonextensionality. We will present an outline of the argument in Part III.

Despite the bewildering diversity of the diagnoses and cures for the paradoxes offered within a type-free setting, all these theories share one common theme, namely, that the mathematics that can be derived within them is rather weak. Because of the missing link between our favored theory of intensional abstracts and an ideal resolution of the paradoxes and because of the limited success of existing predication theories in providing a non-set-theoretic foundation of mathematics, we have forgone a critical review of these theories. Nevertheless, Part III does contain some discussion of the ubiquitous technique in this area, namely, the use of fixed-point constructions to establish the consistency of systems that admit unlimited self-reference in the presence of a principle of full abstraction.

Finally, a word about other research on property theory. Given the purpose of the present volume, we thought it would be valuable to defend and to formulate in detail one particular version of a property theory rather than to attempt a comprehensive overview. The reader will nevertheless find references to competing property theories at various points in our discussion and also in the bibliography.

I. THE ARGUMENT FROM INTENSIONAL LOGIC

The hallmark of extensional logic is the principle that equivalent expressions can be substituted for one another *salva veritate*. That is, whenever two expressions of the same syntactic type apply to exactly the same things – or, in the case of sentences, are alike in truth value – they can be substituted for one another without altering the truth values of the whole sentences in which they occur. The safest general characterization of intensional logic is that it is the part of logic in which there are exceptions, or at least apparent exceptions, to this substitutivity principle.

Sometimes, however, a second criterion is used to characterize intensional logic. According to this criterion, intensional logic is the part of logic in which the rule of existential generalization fails or at least appears to fail. For example, the inference from 'Pythagoras was looking for the rational $\sqrt{2}$' to 'There exists something such that Pythagoras was looking for it' is intuitively invalid; therefore, the occurrence of 'the rational $\sqrt{2}$' in the first sentence would qualify as intensional according to this second criterion. And this is as it should be. However, this criterion is not quite right as it is usually stated, for existential generalization appears to fail in some cases that would not standardly be counted as intensional. For example, the inference from 'The rational $\sqrt{2}$ does not exist' to 'There exists something such that it does not exist' is intuitively invalid, but the occurrence of 'the rational $\sqrt{2}$' in the first sentence would not standardly be counted as intensional.[6]

The first criterion – substitutivity failure – avoids this sort of difficulty. Since 'the rational $\sqrt{2}$' is a vacuous term, the only other terms that apply to the same (real) things must themselves be vacuous; and whenever another vacuous term is substituted for 'the rational $\sqrt{2}$' in 'The rational $\sqrt{2}$ does not exist', the resulting sentence has the same truth value as the original. So according to the substitutivity criterion, 'the rational $\sqrt{2}$' does not occur intensionally in this sentence. And this is the desired result. At the same time, in sentences like 'Pythagoras was looking for the rational $\sqrt{2}$', this vacuous term does occur intensionally according to the substitutivity criterion, for when we put in some other vacuous term, the resulting sentence will often have a different truth value. (For example, 'Pythagoras was looking for the largest integer' is false.) So once again this criterion fits our standard notion of intensional occurrence. It appears, therefore, that failure of substitutivity is indeed the best criterion to use in characterizing intensional logic. This, at least, is what we will assume in the remainder of this paper.

There are a number of interconnected logical phenomena that any adequate formulation of intensional logic ought to accommodate. Although some of them are widely recognized, others are not (for example, the existence of transcendental and self-embeddable predicates). Taken together, these phenomena more or less force one to formulate intensional logic as a certain sort of first-order theory of properties. Our argument for this thesis will be divided into the following sections: (1) generality, (2) 'that'-clauses, gerundive phrases, and infinitive phrases, (3) quantifying-in, (4) learnability, (5) referential semantics for intensional language, (6) what intensional abstracts denote, (7) nominalism, (8) conceptualism, (9) realism, (10) transcendental predicates and type-free languages, (11) self-embeddable properties, relations, and propositions, (12) the first-order vs. higher-order language controversy, (13) names and indexicals, (14) Mates' puzzle, the paradox of analysis, and the need for fine-grained intensional distinctions.[7]

1. GENERALITY

Substitutivity failures typically arise in connection with talk about such matters as intentionality (assertion, belief, desire, intention, perception, etc.), the logical modalities (necessity, possibility, contingency), definition, analyticity, meaning, strict implication, relevant implication, moral obligation, purpose, probability, causation, explanation, epistemic justification, evidence, counterfactuality, etc. However, many philosophers, logicians, and linguists have failed to notice that, when we talk about these matters in a *general* way, our discourse is typically *extensional*. For example, the sentence 'Whatever is necessary is possible' says something general about necessity and possibility, and it is a fully extensional sentence. Any adequate formulation of intensional logic should be able to accommodate, not just intensional talk about intentionality, modality, etc., but also this sort of general extensional talk. In particular, an adequate formulation of intensional logic ought to be able to represent intuitively valid extensional arguments like the following:

(I) Whatever x believes is necessary.
Whatever is necessary is possible.
∴ Whatever x believes is possible.

Suppose that 'is necessary' and 'is possible' are treated as 1-place predicate expressions and 'believes', as a 2-place predicate expression. Then this argument can be represented as valid in a standard quantifier logic:

(I′)
$$(\forall y)(B^2xy \rightarrow N^1y)$$
$$(\forall y)(N^1y \rightarrow P^1y)$$
$$\therefore (\forall y)(B^2xy \rightarrow P^1y).$$

Now in theoretical matters, if a currently accepted theory can be easily and naturally employed to account for new phenomena, then other things being equal it is desirable to do so. In logical theory, the currently accepted theory includes quantifier logic. By treating 'is necessary' and 'is possible' as 1-place predicate expressions and 'believes' as a 2-place predicate expression, we can easily and naturally account for the validity of (I) within this currently accepted theory. Therefore, other things being equal it would seem desirable to do so. Indeed, none of the alternatives appears to be satisfactory.

For example, one alternative is the sentential-operator approach, which posits an open-ended list of special sentential operators (that is, operators that can be applied to sentences to yield new sentences). On this approach, there is a separate operator for each of the topics mentioned earlier – assertion, belief, desire, intention, necessity, possibility, contingency, definition, analyticity, meaning, strict implication, relevant implication, moral obligation, purpose, probability, causation, explanation, epistemic justification, counterfactuality, etc. A major problem with this approach is that it does not provide a *unified* theory of intensionality; it is eclectic and incomplete at best. Furthermore, on the first-order version of this approach, elementary arguments like (I) cannot even be *expressed*. The reason is that in a first-order language sentential operators like 'B_X', '$\Box$', and '$\Diamond$' may only be applied to *specific* first-order sentences. (By 'first-order' we mean syntactically first-order. In a language that is syntactically first-order there are no sentential variables.)[8] But arguments like (I) are *general*; in such arguments expressions like 'x believes', 'is necessary', and 'is possible' are not applied to any specific sentences at all. Plainly, a variable-*cum*-quantifier apparatus – or something comparable – is needed.

Of course, such an apparatus is available on a higher-order sentential-operator approach that contains sentential variables, that is, variables that are themselves counted as sentences and that take other sentences as substituends. (By 'higher-order' we mean syntactically higher-order.)[9] On such an approach (I) would be represented along the following lines:

$$\begin{array}{l} (\forall p)(B_x p \rightarrow \Box p) \\ \underline{(\forall p)(\Box p \rightarrow \Diamond p)} \\ \therefore\ (\forall p)(B_x p \rightarrow \Diamond p). \end{array}$$

On this approach, however, there is no clear distinction between sentential operators, on the one hand, and predicates that take sentences as argument expressions, on the other hand. Consequently, this approach may be viewed as a variant of – rather than a genuine alternative to – our official logical syntax which treats 'believes', 'is necessary', 'is possible', etc. as predicate expressions.

Should the higher-order sentential-operator approach be adopted? In the next paragraph we will give some surface grammatical evidence against this approach. We wish to emphasize that our larger line of argument does not depend on this. So if this grammatical evidence should strike our readers as unconvincing, they should not for this reason be doubtful about our other conclusions. In particular, the argument in the next section that 'that'-clauses should be treated as singular terms will hold with at most minor alterations: on the higher-order sentential-operator approach, sentences are already treated as singular terms, and expressions like 'x believes that p' and 'x believes p' are in effect counted as mere notational variants of one another.

Now although the higher-order sentential-operator approach succeeds in representing arguments like (I), it has *prima facie* implausible side-effects. In particular, it is forced to treat sentences as full-fleged singular terms. Consider the sentence 'There is something that x believes that is different from something that y believes'. On the higher-order approach this sentence would be represented as follows:

$$(\exists p)(B_x p \ \&\ (\exists q)(\neg p = q \ \&\ B_y q)).$$

This shows that these special sentential variables are able to flank the identity symbol as in the (open) sentence '$p = q$'. But since specific (closed) sentences are supposed to be substituends for these variables, specific (closed) sentences may themselves flank the identity symbol, for example, '$5 + 7 = 12 = 7 < 9$', '$2 + 2 = 4 = 3 + 5 = 8$', etc. This, however, is ungrammatical nonsense. The problem seems plain: sentences are not genuine singular terms. But if they are not, then sentential variables – that is, variables for which specific closed sentences are supposed to be substituends – must themselves be illegitimate. It follows that, if a sentential-operator approach makes use of such variables, it too is illegitimate. This, of course, is not to say that the use of a special sort of variable (say, 'p', 'q', 'r', . . .) whose *values* are supposed to be sentences is illegitimate; what is illegitimate is the use of a variable whose *substituends* are supposed to be sentences. But the latter sort of variable is what is needed to enable a sentential-operator approach to represent general sentences and general arguments.

Whether or not this grammatical evidence against the higher-order sentential-operator approach is convincing, this approach does not in any event provide a genuine alternative to our favored approach, which treats 'believes', 'asserts', etc. as 2-place predicate expressions and 'is necessary', 'is possible', etc. as 1-place predicate expressions.[10] This conclusion is all that matters for the rest of our argument.

There are two further alternatives to our favored approach that deserve to be mentioned – the adverbial approach and the adjectival approach. According to the adverbial approach 'x believes that $7 < 9$' would be represented by '((that $7 < 9$)-ly B^1)x', where 'B^1' is a 1-place predicate that expresses the property of believing, '(that $7 < 9$)-ly' is a complex adverbial phrase that expresses a certain "mode of believing", and '(that $7 < 9$)-ly B^1' is a complex 1-place predicate that expresses the property of believing under that mode. According to the adjectival approach, 'x believes that $7 < 9$' would be represented by '$(\exists y)$(x is-in-state y & $B^1 y$ & ((that $7 < 9$)-ish)y)', where 'B^1' is a 1-place predicate that expresses the property of being a belief state and '(that $7 < 9$)-ish' is a complex 1-place predicate that expresses the property of being a state with a "$7 < 9$-ish content."

But how do these approaches represent *general* sentences, for example, the sentence 'x believes something', which we would represent with '$(\exists y)B^2xy$'? Evidently, the adverbial approach must use something like '$(\exists\alpha)((\alpha\text{-ly } B^1)x)$', and the adjectival approach, something like '$(\exists\alpha)(\exists y)$ (x is-in-state y & $B^1 y$ & $(\alpha\text{-ish})y$)'. But notice that on both approaches the everyday use of the verb 'believes' is in effect represented by a complex 2-place predicate expression: either '(②-ly B^1)①' or '$(\exists y)$(① is-in-state y & $B_1 y$ & (②-ish)y)'. Now our thesis is that the verb 'believes' should be treated as a 2-place predicate expression. So, evidently, the adverbial and adjectival approaches just turn out to be complex variants of our logical syntax and, as such, are not genuine alternatives to it. At the same time, these two approaches involve significant additional complexity, and with no apparent gain. Let us take a moment to show that this is really so.

First, the issue of complexity. The adverbial approach requires that we complicate our logical syntax by adding a new syntactic category, namely, the category of predicate adverb and by adjoining an adverb-forming operator '-ly'.[11] In turn, it requires that we devise a new semantical method to deal with these new syntactic structures, a semantical method that inevitably will lead to ontological and ideological complications of its own. And the adjectival approach requires that we complicate our logical syntax by adjoining special vocabulary for dealing with states and by adjoining a predicate-forming operator '-ish'. In turn, it requires that we develop an associated semantical method, which also will carry with it new ontological and ideological complications. [Of course, the operators '-ly' and '-ish' might be contextually defined. For example, $(\alpha\text{-ish})y$ iff_{df} $(\exists z)$ (z is-a-propositional-content-fixing-property-that-corresponds-to α and y is an instance of z). However, such contextual definitions would themselves invoke new logical machinery, for example, the new primitive predicate 'is-a-propositional-context-fixing-property-that-corresponds-to' and a device like 'is an instance of' for attributing properties to their instances, and such machinery would lead to corresponding complications in the semantics. Later on, we will give an argument that devices like 'is an instance of' have no place in intensional logic *per se*. In our introduction, we outlined our reason for thinking that intensional logic is epistemologically more primary than the logic for the instantiation (predication) relation.

Now these complications in syntax and semantics are considerable. Yet they are gratuitous inasmuch as they do nothing whatsoever to advance the formulation of a comprehensive intensional logic, as we will now show in detail. (We will focus on the adverbial approach, but the argument applies *mutatis mutandis* to the adjectival approach.)

Recall that the adverbialist's rendering of 'x believes that $7 < 9$' is '((that $7 < 9$)-ly B^1)x'. In this formula, '$7 < 9$' occurs intensionally, for we cannot replace it with an equivalent sentence – for example, '$7 <$ the number of planets' – without risking an alteration in the truth value of the whole formula. This might lead one to think that the adverb-forming operator '-ly' is what generates the intensionality in sentences concerning intentionality, modality, etc. But this would be an error. We have already seen that sentences like 'x believes something' force the adverbialist to apply the adverb-forming operator '-ly' to a straightforward externally quantifiable free variable like 'α': $(\exists\alpha)(\alpha\text{-ly } B^1)x$. Moreover, kindred sentences show that the adverbialist is forced to use variables like α as terms in elementary identity statements. For example, the adverbialist has no choice but to represent 'x believes something that is different from everything u believes' along the following lines: $(\exists\alpha)((\alpha\text{-ly } B^1)x \;\&\; (\forall\beta)((\beta\text{-ly } B^1)u \rightarrow \alpha \neq \beta))$. Given that variables like 'α' and 'β' may occur as terms in elementary identity statements and given that 'α' and 'β' occur in '$(\alpha\text{-ly } B^1)x$' and '$(\beta\text{-ly } B^1)x$' as straightforward externally quantifiable free variables, the adverbialist has no choice but to accept the following sentence as well-formed and logically true: $(\forall\alpha)(\forall\beta)(\alpha = \beta \rightarrow ((\alpha\text{-ly } B^1)x \equiv (\beta\text{-ly } B^1)x$. Now, we may assume that 'that'-clauses are permissible substituends for the variables 'α' and 'β'. (If the adverbialist were perversely to require instead that the substituends of these variables be in some other syntactic category – say, the category of sentence – the remainder of our argument still would go through *mutatis mutandis*. It is true that intuitively ill-formed expressions – for example, 'The cat is on the mat $= 7 < 9$' or '$7 < 9 = 7 <$ the number of planets' – would result from the adverbialist's requirement, but that would be the adverbialist's responsibility, not ours.) Accordingly, the adverbialist must accept the following sort of instantiation of the above logically true sentence:

$$\text{that } 7 < 9 \;=\; \text{that } 7 < \text{the number of planets} \rightarrow$$
$$(((\text{that } 7 < 9)\text{-ly } B^1)x \;\equiv$$
$$((\text{that } 7 < \text{the number of planets})\text{-ly } B^1)x).$$

However, the original intention of the adverbial theory is that '((that $7 < 9$)-ly B^1)x' and '((that $7 <$ the number of planets)-ly B^1)x)' should be able to differ in truth value. It follows that the adverbialist is forced to accept the truth of the sentence 'that $7 < 9 \neq 7 <$ the number of planets'. (Other things being equal, this outcome should be welcome, for intuitively this sentence is true!) But if this sentence is true, it follows that '$7 <$ the number of planets' occurs intensionally in it. After all, '$7 < 9$' has the same truth values as '$7 <$ the number of planets'. But when we substitute '$7 < 9$' in the true sentence 'that $7 < 9 \neq$ that $7 <$ the number of planets', we obtain a logically false sentence 'that $7 < 9 \neq$ that $7 < 9$'. A clear case of intensionality, a case that does not involve adverbial constructions even implicitly. Now how are such cases of intensionality to be handled logically? All the complicated apparatus of the adverbial theory is of no help whatsoever. Moreover, once we have a theory able to handle complex singular-term cases of intensionality like this, we can easily and economically extend it to handle *all* standard cases of intensionality, and we can do so without recourse to any of the complications of the adverbial theory. For this reason, then, the complications of adverbial theory are gratuitous, having nothing special to contribute to the formulation of a comprehensive intensional logic.

In view of this, the only reasonable decision is to reject the adverbial theory in favor of the essentially simpler theory that treats 'believes', 'asserts', etc. at face value as ordinary 2-place predicates. Furthermore, as we have already indicated, a fully analogous argument can be given against the adjectival theory. And so our original conclusion stands: 'believes', etc. should be treated as ordinary 2-place predicates.

2. 'THAT'-CLAUSES, GERUNDIVE PHRASES, AND INFINITIVE PHRASES

Our conclusion that 'believes', 'asserts', 'is necessary', 'is possible', 'is true', etc. are predicative expressions has an important consequence. Consider the following intuitively valid argument, where A is any formula:

(II) Whatever x believes is possible.
x believes that A.

$\therefore$ It is possible that A.

Suppose, as we have concluded, that one should treat 'is possible' as a 1-place predicate and 'believes' as a 2-place predicate. In this case, we seem to be left with no alternative but to parse the second and third lines of (II) as follows:

$\underline{x}$ $\underline{\text{believes}}$ $\underline{\text{that } A}$

$\underline{\text{It is possible}}$ $\underline{\text{that } A}$

where ⌜that A⌝ is counted as a singular term syntactically. As a notational convenience, let us represent the singular term ⌜that A⌝ by means of ⌜$[A]$⌝.[12] When this bracket notation is adopted, (II) can be naturally represented as follows:

$$\text{(II}'\text{)} \qquad \begin{array}{l} (\forall y)(B^2 x, y \rightarrow P^1 y) \\ B^2 x, [A] \\ \hline \therefore\ P^1[A]. \end{array}$$

The conclusion of (II′) is straightforwardly derivable from the two premises by an application of universal instantiation (UI) and *modus ponens* (MP), two rules of inference valid in standard quantifier logic. Thus, one can bring arguments like (II) within the scope of standard quantifier logic simply by adopting the hypothesis that 'that'-clauses are singular terms representable with the bracket notation. To represent such arguments successfully, one needs no new logical principles, and one needs no knowledge about the logic of expressions occurring within the singular term ⌜$[A]$⌝. It would seem, therefore, this is the simplest way to represent such arguments. Thus, on the assumption that the logic for the new singular terms ⌜$[A]$⌝ can be satisfactorily worked out, we conclude that it is desirable to treat 'that'-clauses as singular terms that may represented by means of the bracket notation.[13]

Now analogous considerations show that certain other complex nominal expressions – for example, gerundive phrases ⌜being something that is F⌝ and infinitive phrases ⌜to be something that is F⌝ – are also best treated as singular terms. An easy extension of the bracket notation provides a natural way to represent these complex singular terms. Accordingly, let A be a formula and $v_1, \ldots, v_m$ be distinct variables where $m \geqslant 1$. Then ⌜$[A]_{v_1 \ldots v_m}$⌝ will be our canonical singular term corresponding to the gerundive phrase ⌜being $v_1, \ldots, v_m$ such that A⌝ and to the infinitive phrase ⌜to be $v_1, \ldots, v_m$ such that A⌝.

We shall see that what is logically distinctive about these singular terms ⌜$[A]$⌝ and ⌜$[A]_\alpha$⌝ is that expressions occurring within them do not obey the substitutivity principles characteristic of extensional logic. That is, when a formula A is enclosed within square brackets (followed by appropriate subscripts), an intensional context is generated. This bracketing operation may therefore be viewed as a generalized *intensional abstraction operation*. Now most types of substitutivity failures result from the fact that the offending expressions explicitly occur within intensional abstracts.

This suggests the general working hypothesis that *all* substitutivity failures can be traced to explicit or implicit occurrences of intensional abstracts. (Consider an example of intensionality that does not involve an explicit occurrence of an intensional abstract, say, 'Pythagoras was seeking the rational $\sqrt{2}$'. The idea would be that such a sentence can be treated as a transform of an underlying sentence that explicitly contains an appropriate intensional abstract, for example, the sentence 'Pythagoras was seeking to find the rational $\sqrt{2}$'. The fact that 'the rational $\sqrt{2}$' occurs intensionally in the transform would then be explained by the fact that it occurs (with narrow scope) in the intensional abstract 'to find the rational $\sqrt{2}$' in the underlying sentence.) Although we need not commit ourselves to this hypothesis, its attractiveness is striking: if true, it would have considerable explanatory power, and it would serve to simplify and unify the entire subject of intensional logic. Indeed, intensional logic could be identified with the logic for intensional abstraction. So we urge it as a working hypothesis.

3. QUANTIFYING-IN

Consider the following argument:

(III) $\quad$ x believes that he believes something.
$\therefore$ There is someone v such that x believes that v believes something.

There is a reading according to which (III) is intuitively valid. This reading provides an example of the logical phenomenon of quantifying-in. It is desirable that all valid cases of quantifying-in should be representable in a comprehensive intensional logic. In the previous two sections we reached these conclusions: 'believes' should be treated as a 2-place predicate; 'is possible', as a 1-place predicate, and 'that'-clauses, as singular terms. These conclusions all but entail an answer to our problem. Consider the following instance of argument (II), which we considered in the previous section:

(IV) $\quad$ Whatever x believes is possible.
x believes that v believes something.
$\therefore$ It is possible that v believes something.

Given our previous conclusions, we must represent (IV) as follows:

(IV′) $\quad$ $(\forall y)(B^2xy \rightarrow P^1y)$
$B^2x[(\exists u)B^2vu]$
$\therefore$ $P^1[(\exists u)B^2vu]$.

And by analogy we must represent (III) as follows:

$$\text{(III}'\text{)}\qquad \frac{B^2x[(\exists u)B^2xu]}{\therefore\ (\exists v)B^2x[(\exists u)Bv^2u].}$$

What is important about this is that the occurrence of 'v' in the singular term '$[(\exists u)B^2vu]$' is an *externally quantifiable* occurrence of a variable.[14] We are thus led to conclude that 'that'-clauses ought to be treated as singular terms which may contain externally quantifiable occurrences of variables.

Now there are several alternate treatments of quantifying-in, but we find none of them acceptable. Before proceeding, let us give a critical survey of these alternatives. Perhaps the most popular one involves multiplying the senses of 'believe' so that, e.g., 'x believes that v believes something' would be represented as '$B^3x,v,[(\exists u)B^2w,u]_w$' (roughly, x believes of v that it has the property of being something w such that w believes something). But on this approach one cannot even begin to represent *mixed* arguments like (IV) – that is, arguments that "mix" the intentional verbs like 'believe' and modals like 'possible' – unless one also multiplies the senses of modals as well. And this is only the tip of the iceberg; senses of all expressions that take 'that'-clauses as arguments must similarly be multiplied – all intentional verbs, modals, 'imply', 'explain', 'justify', 'probable', etc., and even 'true' and '='. Furthermore, this multiple-sense approach is unable to represent formulas that intuitively involve only *one* sense of 'believe' but *two* 'that'-clauses, one containing an externally quantifiable variable and the other containing none: for example, 'x believes *both* that v believes something and that everything is self-identical'. On our approach we would use 'B^2x, $[(\exists u)B^2v,\ u]$ & B^2x, $[(\forall u)u = u]$' to represent this sentence. Evidently, the best someone can do using B^3 is 'B^3x, ,$[(\exists u)(\exists z)B^3w,z,u]_w$ & $B^3x,\phi,[(\forall u)u = u]$' (where ϕ represents the null sequence), thus abandoning altogether the familiar 2-place sense of 'believe'. Moreover, similar examples would then seem to force proponents of the 'B^3'-approach to abandon the familiar 2-place senses of '=', 'assert', 'explain', 'justify', etc., and the familiar 1-place senses of 'true', 'necessary', 'possible', 'contingent', 'probable', etc. For example, whereas we would represent 'x asserted two things; one was that v asserted something and the other was that everything is self-identical' with

$$(\exists y)(\exists z)(A^2x,y\ \&\ A^2x,z\ \&\ y = [(\exists u)A^2v,u]\ \&\ z = [(\forall w)w = w],$$

the opposing approach is forced to use something like

$$(\exists y)(\exists y')(\exists z)(\exists z')(A^3 x,y,y' \ \& \ A^3 x,z,z' \ \&$$
$$y,y' =^4 v, [(\exists u)(\exists u')A^3 v,u,u']_v \ \&$$
$$z,z' =^4 \phi, [(\forall w)(\forall w')w,w' =^4 w,w'],$$

where '$=^4$' is a new 4-place "identity" predicate. Not only are these consequences extremely unintuitive in themselves, but also they evidently make it impossible to represent important "cross-referential" sentences like the following: 'x asserted exactly *one* thing and we have a name for *it*'. Furthermore, given that the familiar 2-place sense of '$=$' must be abandoned on the present approach in favor of '$=^4$', the prospect of any coherent identity theory is seriously threatened. A final problem with these approaches arises in connection with multiple embedding. For example, suppose that someone u is consciously and explicitly thinking that u is consciously and explicitly thinking something; that is, $T^2 u, [(\exists y)T^2 u,y]$. On the 3-place approach, this must be represented along the following lines: $T^3 u,u, [(\exists x)(\exists y)T^3 v,x,y]_v$. However, this is implausible; in the example, u is not consciously and explicitly thinking of u that he has the property of being someone v who is consciously and explicitly thinking of something that it has some property. This thought – and any other one of its ilk – is intuitively different from the thought that u is having; u's thought is simply that u is consciously and explicit thinking something. This subtle difference is just lost on the 3-place approach.

Another approach to 'that'-clauses containing externally quantifiable variables is to associate them with certain *sequences* and to treat 'believe' as a 2-place predicate: for example, 'u believes that Fu' might be represented by '$B^2 u, \langle u, [Fu]_u \rangle$'. However, this seemingly simple idea seems impossible to formulate in a satisfactory general way. Here are some of the problems that confront it.

First, certainly some identities of the following form hold: that Fu = that Gu,v. (For example, is it not true that the following identity holds:

$$\text{That } u \text{ is a Moonie} = \text{that } u \text{ is a follower of } v.$$

where v is Reverend Sun Yen Moon?) However, such identities would be impossible if we were to represent 'that Fu' with '$\langle u, [Fu]_u \rangle$' and 'that Gu,v with '$\langle \langle u,v \rangle, [Gu,v]_{uv} \rangle$' as our sequence theorists would do. For plainly $\langle u, [Fu]_u \rangle \neq \langle \langle u,v \rangle, [Gu,v]_{uv} \rangle$.

Second, we find it extremely counterintuitive that what a person perceives, believes, asserts, hopes, decides, etc. is ever *really* a sequence. How can sequence theorists accept such an implausible theory? (See Section 7 for further discussion of this sort of intuitive objection.)

Third, the sequence approach runs into difficulties in connection with multiple embeddings. For example, it is intuitively possible for someone u to believe that, for every v, that Fv = that Fv and, nevertheless, not believe that, for every v, that $Fv = \langle\langle v\rangle, [Fv]_v\rangle$. In our bracket notation:

$$B^2[(\forall v)[Fv] = [Fv]] \;\&\; \neg B^2[(\forall v)[Fv] = \langle\langle v\rangle, [Fv]_v\rangle].$$

However, on the sequence approach, this would presumably be equivalent to the following contradiction:

$$B^2[(\forall v)\langle\langle v\rangle, [Fv]_v\rangle = \langle\langle v\rangle, [Fv]_v\rangle] \;\&$$
$$\neg B^2[(\forall v)\langle\langle v\rangle, [Fv]_v\rangle = \langle\langle v\rangle, [Fv]_v\rangle].$$

You might try to mitigate this problem by invoking your favorite resolution of the paradox of analysis. However, we believe that such ploys will not succeed. (For more on the paradox of analysis, see Section 13.)

Fourth, by attempting their reduction, the sequence theorists prejudge certain questions concerning the identity conditions for the items denoted by intensional abstracts. For example, on one important traditional conception (dubbed "Alternative (2)" by Alonzo Church)[15] the following principle of identity is valid for any formulas A and B:

> If it is necessary that A and it is necessary that B, then that A = that B.

(Analogous principles hold for intensional abstracts that are gerundive and infinitive in form.) However, for all u and v, it is necessary that u is self-identical and it is necessary that v is self-identical. This fact and the above principle of identity imply:

> That u is self-identical = that v is self-identical.

for all u and v. However, if $u \neq v$, the sequences that would be associated with these 'that'-clauses on the sequence approach would plainly not be identical: that is, $\langle u, [x \text{ is self-identical}]_x\rangle \neq \langle v, [x \text{ is self-identical}]_x\rangle$. In this way, therefore, the sequence approach is incompatible with the above traditional conception. A prudent approach to intensional logic would not prejudge such questions, for it is quite conceivable that our best overall theory of intensionality will invoke this coarse-grained conception perhaps in tandem with various fine-grained conceptions. Indeed, it is quite plausible that, unlike the fine-grained objects of the propositional attitudes, conditions (or states of affairs) in the world conform to the coarse-grained conception. For example, intuitively, the glass's being half empty is the same condition (state of affairs) in the world as the glass's being half full. And this is so despite the fact that someone could be thinking that the glass is half empty without thinking that the glass is half full.

Fifth, the sequence approach runs into difficulty concerning a kind of type-freedom that is called "self-constituency". This kind of type freedom appears to arise in connection with the problem of mutual knowledge.[16] For example, suppose that two enemy soldiers x and y suddenly spot each other in the bush. Conceivably, a full description of this situation should include the following: x sees z and y sees z', where $z = [y \text{ sees } z']$ and $z' = [x \text{ sees } z]$. But what are z and z'? On the sequence approach, $z = \langle\langle y, z'\rangle, [u \text{ sees } v]_{uv}\rangle$ and $z' = \langle\langle x, z\rangle, [u \text{ sees } v]\rangle_{uv}$. Now suppose that sequences are identified with ordered sets. (If instead sequences are ordered properties, one runs into the regress discussed below.) In this case, z and z' would be non-well-founded sets (i.e., $z \in \ldots \in z' \in \ldots \in z$). However, according to the standard conception of sets, non-well-founded sets are impossible. The upshot is this. To represent z and z' on the set theoretical version of the sequence approach, one must revolutionize the standard conception of set. However, such a radical move seems quite unjustified merely to deal with the simple task at hand. No such radical move is required on the more cautious treatment of quantifying-in that we are advocating. (We will return to this topic in Section 11.)

Our sixth objection to the sequence approach is ontological. In Sections (5)–(9) we will argue that intensional abstracts that do not contain externally quantifiable variables denote properties, relations, and propositions. This conclusion, together with the premise that intensional logic is indispensable to any acceptable comprehensive theory of the world, leads to the conclusion that properties, relations, and propositions are indispensable. In view of this, ontological economy demands that we try to replace the more complex ontology consisting of both sets and properties with the simpler ontology consisting of just properties. In fact, this ontological simplification can be easily accomplished within our intensional framework. So on grounds of ontological economy, the set-theoretical verison of the sequence approach to quantifying-in should not be adopted. A related problem with the set-theoretical version of the sequence approach is that intensional abstracts containing externally quantifiable variables are not semantically correlated with properties, relations, or propositions; instead, they are semantically correlated with items from an entirely distinct ontological category, namely, sets. Thus, although intensional abstracts that do not contain externally quantifiable variables (e.g., 'that something is red') are semantically correlated with one kind of thing (namely, propositions), intensional abstracts that do contain externally quantifiable variables (e.g., 'that x is red') are semantically correlated with an entirely different kind of thing (namely, sets). This sort of *ad hoc* theoretical disunity is quite unjustified. Our approach to quantifying-in avoids both the ontological excess and the *ad hoc* theoretical disunity of the set-theoretical version of the sequence approach. The upshot is that the set-theoretical version of the sequence approach should not be adopted.

Our fifth and sixth objections to the sequence approach depended on the identification of sequences with extensional entities (namely, ordered sets). What would happen if the sequence theorist tried instead to identify sequences with intensional entities (namely, ordered properties)? In set theory, sequences are identified with ordered sets; for example, $\langle u, v\rangle$ might be identified with $\{\{u\}, \{u, v\}\}$. (That is, $\{x: x = \{y: y = u\} \vee x = \{y: y = u \vee y = v\}\}$.) By analogy, in a property theory, sequences are identified with ordered properties; for example $\langle u, v\rangle$ might be identified with $[x = [y = u]_y \vee x = [y = u \vee y = v]_y]_x$. So far so good. But notice that this property is, on the face of it, a *de re* property; the intensional abstract that is semantically correlated with it contains externally quantifiable variables (namely, 'u' and 'v'). So advocates of the property-theoretic version of the sequence approach to quantifying-in would be committed to identifying this property with a sequence such as:

$$\langle\langle u, v\rangle, [x = [y = u]_y \vee x = [y = u \vee y = v]_y]_{xuv}\rangle.$$

But this outcome is plainly unacceptable because:

$$\langle u, v\rangle \neq \langle\langle u, v\rangle, [x = [y = u]_y \vee x = [y = u \vee y = v]_y]_{xuv}\rangle.$$

And this is only the beginning. With which property would the latter sequence be identified? On the property-theoretic version of the sequence approach, this sequence would have to be identified with still another property that, on the face of it, is *de re*. In this way, the property-theoretic version of the sequence approach leads to a regress. You cannot eliminate *de re* properties in favor of sequences, for those very sequences must, in turn, be identified with further *de re* properties; and so on *ad infinitum*. To put the point linguistically, suppose that you have a language fitted out with just *de dicto* intensional abstracts and an apparatus for expressing the predication (instantiation) relation. In this language you could never, even in principle, identify the sequences (=properties) with which *de re* properties are supposed to be identical.[17]

Summing up, the overall verdict on the sequence approach to quantifying-in is that it is fatally flawed.

There is a final approach to quantifying-in that should be mentioned, namely, the *self-ascription theory* of externally quantified belief sentences. Such an approach has been advocated independently by David Lewis [1979] and by Roderick Chisholm [1981]. (We will confine our discussion to Chisholm; however, our comments will by and large apply to Lewis as well.) Chisholm's approach is not intended to be a general treatment of quantifying-in; rather, it was developed primarily to help solve certain recalcitrant substitutivity puzzles involving indexicals in propositional-attitude sentences. Chisholm imposes a special ontological constraint on his solution to these and other substitutivity puzzles, namely, that a solution should avoid ontological commitment to *de re* properties, relations, and propositions; for Chisholm, a solution should be ontologically committed to what may be called "pure Platonic" properties, relations, and propositions. For this reason, if his theory is adequate, Chisholm ought to be able to extend it to cover *all* 'that'-clause sentences that on the surface seem to contain externally quantifiable variables. We will give two criticisms of the self-ascription theory – one aimed directly against its treatment of *de re* belief sentences and the other intended to show that it cannot be generalized to yield a uniform treatment of 'that'-clause sentences containing externally quantifiable variables.

According to Chisholm, a person x directly believes that Fx if and only if x self-ascribes the property of being F.[18] (In symbols. A^2x, $[Fv]_v$). And x indirectly believes that Fy if and only if, for some relation R, y is the unique item bearing R to x and x self-ascribes the property of being something v such that there is a unique item u bearing R to v and Fu. (In symbols, $(\exists R)(R!y,x \;\&\; A^2x, [(\exists!u)(Ru, v \;\&\; Fu)]_v$.) Finally, x believes that Fy if and only if x directly believes that Fy or x indirectly believes that Fy. The first problem is that these three biconditionals do not yield the right results for the belief sentences they are intended to cover. Specifically, the second biconditional (and, in turn, the third biconditional) is far too weak. For example, suppose that 'Ry,x' is 'y = the tallest man & $x = x$'. Suppose x self-ascribes the property of being something v such that there is a unique item u bearing R to v and Fu. That is, x self-ascribes $[(\exists!u)(Au, v \; \& \; Fu)]_v$. The problem is that in a typical situation this would not be sufficient for believing *of* the tallest man that *he* is F. On the contrary, this self-ascription would constitute a run-of-the-mill *de dicto* belief to the effect that the tallest man is F. Without stricter constraints on R, the proposal virtually erases the distinction between genuine *de re* beliefs and run-of-the-mill *de dicto* beliefs. Now there might be ways to impose suitably strict constraints on R *on a case-by-case basis*. However, there seems to be no *systematic* way to impose suitable constraints on R.

The second problem is that the self-ascription approach does not mesh well with a *general* treatment of 'that'-clause sentences. To illustrate the problem, consider the following intuitively valid argument form:

$$\begin{array}{l} \text{For all } y\text{, if } x \text{ believes } y\text{, then} \ldots y \ldots . \\ x \text{ believes that } Fx. \\ \hline \therefore \ldots \text{that } Fx \ldots . \end{array}$$

In our notation:

$$\begin{array}{l} (\forall y)(B^2x,y \rightarrow \ldots y \ldots) \\ B^2x, [Fx] \\ \hline \therefore \ldots [Fx] \ldots . \end{array}$$

Suppose that $\ulcorner \ldots y \ldots \urcorner$ is syntactically simple. For example, suppose it is 'y is true', 'y is necessary', 'y is logically true', 'y is probable', 'y is explainable', etc. Then, presumably, self-ascription theorists would then be led to adopt the following representations, respectively: 'x has $[Fy]_y$', 'x necessarily-has $[Fy]_y$', 'x logically-has $[Fy]_y$', 'x probably-has $[Fy]_y$', 'x explainably-has $[Fy]_y$', etc. The idea here is to introduce special new primitive predicates ('has', 'necessarily-has', 'logically-has', etc.) to represent syntactically simple cases of $\ulcorner \ldots y \ldots \urcorner$. However, this pattern of representation breaks down when $\ulcorner \ldots y \ldots \urcorner$ is more complex. For example, suppose $\ulcorner \ldots y \ldots \urcorner$ is 'y implies that Hxz', 'That Hxz explains y', 'y = that Hxz', 'Given the premise that if Hxz then Fx and given the premise that Hxz, then y follows immediately by *modus ponens*, where y = that Fx', etc. There appears to be no systematic way to treat all such cases by straightforward extension of the technique used to represent the syntactically simple cases. It appears, therefore, that the self-ascription theorist has no alternative but to adopt the sequence approach that we discussed a few paragraphs above. (This assessment is fortified by the following consideration. Notice that the three biconditionals that comprise the self-ascription theory do not constitute a *general* analysis (definition) of belief in terms of self-ascription. That is, for an *arbitrary* w, we are not told the conditions under which a person could be said to believe w; we are told merely the conditions under which a person could be said to believe w for certain special w. It appears that the only way to arrive at a *general* analysis would be to utilize the sequence approach.) However, if the sequence approach were adopted, the self-ascription theory would then inherit *all* of the problems inherent in that approach.

Our conclusion, therefore, is that the self-ascription theory is seriously flawed. But this does not mean that the problems that the self-ascription theory was designed to handle (i.e., the explanation of substitutivity failures involving co-denoting indexicals) cannot be solved in some other way. Indeed, they can be solved within the framework for treating quantifying-in that we have advocated. (See Sections 12 and 13 below.) In view of this, it would be a mistake to abandon our treatment of quantifying-in in favor of an alternate treatment inspired by the self-ascription theory.[19]

Our primary conclusion, then, is this: 'that'-clauses are best represented by means of an intensional abstraction operation (such as the bracket notation $\ulcorner[A]\urcorner$) and these intensional abstracts may contain externally quantifiable variables. Now fully analogous considerations lead to the conclusion that gerundive phrases and infinitive phrases likewise are best represented by means of an appropriate intensional abstraction operation (such as our generalized bracket notation $\ulcorner[A]_\alpha\urcorner$) and these abstracts may also contain externally quantifiable variables. Thus, in the formal language for our intensional logic, $\ulcorner[A]_{v_1 \ldots v_m}\urcorner$ will be a well-formed singular term, for any $m \geqslant 0$, even if the formula A contains free variables that are not among the variables $v_1, \ldots, v_m$; such free variables are externally quantifiable.

4. LEARNABILITY

Donald Davidson has argued persuasively that human beings can learn a language only if it contains a finite number of semantical primitives and, hence, that a formal language can serve as an idealized representation of (a fragment of) a human natural language only if it too contains a finite number of semantical primitives.[20]

There has been some confusion about what Davidson's learnability requirement comes to. It does not imply that all learnable languages – and all idealized representations of them – must have a finite number of syntactically primitive constants. This would be too strong. For we humans are able to learn certain specialized languages that have an infinite number of syntactically primitive constants; for example, we could learn a language for arithmetic in which all the numerals are syntactically primitive. But what makes this possible? The explanation, of course, is that each numeral following '0' can be *defined* (e.g., in terms of '0' and '+ 1') and, hence, is not *semantically* primitive. When we generalize on examples of this sort, we arrive at the following highly plausible principle: for any infinitary language L that a human could learn, there must be a finitary language L′ in which all the constants in L (besides those that are already in L′) either could be defined or could be introduced in some comparable manner (for example, by means of Kripkean reference-fixing descriptions). Given this principle, we may infer that, if an infinitary language L is to qualify as an idealized representation of the logical syntax of natural language, there must be an associated finitary language L′ that satisfies the condition just stated. In view of this, the safest and most direct way to insure that a candidate infinitary language L qualifies as an idealized representation of the logical syntax of natural language is just to produce the requisite finitary language L′. Notice, however, that in place of L this finitary language L′ should itself be able to serve as an idealized representation of the logical syntax of natural language. So , in practice, we are entitled to demand from people seeking to construct an idealized representation of the logical syntax of natural language. So, in practice, bill directly with a finitary language like L′. (See Section 11 for further elaboration of this argument.)

The foregoing – or something quite like it – is what Davidson's learnability requirement comes to. And it seems basically right to us. Now the formal language in which intensional logic is formulated should be able to serve as an idealized representation of the logical syntax of the intensional fragment of natural language. Given this fact, together with Davidson's learnability requirement, we conclude that this formal language should have a finite number of primitive constants.

5. REFERENTIAL SEMANTICS FOR INTENSIONAL LANGUAGE

We come now to the question of the semantics for sentences containing 'that'-clauses, gerundive phrases, or infinitive phrases. What, if anything, corresponds semantically to these abstract singular terms? In seeking the answer to this question, we may assume that any adequate semantics either includes an explicit specification of the truth conditions for the sentences of the language or is set up so that these truth conditions can be derived. For simplicity, therefore, let us examine what an explicit specification of truth conditions would have to be like. The crucial issue for us arises in connection with atomic sentences. For example, under what conditions is an atomic sentence of the form $\ulcorner F[A] \urcorner$ true? In a referential semantics, an ordinary atomic sentence $\ulcorner Fa \urcorner$ is true on an interpretation $\mathscr{I}$ iff, according to $\mathscr{I}$, the singular term $\ulcorner a \urcorner$ denotes an item of which the predicate $\ulcorner F \urcorner$ is true. The thesis we will defend is that the truth conditions for atomic sentences containing intensional abstracts must be specified in the analogous way: $\ulcorner F[A] \urcorner$ is true on interpretation $\mathscr{I}$ iff, according to $\mathscr{I}$, the singular term $\ulcorner [A] \urcorner$ denotes an item of which the predicate $\ulcorner F \urcorner$ is true. (In our discussion we will refer to these atomic sentences as *atomic intensional sentences*.)

Our argument will proceed in two steps. First, we will argue that everyone who aspires to an acceptable comprehensive theory of the world must acknowledge the truth of an infinite variety of atomic intensional sentences. Second, we will argue that, besides a referential semantics, there is no viable alternative semantics that will account systematically for the truth of these atomic intensional sentences.

To expedite the first step, let us consider the radical theory that *no* atomic intensional sentences are, strictly speaking, true. This theory is wildly implausible, for true sentences like the following would have to be deemed false: 'It is true that someone has a hand', 'It is true that $7 < 9$', 'It is logically valid that everything is self-identical', 'It is probable that the sun will rise', 'That I have sense experiences has an explanation', and so forth. How much more plausible it would be to accept the truth of these sentences and to try to devise a semantics for them. At this stage, one need not make any assumptions about whether it would have to be a referential semantics.

However, suppose that supporters of the radical theory persist. How could they justify their position? Since it is *locally* so implausible, they have no choice but to try to justify it *globally*, that is, by showing that their best comprehensive theory deems all atomic intensional sentences to be false. But if their theory is truly comprehensive, it must among other things be able to account for its own acceptability (justification). We will argue that the radical theorists cannot show this without resorting to intensional idioms and so their position is essentially self-defeating. As a result, it is not acceptable.

How might the radical theorists try to show that their position is acceptable? The *standard* idiom for discussing acceptability (justification) is intensional. For example, it is standard to say: ⌜It is acceptable that *A*⌝, ⌜The theory that *A* is justified⌝, and so forth. (There is also a metalinguistic idiom for discussing acceptability. This will be considered in a moment.) So if the radical theorists are to defend the acceptability of their theory by this standard means, they will have to make various positive assertions with intensional sentences, sentences which they deem to be not true. Specifically, the conclusion of their argument would be (the proposition expressed by) 'It is acceptable that no atomic intensional sentence is true'. But this sentence is itself an atomic intensional sentence. So the radical thesis (i.e., that no atomic intensional sentence is true) implies that this conclusion is not true. But if this conclusion is not true, then it is not acceptable that no atomic intensional sentence is true. Thus, the radical thesis implies that the radical thesis is not acceptable. The radical thesis is, in this sense, self-defeating.[21]

To avoid this self-defeat, the radical theorists might try to invoke some new, *nonstandard* idiom with which to show that their comprehensive theory is acceptable (justified). However, to succeed at this strategy, they must in addition be able to show that this new, nonstandard idiom is *relevantly like* the standard idiom, for otherwise there would be no reason to think that their argument, which uses a new idiom, has any bearing on *acceptability*. After all, *acceptability*, or something relevantly like it, is what is at issue. There can be many similarities between a standard idiom and a new idiom (e.g., length or sound of constituent expressions, etc.); only some of them are *relevant*. Therefore, it is incumbent on the radical theorists to show that their new idiom is relevantly like the standard one.[22]

(As we indicated above, there is also a metalinguistic idiom for discussing acceptability. For example, someone might say, ⌜The sentence ⌜*A*⌝ is acceptable⌝. However, to the extent that it is standard, this metalinguistic idiom bears the following systematic relation to the standard intensional idiom: the sentence ⌜*A*⌝ is acceptable if and only if it is acceptable that *A*. Suppose that this systematic relationship is affirmed by the radical theorists. In this case, they are led to the same sort of self-defeat described above. On the other hand, suppose that this systematic relationship is not affirmed by the radical theorists. In this case, their use of the metalinguistic idiom would, for all anyone could tell, be nonstandard. That is, for all anyone could tell, it might be just some new idiom. Therefore, if the standard systematic relationship is not affirmed by the radical theorists, they would be obliged to show either that their metalinguistic idiom is, despite this, still the standard idiom or that, if it is not the standard idiom, it nevetheless has bearing on acceptability. In either case, they would need to show that this idiom is *relevantly like* the standard idiom. So no real progress has yet been made.)

The conclusion so far is this. To avoid self-defeat, the radical theorists have no choice but to use an idiom that either appears to be or is nonstandard and then to show that this idiom is relevantly like the standard intensional idiom for talking about acceptability. How might the radical theorists try to show that their idiom is relevantly like the standard intensional idiom? There are two ways. One would be to show that the *meanings* of expressions in the new idiom are relevantly like the meanings of expressions in the standard idiom.[23] The other way would be to show that the *purpose* or *function* of the new idiom is relevantly like that of the standard idiom. (Or radical theorists might try to show that the two idioms share something that is relevantly like meaning, or they might try to show that they share something that is relevantly like purpose or function.) But both ways inevitably fail.

Stated briefly, the problem is this. The standard idioms for talking about meaning, purpose, and function are intensional: $\ulcorner\ulcorner A\urcorner$ means that $A\urcorner$; $\ulcorner$The purpose of F-ing is to $G\urcorner$, $\ulcorner$The function of F-ing is to $G\urcorner$, and so on. So if they use these idioms, the radical theorists once again end up in self-defeat.[24] Moreover, although there are standard extensional idioms for talking about meaning, purpose, and function, they bear systematic relations to the standard intensional idioms for talking about meaning, purpose, and function. (For example, the standard extensional idiom for talking about synonymy bears the following systematic relationship to the standard intensional idiom for talking about meaning: $\ulcorner A\urcorner$ is synonymous to $\ulcorner B\urcorner$ iff $\ulcorner A\urcorner$ and $\ulcorner B\urcorner$ mean the same iff that A = that B.) If the radical theorists affirm these systematic relations, they again end up in self-defeat. If they do not affirm these systematic relationships, then they are obliged to show either that their use of the extensional idiom is standard or, if it is not standard, that it has bearing on meaning, purpose, or function. In either case, they must be able to show that their idiom is relevantly like the standard intensional idiom. Alternatively, the radical theorists could invoke some new, nonstandard idiom for (allegedly) talking about meaning, purpose, or function (or for talking about something that is relevantly like meaning, purpose, or function). But, then, they would once again be obliged to show that their idiom is relevantly like the standard intensional idiom for talking about meaning, purpose, or function. Now how is the required relevant similarity to be shown? Well, by demonstrating that the meaning, purpose, or function of the questionable idiom is relevantly like the meaning, purpose, or function of the standard intensional idiom for talking about meaning, purpose, or function. However, if this demonstration is conducted in the standard intensional idiom, self-defeat results once again. On the other hand, if the demonstration is conducted in the questionable idiom (i.e., an idiom whose relevance to the standard idiom is the very question at issue), this demonstration simply begs the question. For at no stage will it have been established that any conclusion stated in the questionable idiom has any bearing on meaning, purpose, or function (or anything that is relevantly like meaning, purpose, or function).

The overall pattern, then, is this. In the effort to establish the acceptability of their anti-intensionalist theory, the radical theorists get caught either in self-defeat or in begging-the-question. The epistemic situations, if you will, hermeneutical: the standard idioms are intensional, and to show the relevance of a nonstandard idiom, one must use a standard intensional idiom or one must beg the question by using a nonstandard idiom whose relevance is equally in question. There is no epistemically acceptable way to go from where we are to the radical anti-intensionalist theory. And more generally, there is no epistemically acceptable way to make out the possibility of beings who have an acceptable comprehensive theory (or something relevantly like an acceptable comprehensive theory) that includes the radical anti-intensionalist theory.[25]

We have established that every acceptable comprehensive theory of the world must admit a wide variety of atomic intensional sentences as true. In this connection, it would be unacceptable to exclude any part of the standard network of atomic intensional sentences bearing systematic relations to one another; specifically, atomic intensional sentences dealing with acceptability, truth, meaning, purpose, function, definition, intention, belief, causation, explanation, probability, evidence, necessity, and so forth. Given that such a variety of atomic intensional sentences must be counted as true, what semantical theory will account for their truth? As we have indicated, a standard referential semantics provides the most straightforward answer: an atomic intensional sentence $\ulcorner F[A]\urcorner$ is true on an interpretation $\mathscr{I}$ if and only if, according to $\mathscr{I}$, the singular term $\ulcorner[A]\urcorner$ denotes an item of which the predicate $\ulcorner F\urcorner$ is true.

What alternative is there to a standard referential semantics? Evidently, there is only one alternative that is even faintly promising. Namely, the sort of non-referential semantics that *anti*-Meinongian realists often envisage for positive sentences containing ordinary vacuous names, sentences like 'Apollo is a Greek god' and 'Pegasus is a mythical flying horse'. On this theory, such sentences are deemed to be literally true. In this respect, the theory is like Meinong's. However, contrary to Meinong's theory, this theory treats terms such as 'Apollo' and 'Pegasus' as genuinely vacuous. Since these terms refer to nothing, the truth of sentences containing them needs to be explained in some new, nonreferential way. The idea is that, in the semantical description of the truth conditions of these everyday vacuous-name sentences, all purported references to nonactual objects (Apollo, Pegasus, etc.) is to be replaced by references to actual human beings in relevant actual mental states. For example, the truth conditions for 'Apollo is a Greek god' would on this nonreferential approach be characterized in terms of actual religious beliefs (and other mental states) of the ancient Greeks. Now concerning atomic sentences like $\ulcorner F[A] \urcorner$, the proposal would be to characterize their truth conditions along this sort of nonreferential lines.

It is true that ordinary vacuous-name sentences are standardly "backed" by an identifiable body of actual myths, legends, rumors, works of fiction, and so forth. Let us suppose for the sake of argument that the truth conditions for ordinary vacuous-name sentences might be specified in terms of these. (We need not take any stand on whether this sort of nonreferential semantics is really feasible. If it is not feasible even for ordinary vacuous-name sentences, it certainly is not feasible for atomic intensional sentences.) For example, as a first approximation, a nonreferentialist might hold that an ordinary vacuous-name sentence $\ulcorner Fa \urcorner$ is true if and only if it is derivable from a maximal consistent set of sentences extracted from a standard linguistic statement of a community's myths, legends, rumors, and works of fiction. But there is no comparable proposal for atomic intensional sentences. There are at least two decisive reasons.

First, unlike the true atomic vacuous-name sentences, the true atomic intensional sentences are not even recursively enumerable. (Consider sentences of the form $\ulcorner$It is true that $A\urcorner$. $\ulcorner$It is probable that $A\urcorner$, $\ulcorner$It is explainable that $A\urcorner$, $\ulcorner$It is possible that $A\urcorner$, etc.) So there is no body of actual beliefs (and other actual mental states) that could play a role comparable to a community's body of actual myths, legends, rumors, and works of fiction. If there is nothing mental to play this role, the semanticist has no alternative but to posit actual reference to real things.

Second, let us suppose *per impossible* that the true atomic intensional sentences can be fixed on the basis of some relevant body of our beliefs. Which beliefs would these be? They would not be myths, legends, rumors, or works of fiction; rather, they would be straightforward acceptable (justified) theoretical beliefs. For example, as the above argument concerning acceptability indicates, they would include acceptable (justified) beliefs about the acceptability of our overall theory. Such beliefs – unlike myths, legends, etc. – would therefore need to be counted as true in our best overall theory. Now because certain beliefs (myths, legends, etc.) are not true, the nonreferentialist holds that they can "back" the truth of associated vacuous-name sentences without implying thereby that these names are semantically associated with any relevant entity. By contrast, the beliefs that would presumably "back" the truth of our atomic intensional sentences are true. Accordingly, they do imply that there are relevant entities semantically associated with the intensional abstracts occurring in these atomic sentences. After all, the way in which intensional abstracts are used in our acceptable (justified) theorizing is entirely like the way in which standard nonvacuous referring expressions are used in such theorizing. So the nonreferentialists' strategy of likening intensional abstracts to names whose use is sustained by mere myths, legends, rumor, and fiction breaks down. To single out intensional abstracts as vacuous is then nothing but an arbitrary attack. If this were acceptable, it would be equally acceptable to single out any other family of singular terms (e.g., place names, names of people, etc.) as vacuous. And if this were acceptable, it would lead to an absurd form of skepticism that not even our nonreferentialists could tolerate.

Now we submit that, when one surveys alternate ways of characterizing the truth conditions for atomic sentences of the form $\ulcorner F[A] \urcorner$, one will find that they all run into these difficulties or variants of them. If we are right about this, there is no reasonable choice but to give a referential semantics for such sentences.

Given this conclusion, how are we to specify the truth conditions for atomic sentences containing other intensional abstracts, namely, abstracts of the form $\ulcorner [A]_{v_1 \ldots v_n} \urcorner$, for $n \geqslant 1$? Given the conclusions we have just reached, considerations of uniformity support the conclusion that sentences of the form $\ulcorner F[A]_{v_1 \ldots v_n} \urcorner$ are true on an interpretation $\mathscr{I}$ iff, according to $\mathscr{I}$, the singular term $\ulcorner [A]_{v_1 \ldots v_n} \urcorner$ denotes an item of which the predicate $\ulcorner F \urcorner$ is true. Attempts to avoid this conclusion by means of a nonreferential semantics like that considered above only lead to variants of the problems that already undermined that style of semantics. It seems best, therefore, to accept the conclusion that sentences of the form $\ulcorner F[A]_{v_1 \ldots v_n} \urcorner$ have a standard referential semantics.

6. THE DENOTATIONS OF INTENSIONAL ABSTRACTS

Let us now turn to the question of what sorts of things are denoted by (or are semantically correlated with) the singular terms $\ulcorner [A] \urcorner$ and $\ulcorner [A]_{v_1 \ldots v_n} \urcorner$. As we have already indicated, the logically distinctive feature of these terms – and their counterparts in natural language – is that various expressions occurring within them do not obey the substitutivity principle that characterizes extensional logic. For example, neither of the following argument forms is valid:

(V)
$$\frac{\begin{array}{l} G[B] \\ B \leftrightarrow C \end{array}}{G[C]}$$

(VI)
$$\frac{\begin{array}{l} G[B(v_1, \ldots, v_n)]_{v_1 \ldots v_n} \\ (\forall v_1, \ldots, v_n)(B(v_1, \ldots, v_n) \leftrightarrow C(v_1, \ldots, v_n)) \end{array}}{G[C(v_1, \ldots, v_n)]_{v_1 \ldots v_n}}$$

That is, in many arguments having form (V) or (VI), the first two lines are true and the third line is false. Given the conclusion we have reached about the truth conditions for sentences of the form $\ulcorner F[A] \urcorner$ and $\ulcorner F[A]_{v_1 \ldots v_n} \urcorner$ there is only one way in which this pattern of truth values is possible. Consider arguments of form (V). The truth of $\ulcorner G[B] \urcorner$ implies that $\ulcorner [B] \urcorner$ denotes an item of which the predicate $\ulcorner G \urcorner$ is true, and the falsity of $\ulcorner G[C] \urcorner$ implies that $\ulcorner [C] \urcorner$ denotes an item of which the predicate $\ulcorner G \urcorner$ is not true. From these two conclusions it follows that the item denoted by $\ulcorner [B] \urcorner$ and the item denoted by $\ulcorner [C] \urcorner$ must be different. This is so despite the fact that, given the truth of the second line $\ulcorner B \leftrightarrow C \urcorner$, the formulas $\ulcorner B \urcorner$ and $\ulcorner C \urcorner$ are equivalent (in truth value) and, in turn, the items denoted by the terms $\ulcorner [B] \urcorner$ and $\ulcorner [C] \urcorner$ are equivalent (in truth value). Or consider arguments of form (VI). The truth of $\ulcorner G[B(v_1, \ldots, v_n)]_{v_1 \ldots v_n} \urcorner$ implies that $\ulcorner [B(v_1, \ldots, v_n)]_{v_1 \ldots v_n} \urcorner$ denotes an item of which $\ulcorner G \urcorner$ is true, and the falsity of $\ulcorner G[C(v_1, \ldots, v_n)]_{v_1 \ldots v_n} \urcorner$ implies that $\ulcorner [C(v_1, \ldots, v_n)]_{v_1 \ldots v_n} \urcorner$ denotes an item of which $\ulcorner G \urcorner$ is not true. From this it follows that the item denoted by $\ulcorner [B(v_1, \ldots, v_n)]_{v_1 \ldots v_n} \urcorner$ must be different from the item denoted by $\ulcorner [C(v_1, \ldots, v_n)]_{v_1 \ldots v_n} \urcorner$. This is so despite the fact that, given the truth of the second line $\ulcorner(\forall v_1, \ldots, v_n)(B(v_1, \ldots, {}_n) \leftrightarrow C(v_1, \ldots, v_n))\urcorner$, the formulas $\ulcorner B(v_1, \ldots, v_n)\urcorner$ and $\ulcorner C(v_1, \ldots, v_n)\urcorner$ are equivalent (in what they are true of) and, in turn, the items denoted by the terms $\ulcorner[B(v_1, \ldots, v_n)]_{v_1 \ldots v_n}\urcorner$ and $\ulcorner[C(v_1, \ldots, v_n)]_{v_1 \ldots v_n}\urcorner$ are equivalent (in what they are true of).

Thus, to do the semantics for the singular terms $\ulcorner[B]\urcorner$ we need a special category of objects with the following feature: they can be distinct from one another even though in some cases they are equivalent (in truth value). And to do the semantics for the singular terms $\ulcorner[B(v_1, \ldots, v_n)]_{v_1 \ldots v_n}\urcorner$, $n \geqslant 1$, we need a special category of objects with the corresponding feature: they can be distinct from one another even though in some cases they are equivalent (in what they are true of).

Now linguistic entities – sentences and n-ary predicates, respectively – have these special features. And so do certain extralinguistic entities – propositions and n-ary relations (properties if $n = 1$). Should we identify the denotations of the singular terms $\ulcorner[B]\urcorner$ and $\ulcorner[B(v_1, \ldots, v_n)]_{v_1 \ldots v_n}\urcorner$ with linguistic entities or with extralinguistic entities? Nominalists favor the former; conceptualists and realists, the latter. Let us see which theory is better.

7. NOMINALISM

According to the most straightforward version of nominalistic semantics, intensional abstracts denote linguistic expressions. Specifically, 'that'-clauses denote sentences, and infinitive and gerundive phrases denote predicates or open-sentences. The first problem with this sort of theory is that it is extremely counterintuitive. If I see that it is daytime, am I really seeing a sentence? If a prelinguistic child or lower animal knows directly that he is in pain, does he or she know directly a sentence? (If so, how is it possible that he or she should be entirely unfamiliar with the sentence?) If I have an experience of being in pain, do I have an experience of a linguistic predicate? If my dog likes swimming, does he like a predicate? Of course not. Nominalists might reply that this intuitive argument is an instance of the so-called fallacy of incomplete analysis. However, this reply is theoretically weak, for it forces nominalists to hold that the present intuitions cannot be taken at face value. But other things being equal, a theory is superior if it can take relevant intuitions at face value. The traditional realist theory that we advocate permits one to do just this. So, other things being equal, it comes out ahead of the nominalist theory.

Of course, nominalists believe that other things are not equal; specifically, they believe that their ontology is simpler than the traditional realist ontology of properties, relations, and propositions. This belief might be defensible when the debate is restricted to philosophy of mathematics. (An advantage of the argument from intensional logic is that it does not oblige one to take a stand on this issue.) However, the nominalist's belief about the relative simplicity of their ontology is not defensible when the debate is over the semantics for intensional abstracts. The problem here concerns the ontological status of linguistic expressions themselves. Let us explain.

Suppose that our nominalists try to identify linguistic expressions, not with types (e.g., shapes or sound types), but with linguistic tokens or set-theoretical constructs whose ultimate elements are linguistic tokens. Consider sentences of the following form:

It is possible that $F^1 t$.

where $\ulcorner t \urcorner$ is a singular term. In symbols: $P^1[F^1 t]$. Now either $\ulcorner t \urcorner$ has wide scope, or it does not. If it has wide scope, $\ulcorner P^1[F^1 t] \urcorner$ is true if and only if there is something z that is identical to t and it is possible that there is something y such that y is identical to z and $F^1 y$. On the other hand, if $\ulcorner t \urcorner$ does not have wide scope, $\ulcorner P^1[F^1 t] \urcorner$ is true if and only if it is possible that there is something y such that y is identical to t and $F^1 y$. Therefore, whether or not $\ulcorner t \urcorner$ has wide scope, $\ulcorner P^1[F^1 t] \urcorner$ is true only if it is possible that there exists an appropriate item y such that $F^1 y$.[26] Now for the problem. Recall that linguistic tokens are contingent particulars. Indeed, it is possible that there are *no* linguistic tokens at all. (Or it is possible that there are no *relevant* linguistic tokens. The following argument would go through *mutatis mutandis* using this weaker premise.) Accordingly, the following sentence is true:

It is possible that there are no linguistic tokens.

But this sentence is equivalent to the following intuitively true sentence:

It is possible that it is true that there are no linguistic tokens.

This sentence has the form $\ulcorner P^1[F^1 t] \urcorner$, where $\ulcorner F^1 \urcorner$ is the predicate 'is true' and $\ulcorner t \urcorner$ is the singular term '$[\neg(\exists x)\text{Token}(x)]$'. So it follows by the above considerations that, whether or not this singular term has wide scope, this sentence is true only if it is possible that there is an appropriate item y such that y is true. But what could this true item y be? According to the nominalist semantics for intensional abstracts, this true item *y* would be a linguistic expression (or a set built up somehow from linguistic expressions). However, in the envisaged possible circumstance in which there is such an item that is true, there would be *no* linguistic tokens. So if linguistic expressions were identified with linguistic tokens or sets built up somehow from linguistic tokens, then in the envisaged circumstance there would not be any linguistic expression *y*. Therefore, given a nominalist semantics for intensional abstracts, the sentence 'It is possible that it is true that there are no linguistic tokens' would be false. But it is true. Therefore, given the nominalist semantics, it follows that linguistic expressions cannot be identified with linguistic tokens or sets built up somehow from linguistic tokens.[27] (In what follows, we will call this *the problem of necessary existence*. There is of course an analogous problem of eternal existence.)

The only way for our nominalists to get out of this problem of necessary existence is to refrain from identifying linguistic expressions with (items that ontologically depend on contingent) linguistic tokens and, instead, to identify them with shapes or sound types, which are entities that exist necessarily. But shapes and sound types are properties *par excellence*. So the problem of necessary existence (and the analogous problem of eternal existence) is avoided only by invoking the ontology of properties. However, if the ontology of properties is admitted to solve this problem, it would be uneconomical not to make full, systematic use of this ontology in giving the semantics for intensional abstracts. Doing so would lead one simply to drop the nominalistic semantics for intensional abstracts and to adopt instead a straightforward realist semantics.

Indeed, the perversity of the nominalistic semantics can now be brought out with special poignancy. For nominalists who accept the ontology of shapes presumably would hold that the gerund 'being square' denotes, say, the complex shape consisting in order of the shapes 's', 'q', 'u', 'a', 'r', 'e' (or some set-theoretical construct built up from the shapes 's', 'q', 'u', 'a', 'r', 'e') as opposed to simply the shape square. There is no ontological gain in this position, and it is, on its face, incredible.

Now our nominalists might reject the above argument by denying the correctness of the intuitions upon which it is based. However, to press such a counterintuitive position is to press a mere bias. Basing one's theories on a mere bias cannot be acceptable even to the nominalist, for anyone who adopts this way of proceeding loses the ability to refute opponents whose biases favor some other arbitrary (perhaps anti-nominialistic) theory. The only way out of this difficulty is to honor our intuitions as evidence in such controversies. But if intuitions are honored here, consistency demands that they be honored elsewhere. When they are, the nominalist semantics is seen to be inferior. For, as we have seen, intuitions support the argument from necessary existence (or external existence). That argument shows that nominalist semantics is no more economical than a traditional realist semantics. However, the latter semantics, unlike the nominalist semantics, permits us to take at face value our intuitions about the identity of the primary objects of perception, belief, and so on. So, by comparison with the traditional realist theory, the nominalist theory is not acceptable.

We believe that this argument is decisive. However, our positive view can be made more convincing by laying bare the defects in the various specific versions of the nominalist semantics. This is the purpose of the remainder of this section.

According to the most common version of nominalist semantics for intensional abstracts, a 'that'-clause is taken to denote the complement sentence contained within the 'that'-clause itself: for example, the intensional abstract 'that man is a rational animal' is taken to denote its complement sentence 'man is a rational animal'. This nominalist theory has the greatest intuitive appeal in connection with indirect discourse. On the simplest version of the theory, the verb 'say' of indirect discourse is just identified with the verb 'say' of direct discourse. Thus,

(1) Seneca said that man is a rational animal.

is taken to be equivalent to

(2) Seneca said 'man is a rational animal'.

However, this clearly is wrong. Whereas (1) is true, (2) is false: Seneca never spoke English.

This difficulty can be overcome by giving the 'say' of indirect discourse a more sophisticated analysis. For example, Carnap[28] would have analyzed (1) as follows:

(3) There is a language such that Seneca wrote as a sentence of L words whose translation from L into English is 'Man is a rational animal'.

However, sophisticated analyses like this are beset with fatal flaws of their own. Consider, first, Alonzo Church's famous criticism:

> For it is not even possible to infer (1) as a consequence of (3), on logical grounds alone – but only by making use of the item of factual information, not contained in (3), that 'Man is a rational animal' means in English that man is a rational animal.
>
> Following a suggestion of Langford we may bring out more sharply the inadequacy of (3) as an analysis of (1) by translating into another language, say German, and observing that the two translated statements would obviously convey different meanings to a German (whom we may suppose to have no knowledge of English). The German translation of (1) is (1′) *Seneca hat gesagt, dass der Mensch ein vernünftiges Tier sei*. In translating (3), of course 'English' must be translated as 'Englisch' (not as 'Deutsch') and '"Man is a rational animal"' must be translated as '"Man is a rational animal"' (not as '"*Der Mensch ist ein vernünftiges Tier*"').[29]

Another difficulty with the more sophisticated nominalist analysis is that it does not carry over to belief sentences in the way Carnap hoped. Carnap proposed to analyze belief behaviorally in terms of dispositions to assent. However, standard criticisms of behaviorism show that this kind of analysis is mistaken. Dispositions to assent are not correlated with beliefs taken singly; instead they are correlated with the body of a person's beliefs and desires. As a result, they cannot be used to analyze any single belief.

A related problem with the nominalist analysis is that it fails to mesh with a general theory in which 'that'-clauses and other intensional abstracts are treated as singular terms and in which 'says', 'believes', 'perceives', etc. are treated as standard two-place predicates that take 'that'-clauses as arguments and 'is necessary', 'is possible', 'is true', etc. are treated as standard one-place predicates that take 'that'-clause as arguments. This problem is dramatized by the fact that the analysis provides no clue about how to identify what it is that 'that'-clauses actually denote.

A final difficulty with this nominalist analysis is hidden in its use of the phrase 'as a sentence of L'. This restriction is needed; for, without it, the speaker (Seneca in the present example) could utter the sentence without any of the relevant linquistic intentions. For example, the speaker might utter the sentence merely *as* a pleasant sound; in this case, the speaker would not even have made a statement. Or the speaker might utter the sentence *as* a sentence of some phonologically equivalent but semantically different language; in this case, the speaker would not have made the relevant statement (i.e., that man is a rational animal). So the qualifying phrase 'as a sentence of L' is needed. The problem for our nominalist is that this phrase is a covert intensional qualifer with something like the following force: x utters A as a sentence of L iff x utters A and x intends to speak L when x utters A. However, as we have seen, an infinitive phrase such as $\ulcorner$to speak L when x utters $A\urcorner$ is an intensional abstract. So, by employing the phrase 'as a sentence of L', the sophisticated nominalist analysis only sweeps this inherent intensional aspect of indirect discourse under the rug.

Specific difficulties like these spell defeat for all natural versions of the nominalist theory. Nevertheless, there are some quite unnatural versions of nominalism that avoid these difficulties. But they all run into special new difficulties of their own.

According to one of these unnatural versions of nominalism, the denotation of a 'that'-clause is identified with the equivalence class of all sentences synonymous to the complement sentence contained within the 'that'-clause, and the denotation of a gerundive or infinitive phrase is identified with the class of all predicates (or open-sentences) synonymous to the predicate (open-sentence) that generates the gerundive or infinitive phrase. (So, for example, 'that man is a rational animal' would denote the class $\{S$: for some actual language L, the sentence S in L is synonymous to 'man is a rational animal' in English$\}$, and the gerund 'swimming' would denote the equivalence class $\{F$: for some actual language L, predicate F in L is synonymous to 'swim' in English$\}$.) On this approach, such equivalence classes of synonyms are then identified as the primary bearers of truth, necessity, logical truth, probability, etc. and as primary objects of perception, belief, desire, moral obligation, explanation, etc.

The first problem with this sort of nominalist theory is that it too is extremely counterintuitive. If I see that it is daytime, am I really seeing a set of sentences? If a prelinguistic child or lower animal knows directly that he is in pain, does he know directly a set of sentences? (If so, how is it possible that he or she should be entirely unfamiliar with every single sentence in the set?) If I have an experience of being in pain, do I have an experience of a set of predicates? If my dog likes swimming, does he like a set of predicates?

Another problem with the present nominalist theory is that it does not mesh with a satisfactory general explanation of how cognitive states succeed in *representing* things, in being *about* things. To dramatize this point, let us consider a hypothetical situation in which no one ever speaks any of the languages that, as a matter of fact, we actually speak. In such a situation, however, people still would be able to have a wide range of cognitive states, states whose objects in many cases would be the same as objects of our cognitive states. For example, in the indicated situation someone could believe *that someone feels pain*. But if the present nominalist theory were correct, the object of such a person's belief (namely, the object denoted by the 'that'-clause 'that someone feels pain') would be a class of sentences belonging to languages *we* happen actually to speak. Accordingly, the object of such a person's belief would be a class of shapes and/or sounds having nothing to do with the person (or anyone else in the hypothetical situation) and having no relevant relation to what the person's belief is about, namely, pain. On the present nominalist theory, therefore, it would be competely fortuitous that in the hypothetical situation the person's belief is *about* pain rather than some other arbitrary item.

To avoid this outcome, why not allow 'that'-clauses to denote sets of synonyms belonging to *possible*, as well as actual, languages? That is, why not identify the denotation of $\ulcorner$that $A\urcorner$ with the class $\{S$: for some possible language L, the sentence S in L is synonymous to A in English$\}$? The answer is that *all* 'that'-clauses would, wrongly, turn out to be co-denoting. After all, for *every* sentence S, there is some possible language L such that S in L is synonymous to A in English. A similar problem confronts the slightly more sophisticated nominalist semantics in which the denotation of $\ulcorner$that $A\urcorner$ is identified with the class of all possible synonym/language pairs, i.e., the class $\{S, \mathrm{L}$: S in L is synonymous with the sentence A in English, where S is some sentence in some possible language L$\}$. For if one follows the standard extensionalist practice of identifying a language L with an ordered-pair consisting of a set of well-formed expressions and a function that assigns extensional semantical values to those expressions, then on the present more sophisticated nominalist semantics, the extensional semantical value of various intuitively non-codenoting 'that'-clauses would turn out to be the *same* set of possible synonym/language pairs.[30] Another alternative would be to identify the denotation of $\ulcorner$that $A\urcorner$ with a function that assigns to each possible world w a class $\{S$: S in L is synonymous to A in English, where L is some language that is spoken in $w\}$. However, this theory would not be acceptable to the nominalists inasmuch as it relies on an ontology of possible worlds. (For a critique of possible-worlds semantics, see Section 9 which deals with nontraditional realist semantics.)

Another problem with the equivalence-class approach is that it employs the predicate 'is synonymous' in the metalanguage. But what is the status of this predicate? According to our best theory, synonomy is to be defined in a broadly Gricean way in terms of certain complex conventional intentions of speakers. In the specification of these intentions, however, we would use 'that'-clauses. Thus, in the statement of our metatheory, we would identify the nominalists' equivalence classes in terms of certain speaker intentions that are identified by means of 'that'-clauses. So far, then, one does not end up with a purely nominalist specification of the denotation of intensional abstracts: the specification of the denotation of intensional abstracts in the object language involves the use of intensional abstracts in the metalanguage. But given their view of things, one would think that nominalists would be able to state their position without this recourse to explicit intensionality in the metalanguage. For the point of the nominalist semantics is to have syntactic entities take the place of traditional intensional entities (properties, relations, and propositions), and it should be possible to give a purely extensional description of these syntactic entities and of the key relations (e.g., synonymy) holding among them. Of course, nominalists might try to achieve such a description by insisting that 'is synonymous' is undefinable. But this claim would contradict our best theory of synonymy.

A way of trying to circumvent this difficulty is to try to define synonymy within the framework of a "language-of-thought" treatment of the propositional attitudes. The idea would be to define synonymy in a broadly Gricean way in terms of the propositional attitudes and then to identify the objects of the propositional attitudes with sentences in an ideal language-of-thought. But if one adopts this approach to synonymy, one is forced to give up the equivalence-class-of-synonyms semantics for intensional abstracts. For, as we have seen, the objects of the propositional attitudes are paradigmatic examples of the sort of items denoted by intensional abstracts, and on the language-of-thought theory these items are sentences in an ideal language-of-thought, not equivalence classes of synonyms in natural languages. So even if the language-of-thought theory were successful, it would be of no help to the equivalence-class theory.

Let us now examine this sort of language-of-thought semantics for intensional abstracts. According to the most straightforward formulation of this theory, there is a single universal ideal language that underlies all possible cognition and all possible natural languages, and intensional abstracts denote expressions in this ideal language. We have seen that the denotata of intensional abstracts are the primary bearers of truth, necessity, possibility, definition, probability, etc. and are the primary objects of belief, perception, hope, moral obligation, explanation, causation, etc. On the language-of-thought theory, therefore, it follows that expressions in this ideal language are the primary bearers of truth, necessity, possibility, definition, probability, etc. and are the primary objects of belief, perception, hope, moral obligation, explanation, causation, etc.

Like the previous nominalist semantics, this one is extremely counter-intuitive. When I see that it is daytime, do I really see a sentence in some ideal language? When I have an awareness of being in pain, do I have an experience of some hypothetical linguistic shape or sound? Certaintly not. As with the previous nominalist theories, the present one appears to be driven by a mere bias for nominalism. Moreover, as with all nominalist approaches, the interface between the sensation of phenomenal qualities (e.g., the quality of being in pain) and the cognition of phenomenal qualities is a "representationalist mystery" on the language-of-thought theory. The straightforward way to solve this mystery is with a full-fledged realist theory in which the objects of the propositional attitudes (i.e., propositions) are built up, by means of fundamental logical operations, from basic properties (including, in particular, phenomenal qualities), basic relations, and perhaps subjects of singular predications.[31]

A further difficulty with this sort of nominalist semantics is that, like the previous one, it too fails to mesh with a satisfactory general explanation of how cognitive states succeed in representing things, in being about things. The problem here is that the radical language-of-thought hypothesis suffers from the following form of radical arbitrariness: for any candidate universal ideal language L, there are an infinite number of alternative languages L′ that could serve the same theoretical roles attributed to L. Which is *the* right one? Which one provides *the* genuine primary bearers of truth, necessity, etc. and the genuine objects of belief, perception, hope, etc.? The choice is in principle utterly arbitrary. Accordingly, there is no general philosophical explanation of why some linguistic shape or sound S should be taken to represent – or to be about – one thing rather than another. No satisfactory theory can tolerate this degree of arbitrariness.

To avoid this problem of arbitrariness, the language-of-thought theorist could adopt an analysis reminiscent of Carnap's: x believes that A iff x believes a sentence that plays a causal role for x that is analogous to the causal role that the English sentence $\ulcorner A \urcorner$ plays for English speakers. However, this analysis falls prey to difficulties rather like those confronting Carnap's. First, it fails to pass the Langford–Church translation test. Second, because of the phenomenon of fine-grained intensionality, belief cannot be analyzed functionally in terms of the notion of causal role; causal role is simply too coarse a criterion for the identity of belief. (See Bealer [1984].) Finally, this analysis fails to mesh with a general theory in which 'that'-clauses and other intensional abstracts are treated as singular terms and in which 'says', 'believes', 'perceives', etc. are treated as standard two-place predicates that take 'that'-clauses as arguments and 'is necessary', 'is possible', 'is true', etc. are treated as standard one-place predicates that take 'that'-clauses as arguments. This problem is dramatized by the fact that the analysis provides no clue about how to say systematically what it is that 'that'-clauses actually denote. As a result, this analysis suggests no general treatment of sentences in which *non-psychological* predicates take 'that'-clauses as arguments, for example: ⌜It is true that A⌝, ⌜It is possible that A⌝, ⌜It is probable that A⌝, ⌜It is explainable that A⌝, and so forth.

Indeed, a common failing of language-of-thought theories is that they usually disregard the role of intensional abstracts in non-psychological contexts. (For example, many language-of-thought theorists believe that *fine-grained* intensionality arises only in connection with the propositional attitudes. But in fact this phenomenon also arises in connection with familiar logical relations such as following-by-*modus-ponens*. Witness the sentence 'Given the premise that if A then B and given the premise that A, the conclusion that B follows by *modus ponens*'.) What is needed is a unified theory of intensional language, not just a theory that treats intensionality in propositional-attitude sentences. Some language-of-thought theorists might try to respond by claiming that all intensionality (truth, necessity, logical truth, probability, counterfactuality, explainability, etc.) that is not psychological is somehow derivative. However, this kind of metaphysical idealism is a well-known dead-end. (For some of the problems with this kind of metaphysical idealism, see the next section, which deals with conceptualism.)

Another way language-of-thought theorists might try to avoid the problem of arbitrariness is to adopt a theory that is reminiscent of the equivalence-class-of-synonyms theory. Specifically, the intensional abstract ⌜$[A]$⌝ would denote the class $\{S, x$: S is a sentence that has a causal role for an actual cognitive agent x that is analogous to the causal role the sentence ⌜A⌝ has for us$\}$. But this theory runs into the same sort of difficulty that originally plagued the equivalence-class-of-synonyms theory. Namely, it makes it a mystery how, in hypothetical circumstances with altogether different cognitive agents, the mental states of those cognitive agents succeed in representing, or in being about, anything. For if these cognitive agents believe, say, that someone feels pain, the object of the belief would be a set of items having no relevant relation to these cognitive agents. Moreover, modifications of the present proposal that invoke possibility in one way or another fall prey to difficulties quite like those that beset analogous modifications of the equivalence-class-of-synonyms approach.

Before we proceed to a final version of nominalistic semantics, a remark about the merits of the language-of-thought hypothesis in cognitive science is in order. We have seen that a language-of-thought semantics for intensional abstracts is not viable, and we shall soon see that only a traditional realist semantics is defensible. Moreover, relative to the algebraic style of realist semantics that we will present, intensional logic can be given a highly fine-grained formulation, a formulation in which propositions may be treated as entities upon which computations are performed directly, without any linguistic mediation. At the same time, the problem of representationalism, which causes so much trouble for the language-of-thought hypothesis, is solved automatically by our traditional realist theory.[32] In view of these results, why invoke any form of the language-of-thought hypothesis in cognitive science? (For example, some cognitive scientists now advocate treating 'believes' as a three-place relation holding among a cognitive agent, a proposition, and a sentence.) The answer is that there is no good reason whatsoever. On a suitable formulation of fine-grained intensional logic, this residual nominalistic element is entirely extraneous, a mere throw-back to a defunct nominalistic semantics.

There is a final kind of nominalist approach to intensional abstracts that we should mention, namely, the approach of Israel Scheffler [1954]. According to this approach, a singular term $\ulcorner[A]\urcorner$ would be contextually analyzed as follows:

$$\ldots [A] \ldots \text{ iff}_{\text{df}} \ (\exists v_k)(v_k \text{ is-an-}A\text{-inscription} \ \& \ \ldots v_k \ldots)$$

where $\ulcorner$is-an-A-inscription$\urcorner$ is an undefined primitive predicate. On the intended interpretation, this predicate is satisfied by all and only inscriptions synonymous to $\ulcorner A \urcorner$. However, given that, for an infinite number of sentences $\ulcorner A \urcorner$, the sentence $\ulcorner [A] = [A] \urcorner$ is logically valid, this theory implies the actual existence of inscriptions (tokens) of every sentence. But, in fact, there are infinitely many sentences $\ulcorner A \urcorner$ of which there are no actual inscriptions (tokens). Moreover, since there are an infinite number of distinct 'that'-clauses in natural language, Scheffler's approach requires an infinite number of undefined primitive predicates $\ulcorner$is-an-A-inscription$\urcorner$. This fact amounts to a violation of Davidson's learnability requirement. Furthermore, it seems to block the systematization of the internal logic of 'that'-clauses. Finally, as we explained earlier, the need to use the predicate 'is synonymous to' in the metalanguage is inconsistent with the nominalistic point of view. For these reasons, Scheffler's approach does not help to save nominalism.

The above considerations, together with a number of others, lead us to conclude that linguistic expressions, whether types or tokens, are not the sort of entity denoted by intensional abstracts $\ulcorner[A]\urcorner$ and $\ulcorner[A]_{v_1 \ldots v_n}\urcorner$. And the same conclusion goes for sequences or sets of linguistic entities, or indeed any other kind of object that is linguistic in character.

So what sort of entities are denoted by $\ulcorner[A]\urcorner$ and $\ulcorner[A]_{v_1 \ldots v_n}\urcorner$? Given the failure of nominalism, we are left with realism and conceptualism.

8. CONCEPTUALISM

According to both realism and conceptualism, when we use 'that'-clauses and gerundive and infinitive phrases, we denote extralinguistic intensional entities. The difference between realism and conceptualism concerns the ontological character of these entities. Realists hold that they are mind-independent entities whereas conceptualists hold that they are mind-dependent. Mind-dependent in the sense that they depend for their existence on minds or mental activity; they would not exist if there were no minds or mental activity. Contemporary realists tend to call these intensional entities 'properties', 'relations', and 'propositions' (depending on their degree). By contrast, conceptualists usually call them 'concepts' and 'thoughts' (depending on their degree). But this difference is largely terminological. The real difference between conceptualism and realism lies in the alleged ontological status of these intensional entities. Are they mind-dependent or mind-independent?

In our discussion of conceptualism we will confine ourselves to the version that ascribes to intensions an ontological dependence on contingent, finite minds like ours. There is another version of conceptualism, however. On this version, even though intensions are ontologically dependent on mind, they nonetheless exist necessarily; for they exist necessarily in the infinite, necessary mind of God. We will take no stand on this version of conceptualism. Our reason is that, like realism, it implies that intensional entities exist necessarily, and this is what matters most to contemporary realists.

On the most plausible version of conceptualism, there are certain basic intensions (much like Locke's simple ideas) that are simply "given" in ordinary mental activity, and all other intensions are somehow "formed" or "constructed" out of these nonconstructed intensions. At relevant points in our critical assessment, we will focus on this version of conceptualism.

The first difficulty with conceptualism is this. Evidently, there are intensions that have never been "given" in anyone's mental activity and that could not, even in principle, be "formed" or "constructed" from intensions that have been "given". For example, various fundamental physical properties (e.g., quark-theoretic properties such as the property denoted by the intensional abstract 'having spin up') seem to be like this: they appear not to be "constructible" in any way from "given" intensions; rather, they appear to be mere theoretical posits that we can at best describe.[33] Indeed, many physicists believe that there still exist fundamental physical properties and relations (e.g., sub-quark properties, sub-sub-quark properties, etc.) that remain to be described theoretically.

In a related vein, there are no doubt primitive phenomenal qualities that no one has ever experienced (e.g., new shades, fragrances, or tastes) and that, in principle, could not be "constructed" from intensions that have already been "given". Indeed, the taste of pineapple (i.e., the familiar phenomenal quality we know in sensation) once had this ontological status, for there was a time when no one had ever tasted it.

This last example gives rise to a general defect in conceptualism that should have been predictable; namely, conceptualism falls prey to the argument from necessary existence (and also to the analogous argument from eternal existence). (This style of argument was used in the previous section to refute nominalist semantics for intensional abstracts.) Consider sentences of the following form:

It is possible that $F^1 t$.

where $\ulcorner t \urcorner$ is a singular term. In symbols: $P^1[F^1 t]$. We saw earlier that, whether or not $\ulcorner t \urcorner$ has wide scope, $\ulcorner P^1[F^1 t] \urcorner$ is true only if it is possible that there is an appropriate item y such that $F^1 y$. Now, intuitively, the following sentence is true:

It is possible that it is true that there are no finite minds.

This sentence has the form $\ulcorner P^1[F^1 t] \urcorner$, where $\ulcorner F^1 \urcorner$ is the predicate 'is true' and $\ulcorner t \urcorner$ is the singular term '$[\neg(\exists x)$ Finite-mind $(x)]$'. It follows that, whether or not this singular term has wide scope, this sentence is true only if it is possible that there is an appropriate item y such that y is true. But what could this true item y be? According to conceptualists, y would be an intensional entity that is ontologically dependent on finite minds. However, in the envisaged circumstance in which there is an intension y that is true, there would be *no* finite minds. Therefore, in the envisaged circumstance there would not be *any* intensions, and so the sentence 'It is possible that it is true that there are no finite minds' would be false. But it is true. So conceptualism must be mistaken: it cannot overcome the problem of necessary existence. (Some conceptualists might try to escape this conclusion by a "modalizing strategy." We will consider this strategy in a moment.)

We believe that the foregoing intuitive considerations tell decisively against conceptualism. However, to remove lingering doubts, we move on to a more theoretical line of argument. The problem concerns the infinite. Intuitively, there are infinitely many distinct fine-grained intensions. For example, there are infinitely many nontrivial logical truths: that A_1, that A_2, that A_3, ... (To see this, suppose that the sentence $\ulcorner A_i \urcorner$ expresses the nontrivial logical truth that A_i. Suppose that $\ulcorner A_i \urcorner$ is not in prenex normal form, and suppose that $\ulcorner A_j \urcorner$ is the result of converting $\ulcorner A_i \urcorner$ to prenex normal form. Let $x =$ that A_i, and $y =$ that A_j. Intuitively, it is possible that someone is consciously and explicitly thinking x and not consciously and explicitly thinking y. If so, that $A_i \neq$ that A_j.) The problem facing conceptualists is to explain why there seem to be infinitely many intensions. They are not "given" in anyone's actual mental activity, and we do not actually "construct" them. For doing so would require infinitely many acts of "construction," and our finiteness excludes this. Conceptualists have two ways of trying to solve this problem. The first is to grant that there actually exist infinitely many intensions and to identify intensions that are not "given" in actual mental activity with a certain kind of "extensional complex" (e.g., finite sequences, ordered sets, or abstract trees) whose ultimate elements are intensions that are "given" in actual mental activity. The other strategy is to deny that there actually exist infinitely many intensions and to explain why there seem to be by exploiting the distinction between intensions that have actually been "constructed" and possible acts of "construction". The latter strategy is the *modalizing* strategy.

The first strategy is defeated by considerations of ontological economy. For, on this treatment of intensional entities, conceptualists would have to posit *two* fundamentally dissimilar ontological categories – extensional complexes (finite sequences, ordered sets, abstract trees) and primitive intensional entities (namely those "given" in actual mental activity). Realists, by contrast, need only one ontological category, namely, that of intensions. One ontological category suffices for realists because the theoretical work that can be accomplished with the conceptualists' extensional complexes can be accomplished by appropriate realist intensions. For example, the theory of finite sequences, finite ordered sets, and finite abstract trees can be represented within first-order logic with identity and intensional abstraction. (E.g., the jobs done by the finite sequence $\langle v_1, \ldots, v_n \rangle$ can be done by the realists' intension $[u_1 = v_1 \,\&\, \ldots \,\&\, u_n = v_n]_{u_1 \ldots u_n}$.) And if the conceptualists' theory of extensional complexes is supplemented with a set-membership relation, the realists' theory of intensions may be supplemented with a predication (instantiation) relation. Any theoretical task that can be performed by the conceptualists' theory of membership can then be performed by this realist theory of predication. The upshot is that the conceptualist theory can perform no theoretical task that the realist theory cannot perform. At the same time, the conceptualist theory is in principle ontologically more complex than the realist theory, for it requires two fundamentally dissimilar ontological categories whereas the realist theory requires only one. So the conceptualist theory should be rejected on ontological grounds.

In response, someone might wonder whether the conceptualists' two categories (extensional complexes and actually "given" primitive mind-dependent intensions) really are fundamentally dissimilar. To dramatize the fact that they are, recall that on such a theory there would need to be extensional complexes whose elements would not even be intensions (for example, ordered sets whose elements are physical objects).

Another response to our argument would be to hold that the present version of conceptualism is not really ontologically excessive, for the posited extensional complexes can be eliminated in favor of items that everybody (including realists) already accepts. But which items could they possibly be? In debates about foundations of mathematics the usual candidates put forward at this juncture are linguistic entities. But here we encounter the power of the argument from intensional logic. Perhaps linguistic entities can play the role of sets or other extensional complexes in certain formulations of the foundations of mathematics. However, in the present context, the conceptualists' purpose for introducing extensional complexes is to provide a realm of entities to serve as the denotata of intensional abstracts. But in our critique of nominalism we saw that linguistic entities are wholly inadequate for this purpose. So this escape route is not available to conceptualists.

We will mention three other defects in the present version of conceptualism. First, it is highly unintuitive that ordered sets, sequences, or abstract trees are really the sort of thing that are perceived, believed, etc.

or that are true, necessary, probable, explainable, etc. Advocates of this theory certainly have lost the "naive eye". Second, it is *prima facie* implausible that some intensional abstracts should denote one category of entity (i.e., primitive intensions "given" in our actual mental activity) and that other intensional abstracts should denote ontologically very different sorts of entities (i.e., ordered sets or sequences). Third, by identifying the denotata of infinitely many intensional abstracts with extensional complexes, conceptualists might run into a potentially fatal difficulty in connection with "self-embeddable" intensions. (This general issue shall be discussed three sections below.)

Our overall conclusion, then, is that the first strategy is of no help to conceptualists. This outcome leaves them with no alternative but to try the "modalizing" strategy. The idea behind this strategy is to deny that there are really an infinite number of actual intensions (e.g., the nontrivial logical truths that A_1, that A_2, that A_3, . . .) and to hold instead that there are merely an infinite number of possible ways of thinking (which, if actualized, would generate the intensions that A_1, that A_2, that A_3, . . .). Our reply to the modalizing strategy will be that it does not really avoid ontological commitment to an infinity of actual intensions. To explain this reply, we must spell out the modalizing strategy more fully.

Consider intuitively true sentences of the form ⌜It is logically true that A_i⌝. We have argued that each such sentence is ontologically committed to an intensional entity (i.e., the intension that A_i). According to the modalizing strategy, however, the sentence ⌜It is logically true that A_i⌝ is in most cases not strictly speaking true; what is true is an associated modal sentence something like the following: ⌜It is possible that someone should form the thought that A_i and the resulting thought would be logically true⌝. (On a somewhat related form of the modalizing strategy, although ⌜It is logically true that A_i⌝ would be counted as true, it would be treated as a mere abbreviation of the modal sentence ⌜It is possible that someone should form the thought that A_i and the resulting thought would be logically true⌝.)

The fundamental shortcoming of the modalizing strategy is that it does not really address the problem it was supposed to solve. The problem was to find a way to avoid commitment to an actual infinity of intensional entities. However, the proposal still leaves us with an actual infinitude of such entities, namely, those denoted by the 'that'-clause occurring in the proposed modal sentence ⌜It is possible *that someone forms the thought that A_i and this thought is logically true*⌝. (In symbols, ⌜$P^1[(\exists x)(F^2x, [A_i] \;\&\; LT^1[A_i])]$⌝.) After all, as we showed in earlier sections, the best systematic treatment of intensionality is by means of intensional abstracts and accompanying auxiliary predicates. Just as $\ulcorner x$ believes that $B\urcorner$ is represented as $\ulcorner B^2x, [B]\urcorner$, $\ulcorner$It is possible that $B\urcorner$ is represented as $\ulcorner P^1[B]\urcorner$. The modalizing strategy would require a systematic way of capturing the relevant possibilities of forming thoughts. The way to do this is by means of *further* intensional abstracts, ones that generate their own commitment to an actual infinitude of intensional entities. So the modalizing strategy does not work.

One way of trying to avoid this conclusion is by resorting to primitive operators that are designed to avoid use of the offending intensional abstracts. For example, instead of putting forward the above intensional-abstract sentence, modalizers would put forward the following primitive-operator sentence: $\ulcorner$Possibly someone forms the thought that A_i and this thought is logically true$\urcorner$. (In symbols, $\ulcorner\Diamond(\exists x)(F^2x, [A_i]\ \&\ LT^1[A_i])\urcorner$.) The alleged gain is that the offending intensional abstract does not explicitly occur in this new primitive-operator sentence. But the argument from intensional logic undercuts this move.[34]

There are two sorts of reasons. First, we have already established that $\ulcorner$It is possible that $B\urcorner$ is to be represented as $\ulcorner P^1[B]\urcorner$. However, it is intuitively obvious that it is possible that B iff possibly B. It would be entirely *ad hoc* to deny this obvious equivalence just to save conceptualism. So, on intuitive grounds, the primitive-operator move cannot be used to side-step the ontological commitment to the implicit intensional entity (i.e., that B). Second, we have seen that great theoretical economy can be gained by treating commonplace operator sentences as derived forms of intensional-abstract sentences. For example, by treating $\ulcorner\Diamond B\urcorner$ as a derived form of $\ulcorner P^1[B]\urcorner$. Since the latter form is already required by an acceptable *general* formulation of intensional logic (e.g., one that can represent general relations between belief and possibility), it would be highly unjustified theoretically to insist on representing $\ulcorner\Diamond B\urcorner$ as a primitive-operator sentence, rather than to bring it within a unified, general theory of intensional logic.[35]

Now conceptualists might try to avoid our critique of the modalizing strategy by resorting to other primitive operators (e.g., constructibility operators, constructibility quantifiers, etc.). However, these alternate linguistic forms create the same problems for conceptualism. First, they are intuitively equivalent to linguistic forms involving intensional abstracts and accompanying predicates. Second, these primitive-operator sentences typically generate intensional contexts. Therefore, the canonical representation of them is provided by means of intensional abstracts and accompanying predicates. This way these intensional contexts can be dealt with within a unified, general theory. It is inevitable, therefore, that the modalizing strategy does not successfully avoid the commitment to an actual infinity of intensional entities.

Notice that the foregoing critique of conceptualism did not get us involved in many of the usual worries that characterize contemporary discussions of conceptualism and realism, for example, worries about the law of the excluded middle, impredicativity, and so forth. This is a significant advantage of the argument from intensional logic. Considerations in intensional logic (notably, the treatment of generality and intensional abstraction) just on their own force one to posit an actual infinitude of intensions. When conceptualists try to give an alternate explanation, either they end up positing two fundamentally dissimilar ontological categories (mind-independent extensional complexes and mind-dependent intensions) whereas one ontological category (mind-independent intensions) suffices for realism. Or they offer a modalizing strategy that, upon closer examination, implies the very sort of infinite intensional ontology that it is designed to avoid.

For these and the other reasons we listed, our overall conclusion is that, in comparison with realism, conceptualism is not acceptable.

9. REALISM

Given the failure of nominalism and conceptualism, we are left with realism. According to traditional realism, $\ulcorner[A]\urcorner$ would denote the proposition that A; $\ulcorner[A]_v\urcorner$ would denote the property of being something v_1 such that A; and $\ulcorner[A]_{v_1 \ldots v_m}\urcorner$ would denote the relation holding among $v_1 \ldots v_m$ such that A. There are, however, non-traditional forms of realism according to which PRPs are replaced by or reduced to other sorts of mind-independent extralinguistic entities. For example, according to the possible-worlds approach, propositions are reduced to functions from possible worlds to truth values; properties are reduced to functions from possible worlds to sets of possible individuals; and m-ary relations are reduced to functions from possible worlds to sets of ordered m-tuples of possible individuals. And according to the original version of the Perry–Barwise situation semantics, although properties and relations are taken as primitive traditional realist entities, propositions (or situations) are reduced to ordered sets of primitive properties, primitive relations, and actual individuals, or they are reduced to set-theoretical compounds of such ordered-sets. [On another version of situation semantics, propositions (situations) are not reduced to such set-theoretical constructs. Instead, these constructs are used only for *model* propositions (situations). In the final analysis, propositions (situations) are to be taken as primitive, irreducible entities. Int his vein, Barwise and Perry now seem attracted to a traditional realist theory of properties, relations, and propositions much like that we have defended here and in previous work. At this stage in the history of the subject, calling such a theory 'situation theory' risks terminological confusion; it is so similar to traditional PRP theory. In the ensuing remarks, we address ourselves only to the earlier, reductionistic version of situation semantics. We wish to emphasize that Perry and Barwise no longer hold this theory.] Scott Soames and Nathan Salmon have recently advocated reducing propositions to ordered sets of primitive properties, primitive relations, and real individuals.[36] Finally, the theory developed by Max Cresswell in *Structured Meanings* is a special hybrid reductionistic approach that combines a sequence treatment of propositions and possible-worlds reductionism regarding properties and relations.

These non-traditional forms of realism, however, have several defects not found in our traditional realism. For example, some of them often identify the denotations of intensional abstracts with items whose identity conditions are not right. Possible-worlds semantics provides the most notorious case of this, for on this approach necessarily equivalent PRPs turn out to be identical. The original formulation of situation semantics has a number of equally damaging consequences.[37] Although consequences like this can be tolerated in some parts of intensional logic such as modal logic, they are quite unacceptable in other parts, notably, those dealing with intentional matters. To compensate for this defect, some people (e.g., David Lewis and Max Cresswell) propose to reduce only "coarse-grained" PRPs to sets of possibilia and, then, to reduce "fine-grained" PRPs to sequences of these reduced coarse-grained PRPs. But this revised semantics turns out to be quite flawed (see below); moreover, it turns out to be more complicated technically than our realist semantics which treats coarse-grained and fine-grained PRPs as irreducible primitive entities.

Another difficulty with the reductionistic approaches concerns "self-embeddable" PRPs. For example, both the original possible-worlds semantics and Cresswell's hybrid theory are inconsistent with the plain fact that a person can contemplate the contemplating relation, and they are inconsistent with the plain fact that the relation of being distinct is distinct from the relation of being identical. Other difficulties concerning "self-constituency" threaten the other non-traditional realist semantics, including the reductionistic version of situation semantics and the theories articulated by Soames and by Salmon. (See the section after next for a detailed discussion of self-embeddability and self-constituency.) A traditional PRP semantics, by contrast, can easily deal with these phenomena, as we will show later on.

Another critical point is that many of the non-traditional realist approaches are extremely counterintuitive. For example, it is incredible, intuitively, that sets or sequences can ever strictly and literally be the sort of thing that are perceived, believed, and so forth or that are true, necessary, valid, probable, and so forth. People who hold otherwise have lost their "semantic innocence"; they are under the spell of set-theoretical reductionism. Although set-theoretical constructs might in the short run have heuristic value in the model theory for intensional abstracts, in the long run we should like a semantical theory that provides a natural and intuitive semantics for these important expressions. A semantics that takes PRPs at face value as primitive entities does this; possible-worlds semantics, the original version of situation semantics, and the theories of Cresswell, of Soames, and of Salmon plainly do not.

In a related vein, it is doubtful that possible-worlds semantics (and Cresswell's semantics) can be made to mesh with a plausible epistemology. For example, in sense experience we can be directly aware of phenomenal properties – say, the aroma of coffee. But in sense experience can we be directly aware of the function that assigns to each possible world the set of possible individuals that smell like coffee in that world? This is hardly plausible. Here are some related questions. (1) Suppose that the taste of pineapple is a function from possible worlds to sets of possible individuals. Could a person have a sense experience of a function that is identical to this one except for the presence (or absence) of a few possible individuals in one of the sets in the range of this function? Presumably not, for there is nothing such a sense experience could *be like*. But how are we to explain this? (2) Consider two visibly similar but distinct shades of blue b_1 and b_2. Suppose that b_1 is the function from possible worlds to sets of things that are shaded b_1 in those worlds and, likewise, that b_2 is the function from possible worlds to sets of things that are shaded b_2 in those worlds. Given that functions are sets of ordered pairs, b_1 and b_2 would then be sets that have *no* members in common. What makes b_1 and b_2 look so similar? (3) Let the arguments and values of a possible-worlds function b_1' differ from those of the shade b_1 at no points except for the presence (or absence) of one non-actual individual in one of the values. On the possible-worlds theory, b_1' is a property. Does the shade b_1 resemble this property as much as the two shades b_1 and b_2 resemble each other? Presumably, we would answer no. Why? (4) Why does the shade of blue b_1 resemble the other shade of blue b_2 more than it resembles a shade of red r? Perhaps the possible-worlds answer to this question is that the individuals in the ranges of b_1 and b_2 resemble each other more than the individuals in the ranges of b_1 and of r resemble each other. But if this is the answer, a vicious regress results. What is it about the individuals in the ranges of b_1 and b_2 that makes them resemble one another more than the individuals in the ranges of b_1 and r? The answer, of course, is that the shades (b_1 and b_2) of the former individuals resemble each other more than the shades (b_1 and r) of the latter individuals resemble each other. But why? This is the question with which we started. Now, all these questions can be answered satisfactorily, but only with a traditional realist theory that takes properties and relations as primitive, irreducible entities.[38]

Moreover, possible-worlds theories (and Cresswell's theory) are beset with insurmountable epistemological problems concerning the individuation of "nonactual individuals". For example, suppose that I form a thought that is (allegedly) about a particular "nonactual individual". (If one cannot form such a thought about *any* item in the category of "nonactual individual", that is itself a count against the ontology; for no other ontological category is like this.) Suppose that years later, after forgetting all about this earlier episode, I form a thought that is qualitatively indistinguishable from my earlier one. Is the "nonactual individual" I first thought about identical to the one I thought about on the second occasion, or are they nonidentical items that are only qualitatively alike? There is in principle no way to tell! For another example, suppose that two causally separated people form thoughts that are (allegedly) about "nonactual individuals", and suppose that their thoughts are qualitatively indistinguishable. Are they thinking about the same "nonactual individual", or are they thinking about distinct "nonactual individuals" that are only qualitatively alike? Again, there is in principle no way to tell. A theory with this kind of epistemological indeterminacy is, other things being equal, unacceptable.

And then there is the problem of ontological economy. According to the reductionistic version of situation semantics and the positions advocated by Soames and by Salmon, propositions (situations) are to be reduced to set-theoretical constructs whose ultimate elements are actual individuals, primitive properties, and primitive relations. However, these reductionistic theories are guilty of an ontological excess. For they must posit, in addition to individuals, two fundamentally dissimilar ontological categories – intensions and sets. (Relatedly, they must hold that some intensional abstracts denote intensions whereas others denote sets! This kind of *ad hoc* disunity is unacceptable.) On a traditional, nonreductionistic realist theory, by contrast, there is instead only one corresponding ontological category, namely, that of intensions. Sets (i.e., extensions) are just dropped in favor of intensions. Now the main reason these reductionists have resorted to their more complex ontology of both sets and intensions is that it permits them to treat propositions (situations) and other "complex" intensions. However, we will show that this can be accomplished far more simply without resorting to set-theoretical constructs but rather by treating such intensions simply as the result of applying fundamental logical operations (e.g., conjunction, negation, existential generalization, predication, etc.) to other intensions. The result is that the identification of propositions (and other "complex" intensions) with sets is ontologically superfluous.

It might be replied that these reductionistic theories are not ontologically excessive because sets are needed for independent reasons. But this is simply false. As we have already seen in our discussion of conceptualism, the theoretical work done by finite sets and finite sequences can be accomplished within the first-order logic for identity and intensional abstraction. And the theoretical work done by a set theory with a membership relation can be done by a property theory with the predication (instantiation) relation. In foundational matters such as those we are concerned with here, there can be no justification for positing the two fundamentally dissimilar ontological categories of intensions and sets (extensions). Intensions alone suffice: sets are ontologically superfluous, mere hold-overs from the days when it was unknown how to formulate a theory of intensions. It is high time that property theorists acknowledge that they have a thoroughgoing alternative to set theory. From this perspective, it is plain that reductionistic theories that posit both intensions and sets are ontologically unjustified.

The question also arises whether the possible-worlds reduction is guilty of a similar ontological excessiveness. In addition to actual individuals, it posits both sets and non-actual possible individuals. Do the latter constitue a new fundamental ontological category? Many possible-worlds theorists would answer in the negative on the grounds that, ontologically, "nonactual individuals" are just like ordinary actual individuals except that they are nonactual. But this is a very odd statement. For it seems that there could not be a greater difference between two sorts of items, one actual and the other "nonactual". Indeed, a sign that "nonactual individuals" are fundamentally unlike actual individuals is that the former, unlike the latter, present insurmountable epistemological problems of individuation not presented by actual things (whether actual individuals or actual intensions). We have in mind the problems of individuation mentioned a moment ago: there is no way in principle to tell whether, from occasion to occasion or from person to person, you are thinking about the same "nonactual individual". By contrast, actual things (actual individuals and actual intensions) are not by nature like this. (Alternatively, if items in the category of "nonactual individual" cannot be objects of your thought, that would be grounds for deeming them to have a different ontological status than that of actual individual. For you can typically think of particular actual individuals.)

Suppose that this, and other considerations, show that "nonactual individuals" constitute a fundamentally new ontological category. And suppose that the traditional realist theory has no need to posit nonactual things. In this case, the possible-worlds theory would be guilty of ontological excess. For, in addition to actual individuals, it would posit two fundamentally dissimilar categories, namely, sets and "nonactual individuals". By contrast, the traditional realist theory would posit, in addition to actual individuals, only one further ontological category, namely, intensions. In this case, the latter theory would be ontologically more economical than the possible-worlds theory.

On the other hand, suppose that nonactual individuals do not constitute a new ontological category above and beyond actual individuals. Then, ontologically, the two theories would be on a par: the possible-worlds theory would posit individuals and sets; the realist theory would posit individuals and intensions. But in this case, the possible-worlds theory would still be confronted with the insurmountable epistemological problems of identifying nonactual individuals. Moreover, it would be confronted with all the logical and intuitive problems cited earlier. So, even if the two theories were ontologically on a par, the possible-worlds theory would have to be counted as deficient.

The possible-worlds theory is deficient on one further count. There is a compelling list of reasons for thinking that only certain properties are genuine *qualities*. (These reasons are spelled out in *Quality and Concept* [1982] and again by David Lewis, who is perhaps the leading possible-worlds theorist, in "New Work for a Theory of Universals" [1983].) On the traditional realist picture, genuine qualities can be combined, by means of fine-grained logical operations, to form properties that are not genuine qualities (e.g., the property of being grue); but properties that are not genuine qualities cannot be combined, by means of fine-grained logical operations, to form genuine qualities. So, on the traditional realist picture, qualities are *logically* distinctive. Indeed, this logically distinctive feature can be used as the basis of a definition of the notion of a genuine quality. By contrast, qualities (or "natural properties", as David Lewis calls them) are not logically distinctive on the possible-worlds theory even though they are ontologically distinctive. To deal with them, therefore, Lewis is forced simply to introduce a new undefined primitive predicate 'natural property'. Accordingly, what it is to be a quality (natural property) remains an unanswerable mystery on the possible-worlds theory. So on this score, too, the traditional realist theory comes out ahead of the possible-worlds theory.

Our overall conclusion is that the various nontraditional (reductionist) versions of realism do not compare with traditional realism. The best semantics for intensional abstracts is that based on the traditional realism.

10. TRANSCENDENTAL PREDICATES AND TYPE-FREE LANGUAGES

There are in natural language a great many "transcendental" predicates, that is, predicates that we apply freely across ontological categories. The 2-place predicate 'contemplate' is an example, for items from any ontological category can be contemplated by someone or other. The 2-place predicate 'distinct' (or '$\neq$') is another example because items belonging to any two distinct ontological categories are themselves distinct. (For example, for any universal x and particular y, $x \neq y$.) It turns out that the existence of transcendental predicates provides compelling evidence for the thesis that, syntactically, the formal language for intensional logic should *not* be a type-restricted (or categorial) language.

Consider first the matter of predicates. Suppose that all the predicates in this formal language were syntactically type-restricted. Then for every transcendental predicate in natural language – let us take 'contemplate' as our example – there would in the formal language need to be infinitely many distinct primitives $\ulcorner \text{contemplate}_{(o\iota)\alpha} \urcorner$, one for each distinct syntactic type (or category) α in the formal language. But this outcome conflicts with our previous conclusion concerning Davidson's learnability requirement. Therefore, there is no choice but to allow the transcendental predicates to be syntactically unrestricted in this formal language.

Consider next the issue of variables. If all the variables in the formal language were syntactically restricted according to ontological type, then the formal language would not be equipped to express various *general propositions* that are expressible in the natural language. (For example, the proposition that, for any item, it is possible that someone contemplates it; the proposition that, for any item, there is something that is distinct from it; etc. In our notation:

$$[(\forall x)\mathrm{Possible}[(\exists y)y \text{ contemplates } x]];$$

$$[(\forall x)(\exists y)y \neq x]$$

etc.) This implies that the formal language should contain a sort of variable that ranges over all items regardless of ontological type.

Now suppose that certain type-restricted variables or certain type-restricted predicates (i.e., predicates whose argument expressions must belong to some preferred syntactic type) are needed for some purpose or other in an idealized representation of natural language. We have determined that transcendental predicates – and general statements we can make with them – force us to include in our formal language various type-free predicates and a syntactically unrestricted sort of variable. However, once we have adopted a syntax with such devices, the simplest and most economical way to deal with type-restricted variables and type-restricted predicates is to define them contextually by standard techniques using the unrestricted sort of variable and appropriate syntactically type-free predicates. In view of this, it would seem that the simplest and most economical construction of intensional logic would be formulated in a one-sorted, syntactically type-free language, that is, a language in which there is only one sort of variable, which ranges freely over all ontological categories, and in which all predicates are free of syntactical type-restrictions inasmuch as they all may take this single sort of variable as arguments.

11. SELF-EMBEDDING AND SELF-CONSTITUENCY

As we have seen, transcendental predicates express properties and relations that apply freely across ontological categories. An important special case of this occurs in connection with *self-embeddable predicates*, that is, predicates that take as arguments intensional abstracts in which the very same predicate occurs. Consider an example: x contemplates contemplating. (In our notation, C^2x, $[C^2u, v]_{uv}$.) Does the occurrence of the verb 'contemplates' express the very relation of contemplating that is denoted by the gerund 'contemplating'? Intuitively, the answer is that it does: one of the things you can contemplate is the relation of contemplating.

Self-embedding arises in connection, not just with transcendental predicates, but with many others as well. For example, in a sentence like 'It is necessary that something is necessary', 'necessary' hardly seems to occur ambiguously. Intuitively, its two occurrences express the very same property, necessity. Thus, when we assert this sentence, we are ascribing this property to a proposition that "involves" this very property. Similarly, in the sentence 'Someone believes that someone believes something', the verb 'believes' certainly does seem not to occur ambiguously; intuitively, both occurrences of 'believe' express the same relation, namely, believing. The sentence is true if and only if someone stands in this relation to the proposition that someone believes something, a proposition that "involves" the very same relation of believing.

With these observations in mind, let us extend our use of 'self-embeddable' from predicates – e.g., 'contemplate', 'necessary', 'believe', 'identical', 'distinct', etc. – to the corresponding properties and relations that they express. Accordingly, we will say that a property or relation is *self-embeddable* if and only if it applies either to itself or to a PRP that "involves" it. (We should emphasize that this talk of PRP's "involving" one another is heuristic only; on the algebraic semantics we advocate this heuristic talk gives way to fully literal talk of fundamental logical operations such as conjunction, negation, existential generalization, and predication.)

Of course, ramified type theorists such as Russell, Whitehead, and Church would hold that there really are no self-embeddable properties or relations and that our ordinary uses of self-embeddable predicates are instead to be explained in terms of "typical ambiguity". However, there are two sorts of considerations that count decisively against these type theorists.

First, since in ordinary English a predicate like 'believe' can be embedded *any* finite number of times within its own scope – e.g., 'Someone believes that someone believes something', 'Someone believes that someone believes that someone believes something', etc. – ramified type theorists like Russell, Whitehead, and Church must hold that 'believe' and kindred predicates actually are *infinitely* ambiguous. But such infinite ambiguity entails a violation of Davidson's learnability requirement. The natural, intuitive way around this difficulty is to admit that in belief sentences, like those above, 'believe' does not occur ambiguously and, hence, that the familiar belief relation is in fact self-embeddable. Of course, type theorists could posit a relation R^2 that holds between the infinitely many alleged belief relations B_0, B_1, B_2, . . . ; that is, they could hold that $R^2(B_\alpha, B_{\alpha+1})$, for ordinals α. The problem is that R^2 would fall outside the hierarchies with which type theory deals. As a result, type theory would not provide a theory for this logically fundamental relation that presumably holds among the very entities type theory is designed to treat, a relation that is needed simply to explain how ordinary people learn to use the predicate 'believe'. If, to deal with this problem, type theorists try to explicitly incorporate 'R^2' into their logical theory, they face a fatal dilemma. Either they subject 'R^2' itself to infinite typical ambiguity and non-self-embeddability, thereby violating the learnability requirement once again. Or they treat 'R^2' as a type-free, self-embeddable predicate. In the latter case, however, they have just come around to our way of doing things except that they do so in an *ad hoc*, disunified way; for they would still treat familiar predicates like 'believe', 'necessary', 'identical', etc. as infinitely ambiguous and non-self-embeddable. On either horn of the dilemma, therefore, the 'R^2'-approach is unacceptable.

Second, ramified type theorists seem unable to explain satisfactorily little dialogues like the following:

> A: I believe many things.
>
> B: So do I; in fact, what you have just asserted is one of them.

In this dialogue, A asserts a proposition "involving" the relation of believing, namely, the proposition that he believes many things. Then B affirms the corresponding proposition about himself, namely, that he [i.e., B] believes many things, too. And then B goes on to provide an example of one of the things to which he stands in this relation of believing, namely, the original proposition A asserted, which, as we saw, is a proposition "involving" this very relation of believing. Or consider the following little dialogue:

> A: Some things are necessary.
>
> B: I agree; in fact, what you have just asserted is an example of one of them.

Here A asserts a proposition "involving" the property of being necessary, namely, the proposition that some things are necessary. B affirms the proposition and then goes on to indicate a proposition having this property, namely, A's original proposition, which "involves" this very property. Now these little examples are not at all exceptional; they are entirely typical of our everyday thought and discourse about belief, necessity, possibility, epistemic justification, etc. Yet they would make no sense if these properties and relations were not self-embeddable. To rule out the self-embeddability of these central properties and relations would be to undermine one of the primary functions of intensional language. Indeed, we submit that it is impossible to formulate an acceptable comprehensive theory of the world unless one makes liberal use of self-embeddable properties and relations. Such a theory must include, among other things, an epistemology (and a methodology and philosophy of science) that can account for its own acceptability. Here self-embeddability is inevitable.[39] Any intensional logic that does not deal with this phenomenon cannot be deemed acceptable, even provisionally.[40]

We have seen that self-embeddability causes trouble for type-theoretical intensional logics like those of Russell and Whitehead and of Church. But it causes equally serious trouble for many other approaches to intensionality. The possible-worlds approach is a case in point. (Related problems involving propositions that are "constituents" of themselves confront the original version of the Perry–Barwise situation semantics and also the theories of Cresswell, Soames, and Salmon; see below.) According to the possible-worlds approach, all PRPs are identified (at least in the semantical model) with sets constituted or formed ultimately from actual individuals and "nonactual individuals" (and the real world and "nonactual possible worlds"). For example, a property is identified with a function (i.e., a set of ordered pairs) from possible worlds to sets of possible individuals (intuitively, the possible individuals that have the property in that possible world). And an m-ary relation is identified with a function from possible worlds to sets of ordered m-tuples of possible individuals (intuitively, the possible individuals that stand in the relation to one another in that possible world).

Most possible-worlds theorists seem unaware that this theory implies a rigid type theory. To see why, consider the transcendental predicate 'is self-identical'. (Any other transcendental predicate would do, e.g., 'contemplate', 'think of', 'identical', 'distinct', etc.) On the possible-worlds theory, the property of being self-identical is the function from possible worlds to the domain of *individuals* existing in those worlds. As a result, the intuitively true sentence 'Being self-identical is self-identical' could be counted as true, but only at the price of treating 'self-identical' as typically ambiguous. All the fatal difficulties associated with this kind of type theory follow in train.

Now possible-worlds theorists might try to escape these fatal problems by admitting properties and relations right into the domains of things existing in possible worlds. So, for example, the property of being self-identical would then be the function from possible worlds to the set of things, including the property of being self-identical itself, that exist in the possible world. But since on the possible-worlds theory a function is a set of ordered pairs, this move would imply the following: being self-identical $\in \ldots \in$ being self-identical. An ill-founded set, which is an impossibility on the standard conception of set. In a moment, we will elaborate reasons why possible-worlds theorists should be unwilling to posit ill-founded sets. However, before doing so, it would be good to show that the problem of self-embeddability in possible-worlds semantics is even more pervasive than one would initially think.

On the original formulations of possible-worlds theory, propositions were identified with functions from possible worlds to truth values (intuitively, the truth value that the proposition has in that possible world). However, this treatment of propositions has the disastrous consequence of making all necessarily equivalent propositions identical. This is plainly wrong. (For example, most uneducated people believe that $2 + 2 = 4$ and fail to believe that arithmetic is essentially incomplete, even though it is necessary that $2 + 2 = 4$ iff arithmetic is essentially incomplete.) It follows that the objects of belief – the semantical correlates of the kind of 'that'-clauses occurring in ordinary belief sentences – cannot be identified with functions from possible worlds to truth values; they must be identified with another sort of 0-ary intension, which we call *fine-grained propositions*.

If you were a possible-worlds theorist, with what possible-worlds set-theoretical construct would you identify fine-grained propositions? The most popular answer among sophisticated possible-worlds theorists (Max Cresswell, David Lewis, etc.) is that fine-grained propositions should be identified with certain ordered sets or abstract trees whose elements or nodes are either possible individuals or sets constructed ultimately from possible individuals. So, for example, when someone believes that F^2x, y, the fine-grained proposition believed would be the ordered set $\langle f, x, y\rangle$, where f is a function from possible worlds to sets of ordered pairs of possible individuals (intuitively, the possible individuals that stand in the relation F^2 to one another in that world). Or when someone believes that $(\exists y)F^2x, y$, the fine-grained proposition would be $\langle EG, \langle f, x\rangle\rangle$, where EG is some set-theoretical item selected to play the role of the operation of existential generalization. (Which item should play this role? The choice seems utterly arbitrary unless the real logical operation of existential generalization is chosen. But in this case the possible-worlds theory would have drifted very close to the traditional realist algebraic semantics we advocate. The same problem of chosing the fundamental logical operations confronts the theories of Soames and of Salmon: the choice is arbitrary unless it buys into our traditional realist algebraic semantics. But, then, why not just adopt the traditional realist algebraic semantics?)

Now, as we have already noted, it is hardly plausible that, strictly and literally, people believe, hope, or perceive ordered sets such as $\langle f, x, y\rangle$ or $\langle EG, \langle f, x\rangle\rangle$. (See Section 9 above.) But this problem, although severe, is not the one we are concerned with here. The problem, rather, concerns self-embeddability. With which set-theoretical construct is the ordinary belief relation to be identified according to the present sophisticated possible-worlds theory? The answer is that it must be some function b (i.e., a set of ordered pairs) from possible worlds to sets of ordered pairs each of which consists of (i) a possible believer and (ii) a fine-grained proposition (intuitively, a fine-grained proposition that the believer believes in that world). However, on the standard conception of set, there exists no such function b that behaves in anything like the way that the ordinary belief relation behaves.

To see why, recall that, on the standard conception of set, all sets are constituted (or formed) ultimately from ontologically primitive entities that are not sets. At the lowest ontological level, there are just the primitive non-sets (both actual individuals and non-actual individuals, if the possible worlds theory is right). At the next level come sets of these non-sets. (The null set is the degenerate case of the set of non-sets that are not self-identical.) Following that, there are sets whose elements are non-sets and/or sets of non-sets. And so on. That is, at any given level, we find sets whose elements are either sets constituted at some lower level or the non-sets given at the lowest level. Thus, on the standard conception of set, every set "has its being in" ontologically prior entities – either entities constituted at some lower level or primitive entities given at the lowest level. Consequently, nowhere in the hierarchy of sets is there a set that contains itself as an element; nor is there a set that contains a second set that contains the first set as an element, and so on. That is, the following pattern never holds: $u \in \ldots \in u$.

It is now easy to see why, on the standard conception of set, the ordinary belief relation cannot be identified with any of the set-theoretical constructs postulated in the sophisticated possible-worlds theory. Consider the little dialogue discussed earlier: A asserts a fine-grained proposition "involving" the belief relation, namely, the fine-grained proposition that he [i.e., A] believes something, and then B affirms that he [i.e., B] believes this very proposition. Given B's remark, B stands in the belief relation to a fine-grained proposition that "involves" the belief relation itself. Belief is a self-embeddable relation. On the sophisticated possible-worlds theory we are discussing, this fine-grained proposition is identified with an ordered set, say, $\langle EG, \langle b, A\rangle\rangle$, where b is the possible-worlds function with which the belief relation is to be identified. But this function b is itself only a set of ordered pairs, one of whose elements would have to be the ordered pair $\langle$the actual world, $\langle B, \langle EG, \langle b, A\rangle\rangle\rangle$. This would imply that $b \in \ldots \in b$. An impossibility on the standard conception of set. Therefore, on this conception – indeed, on any conception according to which sets have their being in ontologically prior elements – the possible-worlds theory is incompatible with the existence of self-embeddable PRPs.

The problem stems from the fact that the possible-worlds theory is *reductionistic*. It tries to reduce all PRPs to sets constituted (or formed) ultimately from actual indivuals and "nonactual individuals". Although this reductionism is formally feasible for some cases, it is not for self-embeddable PRPs, which are so central to thought and speech. At this juncture, unrelenting possible-worlds theorists have two choices. Either they can assert that self-embeddable PRPs are not sets at all but rather that they belong to an entirely new fundamental ontological category. Or they can abandon the standard conception of set and advocate instead a nonstandard conception that permits non-well-founded "sets".

One problem with the first alternative is that it is disunified. How odd that some PRPs (non-self-embeddable PRPs) should be sets and that others (self-embeddable PRPs) should belong to an entirely different, irreducible ontological category. To avoid this problem of disunity, possible-worlds theorists could identify all PRPs with items in this new ontological category. But the resulting theory would have a problem of ontological economy, for it would posit two new categories of entities, namely, the new, irreducible ones that have replaced sets and the highly questionable category of "nonactual individuals". In view of this, why not just drop these questionable new categories and take PRPs at face value as primitive, irreducible entities? The resulting theory would be more unified, more economical, and more intuitive. Furthermore. it would be free of the insurmountable epistemological problems confronting the possible-worlds theory. (How, from occasion to occasion or from person to person, can one ever tell whether one is thinking about the same "nonactual individual" as opposed to a numerically distinct one that is qualitatively like it?).

The second alternative available to the unrelenting possible-worlds theorist is to abandon the standard conception of set and to adopt instead a nonstandard conception that permits non-well-founded "sets", that is "sets" displaying the pattern $u \in \ldots \in u$. However, there are three considerations that weigh heavily against this alternative.

First, the original possible-worlds program sought to reduce PRPs to items that are constituted (or formed) ultimately from actual individuals and "nonactual individuals". A primary goal of this reduction was to provide a metaphysical explanation of PRPs by showing that they "had their being in" ontologically prior entities (namely, actual individuals and "nonactual individuals"). However, the new possible-worlds proposal undermines the prospect of this sort of metaphysical explanation. For on the new proposal self-embeddable PRPs would be identified with non-well-founded "sets", but such "sets" are not constituted (or formed) ultimately from actual individuals and "nonactual individuals". Unlike standard sets, non-well-founded "sets" do not have their being in ontologically prior entities; on the contrary, inasmuch as they have their being "in themselves", they are virtually on a par with individuals. Because such "sets" are ontologically primitive in this way, identifying PRPs with them cannot yield the kind of metaphysical explanation of the being of PRPs that was originally promised by the possible-worlds program. So on this score, we are just as well off taking PRPs at face value rather than as a queer kind of "set".

Second, it is not clear that talk of non-well-founded *sets* is really coherent to begin with. Many people believe that, as a conceptual or metaphysical necessity, all sets must have their being in ontologically prior entities; this is just the kind of thing sets are. According to these people, non-well-founded "sets" are not strictly and literally sets at all; rather, they belong to an entirely new primitive ontological category above and beyond sets. If this is right, people who are favorably inclined toward set theory would have no reason – either ontological or epistemological – to prefer this new "set" theory to a theory that takes PRPs at face value as a basic category of entities. First, there would be no *ontological* gain, for both alternatives must posit a new primitive ontological category. And let us not forget that the possible-worlds theorist has already posited the additional primitive ontological category of "nonactual individuals". Second, there would be no *epistemological* gain. The usual Quinean argument is that sets are epistemologically superior to properties and relations (-in-intension) because sets can be individuated simply by considering their elements. Let u be one of the new non-well-founded "sets" that is an element of itself. If, following Quine's procedure, we try to individuate u by considering its elements, we only get caught in a vicious circle: to individuate u by these means requires that we must already have individuated u. And must the same sort of epistemological difficulty infects all the new "sets" displaying the pattern $u \in \ldots \in u$. (Of course, one could adopt Peter Aczel's bold decision to permit exactly one "set" displaying the pattern $u \in u$ and exactly one "set" displaying the pattern $u \in v \in u \neq v$ and so on. But how could one *know* that there is exactly one "set" displaying the pattern $u \in u$? This is just Quine's worry all over again.) To overcome this sort of difficulty, some other epistemological procedure will be needed. But it would seem that, whatever this further procedure is (for example, systematization of one's first-person introspective reports and/or systematization of one's *a priori* intuitions), it would work at least as successfully on PRPs as it would on the new non-well-founded "sets". Indeed, if this further procedure is available (as of course it is), PRPs are fully as respectable epistemically as ordinary well-founded sets are commonly thought to be. And finally, let us not forget that the possible-worlds theory is beset with a number of absolutely intractable epistemological problems produced by its peculiar ontology of "nonactual individuals".

Methodology provides the third reason not to accept the possible-worlds reduction of PRPs to non-well-founded "sets". The standard view of sets – according to which, sets have their being in their instances – provides an intuitive diagnosis and resolution of the set-theoretical paradoxes. Advocates of set theory should demand a very good reason to give up this secure position. However, Russell-style antinomies are derivable in the naive version of non-well-founded "set" theory. How should these antinomies be resolved? This becomes an absolutely urgent question if PRPs are identified with non-well-founded "sets", for in that case the semantics of intensional logic cannot even be stated without first formulating a non-well-founded "set" theory and thereby taking a strong stand on how to resolve this new family of antinomies. But this situation is methodologically very unsatisfactory. If at all possible, one should find a way to do the semantics for intensional logic without taking a stand on these highly problematic issues. However, we can do just that if we drop the attempt to reduce PRPs to "sets" and instead take PRPs at face value as unreduced entities. By anyone's standard, this is a far wiser way to proceed. To do otherwise is just asking for trouble, and it is doing so with no gain whatsoever.

Our conclusion, therefore, is that one should not invoke a non-well-founded "set" theory to save possible-worlds semantics from the difficulties posed by self-embeddable PRPs. Rather, one should just abandon possible-worlds semantics and, instead, develop a traditional nonreductionist PRP semantics. On this alternative, the above problems simply do not arise.

We have shown how self-embeddable properties and relations produce grave difficulties for possible-worlds semantics (and for Cresswell's semantics). These difficulties can be avoided if one adopts the traditional realist theory that properties and relations are primitive, irreducible entities. Like our own nonreductionistic semantics, situation semantics and the proposals of Soames and of Salmon follow this path.[41] However, the original version of situation semantics and the proposals of Soames and of Salmon are still reductionistic in character, for like possible-worlds semantics, they attempt to reduce 0-ary intensions (what we call propositions and Perry and Barwise call situations) to certain kinds of sets. Predictably, then, propositions (situations) that are "constituents" of themselves produce grave problems for these theories. (It goes without saying that these problems of self-constituency also beset possible-worlds semantics. We should also emphasize again that our remarks apply primarily to the original theory of Perry and Barwise. In their more recent theory, they attempt to deal with self-constituency by adopting a non-well-founded "set" theory.)

The phenomenon of self-constituency seems to arise in connection with such matters as public information, mutual knowledge, reflexive perception, and so on. For example, suppose two opposing soldiers x and y are tracking one another down, and suppose that simultaneously each spots the other and each perceives fully what has happened. There are reasons to think that a complete specification of what has gone on may include the following (and Barwise now seems to agree):[42] x perceives s_1 and y perceives s_2, where s_1 is that y perceives s_2 and s_2 is that x perceives s_1. If this is correct, s_1 is a proposition (situation) that is a "constituent" of s_2, which, in turn, is a proposition (situation) that is a "constituent" of s_1. So s_1 is a proposition (situation) that is a "constituent" of itself. The question is how to develop a semantics to deal with such apparent self-constituency.

(One response is to deny that there really is such a phenomenon as self-constituency. This response might be correct. However, because it is controversial, one would be much better off having a theory that is equipped to handle self-constituency in case it turns out to be a genuine phenomenon. Our traditional realist semantics is like this. The original version of situation semantics and the proposals of Soames and Salmon are not. This is the point.)

The central idea of the original version of situation semantics and the proposals of Soames and Salmon is to reduce propositions (situations) to sets – for example, to ordered sets of properties, relations, and other items.[43] Therefore, in these theories one had no choice but to try to identify s_1 with an ordered set such as $\langle y$, perceiving, $s_2 \rangle$ and s_2 with an ordered set such $\langle x$, perceiving, $s_1 \rangle$. But this implies that $s_1 \in \ldots \in s_1$, and this contradicts classical set theory.

Our traditional realist semantics does not fall into the trap of trying to reduce propositions (situations) to sets. Instead, it just takes them at face value. Consequently, it is able to deal with self-constituency in a direct and intuitive fashion without having to contradict classical set theory. Rather than following this natural course, Barwise now advocates abandoning the standard conception of set and adopting instead a nonstandard conception that permits non-well-founded "sets". However, in the preceding discussion of possible-worlds semantics, we found convincing reasons for not following this kind of radical course, reasons that apply equally to Barwise's proposal. A traditional realist theory is plainly superior. Indeed, it would be unreasonable to decide to revolutionize classical set theory just to save a certain style of semantics (possible-worlds semantics, situation semantics, etc.) when there is a simple and natural alternative that requires no such revolution and that, for the purpose of modeling intensional logic, makes use of a relatively weak, uncontroversial standard set theory.

Our overall conclusion, therefore, is that the phenomena of self-embedding and self-constituency cause serious difficulties for all reductionistic semantics and, hence, that a traditional nonreductionistic semantics is the best one to adopt.

12. THE FIRST-ORDER/HIGHER-ORDER CONTROVERSY

From a linguistic point of view, the upshot of the previous two sections was that the formal language for intensional logic should be a one-sorted, type-free language that may contain unambigous transcendental and self-embeddable predicates. Even though this formal language should be one-sorted – that is, even though it should contain only one sort of variable – that does not tell us whether, syntactically, this language should be first-order or higher-order. For it is possible – though quite unusual – for a one-sorted language to be syntactically higher-order. In such a language strings like '$x(x)$' would be counted as well-formed. (In a syntactically higher-order language, predicates – and perhaps sentences, as well – are counted as singular terms for which quantifiable variables may be substituted. So in a one-sorted higher-order language strings like '$x(x)$' would be well-formed. In a syntactically first-order language, by contrast, neither predicates nor sentences are counted as singular terms, and variables may not be substituted for them. Accordingly, strings like '$x(x)$' would not be well-formed.)

In this section we turn to the question of whether we should adopt this style of higher-order syntax or whether we should instead adopt a standard first-order syntax. We believe that the considerations favoring the first-order syntax decisively outweigh those favoring the higher-order syntax. The arguments are too lengthy to give in full detail here. However, we will touch on two issues, one methodological and one grammatical.

First, the methodological issue. In Part II, we shall see that first-order intensional logic – that is, first-order logic with identity and intensional abstraction – is complete: there is a recursive axiomatization of the logically valid sentences of the language.[44] Nevertheless, by a straightforward adaptation of the proof of Gödel's incompleteness theorem, we can show that first-order logic with identity and a copula is essentially incomplete, and this is so whether this logic is intensional or extensional, that is, whether or not the operation of intensional abstraction is adjoined to the language. What explains these results? From a semantical point of view, a copula ('is', 'has', 'stand in', etc.) is a distinguished logical predicate that permits one to talk in a general way about what items have what properties and about what items stand in what relations. That is, a copula is a distinguished logical predicate that expresses a predication (or instantiation) relation. This suggests the following explanation. Intensionality – the failure of substitutivity – is not responsible for incompleteness in logic. Rather, the responsibility lies with those devices that permit us to talk in a general way about what has what properties or about what stands in what relations, that is, with devices that permit us to talk generally about predication. This explanation is borne out by the fact that both higher-order intensional logic and higher-order extensional logic are essentially incomplete, and each is equipped with devices for talking generally about these matters. (Specifically, each is equipped with linguistic forms like '$u(x)$', '$u(x, y)$', '$u(x, y, z)$', . . . , where 'u' is a variable.).

For terminological convenience, let us call any logic (whether first-order or higher-order) that treats such matters a *logic of predication*. Our goal is to formalize intensional logic, which is the logic for contexts in which the substitutivity principles of extensional logic do not hold. Our goal is *not* to develop the logic of predication. In view of the fact that the logic of predication is essentially incomplete, our goal of formalizing intensional logic is best served if, initially, we separate it from the formalization of the logic of predication. In a first-order setting, this separation is possible. For in a first-order setting intensional logic is just the logic for intensional abstracts, and these terms may be adjoined to first-order quantifier logic with identity prior to singling out the copula as a distinguished logical predicate. When we do this, the result is a complete intensional logic. However, this sort of separation of goals is not feasible in a higher-order setting. For on the intended interpretation of a higher-order language, devices for dealing with the predication relation are present in the syntactic forms '$u(x)$', '$u(x, y)$', '$u(x, y, z)$', etc. right from the start. For this methodological reason, then, it is desirable to develop intensional logic in a first-order setting. (This completeness issue will be discussed further in Section 4 of Part II.)

There is a closely related, but much more important, methodological point that we touched on in our introduction. Not only do devices for dealing with the relation of predication produce incompleteness in logic, but also they invite logical paradoxes. For example, naive predication (or comprehension) principles – both higher-order principles like $(\exists u)(\forall x)(u(x) \leftrightarrow A(x))$ and first-order principles like $(\exists z)(\forall y)(y\,\Delta\,z \leftrightarrow A(y))$ – lead directly to Russell-style paradoxes. ('Δ' is our symbol for the copula.) On the first-order approach to intensional logic, however, the device that generates intensional contexts (namely, intensional abstraction) and the device for dealing with the predication relation (namely, the copula) are independent of one another. Therefore, on the first-order approach, these paradoxes can be avoided simply by not singling out a distinguished logical predicate (e.g., 'Δ') for the predication relation. On the higher-order approach, by contrast, a device for dealing with the predication relation is built into the very syntax of the language on its intended interpretation. As a result, the paradoxes must be confronted from the very start.

Faced with this demand to resolve the paradoxes, higher-order theorists usually adopt a type-theoretical resolution, and often they actually encode this resolution into the syntax of their language by dividing the variables into sorts, one sort for each distinct ontological type (e.g., one sort of variable for individuals, another for properties of individuals, a third for properties of properties of individuals, etc.). However, these type theories have extremely counterintuitive features. For example, they rule out the possibility of transcendental properties and relations – e.g., contemplating, non-identity, etc. – which we discussed earlier. Moveover, a many-sorted syntax rules out the associated possibility of a sort of variable that ranges freely across all ontological types. We conclude, therefore, that this way of responding to the paradoxes is unsatisfactory.

It is safe to say that, as yet, no one really understands the paradoxes. Despite the elegance and ingenuity of the known resolutions, every one of them is unsatisfactory in one crucial way or another, and it seems unlikely that this situation will change anytime soon. In view of this, the breakdown in type-theoretical higher-order intensional logic should not be viewed as an isolated phenomenon. Since every system of higher-order intensional logic is forced to include a resolution of the paradoxes, it is highly likely that every higher-order system of intensional logic developed in the forseeable future will be unsatisfactory in one crucial way or another. The only realistic strategy for developing a satisfactory system of intensional logic is to use a framework that does not force us to include a resolution of the paradoxes. First-order logic is the only framework like this. For in first-order logic, unlike higher-order logic, we can include a device for representing intensionality – namely, intensional abstraction – without also including a device or devices that threaten to generate the paradoxes. At the same time, the first-order strategy of treating intensionality independently of the paradoxes is not at all *ad hoc*. For, as we shall see in a moment, there are independent grammatical considerations that support a first-order logical syntax. Without any hidden costs, the first-order strategy allows us to keep our options open with respect to the paradoxes: we have a strong chance of being able to incorporate an ideal resolution if ever one is discovered, and, in the meantime, we have a wide variety of interim resolutions to choose from depending on the theoretical task at hand. From a methodological point of view, therefore, the first-order approach to intensional logic is vastly superior to the higher-order approach. There is no good reason not to adopt it.

Now for the grammatical considerations. We wish to emphasize at the outset that these considerations do not carry the same weight as the foregoing methodological considerations we have been discussing. Until one has a satisfactory general syntax of natural language, surface syntactical considerations like those we will discuss are only provisional. Nevertheless, they do suffice to show that the first-order approach is not *ad hoc*. This is all that is needed for our overall argument.

The first-order approach honors the traditional linguistic distinction between subject and predicate, between noun phrase and verb phrase; the higher-order approach does not. That is, on the first-order approach an absolute distinction is made between linguistic subjects and linguistic predicates such that a linguistic subject (noun phrase) cannot, except in cases of equivocation, be used as a linguistic predicate (verb) and conversely. The higher-order approach does not recognize this distinction.[45] On the contrary, it treats linguistic predicates (verb phrases) as substituends for variables and, hence, as a sort of subject expression. Accordingly, strings like $\ulcorner(\exists u)\, u = R\urcorner$, where $\ulcorner R\urcorner$ is a linguistic predicate, are treated as well-formed and valid. But these linguistic forms do not match the surface syntax of anything in natural language, for in natural language linguistic predicates may not (without equivocation) be used as linguistic subjects. For example, 'There is something such that it is identical to runs' makes no sense at all. Of course, we can say 'There exists something such that it is identical to running'. But here the linguistic predicate 'runs' is replaced by the linguistic subject 'running', which is a nominialization of the linguistic predicate, namely, a gerund. Gerundive phrases have exact couterparts in a first-order language with intensional abstraction. Accordingly, the above sentence would be represented by '$(\exists u)\, u = [Rx]_x$'. In this way, the surface syntax of the above natural-language sentence is directly and faithfully represented in a first-order language.

Many higher-order languages also treat sentences as linguistic subjects. For example, strings like $\ulcorner(\exists s)\, s = A\urcorner$, where $\ulcorner A\urcorner$ is a sentence (open or closed), are often treated as well-formed and valid. But these linguistic forms do not match anything in natural language: in natural language, sentences do not qualify as linguistic subjects. Strings like 'There is something such that it is identical to everyone loves someone' make no sense at all. Of course, *nominalizations* of sentences may be used as linguistic subjects. For example, 'There is something such that it is identical to the proposition that everyone loves someone' is meaningful. But here the sentence 'Everyone loves someone' is replaced by a legitimate linguistic subject 'the proposition that everyone loves someone'. This linguistic subject has an exact counterpart in the sort of first-order language we advocate, namely, the intensional abstract '$[(\forall x)(\exists y)\mathrm{L}xy]$'. Accordingly, the above sentence would be represented in our first-order language by '$(\exists s)\, s = [(\forall x)(\exists y)\mathrm{L}xy]$'. So once again the surface syntax of the natural language is directly and faithfully represented in first-order language but not in higher-order language. And this is the general pattern: in natural language no linguistic predicate or (open or closed) sentence is used (without equivocation) as a linguistic subject. Instead, an appropriate nominalization of the linguistic predicate or sentence plays this role, and such nominalizations are none other than the intensional abstracts we have been discussing in earlier sections.

The next issue concerns higher-order uses of names. First, consider names of propositions. For example, let 'c' name some proposition, say Church's thesis. In a higher-order language, such a name may just on its own be used as a sentence: c. But nothing in natural language corresponds to this. The closest we can come is 'Church's thesis holds' which can be represented in a first-order language by 'Hc'. So the naked higher-order use of a name as a sentence gives way to the use of the name as a subject together with the predicative use of an appropriate verb.

A rather similar pattern emerges for names of properties and relations. For example, let 'b' and 'g' name the colors blue and green, respectively. In a higher-order language, such names may be used both as linguistic subjects and as linguistic predicates. Accordingly, a string like '$g(a)$ & $g \neq b$' is counted as well-formed. However, as in the previous examples, there is no natural language sentence corresponding directly to this higher-order string. The closest we can come is '*a is* green and green $\neq$ blue. But here we have an occurrence of a verb, namely, the copula 'is'.[46] Now the most direct way to represent the copula is by means of a corresponding primitive 2-place predicate, say, 'Δ' or simply 'is' itself. With this predicate available, 'a is green and green $\neq$ blue' would be represented by 'a is g & $g \neq b$' rather than by '$g(a)$ & $g = b$', and hence the intuitively ungrammatical predicative use of the linguistic subject 'g' drops out of the picture. And this pattern generalizes: once a primitive copula is available, all predicative uses of property and relation names drop out; such names are used exclusively as linguistic subjects, as in first-order formulas like 'a is g' and '$g \neq b$'.

Much the same verdict holds for higher-order uses of variables. In a higher-order language, a variable, say, 'u' may be used as a linguistic predicate; however, a higher-order string like '$(\exists u)u(x)$' corresponds directly to no natural-language sentence. The closest we can come is 'There is something that x *is*'. But here again we have an occurrence of the copula 'is', a verb which we may represent by a primitive 2-place linguistic predicate. With this primitive predicate available, the natural-language sentence would be represented by '$(\exists u)x$ is u' rather than by '$(\exists u)u(x)$', and, thus, the predicative use of the variable 'u' drops out of the picture. (Indeed, when people introduce novices to higher-order languages, they usually say that '$f(x)$' is to be read as 'x *is* f'. Presumably, then, this is what people actually understand when they grasp a higher-order formula. Is there any reason to think otherwise?) As before, the pattern generalizes: once a primitive copula is available, all predicative uses of variables become gratuitous; the use of variables is confined to their use as linguistic subjects, as in first-order formulas like 'x is u'. Finally, consider the higher-order use of variables as sentences. For example, a string like '$(\forall s)(s \rightarrow s)$' is well-formed in higher-order languages. But, as in previous examples, this string corresponds to no sentence in natural language. The closest we can come in English is something like 'whatever holds holds' or 'whatever is true is true', which can easily be represented in a first-order language by '$(\forall x)(Hx \rightarrow Hx)$' and '$(\forall x)(x$ is $t \rightarrow x$ is $t)$', respectively. So the higher-order use of a variable as a sentence gives way to the use of the variable as a linguistic subject in tandem with a predicative use of an appropriate predicative expression.

Our overall conclusion is this. First, there are decisive methodological grounds for favoring a syntactically first-order approach to intensional logic. Second, there is grammatical evidence that a first-order language with intensional abstraction and the copula (and perhaps other auxilary logical predicates) more directly and faithfully represents the syntax of natural language. In view of these considerations, there seems to be no reasonable choice but to take the first-order option.

13. NAMES AND INDEXICALS

There are many varieties of substitutivity failures. Not only are there the standard substitutivity failures involving materially equivalent formulas, but also there are ones involving necessarily equivalent formulas. There even seem to be substitutivity failures involving synonymous formulas; we have in mind those associated with the paradox of analysis and Mates' puzzle.[47] (See the next section for a discussion of these puzzles.) Finally, there are puzzles involving, not only co-referring definite descriptions, but also co-referring proper names and co-referring indexicals – expressions that may well be lacking in descriptive content, at least if the "direct reference" theory is right.

At this stage of research it is desirable to have a general technique for constructing a spectrum of intensional logics ranging from systems that treat PRPs as relatively coarse-grained to systems that treat them as extremely fine-grained. After all, it is plausible that different kinds of PRPs are responsible for different kinds of intensional phenomena. (In Part II we will develop this sort of general technique and then use it to construct in detail both a coarse-grained intensional logic and a fine-grained intensional logic.) At the same time, we should not *commit* ourselves to the strategy of always explaining new substitutivity puzzles in terms of ever more fine-grained distinctions among PRPs. In this connection, we should not rule out the possibility that some of these puzzles (perhaps Mates' puzzle or puzzles involving co-referring proper names or indexicals) are a special kind of pragmatic phenomenon to be explained, not in terms of ultra-fine-grained distinctions among PRPs, but rather in terms of subtle shifts of interest in the conversational context.

For a case in point, consider the substitutivity puzzles involving co-referring proper names. (Co-referring indexicals would be handled analogously.) There are at bottom two theories about the content of ordinary proper names, Frege's theory and Mill's theory. According to Frege's theory, each name has associated with it a descriptive content that determines the name's nominatum; according to Mill's theory, names lack such a content.

Let us suppose that the Fregean theory is right. In this case, we would treat each ordinary proper name 'a' as an abbreviation for a definite description: $a =_{\mathrm{df}} (\iota v)F(v)$. Here $\ulcorner F \urcorner$ is a new predicate interpreted so as to capture the descriptive property Fregeans would associate with the name $\ulcorner a \urcorner$. Substitutivity failures involving co-referring definite descriptions can be explained by the fact that the associated descriptive properties are distinct. Therefore, given that ordinary proper names can be treated as abbreviated definite descriptions, substitutivity failures involving ordinary proper names can be explained by the fact that the underlying descriptive properties are distinct.

But how are definite descriptions to be treated? One way would be to treat them as contextually defined expressions much as Russell does. A standard objection to this treatment is that there are several candidate Russellian analyses, and there seems to be no way to decide which one is "the" correct one. This problem brings us close to the paradox of analysis. Consider an analogy. There are several candidate definitions of circularity (e.g., locus of coplanar points equidistant from a common point, closed plane figure every segment of which is equally curved, etc.), and there seems to be no way to decide which one is "the" correct one. However, it seems mistaken to say that circularity is simply not definable because of this. A more reasonable response would be to say that there are several correct definitions. Indeed it is easy to develop an algebraic semantics that accommodates this view and that, at the same time, provides the sort of highly fine-grained distinctions needed for the treatment of the propositional attitudes. (This solution is sketched at the end of Chapter 3 of *Quality and Concept*.) It turns out that this kind of semantics can also be adapted to solve the multiple-analyses problem that arises in connection with definite descriptions. Thus, a Russellian approach to definite descriptions can be saved after all.

Another way to deal with the multiple-analyses problem is just to treat 'ι' as a primitive, undefinable operator. This can be done in various ways. One way is to treat it as a primitive binary quantifier. Evans [1977 and 1982] gives persuasive linguistic evidence for this treatment. Moreover, this approach is extremely easy to implement within the algebraic semantic method we will present in Part II. (We simply add to our model structures a logical operation **the** that corresponds to the primitive binary operator 'ι'. The action of **the** is just what one would expect:

$$\mathbf{the}([Fu]_u, [Gu]_u) \;=\; [G(\iota u)Fu]$$

and

$$H(\mathbf{the}(x, y)) \;=\; T \textit{ iff } (\exists! u)(u \in H(x))\ \& \\ (\forall u)(u \in H(x) \rightarrow u \in H(y))$$

for all $H \in K$.)[48] Other syntactical treatments of definite descriptions can also be accommodated by the algebraic semantic method. For example, a treatment that counts 'the F' as a restricted unary quantifier (on a par with 'an F', 'no F', 'every F', etc.), and also a treatment, like Frege's, that counts 'the F' as an ordinary singular term.

For the present purposes, we need not make a choice about which of these treatments of definite descriptions is best. The point is that one of them is bound to be acceptable. Given this and given the Fregean supposition that ordinary proper names have descriptive contents, substitutivity problems involving co-denoting names can be solved within our general framework.

On the other hand, suppose with Mill that ordinary proper names do not have descriptive contents. In this case, we would treat them as primitive singular terms whose semantical behavior (namely, "rigid designation") is like that of free variables with fixed assignments. So, if the Mill theory is right, names can easily be incorporated as long as we have a theory that permits free variables with fixed assignments to occur in any context, including contexts that are otherwise intensional. However, on our approach to intensional logic, this condition will be met automatically, for our intensional language is expressly designed to permit this kind of unrestricted quantifying-in.

But how on this Millian approach do we explain *prima facie* substitutivity failures involving co-referring proper names? On this approach, names behave semantically like free-variables with fixed assignments. Therefore, strictly and literally, co-referring names may always be substituted for one another *salva veritate*. Consequently, *prima facie* substitutivity failures involving these expressions cannot be semantic phenomena. They must, therefore, be pragmatic phenomena. That is, in actual contexts of conversation, what one means by uttering sentences that arise from one another by replacement of co-referring names can be quite different things, and *prima facie* substitutivity failures may be traced to such differences in pragmatic meaning. More specifically, in certain conversational contexts the use of one name will (by Gricean mechanisms) implicate a descriptive content not implicated by the use of a co-referring name, and *prima facie* substitutivity failures in such contexts may be traced to these differences in implicated descriptive content.[49] (For further discussion of pragmatic solutions to substitutivity puzzles, see the next section.) So on Mill's theory we are also able to explain *prima facie* substitutivity failures involving ordinary proper names.

Until we have a final resolution of the Mill/Frege controversy, the best strategy for us is to set up an intensional logic that is neutral with respect to the two theories and yet that can be easily extended to accommodate either theory. The way to do this is as follows. First, we should construct a neutral language to which names (and indexicals) can be adjoined either as abbreviated definite descriptions or as primitive rigid designators lacking descriptive content. Second, within this language we should construct a general intensional logic that can accommodate both the sort of intensional entities posited in a Fregean semantics and the sort posited in a Millian semantics (and the pragmatics that accompanies it). Our algebraic semantic technique permits this two-step approach.

14. MATES' PUZZLE, PARADOX OF ANALYSIS, AND THE NEED FOR FINE-GRAINED INTENSIONAL DISTINCTIONS

In Part II we will present a general technique for constructing a spectrum of intensional logics ranging from coarse-grained to extremely fine-grained, and we will illustrate this technique by presenting in detail a representative coarse-grained theory and a representative fine-grained theory. However, there are certain outstanding substitutivity puzzles that seem initially to call for intensional distinctions that are even more fine-grained than those treated in this fine-grained theory.

The original formulation of Mates' puzzle is a case in point.[50] Mates holds that, for any distinct sentences D and D',

(1) Nobody doubts that whoever believes that D believes that D.

and

(2) Nobody doubts that whoever believes that D believes that D'.

can always diverge in truth value. However, let D be 'Somebody chews' and D' be 'Somebody masticates'. Then, given that the property of chewing and the property of masticating are identical, it will follow in our fine-grained theory that (1) and (2) must be equivalent, contradicting Mates.

There are two reasonable responses to this outcome. The first is to accept this outcome and to explain Mates' intuition *pragmatically*. Accordingly, sentences (1) and (2) would be deemed semantically equivalent. Nevertheless, utterances of (1) and (2) in an appropriate conversational context could express non-equivalent propositions. To determine exactly which propositions, we would appeal, not only to the semantics of the language, but also to Gricean pragmatic rules of conversation.[51]

The second response to the problem would be to construct a new theory that admits even more fine-grained intensional distinctions than our fine-grained theory. In particular, even though the propositions denoted by '$[(\exists x)Cx]$' and '$[(\exists x)Mx]$' would still be identical according to the new theory, the propositions denoted by the following more complex intensional abstracts would not:

$$[(\forall u)(B^2 u, [(\exists x)Cx] \rightarrow B^2 u, [(\exists x)Cx])]$$

and

$$[(\forall u)(B^2 u, [(\exists x)Cx] \rightarrow B^2 u, [(\exists x)Mx])].$$

How can this be? The idea, (once suggested by Putnam)[52] is to exploit the differences in *syntactic form* between these two complex abstracts. Specifically, the predicate 'C' is *repeated* in the former abstract but not in the latter; so the former has the form $\ulcorner[(\forall u)(\ldots 1 \ldots \rightarrow \ldots 1 \ldots)]\urcorner$ whereas the latter has the form $\ulcorner[(\forall u)(\ldots 1 \ldots \rightarrow \ldots 2 \ldots)]\urcorner$. The new theory, then, is built around the following general principle: two abstracts are to be codenoting only if they have exactly the same syntactic form. It turns out that such a theory is easy to formulate within our general algebraic approach.[53]

How are we to choose between these two responses to Mates? The second response is initially very appealing because it is systematic. But there is reason to question it. True, this response solves the original puzzle given by Mates. But, ironically, there are *simpler* versions of the puzzle that cannot be solved no matter how fine-grained we allow PRPs to be. Consider any two predicates that express the same property, for example, 'chew' and 'masticate'. (Or choose some predicate 'C' and then just stipulate that a new predicate 'M' expresses the same property as the one expressed by 'C'.) Consider someone "halfway" along in the process of picking up the use of 'masticate' by hearing others use it. There are conversational contexts in which such a person could assert something true by saying, "I am sure that whatever masticates chews, but I am not sure that whatever chews masticates." In this example, the two intensional abstracts have the same syntactic form: '$[(\forall x)(Mx \rightarrow Cx)]$' and '$[(\forall x)(Cx \rightarrow Mx)]$' are perfectly isomorphic syntactically. Thus, the second response to Mates will not allow us to hold that these two abstracts denote distinct propositions, and, therefore, it cannot be used to solve this instance of the puzzle. Consequently, there is no choice but to invoke the first response, that is, to solve the puzzle pragmatically. (See below for details.) However, if we must resort to a pragmatic solution of this simple version of Mates' puzzle, why not use it to solve the original, more complex version? If the pragmatic solution is adequate, the second response, which involves positing ultra-fine-grained intensional distinctions, would then appear to be extraneous.

This prospect raises a general methodological question. How are we to decide which types of fine-grained distinctions to admit in intensional logic? Principles of ontological economy would seem to suggest that we should admit those distinctions that are needed to explain substitutivity failures. But we have just seen that a pragmatic solution to at least some substitutivity puzzles is inevitable and that, once this style of explanation is available, ultra-fine-grained intensional distinctions might not be needed to explain the versions of Mates' puzzle for which they were designed. If so, ontological economy would lead us to reject such distinctions.

Let us suppose for a moment that this is right. One wonders how far this sort of elimination can go. What types of fine-grained intensional distinctions, if any, survive a systematic attempt to explain substitutivity failures pragmatically? A transcendental argument yields a partial answer to this question: the fine-grained distinctions that survive must include at least those that are needed to spell out satisfactory pragmatic explanations; it turns out that very fine-grained intensional distinctions are needed for this purpose. Here is the argument.

Consider the person "half-way" along in the process of learning to use the predicate 'masticate'; he says. "I am sure that whatever masticates chews, but I am not sure that whatever chews masticates". In an actual conversational context the person certainly would assert something true. The problem is to identify what true proposition it is that the person would assert. The ultra-fine-grained theory, developed in response to Mates' original puzzle, is forced to identify the proposition that whatever masticates chews and the proposition that whatever chews masticates, for the two abstracts 'that whatever masticates chews' and 'that whatever chews masticates' have exactly the same syntactic form: $[(\forall x)(1(x) \rightarrow 2(x)]$. So this theory implies that the sentence uttered by our person expresses something that is strictly and literally false (indeed, something that is formally contradictory). Since on the ultra-fine-grained theory the sentence expresses something that is strictly and literally false and since our person has asserted something that is true, what he has asserted must be something other than what the sentence strictly and literally expresses. Therefore, the problem of identifying what he has asserted cannot be solved semantically; it must be solved pragmatically. In pragmatics, we may take into account, not only the syntactic and semantic features, but also features of the conversational context and the Gricean rules of conversation. Given all this information, one might identify the person's true assertion with something like the following:

> I am sure that whatever satisfies the predicate 'masticate' also chews, but I am not sure that whatever chews also satisfies the predicate 'masticate'.

This pragmatic solution is a good first try, but there is a problem with it.[54]

Suppose that the person who utters the sentence is a child (or a slow-learning adult) who appears to have no command of the metalinguistic concepts we take for granted. In particular, he appears to be unfamiliar with any device (e.g., quotation names) for naming expressions, and he appears to have no articulated concepts from linguistic theory such as the syntactic concept of a linguistic predicate or the semantical concept of satisfaction(-in-English). Furthermore, when we try to teach him these bits of linguistic theory, he has great difficulty learning them. (He learns the new predicate 'masticate' much more readily.) However, a few years later when we try again to teach him these things, he learns them quickly. This shows, so the worry goes, that the above pragmatic analysis of his assertion represents him as having reached a stage of conceptual development beyond what we can plausibly attribute to him. If so, the pragmatic analysis is mistaken; the fellow's assertion could not have involved the specific metalinguistic concepts attributed to him by this analysis.

There appears to be only one successful way out of this problem, and that is to treat our fellow's apparent ignorance of metalinguistic concepts as a species of the kind of ignorance involved in the paradox of analysis. Consider two analogies. First, suppose a child can sort variously shaped objects so well that it becomes plain that he recognizes, say, the circular objects *as* circular and, therefore, that he has command of the concept of circle. However, suppose that the child displays no particular behavior to indicate that he has command of the concept of a (mathematical or physical) point, the concept of a locus of points, the concept of a (mathematical or physical) plane, the concept of degree of curvature, etc. When we try to teach him geometric theory – with its definition of circle as a locus of points in the same plane equidistant from a common point – we get nowhere. (If he were a few years older, he would be able to learn this readily.) In this situation it is natural to characterize the child as follows: he has an unanalyzed concept of circle (i.e., an unanalyzed concept of being-a-locus-of-points-in-the-same-plane-equidistant-from-a-common-point); however, he lacks the theoretical concepts (points, locus, plane, etc.) that someone might use to analyze this unanalyzed concept.

Second, consider someone who can reliably tell whether a middle-sized object *comes to a halt smoothly*. But this person seems to have no grasp of the sophisticated concepts of calculus that would be used to spell out what it is for an object to come to a halt smoothly; indeed, if the person has limited mathematical appitude, he might never be able to grasp these theoretical concepts. It would be natural to say of this person that he has an unanalyzed concept of coming-to-a-halt-smoothly but that he lacks the specific theoretical concepts needed to unpack this unanalyzed concept.[55]

With these geometry and calculus examples in mind, let us consider a linguistics example. Suppose that a child is not yet in command of various theoretical concepts from linguistics, concepts such as satisfaction-in-English, linguistic predicate, and quotation. Despite this, it should still be possible for the child to have an unanalyzed concept whose analysis involves such theoretical concepts from linguistics. This would be quite analogous to the child's having an unanalyzed concept of circle (i.e., an unanalyzed concept of being-a-locus-of-points-in-the-same-plane-equidistant-from-a-common-point) and yet not being in command of the theoretical concepts from geometry (point, locus, plane, etc.) that someone would use to analyze this concept. And it would also be analogous to the child's having an unanalyzed concept of coming-to-a-halt-smoothly and yet not being in command of the theoretical concepts from calculus that someone would use to analyze this concept. Surely nothing can prevent this sort of thing from happening in linguistics examples, too. We submit that this is exactly what is going on in the case of the child who is "half-way" along in the process of learning to use the predicate 'masticate': he has an unanalyzed concept of satisfying-the-predicate-'masticate', but he is not in command of the theoretical concepts (satisfaction, predicate, quotation) that someone would use to analyze this unanalyzed concept. If this is right, then we have the makings of a solution to the problem confronting the pragmatic analysis of what the child asserted when he said, "I am sure that whatever masticates chews, but I am not sure that whatever chews masticates". The child's assertion comes to something like this:

> I am sure that whatever satisfies-the-predicate-'masticate' also chews, but I am not sure that whatever chews also satisfies-the-predicate-'masticate'.

Since this analysis attributes to the child the unanalyzed concept of satisfying-the-predicate-'masticate', it avoids the problem of mistakenly attributing to the child theoretical concepts that he will acquire only at a more advanced developmental stage. And this is what we needed.

Notice, however, that this way of salvaging the pragmatic solution of our 'chew'/'masticate' substitutivity puzzle is based on a very fine-grained intensional distinction, namely, the distinction between the unanalyzed concept of satisfying-the-predicate-'masticate' and the analyzed concept of satisfying the predicate 'masticate'.

Now, as far as we can tell, this outcome is unavoidable. That is, there is no satisfactory way of salvaging the pragmatic solution of the original substitutivity puzzle that does not somehow invoke antecedently given intensional distinctions that are very fine-grained. Here, then, is a place where very fine-grained intensional distinctions cannot, even in principle, be eliminated by the technique of pragmatic explanation. If this is right, a very fine-grained intensional logic is inevitable.[56]

II: THE FORMULATION OF INTENSIONAL LOGIC

Using the above guidelines, we are finally ready to present our formal intensional logic. We proceed in three standard stages: (1) syntax, (2) semantics, (3) axiomatic theory. Following this, we will close with some remarks about the significance of completeness results in first-order intensional logic.

1. SYNTAX

We now construct our first-order intensional language L_ω.
Primitive symbols:

Logical operators: &, $\neg$, $\exists$,

Predicate letters: $F_1^1, F_2^1, \ldots, F_s^r, \ldots$ (for $r, s \geqslant 1$),

Variables: $x, y, z, x_1, y_1, z_1, \ldots,$

Punctuation: (,), [,].

Simultaneous inductive definition of *term* and *formula* of L_ω:

(1) All variables are terms.

(2) If $t_1, \ldots, t_j$ are terms, then $F_i^j(t_1, \ldots, t_j)$ is a formula.

(3) If A and B are formulas and v_k is a variable, then $(A \,\&\, B)$, $\neg A$, and $(\exists v_k)A$ are formulas.

(4) If A is a formula and $v_1, \ldots, v_m$ (for $m \geqslant 0$) are distinct variables, then $[A]_{v_1 \ldots v_m}$ is a term.

In the limiting case where $m = 0$, $[A]$ is a term. On the intended informal interpretation of L_ω, $[A]_{v_1 \ldots v_m}$ denotes a proposition if $m = 0$, a property if $m = 1$, and an m-ary relation-in-intension if $m \geqslant 2$.

The following are auxiliary syntactic notions. Formulas and terms are *well-formed expressions*. An occurrence of a variable v_i in a well-formed expression is *bound* (*free*) if and only if the expression is (is not) a formula of the form $(\exists v_i)A$ or a term of the form $[A]_{v_1 \ldots v_i \ldots v_m}$. A term t is said to be *free for* v_i *in* A if and only if, for all v_k, if v_k is free in t, then no free occurrence of v_i in A occurs either in a subcontext of the form $(\exists v_k)(\ldots)$ or in a subcontext of the form $[\ldots]_{\alpha v_k \beta}$, where α and β are sequences of variables. If v_i has a free occurrence in A and is not one of the variables in the sequence of variables α, then v_i is an *externally quantifiable variable* in the term $[A]_\alpha$. Let δ be the sequence of externally quantifiable variables in $[A]_\alpha$ displayed in order of their first free occurrence; $[A]_\alpha$ will sometimes be rewritten as $[A]_\alpha^\delta$. Let $A(v_1, \ldots, v_p)$ be any formula; $v_1, \ldots, v_p$ may or may not occur free in A. Then we write $A(t_1, \ldots, t_p)$ to indicate the formula that results when, for each k, $1 \leqslant k \leqslant p$, the term t_k replaces each free occurrence of v_k in A. Terms $[A(u_1, \ldots, u_p)]_{u_1 \ldots u_p}^\delta$ and $[A(v_1, \ldots, v_p)]_{v_1 \ldots v_p}^\delta$ are said to be *alphabetic variants* if and only if, for each k, $1 \leqslant k \leqslant p$, u_k is free for v_k in A and conversely. Terms of the form $[F_i^j(v_1, \ldots, v_j)]_{v_1 \ldots v_j}$ are called *elementary*. A term $[A]_\alpha$ is called *normalized* if and only if all variables in α occur free in A exactly once and α displays the order in which these variables occur free in A. The logical operators $\forall$, $\supset$, $\equiv$, $\equiv_{v_1 \ldots v_j}$ are defined in terms of $\exists$, &, and $\neg$ in the usual way. Finally, F_1^2 is singled out as a distinguished logical predicate, and formulas of the form $F_1^2(t_1, t_2)$ are rewritten as $t_1 = t_2$.

Notice that L_ω is just like a standard first-order language except for its singular terms $[A]_{v_1 \ldots v_m}$, which are intended to be intensional abstracts that denote propositions, properties, or relations, depending on the value of m.

2. SEMANTICS

Since the aim is to simply characterize the logically valid formulas of L_ω, it will suffice to construct a Tarski-style definition of validity for L_ω. Such definition will be built on Tarski-style definitions of the truth for L_ω. These definitions will in turn depend in part on specifications of the *denotations* of the singular terms in L_ω. As already indicated, every formula for L_ω is just like a formula in a standard first-order extensional language except perhaps for the singular terms occurring in it. Therefore, once we have found a method for specifying the denotations of the singular terms of L_ω, the Tarski-style definitions of truth and validity for L_ω may be given in the customary way. What we are looking for specifically is a method for characterizing the denotations of the singular terms of L_ω in such a way that a given singular term $[A]_{v_1 \ldots v_m}$ will denote an appropriate property, relation, or proposition, depending on the value of m.

Since L_ω has infinitely many singular terms $[A]_\alpha$, what is called for is a recursive specification of the denotation relation for L_ω. To do this, we will arrange these singular terms into an order according to their syntactic kind and complexity. So, for example, just as the complex formula $((\exists x)Fx\ \&\ (\exists y)Gy)$ is the conjunction of the simpler formulas $(\exists x)Fx$ and $(\exists x)Gx$, we will say that the complex term $[((\exists x)Fx\ \&\ (\exists y)Gy)]$ is the *conjunction* of the simpler terms $[(\exists x)Fx]$ and $[(\exists y)Gy]$. Similarly, just as the complex formula $\neg(\exists x)Fx$ is the negation of the simpler formula $(\exists x)Fx$, we will say that the complex term $[\neg(\exists x)Fx]$ is the *negation* of the simpler term $[(\exists x)Fx]$. The following are other examples: $[(\exists x)Fx]$ is the *existential generalization* of $[Fx]_x$; $[Fy]^y$ is the *predication* of $[Fx]_x$ of y; $[F[Gy]_y]$ is the *predication* of $[Fx]_x$ of $[Gy]_y$; $[F[Gy]^y]_y$ is the *relativized predication* of $[Fx]_x$ and $[Gy]_y$; $[Rxy]_{yx}$ is the *conversion* of $[Rxy]_{xy}$; $[Sxyz]_{xzy}$ is the *inversion* of $[Sxyz]_{xyz}$; $[Rxx]_x$ is the *reflexivization* of $[Rxy]_{xy}$; $[Fx]_{xy}$ is the *expansion* of $[Fx]_x$.

The complex singular terms of L_ω that are syntactically simpler than all other complex singular terms are those whose form is $[F^m(v_1, \ldots, v_m)]_{v_1 \ldots v_m}$. These are called *elementary*. The denotation of such an elementary complex term is just the property or relation expressed by the primitive predicate F^m. The denotation of a more complex term $[A]_\alpha$ is determined by the denotation(s) of the relevant syntactically simpler term(s). But how in detail does this work? The answer is that the new denotation is determined *algebraically*. That is, the new denotation is determined by the application of the relevant *fundamental logical operation* to the denotation(s) of the relevant syntactically simpler term(s). Let us explain.

Consider the following three propositions: $[(\exists x)Fx]$, $[(\exists y)Gy]$, $[(\exists x)Fx\ \&\ (\exists y)Gy]$. (Note: in this paragraph we will be *using* – not mentioning – terms from L_ω.) What is the most obvious relation holding among these propositions? Answer: the third proposition is the *conjunction* of the first two. And what is the most obvious relation holding between the propositions $[(\exists x)Fx]$ and $[\neg(\exists x)Fx]$? Answer: the second is the *negation* of the first. Similarly, what is the most obvious relation holding between the properties $[Fx]_x$ and $[\neg Fx]_x$? As before, the second is the *negation* of the first. In a similar manner we arrive at the following similar relationships: $[(\exists x)Fx]$ is the *existential generalization* of $[Fx]_x$; $[Fy]^y$ is the result of *predicating* $[Fx]_x$ of y; $[F[Gy]_y]$ is the result of *predicating* $[Fx]_x$ of $[Gy]_y$; $[F[Gy]^y]_y$ is the result of a *relativized predication* of $[Fx]_x$ of $[Gy]_y$; $[Rxy]_{yx}$ is the *converse* of $[Rxy]_{xy}$; $[Sxyz]_{xzy}$ is the *inverse* of $[Sxyz]_{xyz}$; $[Rxx]_x$ is the *reflexivization* of $[Rxy]_{xy}$; and $[Fx]_{xy}$ is the *expansion* of $[Fx]_x$. These fundamental logical operations, of course, correspond to the syntactic operations listed earlier.

Now choose any complex term t in L_ω that is not elementary. If t is obtained from s via the syntactic operation of negation (conversion, inversion, reflexivization, expansion, existential generalization), then the denotation of t is the result of applying the logical operation of negation (conversion, inversion, reflexivization, expansion, existential generalization) to the denotation of s. The same thing holds *mutatis mutandis* for complex terms that, syntactically, are conjunctions, predications, or relativized predications. In this way, therefore, these fundamental logical operations make it possible to define recursively the denotation relation for all of the complex intensional terms t in L_ω.

The algebraic semantics for L_ω is thus to be specified in stages. (1) An algebra of properties, relations, and propositions – or an *algebraic model structure* is posited. (2) Relative to this, an intensional *interpretation* of the primitive predicates is given. (3) Relative to this, the *denotation* relation for the terms of L_ω is recursively defined. (4) Relative to this, the notion of truth for formulas is defined. (5) In the customary Tarski fashion, the notion of *logical validity* for formulas of L_ω is defined in terms of the notion of truth.

Omitting certain details for heuristic purposes, we may characterize an algebraic model structure as a structure containing (i) a domain $\mathcal{D}$ comprised of (items playing the role of) individuals, propositions, properties, and relations, (ii) a set $\mathcal{K}$ of functions that tell us the actual and possible extensions of the items in $\mathcal{D}$, and (iii) various fundamental logical operations on the items in $\mathcal{D}$. (All items in $\mathcal{D}$ are treated on a par as primitive entities; none is constructed from the others by means of set-theoretical operations.) Once the general notion of an algebraic model structure is precisely defined, we may then go on to define a spectrum of different *types* of algebraic model structure; these types are distinguished one from another by the strictness of the identity conditions imposed on the PRPs in the domain of the various model structures. It is in this way that the algebraic method is able to provide a *general* technique for modeling any type of PRP, from coarse-grained to very fine-grained.

Our algebraic method also allows us to model transcendental and self-embeddable PRPs. To see what makes this possible let us consider the differences between algebraic model structures, on the one hand, and the usual possible-worlds model structures, on the other. Algebraic model structures contain (i) a domain consisting of individuals, properties, relations, and propositions, (ii) a set of functions that tell us the actual and possible extensions of items in the domain, and (iii) various fundamental logical operations on the items in the domain. In a possible-worlds model structure, on the other hand, (i) is typically replaced by a domain consisting of actual individuals and "nonactual individuals"; then PRP-surrogates are constructed from these items by means of set-theoretical operations; and (ii) and (iii) are omitted. The reason that (ii) can be omitted in a possible-worlds model structure is that each PRP-surrogate is a set that, in effect, encodes its own actual and possible extensions. The reason (iii) can be omitted is that the sets that play the role of "complex" PRPs are formed from other PRP-surrogates by wholly standard set-theoretical operations (like intersection, complementation, etc.), so there is no need to build these operations into the model structure itself. But notice that, if the set-theoretical construction of these PRP-surrogates is done in a standard set theory, these PRP-surrogates must form an hierarchy of well-founded sets; consequently, there are no sets in the construction that can serve as surrogates for transcendental or self-embeddable properties and relations.

In an algebraic model structure, by contrast, there can be such PRPs. The reason there can be transcendental properties and relations is that properties and relations are included in the domain as primitive entities that do not encode their own extensions; their extensions are instead specified by independent extension functions. Consequently, these functions can map items in the domain to any subset of the domain, even subsets that happen to contain the original items. For example, let s be (the element in the domain $\mathscr{D}$ that plays the role of) the transcendental property self-identity. Then each extension function H just maps s to the domain itself; that is, $H(s) = \{x \in \mathscr{D}: x = x\} = \mathscr{D}$. After all, everything in the domain, including s, is self-identical. No ill-founded set theory is involved here: s is just a primitive entity in $\mathscr{D}$ on a par with an individual, and H is just a standard well-founded function that maps s to a set that contains s.

The reason that algebraic model structures can model self-embedded PRPs is that the (items in the domain that play the role of) PRPs are not set-theoretical constructs but rather are primitive entities. So if a given PRP is self-embedded, this will be exhibited exclusively through its behavior with respect to the fundamental logical operations, in particular, the predication operations. Consider, for example, the proposition $[F[Fx]_x]$. This proposition is the result of applying the 2-place logical operation of singular predication to the property $[Fx]_x$ taken as *both* first and second argument. That is, $[F[Fx]_x] = \text{Pred}_0([Fx]_x, [Fx]_x)$. No ill-founded set theory is involved here; on the contrary, this pattern is fully analogous to an application of a standard (set-theoretically well-founded) substitution operation from formal syntax: $\ulcorner F(\ulcorner Fx \urcorner)\urcorner = \text{sub}(\ulcorner Fx\urcorner, \ulcorner Fx \urcorner)$.[57]

Having made these heuristic points, we are ready to state the semantics for L_ω. We begin by defining the general notion of an algebraic model structure. An *algebraic model structure* is any structure $\langle \mathscr{D}, \mathscr{P}, \mathscr{K}, \mathscr{G}, \text{Id}, \mathscr{T}, \text{Conj}, \text{Neg}, \text{Exist}, \text{Pred}_0, \text{Pred}_1, \text{Pred}_2, \ldots, \text{Pred}_k, \ldots \rangle$ whose elements satisfy the following conditions. $\mathscr{D}$ is a nonempty domain. $\mathscr{P}$ is a prelinear ordering on $\mathscr{D}$ that induces a partition of $\mathscr{D}$ into the subdomains $\mathscr{D}_{-1}, \mathscr{D}_0, \mathscr{D}_1, \mathscr{D}_2, \mathscr{D}_3, \ldots$. The elements of $\mathscr{D}_{-1}$ are to be thought of as particulars; the elements of $\mathscr{D}_0$ as propositions; the elements of $\mathscr{D}_1$ as properties, and the elements of $\mathscr{D}_i$, for $i \geqslant 2$, as i-ary relations-in-intension. Although $\mathscr{D}_i$, $i \geqslant 0$, must not be empty, we do permit $\mathscr{D}_{-1}$ to be empty. $\mathscr{K}$ is a set of functions on $\mathscr{D}$. For all $H \in \mathscr{K}$, if $x \in \mathscr{D}_{-1}$, then $H(x) = x$; if $x \in \mathscr{D}_0$, then $H(x) = T$ or $H(x) = F$; if $x \in \mathscr{D}_1$, then $H(x) \subseteq \mathscr{D}$; if, for $i > 1$, $x \in \mathscr{D}_i$, then $H(x) \subseteq \mathscr{D}^i$. These functions $H \in \mathscr{K}$ are to be thought of as telling us the alternate or possible extensions of the elements of $\mathscr{D}$. $\mathscr{G}$ is a distinguished element of $\mathscr{K}$ and is to be thought of as the function that determines the *actual* extensions of the elements of $\mathscr{D}$. Id is a distinguished element of $\mathscr{D}_2$ and is thought of as the fundamental logical relation-in-intension *identity*. Id must satisfy the following condition: $(\forall H \in \mathscr{K})(H(\text{Id}) = \{xy \in \mathscr{D}: x = y\})$. In order to characterize the next element $\mathscr{T}$, consider the following partial functions on $\mathscr{D}$: Exp_i, defined on $\mathscr{D}_i$, $i \geqslant 0$; Ref_i, defined on $\mathscr{D}_i$, $i \geqslant 2$; Conv_i, defined on $\mathscr{D}_i$, $i \geqslant 2$; Inv_i, defined on $\mathscr{D}_i$, $i \geqslant 3$.[58] For all $H \in \mathscr{K}$ and all $x_1, \ldots, x_{i+1} \in \mathscr{D}$, these functions satisfy the following conditions:

a. $x_1 \in H(\text{Exp}_1(u))$ *iff* $H(u) = T$ (for $u \in \mathscr{D}_0$).
$\langle x_1, \ldots, x_i, x_{i+1}\rangle \in H(\text{Exp}_i(u))$ *iff* $\langle x_1, \ldots, x_i\rangle \in H(u)$ (for $u \in \mathscr{D}_i$, $i \geqslant 1$).

b. $\langle x_1, \ldots, x_{i-2}, x_{i-1}\rangle \in H(\text{Ref}_i(u))$ *iff*
$\langle x_1, \ldots, x_{i-2}, x_{i-1}, x_{i-1}\rangle \in H(u)$ (for $u \in \mathscr{D}_i$, $i \geqslant 2$).

c. $\langle x_i, x_1, \ldots, x_{i-1}\rangle \in H(\mathrm{Conv}_i(u))$ *iff*

$\langle x_1, \ldots, x_{i-1}, x_i\rangle \in H(u)$ (for $u \in \mathscr{D}_i$, $i \geqslant 2$).

d. $\langle x_1, \ldots, x_{i-2}, x_i, x_{i-1}\rangle \in H(\mathrm{Inv}_i(u))$ *iff*

$\langle x_1, \ldots, x_{i-2}, x_{i-1}, x_i\rangle \in H(u)$ (for $u \in \mathscr{D}_i$, $i \geqslant 3$).

A proto-transformation is defined to be a function that arises from composing a finite number of these functions in some order (repetitions permitted). A proto-transformation τ is said to be degenerate if and only if $\tau(x) = x$ for all $x \in \mathscr{D}$ for which τ is defined. A function τ is said to be *equivalent* to a proto-transformation τ' if and only if, for all $H \in \mathscr{K}$ and for all $x \in \mathscr{D}$ for which τ' is defined, $H(\tau(x)) = H(\tau'(x))$. Now $\mathscr{T}$ is a set of partial functions on $\mathscr{D}$: for every nondegenerate proto-transformation, there is exactly one equivalent function in $\mathscr{T}$, and nothing but such functions are in $\mathscr{T}$. The functions in $\mathscr{T}$ are called *transformations*. The remaining elements in a model structure are partial functions on $\mathscr{D}$. Conj is defined on each $\mathscr{D}_i \times \mathscr{D}_i$, $i \geqslant 0$; Neg, on each $\mathscr{D}_i$, $i \geqslant 0$; Exist, on each $\mathscr{D}_i$, $i \geqslant 1$; Pred_0, on each $\mathscr{D}_i \times \mathscr{D}$, $i \geqslant 1$; Pred_k, on each $\mathscr{D}_i \times \mathscr{D}_j$, $i \geqslant 1$ and $j \geqslant k \geqslant 1$. These functions satisfy the following, for all $H \in \mathscr{K}$ and all $x_1, \ldots, x_i, y_1, \ldots, y_k \in \mathscr{D}$:

1. $H(\mathrm{Conj}(u, v)) = T$ *iff* $(H(u) = T \;\&\; H(v) = T)$

(for $u, v \in \mathscr{D}_0$).

$\langle x_1, \ldots, x_i\rangle \in H(\mathrm{Conj}(u, v))$ *iff* $(\langle x_1, \ldots, x_i\rangle \in H(u)\;\&$

$\&\; \langle x_1, \ldots, x_i\rangle \in H(v))$ (for $u, v \in \mathscr{D}_i$, $i \geqslant 1$).

2. $H(\mathrm{Neg}(u)) = T$ *iff* $H(u) = F$ (for $u \in \mathscr{D}_0$).

$\langle x_1, \ldots, x_i\rangle \in H(\mathrm{Neg}(u))$ *iff* $\langle x_1, \ldots, x_i\rangle \notin H(u)$

(for $u \in \mathscr{D}_i$, $i \geqslant 1$).

3. $H(\mathrm{Exist}(u)) = T$ *iff* $(\exists x_1)(x_1 \in H(u))$ (for $u \in \mathscr{D}_1$).

$\langle x_1, \ldots, x_{i-1}\rangle \in H(\mathrm{Exist}(u))$ *iff*

$(\exists x_i)(\langle x_1, \ldots, x_{i-1}, x_i\rangle \in H(u))$ (for $u \in \mathscr{D}_i$, $i \geqslant 2$).

4.0 $H(\mathrm{Pred}_0(u, y_1)) = T$ *iff* $y_1 \in H(u)$ (for $u \in \mathscr{D}_1$).

$\langle x_1, \ldots, x_{i-1}\rangle \in H(\mathrm{Pred}_0(u, y_1))$ *iff*

$\langle x_1, \ldots, x_{i-1}, y_1\rangle \in H(u)$ (for $u \in \mathscr{D}_i$, $i \geqslant 2$).

4.1. $\langle x_1, \ldots, x_{i-1}, y_1 \rangle \in H(\mathrm{Pred}_1(u, v))$ *iff*

$\langle x_1, \ldots, x_{i-1}, \mathrm{Pred}_0(v, y_1) \rangle \in H(u)$

(for $u \in \mathscr{D}_i$, $i \geqslant 1$, and $v \in \mathscr{D}_j$, $j \geqslant 1$).

4.2. $\langle x_1, \ldots, x_{i-1}, y_1, y_2 \rangle \in H(\mathrm{Pred}_2(u, v))$ *iff*

$\langle x_1, \ldots, x_{i-1}, \mathrm{Pred}_0(\mathrm{Pred}_0(v, y_2), y_1) \rangle \in H(u)$

(for $u \in \mathscr{D}_i$, $i \geqslant 1$, and $v \in \mathscr{D}_j$, $j \geqslant 2$).

. . . .[59]

These functions, together with the transformations in $\mathscr{T}$, are to be thought of as fundamental logical operations on intensional entities. This completes the characterization of what a model structure is.

Now in the history of logic and philosophy there have been two competing conceptions of intensional entities, which we call conception 1 and conception 2. Conception 1 is suited to the logic for modal matters (necessity, possibility, etc.), and conception 2 appears to be relevant to the logic for psychological matters (belief, desire, decision, etc.).[60] According to conception 1, (i-ary) intensions are taken to be identical if they are *necessarily equivalent*. This leads to the following definition. A model structure is *type* 1 *iff*$_{\mathrm{df}}$ it satisfies the following auxiliary condition: $(\forall x, y \in \mathscr{D}_i)((\forall H \in \mathscr{K})(H(x) = H(y)) \rightarrow x = y)$, for all $i \geqslant -1$. This auxiliary condition provides a precise characterization of conception 1. In contrast to conception 1, conception 2 places far stricter conditions on the identity of intensional entities. According to conception 2, when an intension is defined *completely*, it has a *unique, noncircular definition*. (The possibility that such complete definitions might in some or even all cases be infinite need not be ruled out.) This leads to the following definition. A model structure is *type* 2 *iff*$_{\mathrm{df}}$ the transformations in $\mathscr{T}$ and the functions Conj, Neg, Exist, Pred_0, Pred_1, Pred_2, . . . are all (i) one-one, (ii) disjoint in their ranges, and (iii) noncycling. Auxiliary conditions (i)–(iii) provide us with a precise formulation of conception 2.[61]

In order to state the semantics for L_ω, we must define some preliminary syntactic notions. First, we define certain syntactic operations on complex terms of L_ω. These operations have a natural correspondence to the logical operations Conj, Neg, Exist, Pred_0, . . . in a model structure. If $[(A \;\&\; B)]_\alpha$ is normalized, it is the *conjunction* of $[A]_\alpha$ and $[B]_\alpha$. If $[\neg A]_\alpha$ is normalized, it is the *negation* of $[A]_\alpha$. If $[(\exists v_k)A]_\alpha$ is normalized, it is the *existential generalization* of $[A]_{\alpha v_k}$. Suppose that $[F_i^j(v_1, \ldots, v_{m-1}, t_m, t_{m+1}, \ldots, t_j)]_\alpha$

is normalized and that no variable occurring free in t_m occurs in α. Then this normalized term is the *predication*$_0$ of

$$[F_i^j(v_1, \ldots, v_{m-1}, v_m, t_{m+1}, \ldots, t_j)]_{\alpha v_m}$$

of t_m. (v_m is the alphabetically earliest variable not occurring in the normalized term.) Finally, suppose that, for $k \geqslant 1$,

$$[F_i^j(v_1, \ldots, v_{m-1}, [B]_\gamma^\delta, t_{m+1}, \ldots, t_j)]_{v_1 \ldots v_{m-1} u_1 \ldots u_k \alpha}$$

is normalized, that $u_1, \ldots, u_k$ occur in δ, and that no variable in δ occurs in α. Then

$$[F_i^j(v_1, \ldots, v_{m-1}, [B]_\gamma^\delta, t_{m+1}, \ldots, t_j]_{v_1 \ldots v_{m-1} \alpha u_1 \ldots u_k}$$

is the *predication*$_k$ of

$$[F_i^j(v_1, \ldots, v_{m-1}, u_1, t_{m+1}, \ldots, t_j)]_{v_1 \ldots v_{m-1} \alpha u_1}$$

of $[B]_{\gamma u_1 \ldots u_m}^{\delta'}$. ($\delta'$ is the result of deleting $u_1, \ldots, u_m$ from δ.)

Consider the following auxiliary operations on complex terms:

(a) $\quad \exp_i([A]_{v_1 \ldots v_i}) =_{\mathrm{df}} [A]_{v_1 \ldots v_i v_{i+1}}$

(where $i \geqslant 0$ and v_{i+1} is the alphabetically earliest variable not occurring in $[A]_{v_1 \ldots v_i}$).

(b) $\quad \mathrm{ref}_i([A(v_1, \ldots, v_{i-1}, v_i)]_{v_1 \ldots v_{i-1} v_i}) =_{\mathrm{df}}$

$$[A(v_1, \ldots, v_{i-1}, v_{i-1})]_{v_1 \ldots v_{i-1}}$$

(where $i \geqslant 2$ and v_{i-1} is free for v_i in A).

(c) $\quad \mathrm{conv}_i([A]_{v_1 \ldots v_{i-1} v_i}) =_{\mathrm{df}} [A]_{v_i v_1 \ldots v_{i-1}}$

(where $i \geqslant 2$).

(d) $\quad \mathrm{inv}_i([A]_{v_1 \ldots v_{i-2} v_{i-1} v_i}) =_{\mathrm{df}} [A]_{v_1 \ldots v_{i-2} v_i v_{i-1}}$

(where $i \geqslant 3$).

Consider the operations σ that arise from composing a finite number of these operations in some order (repetitions permitted). A relation R_σ is a *term-transforming* relation if it is associated with one of these operations σ as follows: $R_\sigma(r, s)$ *iff* $\sigma(r') = s'$, where r' is an alphabetic variant of r, s' is an alphabetic variant of s, r is either an elementary complex term, a negation, a conjunction, an existential generalization, or a predication$_k$,

$k \geqslant 0$, and s is none of these. Now for any model structure, a term-transforming relation R_σ is *associated* with a transformation τ in the set $\mathscr{T}$ in the model structure *iff*$_{\text{df}}$ (a) for some $\sigma_1, \ldots, \sigma_m$ selected from exp_i, ref_i, conv_i, inv_i, σ is the composition of $\sigma_1, \ldots, \sigma_m$; (b) for some $\tau_1, \ldots, \tau_m$ selected from Exp_i, Ref_i, Conv_i, Inv_i, τ is the transformation in $\mathscr{T}$ equivalent to the composition of $\tau_1, \ldots, \tau_m$; (c) for all k, $1 \leqslant k \leqslant m$, $\sigma_k = \text{exp}_i$ iff $\tau_k = \text{Exp}_i$; $\sigma_k = \text{ref}_i$ iff $\tau_k = \text{Ref}_i$; $\sigma_k = \text{conv}_i$ iff $\tau_k = \text{Conv}_i$; $\sigma_k = \text{inv}_i$ iff $\tau_k = \text{Inv}_i$. With these preliminary notions in hand we are finally ready to state the semantics for L_ω.

Denotation, truth, and validity. An *interpretation* $\mathscr{I}$ *for* L_ω *relative to model structure* $\mathscr{M}$ is a function that assigns to the predicate letter F_1^2 (i.e., $=$) the element $\text{Id} \in \mathscr{M}$ and, for each predicate letter F_i^j in L_ω, assigns to F_i^j some element of the subdomain $\mathscr{D}_j \subset \mathscr{D} \in \mathscr{M}$. An *assignment* $\mathscr{A}$ *for* L_ω *relative to model structure* $\mathscr{M}$ is a function that maps the variables of L_ω into the domain $\mathscr{D} \in \mathscr{M}$. Relative to interpretation $\mathscr{I}$, assignment $\mathscr{A}$, and model structure $\mathscr{M}$, the *denotation* relation for terms of L_ω is inductively defined as follows:

Variables. v_i denotes $\mathscr{A}(v_i)$.

Elementary complex terms. $[F_i^j(v_1, \ldots, v_j)]_{v_1 \ldots v_j}$ denotes $\mathscr{I}(F_i^j)$.

Nonelementary complex terms. If t is the conjunction – or predication$_k$ – of r and s, and r denotes u, and s denotes v, then t denotes $\text{Conj}(u, v)$ – or $\text{Pred}_k(u, v)$. If t is the negation – or existential generalization – of r, and r denotes u, then t denotes $\text{Neg}(u)$ – or $\text{Exist}(u)$. If R_σ is a term-transforming relation associated with a transformation $\tau \in \mathscr{T}$ and $R_\alpha(r, t)$ and r denotes u, then t denotes $\tau(u)$.

The denotation relation is clearly a function. We henceforth represent it with $D_{\mathscr{I}\mathscr{A}\mathscr{M}}$. *Truth* is then defined in terms of $D_{\mathscr{I}\mathscr{A}\mathscr{M}}$ as follows: $T_{\mathscr{I}\mathscr{A}\mathscr{M}}(A)$ *iff*$_{\text{df}}$ $\mathscr{G}(D_{\mathscr{I}\mathscr{A}\mathscr{M}}([A])) = T$.[62] And finally two notions of validity are defined. A formula A is *valid*$_1$ *iff*$_{\text{df}}$ for every type 1 model structure $\mathscr{M}$ and for every interpretation $\mathscr{I}$ and every assignment $\mathscr{A}$ relative to $\mathscr{M}$, $T_{\mathscr{I}\mathscr{A}\mathscr{M}}(A)$. A formula A is *valid*$_2$ *iff*$_{\text{df}}$ for every type 2 model structure $\mathscr{M}$ and for every interpretation $\mathscr{I}$ and every assignment $\mathscr{A}$ relative to $\mathscr{M}$, $T_{\mathscr{I}\mathscr{A}\mathscr{M}}(A)$. This completes the semantics for L_ω.

3. AXIOMATIC THEORY

The logic for PRPs on conception 1. On conception 1 intensional entities are identical if and only if necessarily equivalent. Thus, on conception 1 the following abbreviation captures the properties usually attributed to the modal operator $\Box$: $\Box A$ *iff*$_{\text{df}}$ $[A] = [[A] = [A]]$. (That is, necessarily A *iff* the proposition that A is identical to any trivial necessary truth.) The modal operator $\Diamond$ is then defined in terms of $\Box$ in the usual way: $\Diamond A$ *iff*$_{\text{df}}$ $\neg\Box\neg A$.[63]

The logic $T1$ for L_ω on conception 1 consists of the axiom schemas and rules for the modal logic S5 with quantifiers and identity and three additional axiom schemas for intensional abstracts.

Axiom schemas and rules of $T1$.

A1. Truth-functional tautologies.

A2. $(\forall v_i)A(v_i) \supset A(t)$ (where t is free for v_i in A).

A3. $(\forall v_i)(A \supset B) \supset (A \supset (\forall v_i)B)$ (where v_i is not free in A).

A4. $v_i = v_i$.

A5. $v_i = v_j \supset (A(v_i, v_i) \equiv A(v_i, v_j))$ (where $A(v_i, v_j)$ is a formula that arises from $A(v_i, v_i)$ by replacing some (but not necessarily all) free occurrences of v_i by v_j, and v_j is free for the occurrences of v_i that it replaces).

A6. $[A]_{u_1 \ldots u_p} \neq [B]_{v_i \ldots v_q}$ (where $p \neq q$).

A7. $[A(u_1, \ldots, u_p)]_{u_1 \ldots u_p} = [A(v_1, \ldots, v_p)]_{v_1 \ldots v_p}$ (where these terms are alphabetic variants).

A8. $[A]_\alpha = [B]_\alpha \equiv \Box(A \equiv_\alpha B)$.

A9. $\Box A \supset A$.

A10. $\Box(A \supset B) \supset (\Box A \supset \Box B)$.

A11. $\Diamond A \supset \Box\Diamond A$.

R1. If $\vdash A$ and $\vdash A \supset B$, then $\vdash B$.

R2. If $\vdash A$, then $\vdash (\forall v_i)A$.

R3. If $\vdash A$, then $\vdash \Box A$.

THEOREM (Soundness and Completeness). *For all formulas A in* L_ω, *A is valid$_1$ if and only if A is a theorem of $T1$ (i.e., $\models_1 A$ iff $\vdash_{T1} A$).*[64]

The logic for PRPs on conception 2. On conception 2 each definable intensional entity is such that, when it is defined completely, it has a unique, noncircular definition. The logic $T2$ for L_ω on conception 2 consists of (a) axioms A1–A7 and rules R1–R2 from $T1$, (b) five additional axiom schemas for intensional abstracts, and (c) one additional rule. In stating the additional principles, I will write $t(F^q_p)$ to indicate that t is a complex term of L_ω in which the primitive predicate F^q_p occurs.

Additional axiom schemas and rules for T2.

$\mathscr{A}$8. $[A]_\alpha = [B]_\alpha \supset (A \equiv B)$.

$\mathscr{A}$9. $t \neq r$ (where t and r are nonelementary complex terms of different syntactic kinds.)[65]

$\mathscr{A}$10. $t = r \equiv t' = r'$ (where $R(t', t)$ and $R(r', r)$ for some term-transforming relation R, *or* t is the negation of t' and r is the negation of r', *or* t is the existential generalization of t' and r is the existential generalization of r').

$\mathscr{A}$11. $t = r \equiv (t' = r' \,\&\, t'' = r'')$ (where t is the conjunction of t' and t'' and r is the conjunction of r' and r'' *or* t is the predication$_k$ of t' of t'' and r is the predication$_k$ of r' of r'' for some $k \geqslant 0$).

$\mathscr{A}$12. $t(F^j_i) = r(F^k_h) \supset q(F^j_i) \neq s(F^k_h)$ (where t and s are elementary and r and q are not).

$\mathscr{R}$3. Let F^p_k be a nonlogical predicate that does not occur in $A(v_i)$; let $t(F^p_k)$ be an elementary complex term, and let t' be any complex term of degree p that is free for v_i in $A(v_i)$. If $\vdash A(t)$, then $\vdash A(t')$.[66]

THEOREM (Soundness and Completeness). *For all formulas A in L_ω, A is valid$_2$ if and only if A is a theorem of $T2$ (i.e., $\models_2 A$ iff $\vdash_{T2} A$).*

The logic for PRPs and necessary equivalence on conception 2. Let the 2-place logical predicate $\approx_N$ be adjoined to L_ω. $\approx_N$ is intended to express the logical relation of necessary equivalence. A *type* 2′ model structure is defined to be just like a type 2 model structure except that it contains an additional constituent Eq_N which is a distinguished element of $\mathscr{D}_2$ satisfying the following condition:

$$(\forall H \in \mathscr{K})(H(\mathrm{Eq}_N) = \{xy\colon (\exists i \geqslant -1)(x, y \in \mathscr{D}_i) \,\&\, (\forall H' \in \mathscr{K})(H'(x) = H'(y))\}).$$

Thus, Eq_N is to be thought of as the distinguished logical relation-in-intension *necessary equivalence*. Now an interpretation $\mathscr{I}$ relative to a type 2′ model structure is just like an interpretation relative to a type 1 or type 2 model structure except that we require $\mathscr{I}(\approx_N) = \mathrm{Eq}_N$. Then type 2′ denotation, truth, and validity are defined *mutatis mutandis* as before. The following abbreviations are introduced for notational convenience;

$$\Box A \ \mathit{iff}_{\mathrm{df}}\ [A] \approx_N [[A] \approx_N [A]]$$

$$\Diamond A \ \mathit{iff}_{\mathrm{df}} \neg \Box \neg A.$$

The intensional logic $T2'$ consists of the axioms and rules for $T2$ plus the following additional axioms and rules for $\approx_N$:

$\mathscr{A}13.$ $x \approx_N x.$

$\mathscr{A}14.$ $x \approx_N y \supset y \approx_N x.$

$\mathscr{A}15.$ $x \approx_N y \supset (y \approx_N z \supset x \approx_N z).$

$\mathscr{A}16.$ $x \approx_N y \supset \Box x \approx_N y.$

$\mathscr{A}17.$ $\Box(A \equiv_\alpha B) \equiv [A]_\alpha \approx_N [B]_\alpha.$

$\mathscr{A}18.$ $\Box A \supset A.$

$\mathscr{A}19.$ $\Box(A \supset B) \supset (\Box A \supset \Box B).$

$\mathscr{A}20.$ $\Diamond A \supset \Box\Diamond A.$

$R4.$ If $\vdash A$, then $\vdash \Box A.$

Notice that these axioms and rules for $\approx_N$ are just analogues of the special $T1$ axioms and rules for $=$. Finally, the soundness and completeness of $T2'$ can be shown by applying the methods of proof used for $T1$ and $T2$.

4. THE COMPLETENESS OF FIRST-ORDER INTENSIONAL LOGIC

We have indicated that first-order logic with identity and intensional abstraction is complete relative to certain technical notions of validity that are defined by means of intensional algebraic semantics. Consider the philosophical thesis that these technical notions of validity are in fact the standard notions of validity (or at least they resemble the standard notions in all respects relevant to genuine completeness results). From the technical result and the philosophical thesis it follows that first-order intensional logic is genuinely complete. This argument is parallel to that used to show that elementary first-order logic with identity is genuinely complete: the logic is proved complete relative to a certain defined notion of validity, and this technical result is then combined with the philosophical thesis that this defined notion is the same as (or resembles in philosophically relevant respects) the standard notion. In the case of elementary first-order logic with identity, the philosophical thesis has been subjected to much critical scrutiny, and something like a consensus has emerged in support of it. In the case of first-order intensional logic, the philosophical thesis strikes many people as highly intuitive. Nevertheless, some commentators (for example, Nino Cocchiarella [1985] and I. G. McFetridge [1984]) have expressed doubts. According to such doubts, the technical completeness result might be a mere artifact of a mistakenly narrow definition of validity that results from using an overly liberal definition of a model (in much the same way that Henkin's quasi-completeness result for higher-order extensional logic is a consequence of the liberal notion of a general model). Specifically, if certain plausible auxiliary closure conditions were imposed on the models, perhaps completeness would no longer follow; indeed, perhaps incompleteness could be derived. So go the doubts. However, these doubts are unfounded as we will now explain.

Consider two plausible closure conditions on models (described by Cocchiarella [1985] and McFetridge [1984], respectively). First, the set $\mathscr{K}$ of alternate extension functions must always be maximal, that is, it should not be possible to add further extension functions H to $\mathscr{K}$ without contradicting one of the original conditions in the definition of model. Second, for every subset s of $\mathscr{D}$ and every extension function H in $\mathscr{K}$, there must be an item x in the subdomain $\mathscr{D}_1$ of properties such that $H(x) = s$. (Notice that, if a model satisfies this closure condition, $\mathscr{D}$ must be a proper class. For the closure condition implies that there are as many properties in $\mathscr{D}$ as there are sub*sets* of $\mathscr{D}$. So if $\mathscr{D}$ were a set, the closure condition would contradict Cantor's power-set theorem.)

THEOREM. *First-order intensional logic is complete even if the strong closure conditions are imposed on models.*

In broad outline the proof goes as follows. We follow the Henkin-style proof given in *Quality and Concept* except that a proper class of individual constants are adjoined to the language, and for all distinct individual constants c and d, the sentence $c \neq d$ is adjoined to the theory. In the

Henkin model that results, these individual constants will comprise the subdomain of individuals. To obtain a model meeting the second closure condition, we massage this model in appropriate ways. First, partition this subdomain into denumerably many proper classes $d_{-1}, d_0, d_1, d_2, \ldots$. The first of these proper classes d_{-1} will be the subdomain of individuals in our new model. Then the i-th proper class ($i \geqslant 0$) will be adjoined to the old subdomain $\mathscr{D}_i$ to form the new subdomain of i-ary intensions, and the functions $H \in \mathscr{K}$ and the fundamental logical operations will be adjusted accordingly. When done properly the result is a model of the theory that meets the second closure condition. What makes this construction possible is the fact that our models have a single, unified domain $\mathscr{D}$ in which individuals, propositions, properties, and relations are taken as primitive entities. Finally, concerning the first closure condition, it is straightforward to show that every $\mathscr{K}$-maximal extension of the new model is also a model of the theory. The key to the proof is the fact that, for any algebraic intensional model, the values of all identity and necessity sentences must be the same in any $\mathscr{K}$-extension of a model as they are in the model itself.

This and similar results (given in Bealer [1987]) provide strong evidence that doubts about the genuine completeness of first-order intensional logic are unfounded and that the two-stage methodology (according to which intensional logic is treated prior to treating the logic for the predication relation) is vindicated.

Incidently, Cocchiarella [1985] claims that *incompleteness* can be proven when the first closure condition is imposed. He begins with the premise that every first-order necessity sentence $\ulcorner N[A(F_1, \ldots, F_n)]\urcorner$ – where $\ulcorner A \urcorner$ is a standard first-order extensional sentence and $\ulcorner F_1 \urcorner, \ldots, \ulcorner F_n \urcorner$ are the predicates occurring in $\ulcorner A \urcorner$ – is true in a model if and only if the second-order sentence $\ulcorner(\forall F_1 \ldots F_n)A\urcorner$ is true in the model. Then he claims that this implicit second-order element in first-order intensional logic is enough to prove incompleteness. But his premise rests on a straightforward error. To see why, choose any model in which interpretation $\mathscr{I}$ assigns the property of being self-identical to the primitive predicate 'F'. That is, $\mathscr{I}$('F') $= [x = x]_x$. On this interpretation, the intensional abstract '$[(\exists x)Fx]$' would denote $[(\exists x)x = x]$, i.e., the necessary proposition that something is self-identical. So on this interpretation the first-order intensional sentence '$N[(\exists x)Fx]$' is *true*. However, according to Cocchiarella's premise, this sentence is true on an interpretation if and only if the second-order sentence '$(\forall F)(\exists x)Fx$' is true on the interpretation. But this second-order sentence is *false*, indeed, it is *logically false*. So Cocchiarella's premise does not hold, and his alleged incompleteness proof fails.

The source of Cocchiarella's error is something like this. In PRP semantics, propositions are the primary bearers of necessity; the mere fact that a syntactically simple sentence like '$(\exists x)Fx$' is not true by virtue of its syntactic form tells us nothing about whether the proposition expressed by the sentence (i.e., the proposition that $(\exists x)Fx$) is necessary and, in turn, whether the sentence '$N[(\exists x)Fx]$' is true or false in a given model. Cocchiarella's error seems to arise from a kind of generalized use/mention confusion, that the syntactic form of a linguistic expression of a proposition and the modal status of the proposition should match up. But this only happens in special cases; it does not typically happen.

By the way, incompleteness of first-order intensional logic would follow if one adopted the following premise (which is adapted from an argument that Cocchiarella gives elsewhere about completeness results in first-order modal logic): for any first-order extensional sentence $\ulcorner A \urcorner$, the first-order intensional sentence $\ulcorner N[A] \urcorner$ is true in a model if and only if $\ulcorner A \urcorner$ is a logically valid first-order extensional sentence, i.e., a theorem of first-order extensional logic. From this premise it follows that, for every such sentence $\ulcorner A \urcorner$, the intensional sentence $\ulcorner \neg N[A] \urcorner$ is valid if and only if $\ulcorner A \urcorner$ is not a logically valid first-order extensional sentence. But the sentences $\ulcorner A \urcorner$ that are not logically valid first-order extensional sentences are not recursively enumerable, so the valid intensional sentences $\ulcorner \neg N[A] \urcorner$ are not recursively enumerable. Consequently, first-order intensional logic is incomplete. However, the premise would be based on an error. To see this, let $\ulcorner A \urcorner$ be the invalid first-order extensional sentence '$(\exists x)Fx$', and consider any model like that discussed a moment ago, wherein interpretation $\mathscr{I}$ assigns to 'F' the property of being self-identical, i.e., $[x = x]_x$. In such a model the intensional abstract '$[(\exists x)Fx]$' denotes $[(\exists x)x = x]$, i.e., the necessarily true proposition that something is self-identical. Therefore, in this model the first-order intensional sentence '$N[\exists x)Fx]$' would be *true*. However, according to the premise, this sentence should be *false* because the first-order extensional sentence '$(\exists x)Fx$' is not a logically valid sentence. So the premise does not hold, and thus the alleged incompleteness proof fails. Like the earlier erroneous premise, this erroneous premise seems to be based on a kind of generalized use/mention confusion, that the syntactic form of a linguistic expression of a proposition and the modal status of the proposition should always match up. And of course this is not so.

III: THE PROPOSITIONAL-FUNCTION THESIS, TYPE-FREE PREDICATION THEORIES, AND NONEXTENSIONALITY

In this final part we will take up a few sophisticated issues in property theory, specifically, the propositional-function thesis, type-free theories of the predication relation, and the proof of nonextensionality within such theories.

We have been defending the thesis that properties, relations, and propositions are required by our best comprehensive theory of the world. Part of the defense of the thesis consisted in the rejection of reductionist approaches which would have us believe that intensional entities can somehow be constructed from other primitive material. In compliance with this position we took PRPs at face value as irreducible entities and showed how they could be modeled within the framework of certain intensional algebras.

Formalizations of PRP theory within this framework run into some technical complexities in connection with the treatment of free and bound terms occurring in intensional abstracts. Many of these complexities seem initially to be avoidable by adopting the thesis that properties and propositional functions are structurally indiscernible. We do not mean by this that properties are reducible after all to a different realm of entities. It would be a grave mistake to construe the thesis of the structural indiscernibility as a strong identity thesis. Properties and propositional functions are not the same. But according to the thesis of structural indiscernibility, propositional functions can serve as an external model that displays the structural conditions that are imposed by the characteristic axioms of $T2$.

One aim of this section is to sketch a model for the consistent implementation of the structural propositional function thesis. The model we give in outline is a structural version of one variant of a predication theory of properties. Actually it constitutes an extension of Aczel's Frege Structures.[67] We thus depart from the policy we have adhered to in the main bulk of the paper, namely, to concentrate on problems that arise within the framework of theories of intensional abstracts. The simple reason for this departure is that we do not know how to construct a fine-grained functional model without relying on the expressive power provided by (some analogue of) the predication relation.

Given the many competing versions of predication theories of properties, we thought it appropriate not to present a formal syntax and an axiom system. We have even left open the question whether the model we indicate should be constructed using a classical or an intuitionistic metalanguage. That both approaches are possible is shown by Flagg and Myhill [1987] and by Aczel [1980], respectively. With the addition of the relevant stipulations our proposal could thus be made consistent with Feferman's [1984] observation that the arguments used to derive the paradoxes are already valid constructively.

To give the reader an idea of the reduction of complexity that can initially be achieved by working with a functional structure, let us briefly consider an illustrative example. The property of being believed by John to be a spy would be denoted by the following term of the formal language introduced above:

$$[B^2 j, [S^1 x]]_x.$$

The corresponding polynomial of the intensional algebra is as follows:

$$\mathrm{Pred}_1(\mathrm{Pred}_0([B^2 z, y]_{yz}, j), [S^1 x]_x).$$

In a suitable functional setting, by contrast, B^2 and S^1 would stand for a binary and a unary propositional function, respectively. If the functional setting satisfies a closure condition to the effect that every expression constructed in the expected way from variables, constants, and propositional functions stands for a propositional function, we are assured that the following is a complex propositional function: $B^2(j, S^1(x))$. This function sends an object b into that proposition that is the value of the binary function B^2 on the pair of arguments whose first component is the individual John and whose second component is the proposition that the function S^1 connects with the object b. The last step of the interpretation procedure within a functional framework would depend on the availability of a functional that maps every unary propositional function into a corresponding object. In a Frege Structure, lambda is a map that establishes the required association. Scott aptly summarizes the general idea behind the functional approach ". . . a formula with a free variable is a *mapping* from constants to the corresponding substitution instance. In this way we eliminate all fuss with variables in the formalization – the use of **lambda** does all the work behind the scenes".[68]

As we have noted in Part I, the iterative conception of set is ultimately analysed in terms of membership. This conception, which was introduced by Cantor and formalized by Zermelo, has to be distinguished from the Fregean logical notion of a class that is ultimately to be analyzed in terms of predication. This logical notion of class is usually thought to be incoherent because of the derivability of Russell's paradox within a system that contains an unrestricted comprehension principle. The logical notion of class can be explicated somewhat more precisely as follows. To any formula $\ulcorner A(x) \urcorner$ that has 'x' as its only free variable there corresponds a class $\{x \mid A(x)\}$ such that for any object b the proposition that $b \in \{x \mid A(x)\}$ and the proposition that $A(b)$ are equivalent.[69] If we assume $\ulcorner A(x) \urcorner$ to be the Russell formula '$x \notin x$' and let 'R' denote the class $\{x \mid x \notin x\}$, then the proposition that $R \in R$ is equivalent to the proposition that $R \notin R$.

There have been many attempts to formulate type-free theories that allow the sort of unlimited self-reference implied by a general abstraction principle without suffering from the devastating effect of Russell's paradox. The story is a long and intricate one, and we are not going to recount it here. Feferman [1984] contains some very useful historical remarks. According to these remarks and other written records, it is Fitch [1948] who is to be credited for the renewed interest in type-free theories.

Our idea is to formulate a type-free theory of properties in which the notion of property corresponds closely to the Fregean logical notion of class, except of course that the identity conditions on properties will be intensional. The particular version of a type-free theory of properties on which we will base our discussion in this section introduces a primitive notion of proposition. Intuitively, propositions constitute the category of objects that can be true or false. If such a propositional formulation of the logical class concept does not contain among its theorems the assertion that $R \quad R$ is a proposition, no Russell paradox cna be derived from the axiom schema of full abstraction. This is the style of resolution we will pursue here.

It is well known that Frege espoused an extensionalist position according to which concepts differ only in so far as their extensions are different. This seems to be a basic mistake. We commented above on the intuitive justification of a theory of intensional entities that rejects the principle of extensionality. What we attempt now is to back this intuitive justification by something approximating a proof. Given full abstraction, the principle of extensionality assumes the following form:

$$(\forall x)(A(x) \leftrightarrow B(x)) \rightarrow \{x \mid A(x)\} \quad = \quad \{x \mid B(x)\}$$

(We use $\ulcorner\{x \mid A(x)\}\urcorner$ as a neutral abstract; our purpose is to show that, within a predication theory with an unrestricted abstraction principle, these abstracts *must* denote intensional entities, contrary to what Frege thought.) Let Q be the object $\{x \mid x \in x = \text{FALSE}\}$. $Q \in Q$ certainly should be a proposition because, given full abstraction, it is equivalent to $Q \in Q =$ FALSE, and we have seen above why it is highly commendable to regard identity as a transcategorial relation that can be predicated of any two objects, always yielding a proposition. Now given full abstraction and extensionality, $Q \in Q$ would be identical to the proposition $Q \in Q =$ FALSE. In this case, however, $Q \in Q$ can be neither a true proposition nor a false proposition. $Q \in Q$ cannot be a true proposition because, by this last identity, it follows that $Q \in Q =$ FALSE would be a true proposition. But, in this case, it follows by the principle of full abstraction that $Q \in Q$ is identical to FALSE, and so $Q \in Q$ is a false proposition. On the other hand, $Q \in Q$ cannot be a false proposition, for the principle of extensionality implies that there is only one false proposition, namely, FALSE. So if $Q \in Q$ were a false proposition, it would be identical to FALSE. But, in this case, the principle of full abstraction implies that $Q \in Q =$ FALSE would be a true proposition. But since this proposition is identical to $Q \in Q$, it follows that $Q \in Q$ would be a true proposition.

This argument shows that the principles of full abstraction and of extensionality, combined with the assumption that identity is a transcategorial relation that always yields a proposition when predicated of any pair of objects, leads to contradiction.[70] Since the unlimited self-reference embodied in the principle of full abstraction is part and parcel of a theory of properties, relations, and propositions and since the transcategorial characteristic of identity cannot be given up in a type-free framework, it is the extensionality principle that is at fault here. Our conclusion is that intensionality is a necessary feature of a type-free system that allows for unlimited self-reference in the form of an unrestricted abstraction principle. (Notice, incidentally, that our original derivation of Russell's paradox did not invoke any extensionalist claim. That contradiction cannot be blocked by relying on some high degree of fine-grainedness for the particular proposition presumably denoted by '$R \in R$'.)

If a predication theory of properties is committed to an intensionalist stance, we were well advised to assume the same attitude towards the meaning of intensional abstracts within the weaker framework constructed earlier. Can we provide a similar justification for our decision to regard PRPs as particular types of objects as against, e.g., certain kinds of propositional functions (where propositions would then be treated as constant propositional functions)? The debate around the strict propositional-function thesis – that is, the thesis that properties and propositional functions are identical – has been obscured by a number of confusions. We address some of them in the following discussion in order to illustrate our point of view and to forestall certain objections that might be engendered by these issues.

Frege is even better known for his ideosyncratic ontology than for his extensionalist position. He divides the world into objects and functions and attempts to impute an understanding of the latter notion by resorting to metaphors of unsaturatedness and completability. We think it is perfectly legitimate to speak of "incomplete objects" and other entities of such ilk if one wants to impart an understanding of a category otherwise proved to be ineliminable. One such well known proof runs as follows. Suppose that Fregean concepts are objects and can be designated by proper names. That an object a falls under (or is an instance of) the concept denoted by 'P' is to be indicated by '$P(a)$'. We have formed a new predicate by appending a pair of parentheses to the name of the object that is by supposition identical with the original concept. In sentences such as '$\neg P(P)$', '$\neg Q(Q)$', etc., we can discern a common pattern of predication. Assuming this pattern stands for a concept, we denote it with the proper name 'S' and construct a new predicate '$S(\ )$'. Does the object referred to by 'S' fall under the concept expressed by '$S(\ )$'? According to its definition, '$S(S)$' is true if and only if '$\neg S(S)$' is true, contradiction. A Fregean would conclude from this: "To escape this absurdity, we must deny that any concept is an object or can have a proper name; and the two sorts of quantification that answer to proper names and to predicates must be strictly distinguished".[71] Is there no other way out?

This question is connected with the problem of empty proper names and the acceptability of non-definite concepts, where a concept P is non-definite if the following assertion fails:

$$(\forall x)(P(x) \vee \neg P(x)).$$

We leave open the question whether Frege changed his mind on these issues under the weight of Russell's paradox. At one time he seems to have entertained the idea of allowing denotationless nominalizations into his system. Realists will sense a tension between their basic world-view and the notion of a non-definite concept. The same tension exists between a realist metaphysics and the primitive notion of proposition we mentioned above. But since, to repeat, it is highly unlikely that in the foreseeable future we are really going to understand the pathology of the paradoxes or that we are really going to know what form an ideal resolution of them should take, we feel justified in pursuing the way out indicated by the primitive notion of proposition.

If at the moment there is no proof that both objects and functions have to be counted among the primitive ontological categories, is it then at least technically feasible to work within a framework that models properties as propositional functions? It is possible, and we will show in outline how this can be done. But first we have to explain what we understand by 'technically feasible'.

The coarse-grained and fine-grained algebras above were presented as certain sets. We took for granted the necessary amount of set-theoretical machinery that enabled us to state the definitions and carry through the completeness proof. Nobody will want to identify properties, relations, or propositions with sets. For the same reason it does not make any sense to identify the operation Pred_0 with a particular single-valued set of ordered pairs with respect to every algebraic model. The set-theoretic definition of validity fulfills a useful role, nevertheless. Set-theoretic semantics can serve as a guide in our investigation of the realm of PRPs. This was true in the construction of the intensional algebras in Part II, and it is true in the investigation of type-free theories of predication. In the end, however, the set-theoretic ladder that we climbed has to be kicked away and supplanted by an intrinsic semantics stated wholly in terms of an applied theory of properties. According to the same line of reasoning, it would be completely implausible, for example, to equate sensible properties with functions. Propositional functions are to be invoked for no other purpose than to serve as an external criterion for such formal characteristics as soundness and completeness of a theory of properties. The two types of external modelings cannot claim any advantage over the other from this perspective.

The relevant literature contains an argument that purports to show that propositional functions do not have the right structure for being of much use as a reliable external guide in the realm of PRPs.[72] Stripped of all irrelevant details, the argument boils down to the following steps. The logic of propositional attitudes demands that very fine-grained distinctions be made among propositions. These distinctions are thought to be analogous to the syntactic pattern of expressions that convey propositional content. Let 'R' be a two-place predicate and a and b two different objects. Comprehension allows us to introduce by stipulation two properties $P = \{x \mid R(x, b)\}$ and $Q = \{y \mid R(a, y)\}$. (As before, we are using $\ulcorner\{x \mid A(x)\}\urcorner$ as a neutral kind of abstract.) If properties are indeed identical to propositional functions and if the general characteristics of functions find an adequate expression in the lambda-calculus, then the following equations are instances of the rule of β-conversion and of the intersubstitutivity of definitional identities:

$$P(a) \;=\; \{x \mid R(x, b)\}(a) \;=\; R(a, b) \;=$$
$$\{y \mid R(a, y)\}(b) \;=\; Q(b).$$

To illustrate the fact that these equations model a far too coarse-grained notion of proposition, let R express the relation of following and a and b stand for Jane Fonda and Rajneesh, respectively. Then we can stipulate that

> Being an x such that x rajneeshes = being an x such that x follows Rajneesh. $(= P)$

and

> Being an x such that x fondalees = being an x such that Jane Fonda follows x. $(= Q)$

From these stipulations and the propositional-function thesis we have by β-conversion:

> The proposition that Jane Fonda rajneeshes = the proposition that Rajneesh fondalees.

But this seems wrong. When a person consciously and explicitly thinks that Jane Fonda rajneeshes must that person be consciously and explicitly thinking that Rajneesh fondalees? It certainly does not seem so. The order of predication seems to be a relevant factor of propositional identity.

The same point can be made perhaps even more palatable if we restate the example within a quasi-categorial framework where every element can be required to be either a projection or a constant function of arity $n \geqslant 0$ or a definable function in some suitably restricted sense. Definability is to be understood as closure under application, functional composition of the basic set of constant and projection functions, and an additional set of functionals which counts lambda among its members and where lambda is defined as a map that sends elements of the space of unary functions into the universe of objects. When these closure conditions are made precise, it is not very difficult to check that such a structure of functions is nothing else but a model of the lambda-calculus.[73]

If we interpret the relation R of our example as a two-place function, a and b as zero-place constant functions, and the variables as one-place projections p, we obtain the equations as follows:

$$P \circ a = R \circ (p, b) \circ a = R \circ (a, b) =$$
$$R \circ (a, p) \circ b = Q \circ a$$

where $\circ$ indicates functional composition[74] such that

$$f \circ (g_1, \ldots, g_n)(a_1, \ldots, a_m) =$$
$$f(g_1(a_1, \ldots, a_m), \ldots, g_n(a_1, \ldots, a_m))$$

for f an n-place and $g_i m$-place functions. Something like functional composition cannot be separated from the very notion of a function. Is it logically possible for a function to behave in a way that violates the principle of functional composition we just stated?

It is, of course, possible to construct "intensional" models in which the order of execution of a complex operation determines its meaning. In such models the "subject-first" and the "object-first" R could be distinguished in the sense that, even though for all pairs of objects both operations would have the same value, they are nevertheless not the same:

$$\neg[(\forall z)(\text{“Subject-first”}\ R(z) = \text{“Object-first”}\ R(z)) \rightarrow$$
$$\{z \mid \text{“Subject-first”}\ R(z)\} = \{z \mid \text{“Object-first”}\ R(z)\}].$$

This intensional notion of a function that incorporates aspects of sequential behavior is of no help with respect to our problem of the internal structure of propositions in propositional attitude contexts. If $P(a)$ and $Q(b)$ are the same proposition, then the fact that the dynamic behavior of P is to be analyzed in terms of "Object-first" R and the dynamic behavior of Q is to be analyzed in terms of "Subject-first" R gives us no handle on the resulting proposition unless these sequential stages could be discerned as parts of the final value. But this is denied by the presupposition that the input-output behavior of the two sequential ingredients is the same.

Proponents of the propositional-function thesis seem to be caught in a dilemma. On the one hand, they have an elegant solution to the worries that have plagued those scholars who tried to reconstruct intensional entities out of the primitive elements provided by the possible-worlds approach. They have the option of relying upon a principle of individuation for intensions that is finer than necessary equivalence, the latter notion being reduced to identity of truth value on all possible worlds in the case of propositions. It is certainly true that anything that is triangular in a possible world is trilateral in that same world. This does not imply that for any object b the proposition that b is triangular is the same proposition as that b is trilateral. When a person consciously and explicitly thinks that b is triangular *must* that person be conciously and explicitly thinking that b is trilateral? It certainly does not seem so. The propositional-function thesis does not entail the unwanted identifications implicit in the possible-worlds approach. Thus, on this score, there is no objection against an external semantics that models properties as propositional functions and interprets the operation of predication as functional application.

On the other hand, the propositional-function thesis still seems to imply a sort of unwanted identification, namely, the unwanted identification illustrated in the rajneeshe/fondalee example. Moreover, some of these unwanted identifications that involve an inverted sequence of predications do not even depend upon the availability of a functional that permits one to assert the existence of an object that is specified (or projected or comprehended) by a complex predicate $\ulcorner A(x) \urcorner$. As we have seen, the same problem arises within a functional structure that treats the predication operation as functional composition instead of first forming a new object corresponding to the original one-place propositional function and then combining this new object and its argument with the help of the binary application operation. Both lambda-abstraction and functional composition have a specific intended meaning that is enshrined in the principle of β-conversion and the principle of functional composition that we have stated above. Giving up these basic principles or distorting the intended meaning of either lambda-abstraction or functional composition are but two sides of the same coin. And these two principles are operative in the semantic conflation of syntactically distinct expressions. Since we see no way of tampering with either lambda-abstraction or functional composition and their attending principles without violating in essential respects the basic intuition that guides our understanding of the notion of function, we have to admit that the propositional-function thesis is mistaken.

Or is it? As we have put it, the thesis is that properties can be externally modeled by propositional functions. In our discussion we have made the assumption that this "model-theoretic" decision would entail the identification – again in the sense of an external criterion – of the operation of predication with functional application. The two identificatory decisions together lead to the counterintuitive conclusion that Jane Fonda's rajneeshing and Rajneesh's fondaleeing stand for the same thought. But would it be possible to split the propositional-function thesis into two separate claims and retain the first half of the thesis of external modeling while rejecting the second half? The answer, as we have indicated already, is that a manoeuvre like this can be carried out technically. We will show one way to do this in a moment. There are two purposes in doing this. The first is to construct a "predication structure" over certain functional models; such a structure can serve as a model of PRPs. The second is to provide an illustration of how there could indeed be a distinction between functional application and predication.

To motivate the construction, it is perhaps helpful to look at the problem from a syntactic point of view. If one tries to repeat the story about fondaleeling and rajneeshing within the confines of the expressive power provided by the first-order language L_ω for intensional abstraction, one stumbles upon the problem of how to indicate the predication relation that is to hold between the abstracts $[Rx, b]_x$ and $[Ra, y]_y$ on the one hand and their respective arguments on the other. Abstracts are names, and the result of juxtaposing an abstract and an individual constant does not form a well-formed expression. Would a more liberal syntax that imposed no type restrictions on the concatenation of expressions bring granularity troubles on the semantic level in its wake? Not necessarily. As long as such liberal theories contain no principles to the effect that schemata of the form $\ulcorner[A(x)]_x(t)\urcorner$ have the same meaning as $\ulcorner[A(t)]\urcorner$, we still steer clear of the troubled waters of intensional individuation problems. What this amounts to semantically is that predication and functional application should be different functionals. Our construction will show that there can indeed be a functional model with distinct functionals like this.

Specifically, we are looking for a model that contains a functional Pred that sends pairs of propositional functions and objects into propositions and that satisfies the following principles:

$$\text{Pred}(P, a) = \text{Pred}(Q, b) \rightarrow (P = Q \ \&\ a = b).$$ [75]

$$\text{Pred}(P, a) \leftrightarrow P(a).$$

As we have seen, the justification for a functional satisfying these principles derives from the requirement of fine-grainedness that is generated by propositional-attitude contexts.[76]

The construction of Pred makes use of a standard inductive definability approach. The technique goes back to Fitch as we mentioned above. Closely related ideas have been used by Feferman [1975], Scott [1975], and Aczel [1980].

Let M be a model for the lambda calculus. For definiteness we assume that it belongs to the class of lambda-*systems* or lambda-*families*. Within the model we can define elements like $=$, N, $\neg$, $\&$, $\vee$, $\rightarrow$, $\forall$, $\exists$ *and* Pred in the following way:

$$
\begin{array}{llll}
= & =_{\mathrm{df}} \langle 0, x, y\rangle & N & =_{\mathrm{df}} \langle 1, x\rangle \\
\neg & =_{\mathrm{df}} \langle 2, x\rangle & \& & =_{\mathrm{df}} \langle 3, x, y\rangle \\
\vee & =_{\mathrm{df}} \langle 4, x, y\rangle & \rightarrow & =_{\mathrm{df}} \langle 5, x, y\rangle \\
\forall & =_{\mathrm{df}} \langle 6, \lambda x(fx)\rangle & \exists & =_{\mathrm{df}} \langle 7, \lambda x(fx)\rangle \\
\mathrm{Pred} & =_{\mathrm{df}} \langle 8, \lambda x(fx), y\rangle & &
\end{array}
$$

Since we are working within a lambda-system, the closure conditions ensure that every element actually denotes a function of the model. Furthermore, these functions enjoy a certain independence property. In the special case of Pred this means that the following holds:

$$\mathrm{Pred}(P, a) \;=\; \mathrm{Pred}(Q, b) \rightarrow (P = Q \;\&\; a = b).$$

Based on the above definitions two subsets $\mathscr{T}$, the set of truths, and $\mathscr{P}$, the set of propositions, can be obtained as the least fixed point of a monotone operator on the lambda-system. The clauses of the monotone operator are constituted by a family of $\mathscr{T}$-positive and $\mathscr{P}$-positive formulas that express the expected characterics of the defined elements. Except for Pred these clauses (with minor variations) can be found in the cited papers by Scott, Feferman, Aczel, or Flagg-Myhill. The clause for Pred can informally be stated as follows:

PREDICATION. *If f is a propositional function (i.e., if f is a function all of whose values are propositions), then Pred(f, a) is a proposition such that: Pred(f, a) is true iff $f(a)$ is true.*

From the predication schema it becomes clear that, semantically, the new functional is nothing else but "proper-name" quantification.[77]

To emphassize the main goal of our discussion of propositional functions. It has not been our objective to provide an argument for the propositional-function thesis in its strong form. We wanted rather to make clear that the functional approach, as an external modeling technique, is flexible enough to accommodate any degree of structural discrimination that is deemed necessary.

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NOTES

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[2] We do not rule out the possibility of an argument to the effect that intensional entities are required for acceptable definitions of such notions as evidence (data), explanation, necessity, causation, law of nature, simplicity, and so forth. (See Section 1 of Bealer [1986].) The worry is that opponents of intensional entities might maintain that we should simply take these notions as primitives rather than trying to define them at the cost of adding intensional entities to our ontology. In the text we are seeking an argument that cannot be rebutted in this fashion. The idea is to show that any acceptable comprehensive theory would be self-defeating unless it makes use of a background intensional logic; but such use of intensional logic generates an ontological commitment to intensional entities. The conclusion is that our opponents cannot consistently deny the existence of intensional entities unless their comprehensive theory is unacceptable by their own standards. This style of argument derives from George Myro [1981]. It is developed further in Section 1 of Bealer [1986] and in Section 7 of Bealer [1988].

[3] We believe that any acceptable comprehensive theory must include an intensional semantics for natural language. But in a debate with a diehard opponent of intensional entities, this thesis must be established; it may not be assumed. The way to trap diehard opponents of intensional entities is to catch them in a self-defeat. In their own comprehensive theories opponents of intensional entities must have an account of the acceptability of their own theories. Such account, we will argue, must make use of a background intensional logic.

[4] Since one is justified in believing that one's acceptable comprehensive theory is true, it follows that one is justified in believing that intensional entities exist. Anyone who denies that intensional entities exist does so at the price of not having an acceptable comprehensive theory.

[5] This theme is developed in Chapter 9 of *Quality and Concept*.

[6] Proponents of free logic believe that existential-generalization failures in extensional logic are not confined to extraordinary cases like this one; they hold that even the most ordinary extensional instances of this rule are, strictly speaking, not valid. So free logicians would agree that failure of existential generalization cannot be used as a criterion for distinguishing intensional logic from extensional logic.

[7] Some of these arguments are fuller presentations of arguments already given in Chapter 1 of *Quality and Concept*. It is hoped that these fuller presentations will help to answer certain questions that critics have raised.

[8] In a syntactically first-order language, neither sentences nor predicates are allowed as singular terms; accordingly, one is not allowed to replace them with quantifiable variables.

[9] In a syntactically higher-order language, in contrast to a first-order language, there are variables whose substitutends are predicates and/or sentences.

[10] For other counts against the higher-order approach, see Section 12 below.

[11] We prefer treating predicate adverbs right within our sort of standard logical syntax. For example, 'x is running quickly' can be represented (very roughly) along the following lines: '$(\exists y)(y$ is a running & y is quick & x is doing $y)$'. This approach requires adding no new syntactic categories to our logical syntax; and, semantically, it requires no special ontological

items beyond PRPs, which as we will see are required to deal with 'that'-clauses and gerundive and infinitive phrases. Incidentally, once we are able to represent 'that'-clauses, sentential adverbs are easy to treat. For example, 'Necessarily, $7 < 9$' can be treated as a transformation from '$N^1[7 < 9]$', where '$[7 < 9]$' is a singular term corresponding to the 'that'-clause 'that $7 < 9$'.

[12] This notation is introduced by Quine for somewhat similar purposes (Quine [1960], §35), and it is used throughout *Quality and Concept*. For the moment we leave open what semantical significance the bracket notation shall have, and the possibility of indirectly defining the bracket notion shall also be left open here.

[13] Advocates of free logic might claim that argument (II) is not strictly speaking valid unless it is supplemented with the premise ⌜That A is something⌝ or ⌜There is something that is identical to that A⌝. We need not suppose otherwise. To accommodate the free logician, we would supplement (II′) with the premise ⌜$(\exists x)x = [A]$⌝. The philosophical point is this. We are, at the present stage, arguing merely that 'that'-clauses should be treated as singular terms. This treatment is compatible with free logic. The question of whether 'that'-clauses actually refer to anything and, accordingly, whether they have ontological significance is a separate question. Our argument that they do is given in Section 5.

So far we have established only that 'that'-clauses are singular terms. This thesis implies, for example, that 'that'-clauses cannot be treated in the way explitives are treated, that is, in the way 'it' in 'it is raining' is treated.

We should also make it clear that, strictly speaking, our treatment of 'that'-clauses in this section is consistent with the higher-order theory that entire sentences can occur as singular terms, for nothing we have said prevents a higher-order theorist from treating, say, ⌜$[A] = [B]$⌝ as a notational variant of ⌜$A = B$⌝. However, as we have already indicated, this higher-order theory has the unacceptable consequence that, for example, the grammatical nonsense 'The cat is on the mat $= 7 < 9$' is a well-formed sentence. For this and many other reasons (see Section 12), we do not advocate the higher-order theory.

[14] The possibility of externally quantifiable occurrences of variables is not allowed in Quine's original bracket notation.

[15] See Alonzo Church [1951].

[16] See Sections 1 and 6 of Bealer [1988] for further discussion of self-constituency.

[17] This regress could be avoided if a primitive notation ⌜$\langle v_i, v_j\rangle$⌝ for sequences were adjoined to the language. The problem with this approach is that one should nevertheless be able to say, specifically, what sort of entities are semantically correlated with these singular terms ⌜$\langle v_i, v_j\rangle$⌝. On the one hand, perhaps these entities are sets. But if so, this leads one right back to the ontological excess and theoretical disunity we found in the set-theoretical version of the sequence theory of quantifying-in. On the other hand, perhaps the singular terms ⌜$\langle v_i, v_j\rangle$⌝ are semantically correlated with properties. But if so, with what sort of properties? It would be unacceptably mysterious if you could not say which. However, if you confine yourself to a language fitted out with just *de dicto* intensional abstracts and an apparatus for expressing the predication relation, you get caught in the regress mentioned in the text; you can never complete your answer to the question. The only alternative is to admit into your language intensional abstracts that contain externally quantifiable variables (or some other apparatus with comparable expressive power). In this framework, it is then easy to specify the sort of properties with which the terms ⌜$\langle v_i, v_j\rangle$⌝ are semantically correlated. They are *de re* properties like $[x = [y = v_i]_y \vee x = [y = v_i \vee y = v_j]_y]_x$.

Incidentally, you might favor treating $\ulcorner\langle v_i, v_j\rangle\urcorner$ as an *indefinite description*, perhaps introduced by contextual definition in terms of the predication relation. (See p. 83, *Quality and Concept*, for an illustration.) However, this move would not entitle you to avoid the question of the general sort of entity with which these expressions are semantically correlated. For property theorists, there seems to be no satisfactory answer to this question that does not invoke *de re* properties as unreducible entities.

[18] If $\ulcorner B\urcorner$ has no externally quantifiable individual variables, then x directly believes that B if and only if x self-ascribes the property of being such that B. In symbols: $A^2 x, [Fv]_v$.

[19] In a fully developed solution of the substitutivity problems involving indexicals, we would need to mark a distinction between *convictions in acquaintance* and *cognitive commitments*. (See Section 29 of *Quality and Concept* for discussion of this distinction.) We do not rule out the possibility that the notion of conviction in acquaintance might be elucidated in terms of the notion of self-ascription. The point is that such elucidation does not help to win the *ontological* point Chisholm favors, nor does it support a *logical* theory that eliminates externally quantifiable variables from 'that'-clauses, gerundive phrases, and infinitive phrases. Although the self-ascription theory might be able to make a contribution to epistemology and philosophical psychology, it appears to have little to contribute to metaphysics and logical theory.

[20] D. Davidson [1964].

[21] Indeed, given the radical theory that no atomic intensional sentence is true, it is impossible for there to be any *sound* argument for the radical theorists' conclusion about the acceptability of their theory. To see why, consider any argument whose conclusion is (the proposition expressed by) 'It is acceptable that no atomic sentence in the standard intensional idiom is true'. Since this sentence is itself an atomic sentence in the standard intensional idiom, it is not true. Therefore, (the proposition expressed by) this sentence cannot follow validly from true premises.

Another observation is in order. If (a proposition expressed by) a sentence implies so directly that it is itself not true, then (the proposition expressed by) such a sentence is not acceptable either. It follows that the radical theorists' conclusion about the acceptability of their theory is not acceptable.

[22] To be relevantly like the standard idiom, the new idiom must have systematic relations to such matters as: the simplest explanation of the evidence, the simplest coherent systematization of one's beliefs, a reliably caused body of beliefs, and so forth. However, the standard idioms for discussing these matters (explanation, evidence, belief, causation, etc.) are intensional. For example, 'It is evident to me that I am having sense experiences', 'That I have sense experiences is explained by physiology and psychology', 'I believe that so and so', 'That my brain is in such and such state causes me to have these sense experiences', 'It is causally necessary that, if my brain is in such and such state, I have these sense experiences', etc. (Of course, there are other standard ways of talking about explanation, evidence, belief, and causation. But they have systematic relations to these standard intensional idioms. If they did not, they could, for all we know, be just some new idiom whose relevance to explanation, evidence, belief, and causation would be in question.) Because the standard idioms are intensional, the radical theorist must deem our ordinary uses of it as, strictly speaking, false. For this reason, the radical theorists have no choice but to introduce some new, nonintensional idioms for discussing explanation, evidence, belief, and causation, and they must be able to show that these new idioms are relevantly like the standard intensional idioms. But

this can be done only by showing that the new idioms are relevantly like the standard ones in meaning, purpose, or function (or something relevantly like meaning, purpose, or function).

[23] To show that the meaning of an expression in a new idiom is *relevantly like* the meaning of an expression in a standard idiom, one has four options. First, one can show an actual meaning identity. But we have seen that statements of meaning identities have systematic relations to statements of intensional identities: the meaning of $\ulcorner A \urcorner$ = the meaning of $\ulcorner B \urcorner$ if and only if that A = that B. So intensionality enters in here. Second, one can show that the two expressions are definitionally related. However, the standard devices for indicating definitional relationships are intensional, for example: 'iff$_{df}$', '$=_{df}$', 'It is definitionally true that', and so forth. So this option does not lead to the elimination of intensionality. Third, one can show that the purpose or function served by the meanings of the two expresions is the same. However, our standard idiom for discussing purpose and function is also intensional. For example. $\ulcorner$The purpose of F-ing is to $G\urcorner$, $\ulcorner$The function of F-ing is to $G\urcorner$, etc. contain gerundive and infinitive phrases, which are intensional abstracts. Fourth, one can show that the two meanings are inherently similar. However, to show that two items are inherently similar, one must show that they share fundamental qualities and relations. But a general theory of fundamental qualities and relations is already a property theory; indeed such a theory is, on its own, sufficient for the construction of intensional logic. (See Chapter 8 of *Quality and Concept* for an elaboration of this argument. See also David Lewis [1983].) On all four options, therefore, intensionality – or a framework that implies it – plays a central role.

[24] To avoid this self-defeat, the radical theorists might try to define meaning (or something relevantly like it) in terms of Gricean intentions, or they might try to define purpose or function (or something relevantly like them) in terms of intention or causation. However, our standard idioms for giving definitions are intensional, for example: 'iff$_{df}$', '$=_{df}$', 'It is definitionally true that', and so forth. Moreover, our standard idioms for talking about intention and causation are also intensional: $\ulcorner x$ intends to $F\urcorner$, $\ulcorner x$'s F-ing caused y to $G\urcorner$, $\ulcorner$It is causally necessary that if Fx then $Gy\urcorner$, and so forth. So intensionality is not avoided. (Of course, there is a standard extensional idiom for talking about causation. However, it bears systemantic relationships to the standard intensional idiom. If the radical theorists do not affirm these systematic relationships, they are obliged to show either that their extensional idiom is still the standard one or that, if it is nonstandard, it is relevantly like the standard one.)

If, to avoid this intensionality, the radical theorists dropped these standard intensional idioms for definition, intention, and causation and if they put some new, nonstandard idioms in their place, they would then be obliged to show that the new idioms are relevantly like the standard intensional ones. But this can be done only by showing that they are relevantly like the standard ones in meaning, purpose, or function (or something relevantly like meaning, purpose, or function). To do this, the radical theorists are pushed right back into the problem.

[25] Of course, a radical theorist might simply present us with some novel scheme for determining acceptability. (This is pretty much what is done in Paul Churchland [1979] and [1981] and in Patrica Churchland [1986]. The problem is that there is no reason to pay any attention to clains made within the new scheme unless this scheme can be shown to be relevantly similar to the standard scheme for determining acceptability. We have seen that, to do this, either the proponents of the new scheme must use some standard intensional idiom, or they

must use an alternate idiom that can be shown to be relevantly similar to a standard one. And we have seen that latter option leads to a vicious regress unless at some stage the proponents of the new scheme invoke some standard intensional idiom to stop it. If they do not do this and if they persist in holding their position, they end up in groundless dogmatism. They would be like people with a magic device that purports to tell them when a candidate comprehensive theory is acceptable; the device does this by flashing the word 'acceptable' or 'not acceptable' when (the linguistic expression of) various candidate theories are typed in. Regarding the acceptability of their own comprehensive theory (which includes a theory of acceptability based on the device), they declare that their theory is acceptable on the grounds that the device has flashed the word 'acceptable' when they type in (a linguistic expression of) their theory. Plainly, their theory is not acceptable, nor is it relevantly like an acceptable theory.

[26] Meinongians might try to avoid this conclusion by invoking their (alleged) distinction between being and existence. However, to develop their views formally, Meinongians already admit the ontology of PRPs and would therefore have no good reason not to accept the natural style of PRP semantics for intensional abstracts that we are defending in the text.

[27] There is another problem with identifying linguistic expressions with linguistic tokens, namely, that it does not provide enough items for a general theory of language. A general theory of language must hold for the infinitely many expressions in a language, not just for the finitely many expressions that happen actually to be uttered or written by speakers. Because there are only finitely many (actual) linguistic tokens, tokens cannot play the role of linguistic expressions in a general theory of language. One way of trying to overcome this cardinality problem is to identify linguistic expressions with regions of physical space. Another way to overcome the problem is to identify linguistic expressions with certain set-theoretical constructs whose ultimate elements are (actual) linguistic tokens. (Quine, for example, identifies a primitive linguistic expression 'p' with the set of actual tokens of 'p', the primitive linguistic expression '$-$' with the set of actual tokens of '$-$', and the complex expression '$-p$' with the ordered set consisting of the set of actual tokens of '$-$' and the set of actual tokens of 'p'.) Three observations are in order. First, both the regions-of-physical-space treatment and the set-theoretical treatment run into the problem of contingent existence, which we are discussing in the text. For neither regions of physical space nor sets that depend on linguistic tokens *necessarily* exist. Second, regions of physical space are particularly implausible candidates for being the primary bearers of truth, necessity, logical truth, etc. and the primary objects of mental representation, explanation, etc. How, for example, do regions of space succeed in *representing* things in the world? (A kindred problem besets the Quinean alternative. For more on this problem of explaining representation, see the discussion that follows shortly in the text.) Third, the set-theoretical treatment requires positing two distinct ontological categories – particulars and sets. From the point of view of pure ontological economy, this is no better than positing the two categories of particulars and properties. At the same time, the latter ontology has a clear intuitive advantage: it enables one to adopt the intuitive theory that linguistic expressions are just shapes or sound types. For shapes and sound types are properties *par excellence*.

[28] R. Carnap [1947].

[29] A. Church [1950]. For expository convenience, we have renumbered the sentences mentioned in Church's argument.

[30] These "sets" would also be non-well-founded. See section 11 for a critique of treatments of intensional logic that posit non-well-founded "sets".
[31] This theme is developed in Section 42, "Realism and Representationalism", in *Quality and Concept.*
[32] For more on this, see Section 42, *Quality and Concept.*
[33] On the Kripke-Putnam view, names of natural kinds are introduced by means of "reference-fixing descriptions" that apply only contingently to their bearers. Such descriptions therefore do not qualify as definitions and, hence, cannot double as "constructions" of these fundamental properties from intensions that are "given" in our earlier mental activity.
[34] Another problem with this move concerns quantifying-in. Suppose, for the sake of argument, that $\ulcorner$Possibly $B\urcorner$ is not equivalent to $\ulcorner$It is possible that $B\urcorner$. Nevertheless, $\ulcorner\Diamond(\exists x)(F^2x, [A_i] \,\&\, LT^1[A_i])\urcorner$ intuitively entails $\ulcorner(\exists u)(u = [A_i] \,\&\, \Diamond(\exists x)(F^2x, u \,\&\, LT^2u)]\urcorner$. If this is right, the primitive-operator modalizing strategy does not even begin to avoid the ontological commitment to the intensional entity that A_i.
[35] This sort of unified intensional logic is needed to formulate a general epistemological account of why we are justified in our modal beliefs. For example, an account of what would make a person justified in believing that possibly A would go by way of an account of what "akes a person justified in believing that a proposition is possible, and then it would show that the proposition that A has these features.
[36] Soames [1985], Salmon [1986].
[37] Soames, *ibid.*, discusses some of these.
[38] Specifically, one needs the notions of quality and connection (i.e., the notions of "natural property" and "natural relation"). See Chapter 8, *Quality and Concept*, and David Lewis [1983]. More will be said about this topic at the close of this section.
[39] This argument is developed in Section 7 of Bealer [1988].
[40] Further support for the existence of self-embeddable properties and relations can be extrapolated from the arguments Kripke gives against Tarski's infinite hierarchy of distinct truth concepts for English (Sections 1 and 2, Kripke [1975]).
[41] This is not quite accurate. In situation semantics a distinction is made between *basic* properties and relations, on the one hand, and *complex* properties and relations, on the other hand. Basic properties and relations are treated as unreduced entities. However, complex properties and relations are reduced to certain kinds of sets (dubbed "event types"). As a result, self-embeddable complex properties and relations create a difficulty for situation semantics that is fully analogous to the self-embeddability problem in possible-worlds semantics that we have been discussing. Furthermore, isn't it odd that in situation semantics some properties and relations are supposed to be sets whereas others belong to an entirely different ontological category? This sort of disunity is undesirable even in reductionistic theories.
[42] See Jon Barwise [1985].
[43] Strictly speaking, the reduction is more complicated in situation semantics. But the complications do not affect the philosophical issue we are discussing, so it is convenient to suppress them.
[44] Nino Cocchiarella [1985] has claimed that this kind of first-order intensional logic can be shown to be incomplete if the semantics is modified only slightly. However, his argument is based on an elementary technical error. When the semantics is modified in the way Cocchiarella suggests, completeness still can be shown. For a discussion of this issue, see Part II, Section 4.

[45] We use 'linguistic subject' and 'linguistic predicate' to contrast with 'ontological subject' and 'ontological predicate'. Our use comes close to Strawson's use of 'logical subject' and 'logical predicate'; see Chapter 8 of his *Individuals*, for example.

[46] Linguists have assembled independent syntactic evidence for the existence of a primitive copula in English. See, for example, Williams [1980] and Higginbotham [1984].

[47] See p. 215 in Mates [1950].

[48] In the general case,

$$\mathbf{the}([F^m(u_1, \ldots, u_m)]_{u_1 \ldots u_m}, [G^n(v_1, \ldots, v_n)]_{v_1 \ldots v_n}) =$$
$$= [G^n(v_1, \ldots, v_{n-1}, (\iota u_1)(F^m(u_1, \ldots, u_m))]_{v_1 \ldots v_n u_2 \ldots u_m}.$$

and

$$\langle v_1, \ldots, v_{n-1}, u_2, \ldots, u_m \rangle \in H(\mathbf{the}(x, y)) \textit{ iff}$$
$$(\exists! u_1)(\langle u_1, u_2, \ldots, u_m \rangle \in H(x)\ \&$$
$$(\forall u_1)(\langle u_1, u_2, \ldots, u_m \rangle \in H(x) \rightarrow \langle v_1, \ldots, v_{n-1}, u_1 \rangle \in H(y)).$$

[49] This general approach to substitutivity failures is discussed in §39 'Pragmatics' in *Quality and Concept*, and a concrete example of the conversational pragmatics is traced out on pp. 172–4.

[50] P. 215, Mates [1950].

[51] See Section 39 in *Quality and Concept* for further discussion of this sort of pragmatic explanation.

[52] Putnam [1954].

[53] In the semantics, for example, we need only define a new type of model structure in which there is a primitive logical operation for each different syntactic form. These operations will be 1–1; their ranges will be disjoint, and their behavior with respect to the extension functions H in $\mathscr{H}$ will be just what one would expect.

[54] Tyler Burge expresses a closely related worry; see pp. 127ff., Burge [1978]; p. 97, Burge [1979], and Burge [1975]. The issue here dramatizes the fact that any adequate theory of language learning must incorporate a resolution of the paradox of analysis.

[55] A formal semantics that deals with fine-grained distinctions like this may be developed along the lines suggested in *Quality and Concept*, p. 257, n. 17.

[56] The only other known way of trying to solve this sort of substitutivity puzzle is by treating the standard propositional attitudes as three-place relations holding among a person, a proposition, and a "mode of presentation". However, there are a host of problems with this proposal. For example, it runs into trouble with iterated propositional attitudes, quantifying-in, and general sentences that mix intentional and nonintentional predicates. We plan to spell out these difficulties in a future publication.

[57] The self-embedded propositions s_1 and s_2 involving the two soldiers we discussed at the close of Section 11 are dealt with as follows:

$$s_1 = \mathrm{Pred}_0(\mathrm{Pred}_0(\text{perceiving}, s_2), y)$$
$$s_2 = \mathrm{Pred}_0(\mathrm{Pred}_0(\text{perceiving}, s_1), x).$$

Once again, no ill-founded sets are involved; indeed, this pattern is comparable to:

$$2 = (0 + 3) + -1$$

$$3 = (0 + 2) + 1$$

where + corresponds to Pred_0; 2 to s_1; 0 to perceiving; 3 to s_2; -1 to y; 1 to x.

[58] These functions – along with Conj, Neg, and Exist – are closely related to the operations Quine introduces in Quine [1966]. See also Quine [1981].

[59] In general,

$$\langle x_1, \ldots, x_{i-1}, y_1, \ldots, y_k \rangle \in H(\text{Pred}_k(u, v)) \text{ } \textit{iff}$$

$$\langle x_1, \ldots, x_{i-1}, \text{Pred}_0(\ldots \text{Pred}_0(\text{Pred}_0(v, y_k), y_{k-1}), \ldots, y_1)\rangle \in H(u)$$

where $u \in \mathscr{D}_i$, $i \geqslant 1$, and $v \in \mathscr{D}_j$, $j \geqslant k \geqslant 1$. The following examples help to explain the predication functions Pred_0, Pred_1, Pred_2, Pred_3, . . . :

$$\text{Pred}_0([Fxyz]_{xyz}, [Guvw]_{uvw}) = [Fxy[Guvw]_{uvw}]_{xy},$$

$$\text{Pred}_0([Fx]_x, [Guvw]_{uvw}) = [F[Guvw]_{uvw}],$$

$$\text{Pred}_1([Fx]_x, [Guvw]_{uvw}) = [F[Guvw]^{w}_{uv}]_w,$$

$$\text{Pred}_2([Fx]_x, [Guvw]_{uvw}) = [F[Guvw]^{vw}_{u}]_{vw},$$

$$\text{Pred}_3([Fx]_x, [Guvw]_{uvw}) = [F[Guvw]^{uvw}]_{uvw},$$

$$\text{Pred}_k([Fx]_x, [A]_{v_1 \ldots v_m u_1 \ldots u_k}) = [F[A]^{u_1 \ldots u_k}_{v_1 \ldots v_m}]_{u_1 \ldots u_k}.$$

(Note that we have just *used*, not mentioned, intensional abstracts from L_ω.) For further clarification of these predication functions Pred_0, . . . , see the definition of the associated syntactic operations given on page 220.

[60] On conception 1, PRPs are thought of as the actual qualities, connections, and conditions of things; on conception 2, PRPs are thought of as concepts and thoughts. (See §2 in Bealer [1979] and §§40–41 in *Quality and Concept* for discussion of these distinctions.) Conception 1 and conception 2 correspond very closely to what Alonzo Church calls, respectively, Alternative 2 and Alternative 0 (pp. 4ff. in Church [1951] and pp. 143ff. in Church [1973 & 1974]). Church states that he ". . . attaches greater importance to Alternative 0 because it would seem that it is in this direction that a satisfactory analysis is to be sought of statements regarding assertion and belief". (P. 7n., Church [1951]) A fuller defense of his approach to the logic for psychological matters is given in Church [1954], where he develops the criterion of strict synonymy upon which he bases Alternative 0. The importance of conception 2 is discussed at length in *Quality and Concept*, §§2, 4, 6–11, 18–20, 39.

For the present purposes, we advocate developing *both* conception 1 and conception 2 side by side without attaching greater importance to one over the other. An advantage of such a dual approach is that, once those two conceptions are well developed, it is relatively straightforward to adapt our methods to handle intermediate conceptions in the event that they should prove relevant. Consider two examples. First, according to the construction of conception 2 presented in the text, the proposition $\text{Pred}_0(\text{Pred}_0([Lxy]_{xy}, a), b)$ is treated as distinct from the proposition $\text{Pred}_0(\text{Pred}_0([Lxy]_{yx}, b), a)$. If this distinction seems artificial, then along the lines of p. 54, *Quality and Concept* one can relax the identity conditions on PRPs within a type 2 model structure so that these two propositions are treated as identical.

Secondly, there are instances of the paradox of analysis involving analyses of the logical operations themselves. (E.g., despite the usual definition of conditionalization in terms of negation and conjunction, someone might doubt that $(A \rightarrow B) \equiv \neg(A \,\&\, \neg B)$ and yet not doubt that $(A \rightarrow B) \equiv (A \rightarrow B)$.) Such puzzles can be easily resolved along the lines of Chapter 3 in *Quality and Concept* once one enriches model structures with appropriate additional logical operations (including a primitive operation for conditionalization): e.g., for each nondegenerate finite composition of the present logical operations, one might add a primitive operation that is equivalent to it in H-values. The broader philosophical point is that, if there is artificiality in the construction given in the text, it appears not to be inherent in the general algebraic approach; evidently it can be removed by some combination of the above methods. It does not follow, of course, that these methods can be used to rid other approaches to intensional logic of their forms of artificiality.

61 Taken together, (i) and (ii) guarantee that the action of the inverses of the $\mathcal{T}$-transformations and Conj, Neg, . . . in a type 2 model structure is to decompose each element of $\mathcal{D}$ into a unique (possibly infinite) complete tree. (A decomposition tree is *complete* if it contains no terminal node that could be decomposed further under the inverses of the $\mathcal{T}$-transformations and Conj, Neg, . . .). Notice that without condition (iii) unwanted identities such as $[Fx]_x = [A \,\&\, Fx]_x$ could arise. For, as far as conditions (i) and (ii) are concerned, the property $[Fx]_x$ can have a unique complete decomposition tree in which $[Fx]_x$ occurs (denumerably many successive times) on a path descending from $[Fx]_x$. Condition (iii) rules out such a tree.

Examples of type 1 and 2 model structures are easily constructed. E.g., a type 1 model structure can be constructed relative to a model for first-order logic with identity and extensional abstraction, and a type 2 model structure can be constructed relative to a model for first-order logic with identity, extensional abstraction, and Quine's device of corner quotation.

62 Meaning may also be defined: $M_{\mathcal{I},\mathcal{A},\mathcal{M}}(A) =_{\mathrm{df}} D_{\mathcal{I},\mathcal{A},\mathcal{M}}([A])$.

63 These notational conventions are adopted for convenience only. We are not reversing our earlier position on the correct parsing of natural language sentences such as ⌜It is necessary that A⌝. We would represent this sentence as ⌜$N^1[A]$⌝. The 1-place predicate 'N' may on conception 1 be defined as follows: $N^1 x$ iff$_{\mathrm{df}}$ $x = [x = x]$.

64 Proofs of this and the succeeding theorems are given in Bealer [1983]. A corollary of the present theorem is that first-order logic with identity and extensional abstraction (i.e., class abstraction) is complete. Notice also that, in view of the defintitions of $\Box$ and $\Diamond$ in terms of identity and intensional abstraction, modal logic may be thought of as part of the identity theory for intensional abstracts.

65 That is, t and r are not in the range of the same term-transforming relation, nor are they in the range of the same syntactic operation – conjunction, negation, existential generalization, predication$_0$, predication$_1$, ...

66 $\mathcal{A}8$ affirms the equivalence of identical intensional entities. Schemas $\mathcal{A}9$–$\mathcal{A}11$ capture the principle that a complete definition of an intensional entity is unique. And schema $\mathcal{A}12$ captures the principle that a definition of an intensional entity must be noncircular. The following two instances of $\mathcal{A}12$ should help to illustrate how it works:

$$[Fxy]_{xy} = [Gxy]_{yx} \rightarrow [Fxy]_{yx} \neq [Gxy]_{xy}$$

$$[Fx]_x = [\neg Gx]_x \rightarrow [\neg Fx]_x \neq [Gx]_x.$$

Finally, $\mathscr{R}3$ says roughly that, if $A(t)$ is valid$_2$ for any arbitrary elementary p-ary term t, then $A(t')$ is valid$_2$ for any p-ary term t'.

[67] The idea was presented in a talk by the second author at Augsburg (February 1987). In the meantime E. Klein informed us that a similar construction is carried through in the Edinburgh dissertation by F. D. Kamareddine. Scott's treatment of class abstraction using lambda-abstraction (Scott [1975]) must be mentioned as well. In the inductive truth definition of that paper he represents $a \in b$ as a defined formula and identifies it with $b(a)$. He comments on his choice of the particular representation and the loss of fine-grainedness caused by it: ". . . as long as we do not use the lambda-notation, the representation of formulas by elements is *unique*, because the whole system is based on tuples. (If we wanted full uniqueness, we could make the $\in$-combination a primitive . . . and save the $b(a)$-part for the truth-definition)". (Scott [1975], p. 7.) It is exactly this strategy that we will use below. We will introduce a primitive operation Pred. This allows us to abide by the requirements on the internal structure of propositions that characterize propositional-attitude contexts.

[68] Scott [1975], p. 8. Lest the reader accuse us of confusing use and mention we would perhaps observe that the cited passage appears in a context where the inductive truth definition is still to come.

[69] We want to be neutral at this stage of the discussion with respect to the proper reading of the schema $\ulcorner u \in \{x \mid \ldots x \ldots\}\urcorner$. '$\in$' may indicate functional application and the abstract $\ulcorner\{x \mid \ldots x \ldots\}\urcorner$ may accordingly stand for a propositional function. Another construal would interpret '$\in$' as a symbol for the binary application operation of a combinatory structure. Finally, '$\in$' could be read as a binary predication relation.

[70] The argument is due to Aczel [1980]. The special form in which we have presented it is taken from Flagg and Myhill [1987].

[71] Geach [1972], p. 229.

[72] Bealer [1988], Section 4.

[73] Spelled out in more detail, the functional structure adumbrated above corresponds to what Barendregt calls a *lambda-family* (Barendregt [1981], p. 110) and Aczel [1980] a *lambda-system*. The notion seems due to H. Volken.

[74] We have been "cheating" in stating the equations. R as a two-place function cannot be composed with a pair of functions one of whose components is a one-place (p) and the other component is a zero-place (a or b) function. Therefore, the constants in the argument-positions of 'R' have to be read as actually standing for the composition of the unique function which maps the universe of objects into the terminal element with a zero-place function.

[75] This is a particular case of the axiom schema 11 of $T2$.

[76] It may be questioned whether this requirement alone can provide a firm basis on which to erect a defensible, non-*ad hoc* theory of a notion of predication within a functional approach. We do not rule out there being good reasons of a different sort that would finally vindicate the introduction of a fine-grained doppelgänger of functional application.

It should be mentioned that Aczel [1985] introduces a related distinction between functional application and predication. Since he retains both properties and propositional functions within the framework he sketches, he needs a second operation pred′ and a corresponding principle which allows him to derive a bijective correspondence between properties and propositional functions. On philosophical grounds a system that encompasses both properties and propositional functions may have certain advantages. For our limited objective, though, the defense of propositional functions as external modeling objects, we thought it appropriate not to complicate the technical issue by admitting properties and propositional functions side by side into our model.

[77] We do not want to defend our choice of the particular propositional part of the schema. It would have been possible to simply stipulate that Pred(f, a) be a proposition if $f(a)$ is a proposition. Irrespective of the propositional condition, the extensional equivalence between predication and functional application provides a vivid illustration of our misgivings concerning the ultimate appropriateness of this version of a fine-grained functional structure.

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