DOWN TO EARTH UNDERDETERMINATION[†][‡]

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> *Geologists, chemists, biologists, and many physicists tend to be impatient when they hear about the problem of underdetermination of theory by evidence. A common response is to declare that this is simply a philosopher's problem (in the pejorative sense), a conundrum that people with a certain quirky intelligence might play with, but something of no relevance to the sciences. That response is overblown. ... Yet the sound instinct expressed in quick dismissal is a legitimate wish to be shown convincing examples across the range of scientific disciplines.*
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> Philip Kitcher
> *Science, Truth, and Democracy*

> *Finally, the fact that solving the inverse problem yields a set of model parameters that describe the observations well does not necessarily mean that the resulting model actually reflects physical reality. ... In fact, we often have no way of determining what the reality is. For example, we will never truly know the composition and temperature of the earth's core because we cannot go there. This limitation remains in spite of the fact that over time our models of the core have become increasingly consistent with seismological data, experimental results about materials at high pressure and temperature, and other data including inferences from meteorites about the composition of the solar system.*
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> Seth Stein and Michael Wysession
> *An Introduction to Seismology, Earthquakes, and Earth Structure*

1. INTRODUCTION

Underdetermination arguments against scientific realism have a familiar structure. First it is argued that underdetermination of theory by evidence is endemic: for any scientific theory, it would be unsurprising were there to exist a theory empirically equivalent to it (i.e., a theory that makes identical predictions about all observable matters of fact). Then it is argued that such underdetermination undermines scientific realism: from the (not unlikely)

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existence of empirically equivalent alternatives to extant theories, it follows that we should have little confidence in even the approximate truth of our current theories, no matter how well they fit all available evidence. Of course, such arguments rest on the presupposition that there is some fixed salient distinction between the observable and the unobservable.[1]

My interest here is in the first step. Realists tend to see the burden of proof in this matter as lying with anti-realists.[2] It is not enough to point to examples of pairs of physical theories that are empirically equivalent relative to some criterion of observability (Ptolemy and Copernicus, suitably restricted); nor to give general recipes for producing a theory empirically equivalent to a given theory that invoke concepts with dubious scientific credentials ("just allow miniature blue gnomes to do whatever work electrons are supposed to do"); nor to offer a general recipe that takes as input not genuine scientific theories, but rather formal surrogates for such theories.[3] What is wanted is a more or less general recipe for reliably generating an empirically equivalent alternative to a given theory—a recipe that takes as input and gives as output things that look like actual scientific theories. Absent that, it is widely felt, realists are entitled to assume that the existence of theories empirically equivalent to our best theories is an idle philosophical fantasy.

I do not have such a recipe to offer. But I think that reflection on some down to earth considerations shows that realists are mistaken about where the burden of proof lies.

2. UNDERDETERMINATION IN GEOPHYSICS

We can investigate the internal structure of the Earth by making measurements at its surface.

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[1] For an influential attack on all three aspects of underdetermination arguments, see Laudan and Leplin (1991).

[2] For discussion and references, see Stanford (2013, §3.2).

[3] See Earman (1993) for a sophisticated and illuminating implementation of this last strategy.

GRAVIMETRY. The problem of *gravimetry* is to reconstruct the distribution of mass density within the Earth from knowledge of the gravitational field at points on the surface of the Earth.

TRAVEL-TIME TOMOGRAPHY. Earthquakes, volcanic eruptions, nuclear detonations, etc., send compression waves through the interior of the Earth. Like light rays travelling through a medium of variable refractive index, these can be thought of as travelling along lines whose departure from straightness is caused by variations in the structure of the material through which they propagate.[4] The problem of *travel-time tomography* (also known as the *kinematic inverse problem*) is to reconstruct the way that the index of refraction (speed of waves) varies within the Earth from knowledge of how long it takes news of a seismic disturbance at certain points on the Earth's surface to reach certain other points on the Earth's surface.[5]

NEUTRINO TOMOGRAPHY. To an excellent first approximation, neutrinos travel through the Earth along straight lines, with the chance of a given neutrino being absorbed being proportional to the nucleon density in the matter through which it passes. The problem of *neutrino tomography* is to reconstruct the pattern of nucleon density within the Earth from knowledge of the rate of attenuation of beams of neutrinos passing between given pairs of points at the surface of the Earth.[6]

For problems of these sorts, the family of possible internal structures of the Earth under consideration will be vast: completely specifying the internal state of the planet would involve specifying the values of infinitely many quantities (e.g., one might specify the density at each of an infinite set of points internal to the planet). Performing a finite number of measurements external to the planet will allow one to determine the values of only finitely many such quantities. So

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[4] Here we pretend, somewhat unrealistically, that the Earth is an isotropic elastic solid and focus only on one of the many types of waves generated by seismic events.

[5] See Uhlmann (2001) for an introduction to the mathematics of travel-time tomography.

[6] This problem is analogous to the problem of x-ray tomography and other problems of medical imaging—for a survey, see Deans (2007, Chapter 1). The idea of neutrino tomography, which has not yet been put into practice, seems to have originated in a science fiction novel, Clement (1971). For a recent discussion of this technique, see, e.g., Winter (2006).

the family of models consistent with any finite set of measurements will still be vast (infinite-dimensional). It is standard to appeal to considerations of simplicity and general plausibility in cutting down this underdetermination.[7]

Short-term underdetermination of theory by data is only to be expected—few interesting scientific questions can be settled absolutely definitively by looking at a finite amount of data.[8] What about underdetermination in the infinite long-run in which complete data sets become available? Here we find interesting differences between methods of investigating the Earth's internal structure. For neutrino tomography, underdetermination evaporates in the limit of complete data: if the rate of attenuation for neutrino beams between each pair of points at the Earth's surface is fixed, then the relevant aspects of the internal structure of the Earth are fixed.[9] The situation is quite different in the case of gravimetry: corresponding to any given configuration of the gravitational field external to the Earth is a vast (infinite-dimensional) family of possible internal structures (some of which will differ very dramatically from one another).[10] Travel-time tomography constitutes an interesting intermediate case: for some internal structures the Earth might have, underdetermination would evaporate in the limit of complete data; for others, even complete data would be consistent with a vast and various family of possibilities.[11]

Of course, in the case of the Earth and other planets, we have ways of finding out about internal structure other than by making measurements at the surface (in principle, if not in practice). But the techniques considered above can be used to study the internal structure of the Sun and other stars (in principle and, in some cases, in practice—see, e.g., Nolet 2008). And in the stellar case there is

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[7] For discussions of several popular approaches, see Parker (1994) and Tarantola (2005). See also the philosophical literature on this topic: Miyake (2013). Note that very similar problems arise in medical imaging; see, e.g., Smith *et al*. (1977) and Helgason (2011, §I.7.B).

[8] For a dramatic illustration of this point, see Theorem 4.2 in Smith *et al.* (1977).

[9] See, e.g., Smith *et al.* (1977, §4) or Helgason (2011, Chapter I).

[10] For helpful discussions, see, e.g., Anger (1990, §§3.2, 4.2, and 5.3), Michel and Fokas (2008), or Sansò and Tscherning (1989).

[11] For relevant results, see, e.g., Croke (1991) and Pestov and Uhlmann (2005).

considerable plausibility to the idea that we never will (and never could) have any way of investigating internal structure directly.

So there is a sense in which underdetermination of scientific theory by (all possible) evidence is not only possible but actual. I suspect that most realists will be unfazed by this observation. For consider a widely-discussed case that is in some ways analogous to our examples. In general relativity, even infinitely long-lived observers cannot in general determine the topology of the cosmos in which they live.[12] Many realists accept that this is a case of underdetermination that points up a limit to the reach of scientific reason.[13] But few see here much of a threat to the project of scientific realism.[14] It is, for instance, sometimes asserted that the real battlefield in the debate over scientific realism is the status of theories—rather than that of particular facts, such as the topology of our cosmos (or, presumably, the internal structure of the Earth or the Sun).[15]

However, it is not clear to me that there is a stable form of scientific realism that concedes so easily that science is incapable of resolving some questions that are ordinarily regarded as scientific.

(a) I suspect that many realists will agree that Fine puts his finger on an important sense in which, from the point of view of scientific practice, constructive empiricism and other anti-realisms ask us to make an epistemically invidious distinction:

> [Constructive empiricism] can follow the usual lattice of inferences and reasons that issues in scientific beliefs only until it reaches the border of the observable, at which point the shift is made from belief to acceptance. But the inferential network that winds back and forth across this border is in no way different from that on the observable side alone. (1986, 167)

But if *that* is what bothers realists about constructive empiricism, then they should also be bothered by the suggestion that belief should be withheld in cases of infinite long-run underdetermination. After all, in the short run the lattice of

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[12] See Glymour (1977), Malament (1977), and Manchak (2009). For the role of this problem in the genesis of constructive empiricism, see van Fraassen (1985, Postscript §1).

[13] See, e.g., Earman (1993, §9), Glymour (1980 354 ff.), and Stanford (2006, 13 f.). For a dissenting voice, see Norton (2011).

[14] Earman (1993) is perhaps an exception.

[15] For this viewpoint, see Stanford (2006, 13 f.) and Norton (2011, §3).

inferences concerning, e.g., the internal structure of the Earth will look much the same whether we are working with a technique for which underdetermination vanishes in the infinite long run (such as determination of the internal magnetic field from the external magnetic field) or one such for which it does not (such as gravimetry).[16]

(b) Consider the proposal floated by Earman (1993, §11): Bayesian agents should distance themselves from their judgements of the relative plausibility of pairs of hypotheses just in case convergence of their own opinion to the true hypothesis cannot be expected even in the infinite long-run. Following this advice can lead to disaster—e.g., I may end up being thoroughly confident that the relation between the variables that I am investigating is given by a certain function, but be unwilling to pronounce on whether that function is or is not continuous (see Belot 2013).

3. UNDERDETERMINATION AND INVERSE PROBLEMS

I contend that, seen in the proper light, examples like those from geophysics discussed above serve to undermine in a quite radical way the orthodox perception of the dialectic between realists and anti-realists regarding the question of the existence of empirically equivalent rivals to scientific theories.

The geophysical problems discussed above are examples of a type of problem that is endemic in the mathematical sciences. One has a space, X, of possible states of some system. There is some interesting class of observations that can be made on this system. There is some space, Y, whose points correspond to possible (joint) outcomes of this family of observations. Some background

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[16] On the uniqueness of magnetic field reconstruction, see, e.g., Gubbins (2007). Authors of textbooks on geophysics will often note that there is an in-principle problem of underdetermination in gravimetry that is absent in some other problems. But this observation has no apparent impact on the advice offered to practitioners facing problems of either sort—see, e.g., Parker (1994, 241). Indeed, some authors are explicit about the practical irrelevance of infinite long-run considerations: e.g., Scales and Snieder remark that while the problem of infinite long-run uniqueness is "hotly debated in the mathematical literature on inverse problems, it is largely irrelevant for practical inverse problems" (2000, 1708). For what it is worth: so far as I can tell, the authors of the second epigraph above do not take the distinction between those problems in which underdetermination evaporates in the infinite long run and others to be epistemologically salient.

theory tells one, for each possible state x in X, what evidence y in Y would be gathered were the system in that state and one were to perform each of the possible observations on that system. From this perspective, the work done by this theory can be summed up in a function K that maps states in X to evidence sets in Y that takes as input a possible state of the system and gives as output the corresponding body of outcomes of observations. This is a so-called *direct problem*. Corresponding to each direct problem is an *inverse problem*: given a body of evidence y in Y, find those states x in X such that $y = K(x)$. Each of the geophysical problems considered above is an inverse problem of this kind.

The question whether or not, for each body of observation outcomes, there is a unique compatible state of the system is the question whether K is or is not an invertible function. In our geophysical examples, if our focus is on a finite number of observations performed at the surface of the Earth, then the relevant map K is never invertible: for any y there is a vast number of compatible x. If we focus instead on the class of all possible observations of a given type performable at the surface of the Earth, we find that in the case of neutrino tomography the relevant map K is invertible—but that the maps corresponding to the problems of gravimetry and travel-time tomography are not.

These examples from geophysics are not atypical. Very often, one is interested in systems with infinitely many degrees of freedom. In that case, a finite number of observations will never suffice to determine the state of the system. If one considers an infinite number of possible observations, the underdetermination may or may not disappear—this depends on the details of the theory determining the connection between states and data and on the space of states in question.[17]

Consider now the mother of all inverse problems. The space $\mathcal{X}$ is the space of all possible total theories (laws and initial conditions). The space $\mathcal{Y}$ is the space

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[17] One might be tempted to reason that, at least under ideal circumstances, invertible K *ought* to be typical. After all: if X and Y are vector spaces of the same finite dimension and K is a linear map, then, generically, K is invertible (the set of invertible n by n matrices forms an open and dense subset of the space of n by n matrices). But this result does not carry over to the infinite-dimensional setting, even in the linear case. See Bouldin (1990).

of all possible (joint) observation outcomes. The map $\mathcal{K}$ associates with each theory x in $\mathcal{X}$ the body of evidence y in $\mathcal{Y}$ that would be available were x true.

Clearly there are difficulties in making this picture precise. The notion of a space of all possible theories is of dubious coherence—e.g., on cardinality grounds.[18] The notion of a space of all possible observations is if anything even more fraught. And the map $\mathcal{K}$ is also beset by difficult questions: in order to make sense of it, one would presumably have to think of the background theory that underwrites the passage from a theory of everything (including initial conditions as well as laws) to an account of what would we would observe were that theory true as including logic together with some account, of the sort sketched in van Fraassen (1980), of how theories internally demarcate the boundary between the observable and the unobservable.

Still, the picture makes rough heuristic sense. And in terms of this picture, the question in dispute between the realist and the anti-realist is whether or not $\mathcal{K}$ is invertible—or, perhaps more plausibly, whether for a typical or generic body of evidence y there is a unique theory x such that $y=\mathcal{K}(x)$.

It is usually held that there is a presumption in favour of an affirmative answer here. Perhaps that seems plausible when anti-realists, after muttering for a while about one or two suggestive examples, assert without further argument that there are likely theories empirically equivalent to our favourite theories.

But the dialectic takes on a quite different cast when one thinks of anti-realists as advancing a claim about a very complicated inverse problem. There is not yet any general theory of inverse problems. One cannot definitively assert: most inverse problems do (or do not) involve underdetermination of theory by

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[18] *Prima facie*, for each cardinal number, there is a theory whose space of states is a Hilbert space with dimension of that cardinality. So the possible theories form a proper class rather than a set—which means that it is difficult to speak of the space of all theories as having a structure. Note that while renormalization group methods in quantum field theory are sometimes said to be set in a "space of all theories," something quite restricted is in fact meant. See, e.g., Costello (2011, Chapter 1).

observation.[19] At the same time, a perusal of the literature on inverse problems that arise in contemporary physical science suggests that it would be, to put it mildly, highly incautious to *suppose* that the map determined by a complicated and ill-understood direct problem was invertible and hence that there was no underdetermination in the associated inverse problem. But that is just what realists are supposing in asserting that the burden of proof lies with anti-realists in their dispute over the first stage of the underdetermination argument against scientific realism.

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[19] In fact, some authors come tolerably close to saying that most inverse problems suffer from nonuniqueness (see, e.g., Anger 1990, 37 and 63 or Parker 1994, 293). I know of no authors who come close to asserting that most inverse problems enjoy uniqueness.

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