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GUPTA'S RULE OF REVISION THEORY OF TRUTH[1]

Gupta's Rule of Revision theory of truth[2] builds on insights to be found in Martin and Woodruff (1975) and Kripke (1975) (who in turn build on Tarski) in order to permanently deepen our understanding of truth, of paradox (and of the absence of it), and of how we work our language while our language is working us. His concept of a predicate deriving its meaning by way of a Rule of Revision ought to impact significantly on the philosophy of language. Still, fortunately, he has left me something to say.

1. TRUTH AS A PREDICATE

My first remark is essentially contextual. Gupta "assumes" that truth is carried by a predicate. Under that assumption, he gives us a theory which is not only beautiful but may also be true. I wish to call attention, however, as Gupta himself does, to the fact that he does not *argue* that we should take as a "fact" that truth in ordinary language functions as a "real" property-ascribing predicate. I continue to believe in the idea of Grover *et al.* (1975), and subsequent papers by Grover (especially Grover, 1977 and Grover, 1981), that truth in ordinary language functions as what she calls an "inheritor", in some respects like a pronoun, picking up its meaning from something else in the discourse. I continue to believe that in ordinary language it does not have a property-ascribing function. It may nevertheless be that for certain technical or philosophical purposes one wants to give it such a role; why should one wish to block the road to inquiry by denying that? In such a case, it seems to me that Gupta's theory is far and away the best account we have of how to deploy truth in a property-ascribing way.[3]

2. BOOTSTRAPPING

Next I want to describe Gupta's Rule of Revision theory of truth from a point of view chosen with an eye on a technical fiddle to be suggested

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below. (Alan Ross Anderson was fond of observing that it is impossible for a logician to hear a formal suggestion without wanting to tinker with it.)

Gupta characterizes his apparatus as providing truth with a "Rule of Revision". I think that is a splendid picture; and the only sense in which I am not prepared fully to endorse it right now is this: Gupta's proposal is sufficiently subtle so that it seems best to think about it for a year or two. But he also sometimes says, in articulating this idea, that each stage provides a "better candidate" for the extension of truth than the previous candidate or that the rule enables us to "improve" a given candidate for the extension of truth. He says that "at limit stages we sum up the effects of all the earlier improvements". Now this language is just slightly inaccurate, as Gupta himself tells us; sometimes these seeming improvements are only "seeming improvements" which "may turn out to be illusory in light of later revision". It can happen that in the process of revision we get a particular sentence right for a while, and then later, perhaps much later, wrong for a while, and then finally it does stabilize to have the earlier value – which indeed is our licence for saying, after the fact, that we were right in the first place. To take a simpleminded example, let '*Ta*' be a truth-teller sentence (so that '*a*' denotes '*Ta*'), and suppose we start with counting '*Ta*' and '$T'T'Ta''$' as in the extension of the truth-predicate '*T*', while counting '$T'Ta'$' as out of the extension of '*T*'. So to begin with we are *right* about '$T'T'Ta''$'; but our first revision is clearly going to get it wrong. Only later (in fact "later" comes very soon) will everything come out all right.

I think the talk of "improvements" derives from Kripke's construction, which indeed gets better and better at every stage, but that is not quite the right picture to have of Gupta's Rule of Revision. Instead the picture to have of Gupta's construction is something like a tennis match, where the ball is driven back and forth between the true court and the false court, maybe forever. Or, under favorable circumstances, maybe one of the players slams it so hard into the grass of the other court that it stays there for good. In the course of the process of "revision", though, our eyes flit back from court to court, in a way which is exactly right for the paradoxes, and *also* for other sentences which do not stabilize until some later stage of the process of revision.

A good way of characterizing Gupta's account is as a *bootstrapping* operation. We start with an absolutely arbitrary extension for truth, and in fact we use this absolutely arbitrary information in constructing the next stage, for we cannot begin to use Tarski to calculate a truth value for sentences involving the truth predicate unless we have some extension for it: a Rule of Revision requires something to revise. Let us call this arbitrary extension a '*Bootstrapper*'. The next stage still retains some arbitrariness, because of the arbitrariness of the Bootstrapper that went into it; but the *mode of its construction* is wholly determined by Tarski's semantic concept of truth, so that this mode of construction is *non*arbitrary: the Rule of Revision at successor stages is a Rule of Reason.

At limit stages, Gupta's Rule of Revision has two quite different aspects, being partly a matter of reason and partly a matter of haphazard bootstrapping. If as a limit is approach some sentence is locally stable as in the extension of '*T*', or locally stable as out of it, then the mode of construction at the limit is not arbitrary: the sentence has at the limit whatever relation to the truth-predicate '*T*' it stably had as the limit was approached. This is a Rule of Reason, and it is an essential part of Gupta's Rule of Revision. But Gupta has taught us that bootstrapping is needed even at the limit for those sentences that are *not* locally stable. We *must* give them a relation to the truth-predicate, either putting them in the extension of '*T*' or not, for that is the nature of bootstrapping: we shall not be able to continue beyond a limit stage with renewed applications of the Tarski part of the Rule of Revision unless we have a fully determinate extension for '*T*' to revise. Whether we put locally unstable sentences in or out of the extension of '*T*' doesn't matter in the least; that is a matter of caprice, not reason. Now it is at this very point of deciding unstable sentences at a limit that Gupta's proposal is an improvement over Herzberger's (wholly independent) proposal; for although Herzberger allows us to start out with an arbitrary set, at a limit Herzberger mechanically throws *out* of the extension of truth all those sentences which have not stabilized within it. Gupta's proposal, in contrast, tells us to decide whether or not a locally unstable sentence is put into the extension of '*T*' at a limit by appeal to a Bootstrapper: we put the unstable sentence in the extension of '*T*' just in case the Bootstrapper tells us to do so. Herzberger's proposal, then, does not do as much justice as does Gupta's to the arbitrary and unsystematic aspect of the bootstrapping operation at a limit.

Let me summarize. There are essentially two cases in Gupta's Rule of Revision: successor stages, and nonsuccessor stages. For successor stages, there is no room for maneuver, so that this aspect of the Rule of Revision is wholly a Rule of Reason: we use the Tarski principle to calculate an extension for '*T*'. For the nonsuccessor stages, we use the outcome of previous stages to the extent that they give us a stable verdict; this aspect, too, is a Rule of Reason. But if for a certain sentence there is no stable verdict issuing from the preceding stages, then we put that sentence in or out of the extension of '*T*' according to some Bootstrapper's instructions; and *that* aspect is sheer caprice.

3. BOOTSTRAPPING POLICIES

On Gupta's proposal, sentences not locally stable are indeed decided arbitrarily. But the bootstrapping principle of arbitrary choice at each limit is exactly the same as the Bootstrapper which we used to get the bootstrapping operation under way in the first place. There is a single set, *U*, to which *all* arbitrary bootstrapping choices are appealed. At the very beginning, and also at each limit stage, Gupta uses the *same* set *U* to settle the truth status of all those sentences which have not been settled by the Rule of Reason part of the mode of construction.

But why? Why should the *same* Bootstrapper govern what we do with the locally unstable sentences at each limit stage as we go up and up and up the ordinals? It seems to me to detract from the very concept of arbitrariness that we should have to use the same Bootstrapper at each stage.

Instead, I ask you to consider a scheme which is just like Gupta's, except that there is a separately defined Bootstrapper for each nonsuccessor ordinal. (I remind you that the construction follows Tarski's Rule of Reason at successor ordinals, so that no Bootstrapper is needed for these.) Having decided that 'Bootstrapper' is a good word for a single principle of arbitrary choice used to decide cases not settled by a Rule of Reason, let us agree to call such a wholesale assignment of Bootstrappers to nonsuccessor ordinals a '*Bootstrapping Policy*'. From this point of view, Gupta's policy, which adverts to a *single* Bootstrapper for all nonsuccessor ordinals, is a "*Constant Bootstrapping Policy*".[4] In contrast, I wish to include Bootstrapping Policies which *vary* their Bootstrappers as the construction ascends the ordinals. The purpose of my suggestion is to make maximally arbitrary those features of the Rule of Revision that require any arbitrariness at all. Only then will we be sure that what *survives* this arbitrariness really fully depends only on the model with which we began together with the Rule of Reason components of the process of revision. The more arbitrary, the more capricious, the more unreasonable, and the less patterned the permitted Bootstrapping Policies, the more interesting is what remains invariant over all permitted Bootstrapping Policies.

There is a rather beautiful feature of Gupta's construction, and Herzberger's, which, if my suggestion is adopted, will be left out. They both point out that for each Constant Bootstrapping Policy (my terminology) their procedures eventually reach a stage after which the extension of the truth predicate cycles.[5] For a while its extension may zig zag around a lot, but eventually it will settle down to what Herzberger wonderfully calls "the Grand Loop". That is a beautiful picture, but I submit that it is an artifact of the construction, due entirely to the fact that the same Bootstrapper is used for each and every limit stage. When we abandon Constant Boostrapping Policies and instead increase the degree of arbitrariness by allowing an independent erratic choice whenever one is called for, the inevitablity of cycles disappears. And I think that is right: I think the Grand Loop is an artifact created by an *ad hoc* decision to adopt always a Constant Bootstrapping Policy where no such constancy is called for.

4. QUANTIFICATION OVER BOOTSTRAPPING POLICIES

Gupta defines several concepts by quantifying over Constant Bootstrapping Policies. Let us see what happens if we modify his definition by quantifying instead over *all* Bootstrapping Policies, Protean as well as constant.

I remind you that, as we now view Gupta, a sentence is stably true in a model relative to a certain Constant Bootstrapping Policy if there is an ordinal such that after that ordinal the sentence is always true. And – key idea – a sentence is *stably true* relative to a model if it is stably true relative to each Constant Bootstrapping Policy. In exact parallel, let us say that a sentence is stably true relative to a model relative to a Bootstrapping Policy (constant or not) if there is an ordinal after which it is always true. And let us say that a sentence is *universally stably* true in a model if it is stably true relative to all Bootstrapping Policies.[6] What I want to say is: it is only those sentences which are universally stably true about which we can say that the process of revision "yields a definite verdict" of truth. We cannot say that we get a definite verdict of truth on a sentence in a Rule of Revision bootstrapping scheme unless we can say that the truth of the sentence survives each and every policy of arbitrary choice made for bootstrapping purposes. It is only when we have universal stability that we can be sure that the model itself, coupled with, and only with, Tarski's semantic conception of truth, gives us a definite verdict totally free of any tinge of arbitrariness. (A concrete example distinguishing Gupta's stability from my universal stability is given below, Section 6).

Another concept that Gupta defines by quantification over Constant Bootstrapping Policies is that of a Thomason model. 'Thomason models' are those such that the revision process arrives at the same fixed point regardless of the choice of Constant Bootstrapping Policy.[7] Surely we obtain a more interesting idea if we characterize a model as Thomasonian if it leads to the same fixed point no matter what entire scheme of arbitrary choices – that is, no matter which Bootstrapping Policy – is employed. Gupta suggests being Thomasonian as an account of being free of vicious paradox. It seems to me that we could not trust a model as being free of vicious self-reference if it is only Thomasonian in the weaker sense and not also Thomasonian in the stronger sense. (I do not know whether these conceptually different notions of 'Thomasonian' are mathematically different.)

Another example, this time from the opposite end of the spectrum. Gupta defines a sentence as "paradoxical in a model" if it never stabilizes relative to any one of his Constant Bootstrapping Policies. Clearly one obtains a deeper sense of paradoxicality if a sentence never stabilizes regardless of which (possibly inconstant) Bootstrapping Policy, is picked. A sentence which fails to stabilize regardless of which Constant Bootstrapping Policy is employed is not as deeply paradoxical as a sentence which fails to stabilize even when you are allowed to try as hard as you can to make it stabilize by some crafty choice of nonrepetitive Bootstrapping Policy. (As before, I can only note a conceptual difference, and not a mathematical one.)

I should note that as far as I can see the *only* fact among those which Gupta establishes that relies on the Bootstrapping Policy being constant is the 'Grand Loop' cycling mentioned above. Since nowhere else does he use the fact that there is but a single way of deciding arbitrary cases to be used at each and every non-successor stage, all his other results go through as before.

5. LIMITED BOOTSTRAPPING POLICIES

Bootstrapping Policies are proper classes. Gupta has pointed out in discussion that if one wished not to quantify over them but instead to confine quantification to sets, one could quantify instead over 'Limited Bootstrapping Policies', where the abritrary aspect of each 'Limited Bootstrapping Policy' is bounded by some ordinal. After that ordinal, one would have to have some kind of nonarbitrary repetitive scheme of Bootstrappers, say a constant function which picks the same Bootstrapper for that and all longer ordinals, or a scheme which endlessly repeats the pattern of the Limited Bootstrapping Policy.[8] This alternative should, I think, make no difference for universal stability and the like; but once again it would appear that Grand Loop Theorems would arise; and they would become available only because of the *ad hoc* limitation on the size of each Limited Bootstrapping Policy, so that, in my view, they would not be interesting. Furthermore, if the policy is required to be constant after some ordinal, then one would continue to obtain the (I think) nonintuitive results reported in Section 6 below about sentences claiming the equivalence of liars.

Perhaps, however, there is some way to invoke a more general notion of Limited Bootstrapping Policy that would avoid all such *ad hoc* features. I hope so, for it would be nice to be able to stay within ZF.

In the meantime, the following re-sorting of concepts might be of some service by directing attention away from unlimited Bootstrapping Policies. Say "A *is universally true relative to a model at an ordinal*" if it is true relative to that model at that ordinal relative to all Bootstrapping Policies limited by that ordinal. Then say that "A *is stably universally true relative to a model*" if there is an ordinal after which it remains universally true relative to each ordinal. These definitions clearly stay within ZF; by quantifying on Bootstrapping Policies *before* quantifying on ordinals, it seems *natural* to invoke only *Limited* Bootstrapping Policies.[9] My thought is that *provided* one wants to stay in ZF, it is probably better to organize the concepts in this way; but I am offering no facts, just definitions, so that there are no grounds for more than a guess.

6. A CONCRETE DIFFERENCE

I have said that I do not know whether the two proposals differ on some of the really interesting questions, such as which sentences are paradoxical; but they do differ. First, however, I will mention two cases of agreement: the truth-teller and the liar.

Truth-tellers

As evidence that Gupta's Constant Bootstrapping Policies work well even if in the end one prefers to admit more general Bootstrapping Policies, one can take the case of the truth-teller: '*a*' denotes '*Ta*', so that '*Ta*' says of itself that it is true. On Gupta's plan, the truth of the truth-teller is certainly not stable, for its truth value depends on the Constant Bootstrapping Policy; but for each Constant Bootstrapping Policy using a Bootstrapper (set) U, the truth-teller sentence *is* stable relative to that Constant Bootstrapping Policy, retaining throughout the construction whatever truth value it was given by the U used as the basis of the Constant Bootstrapping Policy. That sounds intuitively correct; and I wish to point out that exactly the same is true for truth-tellers in the presence of more general Bootstrapping Policies. For each such Bootstrapping Policy, each truth-teller sentence is stable, retaining throughout the construction whatever truth value it was awarded in the very beginning. The possibilities of new choice at later limits offered by inconstant Bootstrapping Policies in no way affect truth-tellers; only the starting point makes a difference, alike on Gupta's plan and on mine.

Liars

Turning now to liar sentences, where '*a*' denotes '$\sim Ta$', the results are also the same: liar sentences are as deeply paradoxical on both plans as it is possible to be, and it is no more possible to use a capricious Bootstrapping Policy to stabilize them than it is possible to stabilize one with a Constant Bootstrapping Policy. The Tarski component of the Rule of Revision is itself enough to destabilize liar sentences.

Equivalence of liars

There is a phenomenon for liar sentences, however, on which the two constructions diagree.

Take two distinct liar sentences: let '*a*' denote '$\sim Ta$' and '*b*' denote '$\sim Tb$', and consider a statement that the two liar sentences are equivalent: '$Ta \equiv Tb$'.[10] It would be dreadful if Gupta's plan counted the two liars as in some sense equivalent – say, if '$Ta \equiv Tb$' were stably true; but that is certainly not the case, since we can choose a Constant Bootstrapping Policy using a U which includes '$\sim Ta$' but excludes '$\sim Tb$'; which is enough to prevent the stable truth of '$Ta \equiv Tb$'.

But now notice this. If any Constant Bootstrapping Policy using U includes both of '$\sim Ta$' and '$\sim Tb$', then throughout the entire construction, '$\sim Ta$' and '$\sim Tb$' will share a like fate, always being either both *in* the extension of 'T' or both *out* of the extension of 'T', both at successor ordinals and at limits. That means that for such a Constant Bootstrapping Policy, '$Ta \equiv Tb$' will be stably true, relative to that Constant Bootstrapping Policy. Contrariwise, if a Constant Bootstrapping Policy using U separates '$\sim Ta$' and '$\sim Tb$', then throughout the entire construction '$\sim Ta$' and '$\sim Tb$' will always be separate, both at successor ordinals and at limits, so that in this case, '$Ta \equiv Tb$' will be stably false. Hence, for every Constant Bootstrapping Policy, '$Ta \equiv Tb$' is either stably true, or stably false, relative to that Constant Bootstrapping Policy. In the terminology of Gupta's Definition 9, '$Ta \equiv Tb$' is "weakly unstable" without being fully unstable in the sense of admitting Constant Bootstrapping Policies relative to which it is unstable. It is therefore in precisely the same category as the truth-teller sentences.

Let us ask how the statement '$Ta \equiv Tb$' of equivalence between two liar sentences fares on the Bootstrapping Policy plan when one does not have to remain constant. Not so well: there will certainly be Bootstrapping Policies which, so to speak, alternate putting the paradoxical sentences '$\sim Ta$' and '$\sim Tb$' together, or apart, at various higher limit ordinals. And as long as a Bootstrapping Policy itself does not "stabilize" in treating these two sentences as either always alike, or always different, we can be sure that the sentence '$Ta \equiv Tb$' does not stabilize as either true or false, relative to that Bootstrapping Policy. Therefore, the sentence which says that two liar sentences are equivalent is not merely "weakly unstable" but worse than that: there is a Bootstrapping Policy on which it is unstable. It is therefore "more paradoxical" than the truth-teller. (It is not of course outright paradoxical, since there are Bootstrapping Policies on which '$Ta \equiv Tb$' stabilizes – namely, those which always put the two paradoxical sentences together, or always separate them. That is clearly as it should be).

Gupta has pointed out in conversation that as a consequence there is a sentence which is stably true in his sense without being universally stably true in mine, namely '$T'Ta \equiv Tb' \equiv (Ta \equiv Tb)$'; this makes good on an earlier promise.

Those are the facts. I do want to conclude that they count in favor of deploying arbitrary Bootstrapping Policies instead of only Constant Bootstrapping Policies: it seems to me nothing but an oddity of the Constant Bootstrapping Policy construction that '$Ta \equiv Tb$' should always stabilize, and a virtue of the more general Boostrapping Policy construction that sometimes it does not stabilize. But permit me to urge that although there is this difference between the two plans, it is a small difference. What is fundamental is Gupta's key idea of taking truth, when construed as a predicate, as obtaining its meaning through a Rule of Revision such as he describes. Admission of capricious Bootstrapping Policies may be interesting – but they are still only a technical fiddle on a subtle and profound conceptual and mathematical analysis.

7. TRANSFER TO OTHER CONCEPTS

It is worth thinking for a bit about how Gupta's Rule of Revision theory of truth transfers to other concepts.[11] Gupta's own application of a Rule of Revision has several special features. In the first place, what he is defining is the extension of a predicate; and one can see immediately that this feature is indeed "special". There is no work at all required in order to define extensions for operators such as "the denotation of". The Bootstrapper assigns some arbitrary function to the operator to get things under way; some Rule of Reason such as Tarski's theory of "the denotation of" provides its extension at successor ordinals; and at the limit, for each argument, if its value for that argument has stabilized, we take that very value, otherwise consulting a Bootstrapper to pick out for us an arbitary value for that argument.

Second, one can see that although Gupta's case involves only a single predicate, there is no bar to using his ideas to give a Rule of Revision account of several predicates and operators simultaneously; the Bootstrapper is up to the job of making all the arbitrary choices required. For example, one could simultaneously give Tarski-based accounts of denotation and of truth.

Third, one can use the general kind of machinery developed by Gupta to treat predicates which derive their meaning not from a Tarski successor rule, but in other ways. Here is an example of some interest. Suppose we include '*ST*' in the language as the 'stable truth' predicate. A Bootstrapper must get '*ST*' (as well as '*T*') started. Then at successor stages, which are not so interesting for stable truth, we treat '*ST*' as constant (its extension at the successor is the same as its extension at the predecessor). But at limit ordinals we use a construction something (but not quite) like Herzbergers's; we put into the extension of '*ST*' all and only those sentences which have stabilized as true (hence have stabilized as in the extension of '*T*'). Observe that, just as in Herzberger's scheme, no appeal to a Bootstrapper is made at the limit in calculating an extension for '*ST*'. Nevertheless, because the extension of '*ST*' depends on that of '*T*', at earlier stages, not on its own extension at earlier stages, non-uniform Bootstrapping Policies will still have their effects, albeit indirectly (no Grand Loops).[12]

Fourth, one sees that one of the glories of Gupta's account is that it allows us a theory of truth based on ordinary two valued semantics, with no fiddles whatsoever (no Kleene-strong or Kleene-weak three valued tables, no super-valuations, just the absolutately classical two valued semantics). One nevertheless *also* sees that the two valuedness (for instance) of the semantics plays no essential role. One can as well use Gupta's ideas for giving an account of the truth predicate (or of the "the truth value of" operator) in many valued semantics. Or one could give an account of the "the proposition expressed by" operator in modal semantics, or of a Kaplan "character of" operator in indexical semantics. Etcetera, etcetera, etcetera.

Fifth, Gupta restricts himself to closed sentences: he defines truth, not satisfaction. What would be required to give a Rule of Revision account for satisfaction (I take this as a paradigm for an account of semantic vocabulary applying to open formulas)? First we must say in which way we are conceiving satisfaction: (1) as a relation between sentences and infinite sequences – the most usual way; (2) as a relation between sentences and "assignments", that is, functions from the variables into the domain, which is less disingenuous, since that is anyhow what is and always has been going on; (3) as a relation between sentences and finite sequences, or their "finite assignment" correlates; or (4) as a function which picks out for each sentence the *set* of (say) sequences it satisfies – the "satisfaction-set-of" operator.

I discuss only (1) and (4). In the case (1), to apply Gupta's ideas we shall need to put all the sequences into the domain of the original model, just as Gupta needed to put all the sentences into the domain in order to begin to talk about an extension for the truth predicate. We shall therefore have to be sure that the domain is closed under denumerable sequences of its own members, which first happens after repeating the closing process up to the smallest uncountable ordinal. Let us call such a domain "closed under sequences". It seems to me that provided one conceives of satisfaction according to pattern (1), instead of (say) according to pattern (3), starting with models having domains which are closed under sequences is totally unproblematic and no more of an oddity than requiring that all the sentences be in the domain. One must put all the sentences in the domain to have a theory of truth as a predicate, and one must put all the sequences as well in the domain to have a theory of satisfaction of kind (1).[13]

And there are no other changes to be made: one uses a Bootstrapper to get started with an extension for the (two place) satisfaction predicate, uses Tarski's theory for successor ordinals, and recurs to a combination of a limiting process and appeal to a Bootstrapper at limit ordinals.

The case (4), where one wants an account of the satisfaction *operator*, "the satisfaction-set-of", is more delicate, for the simple and obvious reason that we cannot, by Cantor's theorem, close the domain under subsets, so that we cannot be sure we have all the needed values for the satisfaction operator in the domain by indelicately attempting to close it under subsets. I leave this problem open; perhaps it is the same as the problem of adapting Gupta's ideas to structurally similar problems in set theory.[14]

8. UPSHOT

It all seems so natural that it now becomes hard to envisage alternatives. Gupta has given us a wonderful picture of Truth.

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NOTES

[1] These comments were written to be delivered orally at the meeting of the Eastern Division of the American Philosophical Association in December, 1981. Thanks for help are due Anil Gupta and Kenneth Manders.

[2] This issue of this journal. I shall also refer to Herzberger's article in the same issue, though I have seen only precursors and not the piece itself.

[3] Gupta calls one of his wonderful examples a type of reasoning "that we allow in everyday discourse". It is to be noted that the reasoning he illustrates does not get off the ground unless we already take truth as a property-ascribing predicate. If instead it is viewed as an inheritor, we cannot be sure that his (a3) and (a5) are contradictory,

any more than we can be sure that 'the dog in the manger is not brown' contradicts 'the dog in the manger is brown' without first assuming, say, that 'the dog in the manger' is, contra Russell, a denoting term. I mean to say that it is not an argument against the prosentential theory of truth that it does not allow the reasoning of Gupta's example, any more than it is an argument against Russell that he does not allow "every day" reasoning to the conclusion that the present king of France exists, because everything does. *But* when truth is taken as property-ascribing, it seems to me that Gupta's example is exactly pertinent.

[4] I am going to continue to describe Gupta's scheme as involving Constant Bootstrapping Policies, even though in reality only Bootstrappers (sets) are involved in his scheme. Because I am taking this liberty, the following set-theoretical point wants making: my Bootstrapping Policies are proper classes (they are too big to be sets) that accordingly reside in an NBG set-class theory which permits quantification over them. In contrast, Gupta can make do with ZF and quantification over only sets.

[5] 'Constant Bootstrapping Policy' is not quite right for Herzberger: he uses an arbitrarily chosen Bootstrapper to get started, but at the limit he appeals only to the empty set. Nothing in the present discussion, however, seems to depend on this point.

[6] I introduce "universal stability" as a local expository device to permit comparison with Gupta's concept of stability.

[7] Gupta says "leads to the same results from all starting points", but it must be remembered that Gupta's Bootstrapper *U* is not only a starting point, but a principle of arbitrary choice to which appeal is made again and again. It does not play the same role conceptually as does Kripke's starting point, which is used only once.

[8] Gupta has pointed out in conversation that the first scheme really reduces to just: a starting set for zero, plus a single set for the limit ordinals. He has suggested a third scheme combining the first two: a starting set for zero plus a repeated Limited Bootstrapping Policy for the limit ordinals.

[9] The difference between 'universal stability' and 'stable universality' is supposed to convey the order of the quantifiers. But does it?

[10] Nixon: I continually lie. Dean: I certainly never tell the truth. Robert Redford and Dustin Hoffman: they are two peas in a pod.

[11] I know Gupta has thought about all of these matters, for we have discussed them.

[12] If we add '*SF*' for 'stably false' as well, we can say some things in the language which would otherwise be outside it. For example, if '*a*' denotes '*Ta*' and is thus a truth-teller, '$STa \vee SFa$' is stably true: every truth-teller is either stably true or stably false. And if '*e*' denotes the sentence '$Ta \equiv Tb$' discussed above which states the equivalence of two liars, then if quantification is limited to Constant Bootstrapping Policies, '$STe \vee SFe$' will be stably true, while if we admit inconstant Bootstrapping Policies, then it will not be stably true (or stably false either). And interestingly, if '*a*' denotes a liar '$\sim Ta$', then '$(\sim STa \ \& \sim SFa)$' is stably true, which indicates how deeply different this sort of proposal is from monotonic schemes.

[13] My point is that anyone who thinks according to scheme (1) already has in mind a domain the power of the continuum since that is how many infinite sequences (at least) there are; and closing such a set under sequences does not increase its size. Of course others may prefer to employ scheme (3) just in order to remain with a smaller domain.

[14] One plan – call it 'Plan 1' – would be to enlarge the domain at each successor stage so as to accommodate all the new components of the satisfaction relation which

must be added for that stage. That is, at a successor stage $a + 1$, we are given a domain D_a closed under sequences and an extension Sat_a for the satisfaction operator. Given a sentence A we can compute by Tarski the set X of sequences of members of D_a which satisfy A, but we cannot simply put the pair $\langle A, X \rangle$ in the extension Sat_{a+1} of the satisfaction operator at stage $a + 1$, because we cannot be sure X is in D_a. So what we do is define D_{a+1} as the result of closing the power set of D_a under sequences. (One would not of course need to add *all* subsets of D_a, since the denumerable collection – one for each sentence – required for the values of the satisfaction operator would suffice; but why be stingy?) And at the limit we do the usual thing: the limit domain is the union of all the prior domains, and the extension of the satisfaction operator is given by judicious combination of settling a value for a sentence A if there is one which has stabilized as the limit was approached, and otherwise appealing to a Bootstrapper.

But Plan 1 is No Good: almost nothing stabilizes, since the satisfaction set attached to any sentence A will clearly continue to grow with the universe by plugging in new assignments to variables on the values of which the value of A does not depend.

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