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RESCHER'S HYPOTHETICAL REASONING: AN AMENDMENT

1. Rescher 1964 – henceforth HR – proposes a way of reasoning from a set of hypotheses which may include both some of our beliefs and also hypotheses contradicting those beliefs. The aim of this paper is to point out what I take to be a fault in Rescher's proposal, and to suggest a modification of it, using a nonclassical logic, which avoids that fault. The paper neither attacks nor defends the broader aspects of Rescher's proposal, but merely assumes that it is at least prima facie worthwhile and therefore worthy of amendment; consequently, I shall try to tinker as little as possible. In particular, the use of a nonclassical logic which I propose does *not* replace any use by HR of classical logic – in those places where Rescher is classical, I shall be classical, too. (Instead, the amendment introduces a nonclassical logic at a point where HR uses no logic at all.)

2. I begin with a description of Rescher's proposal. Suppose we have a set of hypotheses P constituted by some of our beliefs together with an additional hypothesis which is inconsistent with those beliefs. We may still want to say something about the consequences of P – such is the topic of getting clear on counter-factual conditionals as addressed hy HR.

The first of three elements of Rescher's proposal is *modal categorization* of all sentences in our language. A *modal family* M is a list $M(1), \ldots, M(n)$ of nonempty sets of sentences, called *modal categories*, (1) each of which is a proper subset of its successors, (2) each of which contains the classical logical consequences of each of its members (but is not necessarily closed under conjunction), and (3) the last of which contains all sentences. This definition is slightly at variance with HR, p. 46, but not (I think) in any way which makes a difference. If each member of a family is also closed under conjunction, I will speak of a *conjunction-closed modal family*; and I note that all modal categories of such, except $M(n)$, are consistent (on pain of violation of *proper* subsethood – see HR, p. 47).

It is part of the proposal of HR that reasoning from a set of hypotheses P shall be carried out in the context of some modal family M. In application to the belief-contravening hypothesis case, we let $M(1)$ be the hypothesis H together with all its consequences, and then sort our beliefs into the remaining categories $M(2), \ldots, M(n)$ according as to how determined we are to hold on

E. Sosa (ed.), The Philosophy of Nicholas Rescher: Discussion and Replies, 19–28.

to them, where a lower index indicates a higher degree of epistemic (or doxastic) adhesion – the beliefs in the lower-numbered categories are those with which we intend to stick, if we can. This sorting is perhaps the critical notion of HR, and a good deal is said there about the principles on which it might be based. But the amendment we have in mind does not pertain thereto, and accordingly we shall say no more about it.

The second element of Rescher's proposal begins to tell us how to put the hypotheses P together with a modal family M in order to tease out the consequences of P. This is done through the instrumentality of "preferred maximally mutually-compatible (PMMC)" subsets of P, relative to M. And these may be defined inductively, by defining PMMC(i) for each i ($1 \leqslant i \leqslant n$), assuming that the work has already been done for $i' < i$. Choose a member X of PMMC(i–1), or let X be the empty set if $i = 1$. If all of the members of $P \cap M(i)$ can be consistently (classical sense) added to X, do so, and put the result in PMMC(i). Otherwise, form *each* result of adding to X as many members as possible of $P \cap M(i)$ without getting (classical) inconsistency, and put *each* such result in PMMC(i). All PMMC(i) having been defined, PMMC ("the PMMC subsets of P") is defined as PMMC(n).

If one wants a more set-theoretical definition, it could go like this. PMMC(i) (for $1 \leqslant i \leqslant n$) is the set of all sets of sentences S such that there is a set U such that $U \in$ PMMC(i–1) (or U is the empty set if $i = 1$) and (a) U is a subset of S, (b) S is a subset of $P \cap M(i)$, (c) S is classically consistent, and (d) no proper superset S' of S satisfies (a)–(c).

The third and last element of the proposal of HR is to define the consequences of P relative to M as those sentences which are (classical) consequences of *every* member of PMMC. For this notion, when M is understood, we use the notation

$$P \rightarrow A$$

of HR, which we can read as 'A is an HR-consequence of P' (relative to M).

It is also convenient to use

$$P \nrightarrow A$$

for the failure of HR-consequence. Evidently $P \nrightarrow A$ holds just in case there is some member of some PMMC(i) which contains or (classically) implies $-A$.

It is useful to have a transparent notation representing how the hypotheses of a set P fall into modal categories of a fixed family M; to avoid endless subscripts, I introduce it by way of example.

$$(A, B, C \,/\, D, E \,/\, F, G)$$

represents that the sentences to the left of a given slash fall into a narrower ('more fundamental', 'more important' – HR, p. 47) modal category than any sentence to the right of the slash, but that sentences unseparated by slashes are themselves modally indistinguishable. (This is related to but distinct from the notation of HR, p. 50.) As a special case we write, for example,

$$(A, B, C, D)$$

(no slashes) to indicate that all members of P are modally indistinguishable.

EXAMPLE 1. $(Cvb \,/\, Fb \supset -Ib,\ Fv \supset -Iv,\ Cvb \supset [(Fb \equiv Fv) \,\&\, (Ib \equiv Iv)] \,/\, Fb, Iv, -Cvb) \rightarrow (Fb \& Fv) \vee (Ib \& Iv))$.

This is about Bizet and Verdi, of whom HR gives a slightly different account on pp. 67–68: under the hypothesis Cvb that they are compatriots, together with strongly held beliefs about disjointness of the French and Italians and about what necessary conditions for being compatriots are, together with more weakly held beliefs about the nationality of Bizet and Verdi, and that they are not compariots, we can HR-conclude that either they are both French, or both Italian. (We can also conclude that they are either both non-French or both non-Italian; but this is less interesting since it does not use statements in the weakest modal category.)

The remaining examples are kept wholly unrealistic in order to make certain points in the simplest possible way.

EXAMPLE 2. $(p, -p \vee q) \rightarrow q$. If P is consistent, its *HR-consequences* (as I shall way) are just its classical ones.

EXAMPLE 3. $(p \,/\, -p, q) \rightarrow p \& q$. HR, p. 53, notes of a similar example that "q is an 'innocent bystander,' not involved in the contradiction at all," and that the modal categorization is irrelevant to getting q (but of course not p). That seems right, and we shall make much of it.

3. I have an objection to the concept of HR-consequence as described in the preceding section: it is entirely too sensitive to the way in which conjunction figures in the description of our beliefs. This complaint must not be taken too far: *some* segregation of our premisses is essential for Rescher's program to get underway at all – certainly the belief-contravening hypothesis must be separated out, and certainly the categorization of our beliefs requires segregation – not everything must be inextricable.

But within categories, Rescher's method gives wildly different accounts depending on just how many ampersands are replaced by commas, or vice versa. It depends too much on how our doxastic subtheory of a certain category is itself separated into sentential bits. The trouble is seen bare in:

EXAMPLE 4. $(p \mid -p\&q) \not\Rightarrow q$, that is, HR does *not* get the 'innocent bystander' q of Example 3 if in describing the relevant beliefs one uses an ampersand instead of a comma. That seems to me wrong. Furthermore, consider:

EXAMPLE 5. Let $P = (p, -p\&q)$, where modal categorization yields $(-p \mid p, -p\&q)$. Here, because $-p$ is bound up with q in P, its narrower modal categorization cannot on Rescher's account come into play. So P has *no* HR-consequences other than tautologies. But a sensible account should let P yield $-p$ because of its membership in a more ferocious category – and of course q because of its not participating in the contradiction at all.

So sometimes HR doesn't get consequences which I think it should. But sometimes it gets too many. Consider the following pair.

EXAMPLE 6. $(p \mid q, -q\&-p) \Rightarrow q$, since one can add q but not $-q\&-p$ consistently to p.

EXAMPLE 7. $(p \mid q, -q, -p) \not\Rightarrow q$. since one can add $-q$ consistently to p, so that at least one PMMC omits having q as a (classical) consequence.

It seems to me that Example 6 only gets q 'deviously,' because its negation $-q$ 'happens' to be tied to $-p$. Example 7 seems to me right.

Here I was looking mostly at examples in which A, B, and $A\&B$ were all modally indistinguishable. I do *not* mean to imply that we can always settle the consequence question for $A\&B$ as a hypothesis in a certain context by looking at the question for A and B separately in that same context; for one or both of A and B might be in a narrower category than $A\&B$. But *if* $A\&B$, A, and B are modally indistinguishable, it seems a hard saying that the consequence question for $A\&B$ should be different from that for A and B separately.

Since different ways of articulating our beliefs (of a single modal category) give different results under Rescher's proposal, and since I do not want this, evidently I have to have some views about which articulations I most want to reflect.

Policy: try to reflect *maximum* articulation. I note that this is a policy and not a whim. For the opposite policy – the agglutinative policy – gives entirely too few interesting results in central cases. Consider the very central case when some finite P is inconsistent. Then if we represent P by a single sentence, the conjunction of its members, evidently we will have *no* HR-consequences beyond tautologies. In contrast, if we maximally articulate P, we may be able to isolate the effect of its contradiction, adding the consistent bits and obtaining something entertaining. Or, which seems just as important, we may be able to block a consequence by freeing for use some conjunct of a conjunction which is itself not consistently available, as in Example 6–7.

4. So much for complaints. My *aim* is to minimally modify HR so as to avoid them. My *strategy* is to amend the definition of HR-consequence at only one place. I am going to keep the first element, the apparatus of modal categorization untouched. I shall also retain the third element, the account of consequence in terms of PMMC: A is to be a consequence of P, relative to M, just in case it is a (classical) consequence of every PMMC.

Further, I am going to keep the outline of the second element, the definition of PMMC. I change it at only one place. Rescher considers the addition, at the i-th stage, of only formulas in $M(i)$; good. But he *also* allows only the addition of formulas which are actually in P. This is what I suggest changing. I suggest allowing also the addition for formulas in a larger set, P^*, which can be thought of as the *articulation* of P, the freeing of its contents from such notational bondage as they might have in P. All of this is to be done before the application of the device of modal categorization to get PMMC.

In what follows I shall experiment with various possible articulations P^*. In all cases, please spare both of us the pains of repetition by picturing the definition of PMMC in Section 2 as containing 'P^*' wherever 'P' occurs. (Hence, the Rescher proposal can be described in these new terms by simply identifying P^* with P.)

5. The first thing one might try is to define P^* as the closure of P under classical consequence, but this is ridiculous; for *typically* P is inconsistent, so that P^* would contain every sentence. It follows that the (amended) HR-consequences of P would be determined entirely by the modal family M and be correspondingly wholly independent of P itself! In short, we would be giving up *all* of Rescher's gains. So much for classical consequence.

6. The second thing one might try is to define P^* as the closure of P under *relevant* consequence, in the sense of the concept of 'tautological entailment' of Anderson and Belnap 1975, or its generalization to quantifiers as in Anderson and Belnap 1963. Please notice that it won't do to count on some kind of relevant idea of entailment to do *all* the work. For it is quite essential, I should say, that in Rescherian consideration of belief-contravening hypotheses we give *consistency* its proper role, not letting in any inconsistent

consequences. But at the level at which we are working, it is not unfair to say that relevant entailment just doesn't care about contradictions at all: (p, $-p$, q) relevantly implies $p\&-p$ as well as q.

So the idea is to use a judicious *combination* of relevance notions and classical notions. First use relevant implication to articulate our hypotheses P; i.e., define P^* as the collection of all relevant consequences of P. Then use modal categorization and plain old classical logic to tease out its (amended) HR-consequences. Since contradictions do not relevantly imply everything, we can at least be sure that this proposal does not have the same defect as that of Section 5.

The proposal gets some examples right. I ignore its virtues, however, because in other cases it gives results which deviate not only from HR-consequence, but from what I think is correct. Consider

EXAMPLE 8. (p / $-p$, q) does not on this proposal yield q, although as indicated in my remark on Example 3, I agree with Rescher that this P *should* give the 'innocent bystander' q. The reason it does not is because the implication from A to $A \vee B$ is relevantly O.K., so that P^* will contain $-p \vee -q$. Since $-p \vee -q$ must be in every modal category containing $-p$, it certainly does not have a weaker modal standing than q. So in its turn it will form with p the basis of a member of PMMC – which, since consistent and having $-q$ as a classical consequence, cannot have q as a consequence.

For a while, after discovering this, I fooled around with some related proposals which paid attention to the fact that $-p \vee -q$ 'threatens' contradiction when put with p in a way that q does not – sense can be made out of this by looking at the four-valued representation of the set (p, $-p \vee -q$) according to the pattern of Belnap 1977a or 1977b. But although there may be something in the vicinity, as I conjectured in Belnap 1977a, p. 50, I do not now know what it is. Instead I think that the trouble lies deeper, and that in fact it is to be found in too free use of the principle of "disjunction introduction," as Fitch 1952 labels the inferences from A (or B) to $A \vee B$.

7. It is not that I have started thinking that the consequence from A to $A \vee B$ is somehow doubtful. But we are not speaking of a matter of consequence; instead, we are searching for principles for articulating sets of hypotheses, and we already know that such principles may be far weaker than consequence.

In any event, consideration of Example 8 makes it plausible to suggest replacing the role of relevance logic in defining the set P^* which articulates P

by the set of implicates of P according to some logic which in a natural way bars disjunction introduction. And there is such a logic: the logic of "analytic implication" of Parry 1933. (See Anderson and Belnap 1975 for a summary and some references; more is forthcoming in Collier, Gasper and Wolf 197+.)

The idea behind Parry's system is that A shall not analytically imply B unless every variable occurring in B 'already' occurs in A – so that in this sense, B does not "enlarge the content" of A. Of course the inference from A to $A \vee B$ fails this test.

But it turns out that although we may be on the right track, Parry's own system is not enough help. For he wishes to *maximize* the implicates of A relative to the above idea of analytic implication, and hence allows the inference from $-p$ and q to $-p \vee -q$ – note that indeed all the variables of the conclusion lie among those already in the premisses. And since this inference is allowed, if we define P^* as the closure of P under Parry's analytic implication, we won't get q from $(p \,/\, -p, q)$, since q will be missing from among the consequences of every consistent extension of the set $(p, -p \vee -q)$, one of which, at least, will be in PMMC – exactly as in Section 6.

The upshot is that for our purposes, analytic implication is No Good.

8. So relevance logic and analytic implication are too strong to give satisfying results in defining P^*. The weakest solution to the problems so far found is just to let P^* be the closure of P under "conjunction elimination" (Fitch 1952), the inference from $A \& B$ to A (or B). But this is *too* weak. At the very least we must allow dissolution of conjunctions inside of disjunctions, as in the following example, which merely adds r as a hypothesis and then uniformly disjoins $-r$ to the elements of Example 4.

EXAMPLE 9. $(r, -r \vee p \,/\, -r \vee (-p \& q))$ does not yield q either as an HR-consequence, or when P^* is defined as the closure of P under conjunction elimination. But it should; just as in Example 4, q is an 'innocent bystander,' which becomes apparent if we put $-r \vee q$ in P^* because $-r \vee (-p \& q)$ is.

Further, any of our other examples can be modified in a parallel routine way to make the same point: if we buy into the principle of dissolution of conjunctions at all, we need it as well for conjunctions lying under disjunctions.

Evidently there are other ways in which conjunctions can be hidden. If we think of our notation restricted to conjunction, disjunction, and negation, then they can lie under double negations as well, or be concealed as denied disjunctions. And the disjunctions under which conjunctions might lie might themselves be hidden or concealed, so that we should be adding further principles of articulation; but we postpone this for a paragraph.

What about "conjunction introduction," the principle that gets $A\&B$ from A and B? Should P^* be closed under conjunction introduction? It does not matter in a direct way, since at any stage of the formulation of PMMC at which $A\&B$ could be added, A and B (which must be in any modal category containing $A\&B$, and which must together be consistent with any set with which $A\&B$ is consistent) could be added instead; and evidently the classical consequences of a set with $A\&B$ are exactly the same as the set with A and B instead of $A\&B$. But on the one hand, it does keep our thinking straight to have P^* closed under conjunction introduction, since it reenforces the doctrine that it is irrelevant whether our hypotheses are articulated with conjunctions or commas; and on the other, it allows us to state the further principles of articulation, needed for hidden conjunctions and the like, in a somewhat briefer manner than would otherwise be possible.

9. What I suggest is that in addition to the principles of conjunction elimination and introduction, we should use as our standard of articulation just the equivalence principles sanctioned by a new logic, one which is stricter than either relevance logic or Parry's analytic implication: the logic of *analytic containment* of Angell 1975. I describe it by reference to the following equivalence principles:

$$A\&B \iff B\&A$$
$$(A\&B)\&C \iff A\&(B\&C)$$
$$A\vee(B\&C) \iff (A\vee B)\&(A\vee C)$$
$$--A \iff A$$
$$-(A\vee B) \iff -A\&-B$$
$$(A\&A) \iff A$$

In the present context, these are to be used to generate further closure conditions on P^* in the following straightforward way: if $(..A...)$ is in P^*, then so is $(..A'...)$ if A is equivalent to A' by any of the above principles. (Evidently lots of other equivalences follow from the six above; see Angel 1976. We do not need them because we get their effect through our closure principle; e.g., given $A\vee B$ we may pass to $--(A\vee B)$ to $-(-A\&-B)$ to $-(-B\&-A)$ to $--(B\vee A)$ to $B\vee A$, even though we cannot write down $A\vee B \iff B\vee A$.)

Let me say just a few words about Angell's system. He sharply distinguishes the concept of *containment* from *deducibility*, and sets out only to formalize the former. Angell accepts the Parry intuitions for containment: A does not contain $A\vee B$. But he goes further, suggesting that it is not enough, as with Parry, to have B's variables occur in A. It must furthermore be the case that variables occurring in B positively also occur in A positively, and those occurring in B negatively also occur in A negatively. This immediately rules out the Parry-acceptable (and relevance-acceptable) inference from $-p$ and q to $-p\vee -q$, since q occurs negatively in the consequence but not in the hypotheses. In this way the problem of Example 8 is avoided. Positively put: if P^* is defined as suggested, then $(p \ / \ -p,\ q) \rightarrow q$, just as in Example 3. Indeed, using the sharp normal form theorem of Angell 1975, we can be sure that P^* contains no formula with a negative occurrence of q, so that q must be consistently addable to every member of each PMMC(i), hence in every member of PMMC.

One equivalence (more accurately: two-way closure principle) deducible from the above six is

$$A\&(B\vee C) \quad <\!\!-\!\!> \quad A\&(B\vee C)\&(A\vee C)$$

by means of which we are led to:

EXAMPLE 10. $(p \ / \ -p,\ q,\ r\vee -q) \Rightarrow q$ when P^* is defined as suggested via analytic containment. (Compare Examples 3 and 8.) Reason: $-p$ conspires with $r\vee -q$ to put $-p\vee -q$ in P^*, via the above equivalence, and the rest of the reasoning is as in Example 8. This is in definite contrast to HR-consequence, which continues to get q even when $r\vee -q$ is added, as above, to the hypotheses of Example 3. So if a case is to be made against my suggestion, perhaps it could be based on this example. For myself, however, I am inclined to think that adding the hypothesis $r\vee -q$, in which q has a negative occurrence, is enough to render q no longer a bystander of shining innocence. And so I stay with the proposed amendment: tinker with the definition of HR-consequence only to the extent of basing the definition of PMMC on P^* instead of P, where P^* is the closure of P under conjunction elimination and introduction, together with the six replacement principles, listed above, of Angell's analytic containment.

10. The present proposal illustrates how classical and nonclassical logic can occasionally be made to *cooperate* in a single venture. The nonclassical logic was quite essential in the role of an articulator of hypotheses, while classical logic, which came in at each of the three stages of Rescher's proposal, played its own distinctive role in carrying for Rescher the ideas of consistency and deducibility. One might well ask about variations on this theme which bring in, say, relevance logic as the standard of deducibility. But in the meantime it seems to me of considerable interest to note how an enterprise need not go to pieces when more than one logic is involved.

A final, anticlimactic word. In Belnap 1977a, I claimed never to have heard of a "practical, reasonable, mechanizable" strategy for giving up information, say to avoid a contradiction. HR-consequence, whether amended or not, does indeed give us a way of giving up information. But I think that it is *not* 'mechanizable'. For whether $P \rightarrow A$ holds depends on *consistency* claims as well as deducibility claims, and of course, outside of elementary propositional logic, consistency is not formalizable. There is, then, no *logic* of $P \rightarrow A$, even for finite P and a decidable modal family M – nor did anyone ever say there was.

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REPLY TO BELNAP

Nuel Belnap's clear and vivid paper manifests in every line the positive tendency and constructive outlook that typifies its author. Nevertheless, I do have one bone to pick with him. I have problems with the motivation for playing down, or even abrogating the difference between commas and ampersands, between the *juxtaposition* of theses and their explicit *conjunction*, between what is said in one breath, and what is said in two, so to speak.

In certain information-processing contexts there is, as I see it, a substantial difference between accepting the *conjunctive* truth of $P \& Q$ and accepting the *distributive* truth of the members of the pair P, Q – a difference which it is well worthwhile to heed and to preserve in our logical operations:

(1) The juxtaposition of theses facilitates preserving the distinction of sources. If Source No. 1 gives us p and Source No. 2 maintains q, then we have the pair of claims p, q. We should not automatically take ourselves to be in possession of the claim $p\&q$ for which nobody has vouched. Thus getting $r\&{\sim}r\&s$ from one (obviously confused) source is from the informative point of view quite a different sort of thing from getting the conflicting reports r and ${\sim}r\&s$ from two sources.

(2) Contexts of probable and inductive reasoning require the distinction between conjunction and juxtaposition as well. If p and q are both individually probable (or both "inductively indicated"), it by no means follows that $p\&q$ is so. And inductive considerations can powerfully substantiate p and q taken separately, without thereby substantiating $p\&q$, which may even fail in self-consistency.[1]

An adjunction principle can take several distinct forms, the following three in particular:

(A) as the *deductive* principle:

$$P, Q \quad P \& Q$$

(B) as the *semantical* principle:

$$t(P), t(Q) \Rightarrow t(P \& Q)$$

(C) as the *metatheorematic* principle:

$$\vdash P, \vdash Q \Rightarrow \vdash P \& Q$$

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Our present approach rejects (B) alone, retaining (A) and (C) intact. We have to do with an unorthodox *semantics*, not an unorthodox *logic*.

Of course, the *converse* of the adjunction principle (B) – namely $t(P \& Q) \Rightarrow t(P), t(Q)$ – obtains unproblematically. But the original principle will require a special additional proviso as to the mutual *cotenability* of P and Q. In general, then, the tenability of a thesis P in one setting or context of our informational terrain and that of Q in another context does not establish the tenability of their *conjunction, P & Q*, because P and Q might fail to obtain in one and the same context, so that their separate or *distributive* tenability does not suffice to assure their conjoint or collective cotenability (which, however, is *automatically* assured in the consistent case).

To be sure, this distinction between juxtaposition and conjunction in information processing contexts only becomes critical in the case of *incompatibility*, when conflicts arise among the theses at issue. As long as only mutually consistent theses are at issue,the difference at issue will not be a telling one. But the theory of *Hypothetical Reasoning* (as well as that of the book on *Plausible Reasoning* which is its successor) explicitly addressed itself to just this inconsistent case. And the pivotal point is that given *distributively* inconsistent truth-claims – such as $t(P)$ and $t(\sim P)$ – there may well be sensible things one can do by way of extracting the plausible consequences of the situation. But given the *collectively* inconsistent truth-claims of the self-contradictory contention $t(P \& \sim P)$, we face a rather more hopeless situation.

Above all, we must distinguish between

(1) $t(P) \& t(\sim P)$

which is a theoretically feasible circumstance in the case of an inconsistent system (and does not – or need not – lead to $t(P \& \sim P)$), and

(2) $t(P) \& \sim t(P)$.

For with (2), a claim that itself takes the form $t(P \& \sim P)$, it is our own discourse that is inconsistent, whereas with (1) we have safely managed to insert another assertor – the target-system of the truth-operator system at issue – between ourselves and the inconsistency. A claim of this second sort would indeed be problematic. But a system can be inconsistent – and can be recognized by one as such, as per (1), without this inconsistency spilling over into our discourse about it as per (2). The inconsistency of the objects of our discussion – or indeed even of the world that we inhabit – need not affect the consistency of our own discourse. A consistent account of an inconsistent object of consideration is perfectly possible.[2]

These brief considerations may at any rate provide some tentative and merely suggestive indications why I am reluctant to subscribe to the usual logicians' failure to distinguish between juxtaposing commas and conjoining ampersands. The difference may be irrelevant in the standard range of issues in deductive logic, but there are other important areas of information processing – plausible and hypothetical inferences included, where, as I see it, the difference becomes important.

NOTES

[1] For an interesting treatment of the relevant issues in the inductive context, see Henry E. Kyburg, Jr., 'Conjunctivitis' in M. Swain (ed.), *Induction, Acceptance, and Rational Belief* (Dordrecht, 1975), pp. 55–82.

[2] On these issues see N. Rescher and R. Brandom, *The Logic of Inconsistency* (forthcoming, Oxford, 1980).