Abstract

This paper examines the distinct nature of Newtonian inductivism and its connection to methodological atomism. According to this interpretation, Newton's Rule III for the Study of Natural Philosophy is a criterion for isolating the primary qualities of the atomic parts. The universal nature of such qualities also raises the laws describing these qualities to the status of laws of nature.

On Newtonian Induction

June 29, 2016

1 Introduction

Newton's commitment to induction is articulated in his Rule 4 for the Study of Natural Philosophy. According to Newton, induction is the method that determines a proposition's scientific status. As he famously argues in the General Scholium, science should not include any hypothesis whatever its merit might be, if it is not gathered from the phenomena by induction (Newton, 1999, p. 943).

Following various criticisms of induction, the most central one being David Hume's, many philosophers of science adopted an Hypothetico-Deductive (HD) account of science or some anti-inductivist line of thinking. Philosophers such as Hanson (1970), Popper (2003), and Stein (1970, 1990) argued that Newton's inductivism functioned more as a rhetorical device, intended to prop up the credentials of Newton's theory of gravity and defend it from the criticism. But his actual science must have relied on hypotheses and conjectures.

Newtonian scholarship in the last few decades has done much to mitigate the reinterpretation of Newton's method along HD lines.[1] The main thrust of this

[1]See Dorling (1973), Glymour and Stalker (1980), Belkind (2012), Harper (2012) and Smith (2002a,b).

scholarship might be summarized as showing that Newton made extensive use of Demonstrative Induction (DI). The notion of a DI argument dates back to Aristotle. According to Aristotle, we can sometimes derive from a particular phenomenon its cause. Aristotle's example is the following:

> All planets do not twinkle.
>
> All non-twinkling bodies are near.
>
> Therefore, all planets are near.

This argument is deductively valid. It takes an observational generalization, and uses another universal claim to derive a further universal claim. Nevertheless, a DI argument is still inductive in character, since it moves from an observed fact to an unobserved fact, that is, from the effect to the cause. In Aristotle's example, the transition is from the observed fact that planets do not twinkle, to the unobserved fact (during Aristotle's time), that planets are near. How is it that a DI inference is possible? In the case of Aristotle's example, the DI inference is made possible by the proposition "All non-twinkling bodies are near". This proposition is true because there is a causal connection between being near and non-twinkling. The following claim describes a causal relation:

> All bodies that are near do not twinkle.

This proposition supports a counterfactual "if a body were far, it would twinkle". And this counterfactual's contrapositive is "if a body does not twinkle, it is not far". The transition from an observed fact to its cause presupposes that at least one of the premises include a law of nature, a causal relation, or a proposition derived from a causal relation.

Newton's application of a DI form of reasoning is perhaps one of the most sophisticated ones in the history of science. For example, in Book III of the

Principia, Newton utilizes Kepler's Area Law (i.e., that the radius from the central body to each satellite covers equal areas in equal amounts of time) and Kepler's Harmonic Rule (that the period of the planets' orbits and the radius of the orbit is related as $T^2 \propto R^3$) to deduce the nature of the gravitational force. Thus, Newton uses a mathematical form of reasoning to derive the nature of the cause (i.e., the gravitational force) from its observed effects (Kepler's Laws). Newton demonstrates that the Area Law implies that the gravitational force is centripetal.[2] Two important premises in this derivation, other than Euclid's postulates, are Newton's first and second laws of motion.

Newton's proof is geometric; he compared the motion that is involved in the Area Law to the motion that *would have resulted* had the body continued in its rectilinear motion without any force applied. Given the necessary connection between change of motion and direction and presence of the force, we can examine the phenomenal law, i.e., Kepler's Area Law, relative to counterfactual scenarios. At the same time that he showed the correlation between the Area Law and the centripetal nature of the force, Newton also showed that were the Area Law not valid, the force would not have been centripetal. Thus the argument has the function of revealing the cause (the centripetal nature of the force) via the presence of the effect (The Area Law).

A similar type of DI argument was utilized by Newton to argue for the inverse-square nature of the force of gravitation. Taking Kepler's Harmonic Rule, Newton showed the inverse-square nature of the force. Were the planets not to observe the Harmonic Rule, the force would not have been inverse-square.

Emphasizing the role of DI arguments in Newton's reasoning bolsters to some extent Newton's assertion that he used induction in his derivation. It was

[2]See Harper (1990, 2002, 2012) and Belkind (2012) for an account of these inferences.

important for Newton to show how phenomenal laws can be used to derive the nature of the force that gives rise to these phenomenal laws. However, for a DI argument to work, it needs to rely on a premise that is derived from a universally valid proposition; i.e., a causal law or a law of nature. Thus, the main contention that can be leveled against accounts that emphasize the role of DI arguments is that they are a variant of the HD method.[3] Without an account of how inductive arguments can ground laws of nature or causal laws, we have no justification in treating background assumptions in DI arguments as anything but hypotheses or conjectures. It is the process of elevating empirical claims to causal laws that provides the hinges on which the DI method turns.

To fully endorse Newton's claim that he is an inductivist, one needs to find an argument that shows how laws of nature can be derived from the phenomena. There is one methodological rule that might explain how such a derivation is possible, and that is Newton's Rule III for the Study of Natural Philosophy:

> **Rule III** The qualities of bodies that cannot be intended or remitted [i.e., qualities that cannot be increased or diminished] and that which belong to all bodies on which experiments can be made should be taken as qualities of all bodies universally. (Newton, 1999, p. 795)

Rule III purports to articulate a criterion for determining which qualities can be deemed *universal*. I.e., Newton is providing us a rule for determining those qualities that are present in all material bodies. Presumably, once we are able to ascertain those qualities as universal, the laws articulating the nature of such qualities would acquire the status of laws of nature that are necessarily valid. For example, if extension is to be determined as a universal quality, the laws determining the nature of extended bodies and their geometric properties, i.e., Euclid's Geometry,

[3]See Worrall (2000) for such a suggestion.

would therefore be determined as laws of nature governing all bodies.

Rule III has the potential of bolstering and supporting the DI method, explaining how certain background assumptions assume the role of laws of nature. However, Rule III has been notoriously difficult to interpret. We find a variety of readings of Rule III in the literature, which can be summarized as follows:

1. Rule III expresses commitment to Baconian **Induction**.

   According to this reading, Rule III argues that a property that observed in all bodies can be universalized to all bodies whatsoever. So Rule III is simply an inductive procedure applied on a massive scale. This reading is natural since Newton asserts that qualities which belong to all bodies on which experiments are made belong to all bodies universally. According to this reading, Rule IV asserts the same methodological rule articulated in Rule III; while Rule III consider the inductive inference itself, Rule IV also explains that induction functions as a criterion for demarcating scientific propositions from hypotheses. Some version of this reading may be found in several commentators, including Whewell (1971, p. 194), Burtt (1954, p. 219), Koyré (1965, p. 267ff), Torreti (1999, p. 73), De Pierris (2012) and Ducheyne (2012, p. 117).

2. Rule III is a form of **transductive** reasoning.

   According to this reading, Rule III argues that there is an inference from the observable qualities to the qualities of their ultimate, atomic parts. The atomic parts are not accessible to the senses as they are too small to leave traces on our senses. Thus a transductive argument follows a more radical inferential transition than induction, in that it is an inference from qualities of observable bodies to bodies that are in principle unobservable. This reading is difficult to make sense of whenever we consider the formal wording

of Rule III, but the history of Rule III, the historical context of Newton's atomism, and Newton's explication of Rule III make this reading plausible. We find the assimilation of Rule III to transdusction in Mandelbaum (1966, pp. 84) and McGuire (1967, 1968, 1970).

3. Rule III uses **invariance** as a criterion of universality.

   According to this reading, Rule III bases the universalization of qualities on qualities of bodies cannot be intended or remitted, i.e., *invariant* qualities. According to this reading, one should consider the parameter characterizing the quality. If we know from experience that the body carries the same parameter, no matter which forces operate on it or what experimental context we measure this parameter, this parameter represents a universal quality. We might consider the distinction between rest mass m_0 and relativistic mass $m_r = \gamma m_0$ as a relevant analogy for understanding Newton's criterion. While rest mass is invariant in all inertial reference frames, relativistic mass is variant.[4] Thus we are comfortable assigning rest mass to bodies themselves. Similarly, Newton is alluding to the invariant nature of the Newtonian mass parameter, and thus thinks of it as an inherent quality that describes the nature of bodies.

   This reading of Rule III can be found in Finocchiaro (1974), McMullin (1978, p. 142), Okruhlik (1989), Hooker (1991), Harper (2012, p. 38) and Janiak (2010, p. 95). The reading renders Newton's Rule III continuous with modern mathematical ideas about representing physical attributes, and with leading interpretations of the theory of relativity.

It is not entirely clear whether the various readings are necessarily exclusive.

[4]See Lange (2002, p. 224) for a view that connects the invariance of a parameter with its status as an objective property that is "real".

Perhaps Newton meant to combine more than one criterion to form Rule III. But a few commentators have argued that reading Rule III as articulating a transductive form of reasoning cannot cohere with the intension/remission criterion. For example, McGuire has suggested that in addition to the inference to the qualities of the atomic parts, Rule III relies on the intension/remission criterion for highlighting the nature of these qualities as primary and essential. For McGuire the purpose of the intension/remission criterion is to demonstrate that these qualities are *necessarily* in the atomic parts of matter. But Okruhlik (1989) criticizes McGuire's reading since Newton argues that gravity is not an essential quality, given that the weight of a body depends on the distance to the central gravitating body. Thus, according to Okruhlik, since gravity cannot be inherent or essential, it also cannot be a primary quality. In Okruhlik's view, Newton was not being very coherent in his methodological remarks. Janiak (2010, p. 93-95) similarly argues that Newton could not have taken gravity to be essential, given the variance of its intensity, and so it cannot be a primary quality. Janiak consequently argues that the intention of Rule III cannot be to articulate a criterion for identifying the primary qualities of the atomic parts. Instead Janiak claims we should think of Rule III as incorporating two distinct criteria, one empirical (Baconian induction), and one conceptual (intension/remission). The conceptual criterion applies only to the standard list of primary qualities; the empirical criterion applies to gravity as well.

In this paper I argue that there is no genuine conflict between the three readings, as long as we properly understand what Newton meant by his intension/remission criterion. According to the reading offered here, the transductive inference is in fact *grounded* by the intension/remission criterion. Given that the qualities were observed in all bodies, and given that the qualities

cannot be intended or remitted, Newton argues that one may conclude that they are present in the atomic parts of matter, and should be deemed as primary and universal qualities. However, in order to see how the intension/remission criterion grounds the transductive inference, we need to revise our understanding of this criterion, and the nature of invariance that Newton has in mind.

According to the account developed here, Newton is committed to thinking about material bodies as composed of indivisible atoms. While atomism, or the corpuscular thesis, is traditionally associated with a predetermined list of primary qualities (namely, size, shape and motion), at Newton's hand the atomist thesis becomes a *methodological* requirement for scientific explanations. Instead of basing the atomist thesis on a predetermined list of primary qualities, in Newton's thinking this list is open-ended, allowing experience to determine which qualities are attributed to the atomic parts. I shall term this Newtonian way of thinking "methodological atomism", to differentiate it from standard atomist theories. Thus my reading of Rule III follows the traditional reading of Mandelbaum and McGuire, and its purpose is to defend this reading, namely, that Newton is thinking of Rule III as an inference from observed qualities to primary qualities of the atomic parts. However, the reading of Rule III as a criterion for distinguishing primary from secondary qualities requires a revision in our understanding of the intension/remission criterion. I argue that this criterion is not meant to articulate a general mathematical criterion of invariance as a mark of universality, but that Newton is thinking of a very specific form of invariance; namely, *invariance under reconfiguration of the atomic parts*. That is, the intension/remission criterion highlights a specific form of invariance that is the counterpart of Newton's methodological atomism. Given that matter is made of atomic parts which are all alike, the qualities that directly arise from the primary qualities would not change

their intensity whenever the configuration of the parts changes, or the spatial arrangement of the atomic parts realigns. On the other hand, secondary qualities are those that vary as a result of the reconfiguration of the parts. Given this reading of the intension/remission criterion, one would be able to accept the gravitational quality as both *primary* and *non-essential*. Indeed, I shall argue that this is the main conceptual innovation that enables Newton to broaden the boundaries of the mechanistic paradigm.

In section 2 I reconstruct and provide some textual evidence for the transductive reading of Rule III. In Section 3 I consider some reasons to doubt the existing readings of Rule III. In section 4 I argue that we ought to revise our understanding of the intension/remission criterion of Rule III, and take it as invariance under changes of configuration of the atomic parts. In section 5 I consider how the reading offered here coheres with the list of qualities deemed as universal by Newton. I conclude in section 6.

2 Some evidence in support of the transductive reading

Before examining the coherence of the various readings of Rule III, we investigate what textual evidence can be given in support of the transductive reading; since the other readings flow more naturally from the wording of Rule III.

After articulating Rule III, Newton adds the following explication:

> For the qualities of bodies can be known only though experiments; and therefore qualities that square with experiments universally are to be regarded as universal qualities; and qualities that cannot be diminished cannot be taken away from bodies. Certainly idle fancies ought not to

> be fabricated recklessly against the evidence of the experiments, nor should we depart from the analogy of nature, since nature is always simple and ever consonant with itself. The extension of bodies is known to us only through our senses, and yet there are bodies beyond the range of these senses; but because extension is found in all sensible bodies, it is ascribed to all bodies universally. We know by experience that some bodies are hard. Moreover, because the hardness of the whole arises from the hardness of its parts, we justly infer from this not only the hardness of the undivided particles of bodies that are accessible to our senses, but also of all other bodies. That all bodies are impenetrable we gather not by reason but by our senses. We find those bodies that we handle to be impenetrable, and hence we conclude that impenetrability is a property of all bodies universally. That all bodies are movable and persevere in motion or in rest by means of certain forces (which we call forces of inertia we infer from finding these properties in the bodies that we have seen. The extension, hardness, impenetrability, mobility, and force of inertia of the whole arise from the extension, hardness, impenetrability, mobility, and force of inertia of each of the parts; and thus we conclude that every one of the least parts of all bodies is extended, hard, impenetrable, movable, and endowed with a force of inertia. And this is the foundation of all natural philosophy. (Newton, 1999, p. 795-796)

The explication of Rule III suggests that there remains in Rule III a connection to Newton's atomistic thesis, since Newton asserts that the Rule allows him to infer the qualities of the ultimate parts of matter from the observable composite bodies. He argues that we universally find certain qualities to exist in observable

macroscopic bodies and that we are "not to recede from the analogy of Nature, which uses to be simple, and always consonant to itself". The analogy of nature is an analogy between various scales of nature, the very large, the medium sized and the microscopic. The reference to analogy of nature suggests that while the atomic parts are inaccessible to our senses, it is reasonable to conclude that they conform to the analogy of nature. The laws of Euclidean geometry are isotropic, thus they are the same laws existing at all observable scales of nature. We apply Euclid's laws at astronomical scales, at the scale of extremely large objects, and at the scale of small objects that we can touch with our hands and see with our eyes. It would therefore be natural to extend the analogy of nature from the observable realm to the microscopic components from which all matter is made. That is, it is reasonable to assume, given that the analogy of nature holds for extension at all observable scales, that it will also hold for the atomic parts. Newton goes on to suggest that the analogy of nature holds for extension, impenetrability, hardness, mobility, force of inertia, and eventually gravity. However, such an analogy of nature does not hold for other qualities, such as color, taste, and perhaps also elasticity and magnetism. Thus we may conclude that extension, hardness, impenetrability, mobility and force of inertia are qualities that describe the least atomic parts, i.e., the smallest scale at which these qualities exist, while other qualities do not.

The nature of the inference is fully brought out the when Newton describes the inference regarding the quality of hardness: "because the hardness of the whole arises from the hardness of its parts, we justly infer from this not only the hardness of the undivided particles of bodies that are accessible to our senses, but also of all other bodies." There is first the claim that the hardness of composite bodies is reduced to the hardness that characterizes their atomic parts. We therefore

conclude that all parts of *observable* matter are hard. The further inference is that such a quality is a primary quality, that is a quality that characterizes atoms in general. And so, we conclude that all material bodies are made of the same kind of atoms that are hard, that is, all material bodies are themselves hard.

Newton makes clear in his explication that the inference from bodies to their ultimate parts governs all known primary qualities: "The extension, hardness, impenetrability, mobility, and force of inertia of the whole arise from the extension, hardness, impenetrability, mobility, and force of inertia of each of the parts; and thus we conclude that every one of the least parts of all bodies is extended, hard, impenetrable, movable, and endowed with a force of inertia. And this is the foundation of all natural philosophy." Newton argues that he has identified the central inference from qualities of observed bodies to qualities of the atomic parts, and that this inference grounds the entire enterprise of natural philosophy.

Further support for the transductive reading can be found in Cotes's Preface to the *Principia*. Cotes was the editor of the second edition, and he took upon himself to summarize the overall structure of Newton's argument, so that we might have an overall view of Newton's main scientific achievement. After rehearsing the central features of the argument, Cotes compares the inference regarding the nature of gravity to other inferences we might have about qualities that are traditionally seen as primary:

> Now, since all terrestrial and celestial bodies on which we can make experiments or observations are heavy, it must be acknowledged without exception that gravity belongs to all bodies universally. And just as we must not conceive of bodies that are not extended, mobile, and impenetrable, so we should not conceive of any that are not heavy. The extension, mobility, and impenetrability of bodies are known only

> through experiments; it is in exactly the same way that the gravity of bodies is known. . . . If anyone were to say that the bodies of the fixed stars are not heavy, since their gravity has not yet been observed, then by the same argument one would be able to say that they are neither extended nor mobile nor impenetrable, since those properties of the fixed stars have not yet been observed. Need I go on? Among the primary qualities of all bodies universally, either gravity will have a place, or extension, mobility, and impenetrability will not. And the nature of things either will be correctly explained by the gravity of bodies or will not be correctly explained by the extension, mobility, and impenetrability of bodies. (Newton, 1999, p. 392-393)

According to Cotes, the evidence Newton points out shows that gravity is a primary quality like any other primary quality, a quality that characterizes all atomic parts. The choice, according to Cotes, is either to accept gravity as a primary quality like other known primary qualities, or deny the status of primary qualities to all the qualities that are deemed primary in the mechanistic tradition.

The atomistic background to Rule III is also evident from the prehistory of the Rule. Some intriguing revisions occur between the first and second editions of the *Principia*. The revisions came in response to severe criticisms that were leveled against the non-mechanical nature of Newton's theory of gravity. In the first edition (1687), Newton included the following hypothesis:

> **Hypothesis III** Every body can be transformed into a body of any other kind and successively take on the intermediate degrees of qualities. (Newton, 1999, Cohn's Introduction, p. 202)

Newton here appears to be committed to an atomistic conception, according to which all parts of matter are of the same kind. According to this view, the

difference between kinds of body is the mere result of a body's transformation. For example it should be possible to take a body made of wood, and reconfigure the parts of the body, i.e., transform it, so as to produce a body that is made of iron. Given that all parts of matter are the same, it is only the distinct configuration of the parts that determines their particular kind. Newton does not seem to place significance on the shapes or contingent properties of the atoms in explaining how qualities of composite bodies are determined, but he seems to argue that no quality (other than the ones to be found in the atomic parts) is inseparable from the body. Any such quality can be made to diminish its amplitude or degree, and assume successively the quality's intermediate degrees. It is also possible to diminish the quality's intensity until it is no longer present in the body. For example, assume that we have a body with the color white. Given the atomistic thesis, the quality of being white belongs to the composite body, and does not stem solely from the qualities present in the atomic parts. When we manipulate the configuration of the parts, e.g. take the body and melt it, or pound it to reduce it to powder, we may introduce via the transformation of the body various manifestations of the color white, in this way diminishing or increasing its intensity. Furthermore, it may be possible to slowly transform the body until it no longer has the quality of being white, i.e., it is transparent. The suggestions is that while bodies appear to have various sensible qualities, such as the color of wood, the smoothness of metals, the acidity of lemon juice, these qualities are partly the result of the contingent configuration of the parts, and can be separated from bodies (or reintroduced) as a result of a process of reconfiguration.

The transductive reading of Rule III becomes plausible when we place Rule III in its proper historical and textual context. Given that Hypothesis III commits Newton to the atomist thesis, and given that Rule III is intended to provide the

same methodological justification as Hypothesis III, it seems plausible to think that the purpose of Rule III is to articulate an inference from the observable qualities to the primary qualities of the atomic parts. In this way, demonstrating which qualities ought to be considered universal qualities.

3 Problems with traditional readings of Rule III

In the next section I shall argue that given the transductive reading of Rule III, a correct analysis of the intension/remission criterion should alter our understanding of the type of invariance Newton has in mind. Before articulating the argument in favor of reinterpreting Newton's intension/remission criterion, it serves us to consider the tensions that each reading of Rule III raises. The most important criteria for evaluating the various readings of Rule III are textual fidelity, historical plausibility, and conceptual coherence. It is important that the reading of Rule III will cohere with Newton's wording of Rule III and its explication. Furthermore, a plausible reading of Rule III will explain how Newton thought it was legitimate to replace Hypothesis III, which includes a commitment to the thesis of transmutation, with Rule III, which combines the intension/remission criterion together with proper empirical grounding. It is evident, from the specific ways in which Hypothesis III and Rule III are used in Newton's argument, that they serve the exact same methodological role. Thus, a reading that is faithful to Newton's text would explain the continuity between Hypothesis III and Rule III. Such a reading would also cohere with the entire explication offered for Rule III in the second and third editions of the *Principia*. The criterion of conceptual coherence requires that the reading explains how Rule III applies to the specific list of

universal qualities Newton mentions. In particular, this should be demonstrated for the quality of gravity, since Rule III was specifically designed to show that gravity ought to be taken as a universal quality.

The main problem with assimilating Rule III to simple Baconian induction is that it largely ignores the intension/remission criterion. This reading therefore has a serious problem with being faithful to the text. More seriously, this reading is unable to account for the continuity between Hypothesis III and Rule III, because it is unable to show why a largely metaphysical statement is equivalent in Newton's eyes to a rule emphasizing an inductive procedure. Such a reading makes Newton a poor philosopher or an opportunistic thinker. While it is legitimate to accuse Newton of being an imperfect philosopher, one does need to consider the exactness with which almost all other propositions in the *Principia* are articulated, and the care which Newton had taken in forming various drafts for Rule III, before inserting the final draft into the second edition of the *Principia*, some 28 years after the first edition was published.

The benefit of assimilating Rule III to a transductive inference is that it is much more faithful to Newton's text and Newton's historical context. The doctrine of transmutability is essentially a commitment to methodological atomism, and so it is reasonable to read Rule III as differentiating the qualities which are primary, i.e., those qualities that characterize all atomic parts, from secondary qualities which arise from the primary qualities and the configuration of those parts. However, there is a problem with the conceptual coherence of this reading, at least according to some common criteria for distinguishing between primary and secondary qualities. It is ordinarily understood during the 17th century that primary qualities are essential and inherent to bodies, as they provide the conceptual bedrock for the independent existence of atoms. If we take McGuire's suggestion,

that the intension/remission criterion provides a criterion for the essential nature of primary qualities, an immediate tension is formed with Newton's assertion that gravity is not essential. If the intension/remission criterion is not a criterion for the essential nature of primary qualities, then it is not clear what distinguishes the transductive reading from the inductive reading. In what sense is an inference to the primary qualities of the ultimate parts of matter anything more than an inductive inference from observed bodies to all bodies in nature?

The main benefit of taking Rule III as articulating a connection between the invariant nature of a quality and its universality, is that it brings to bear Newton's skillful use of mathematical methods for analyzing physical quantities. It is the invariant nature of mass, for example, in various physical and chemical processes, and in mathematical representations of such processes, that makes this quality a very good candidate for a universal quality that exists in all bodies. This reading's drawback is that it is difficult to make it cohere with the list of qualities that are mentioned in Rule III. For example, how is force of inertia shown to be a universal quality, given that the force of inertia can be intended or remitted? Given that the force of inertia depends on the body's velocity, it clearly is not an invariant quality of the body, and it would be strange if one could think of it as essential in any way or primary.[5] More importantly, how can Newton think that invariance is a mark of the universal nature gravity, given that its intensity is not invariant? In response to this worry one often finds the claim that gravity is always proportional to the body's mass, and so is a function of an invariant parameter. If a force law depends on invariant parameters of bodies, so the argument goes, the force is a universal force. But this is a strange reading of the *wording* of Rule III, because Newton has never said that the *mass* parameter *causes* both the force of inertia and the force of

[5]See Finocchiaro (1974, p. 68) for raising this issue with the force of inertia.

gravity, and that since an invariant quality *causes* the force of inertia and the gravitational force, that the latter are deemed universal. The language that treats the mass parameter as some cause is entirely absent from Newton's explication of Rule III.

Another problem with the invariance reading of Rule III is that it seems unfaithful to Newton's text, since it is not clear at all how the intension/remission criterion is related to Hypothesis III and the doctrine of transmutation. In Hypothesis III, Newton argues that *all* qualities of bodies can be intended and remitted, and a body of any kind can be transformed into a body of any other kind. How does Newton move from the doctrine of transmutation and the varying intensities of qualities to mathematical invariance as a mark of universality?

It is clear that each of the available readings of Rule III have serious drawbacks of either being unfaithful to Newton's text, lacking historical plausibility, or lacking coherence. The suggestion in what follows is that one needs to combine elements of these various readings into one coherent reading, that will remove the drawbacks of each specific reading.

4 The nature of invariance in Newton's intension/remission criterion, and the character of Newtonian inductivism

The various drafts Newton has written down in preparation of the second edition of the *Principia* (1713) suggest that Hypothesis III is an earlier version of Rule III which appears in the second edition. While Hypothesis III and Rule III are very different in formulation, it is clear from the drafts that Newton had erased the

word "Hypothesis" where Hypothesis III was written, and introduced the word "Rule" (McGuire, 1967, p. 235). He also articulated various versions of this Rule, ending with the final version inserted into the second edition. That they are intended as different expressions of the same idea is made clear by the fact that Rule III is cited at the same place within Book III, Proposition VII, Corollary II of the *Principia*, where previously Hypothesis III was cited.[6]

A comparison between Hypothesis III and Rule III suggests that a quality that cannot be intended or remitted is what Boyle and Locke have referred to as a primary qualities, while those that are intended and remitted are secondary qualities. Thus, Hypothesis III refers to *secondary* qualities, which can assume various degrees when a body undergoes transformation. Rule III, on the other hand, refers to *primary* qualities which do not assume various degrees when a body is transformed.

The atomist thesis is therefore implicitly committed to the existence of two kinds of qualities. Those qualities that are present in the atomic parts, and cannot be separated from them (i.e., primary qualities), and qualities that are attributed to composite bodies, that are nevertheless separable from them (i.e., secondary qualities). Since he asserts that every body can be transformed to a body of any other kind, Newton is committed to the notion that the real difference between bodies of distinct kinds amounts to differences of configuration, i.e., differences in how the atomic parts are put together. Boyle's technical term for the notion of configuration is "texture." According to Boyle the texture of a body includes the

[6]Further evidence that he viewed Rule III as a reformulation of Hypothesis III is a draft of Corollary II written after the first edition where Newton argued that the fact that gravity is a quality that cannot be remitted is a property of all bodies, according to *Hypothesis III*. See McGuire (1967, p. 234).

order and *posture* of the different corpuscles, i.e., order consists of the relative positions between the parts and posture includes the spatial orientation of each of the parts. Thus for Newton whenever we find a quality of a composite body, which has an intensity that varies as a result of changing the body's texture, the quality can assume every degree of its intensity, until a body is transformed into a different kind of body. Thus in Hypothesis III Newton commits himself to the claim that qualities variant under changes of texture are secondary qualities, and are separable from bodies.

We are not using the distinction between primary and secondary qualities in Descartes' or Locke's sense, who claim that ideas of primary qualities resemble those qualities while ideas of secondary qualities do not. In the context of Rule III, the distinction amounts to that between fundamental and derived; the primary qualities are those that form the basis of a scientific theory (and are present in all the atomic parts) and secondary qualities are those are derived from the fundamental qualities of the parts, together with the body's texture.[7]

The continuity between Hypothesis III and Rule III can also be seen in one of the drafts of Rule III, added as an Axiom 4 to one of Newton's revisions of the first edition:

> Qualities which are intended (and remitted such as motion and rest,) heat and cold, wet and dry, light and darkness, ..., these things do not come in for consideration here. The things which cannot be intended and remitted such as (bulk) impenetrability (solidity) and motion

[7]Notice that this atomist view is consistent with Newton's agnosticism about whether atoms are divisible or not. While the ultimate parts of matter might be divisible, the point is that however finely the parts are divided, they will carry the universal qualities at stake. See Janiak (2010, p. 109).

> . . . and that inertia which causes a resistance to motion and to changes of motion (solidity and extension) are usually considered to be the properties of all bodies. And the reason is because a quality which cannot be remitted cannot be taken away (from the whole); and on the other hand that which can be taken away, if it were to be taken away from some parts of the whole, it could be remitted in the whole. Impenetrability indeed is usually described as the essence of bodies and is hence attributed to all of them; but the essential properties of bodies do not become known by the light of nature. We gather (only) from the senses the fact that the bodies we touch (all tangible things) are impenetrable and we (conclude that) we attribute to all bodies alike, no less to the heavenly bodies and to bodies imperceptible to the senses than to those bodies which we touch: and that too with an argument which we think so strong (that nothing in the whole nature of things) that this quality is taken to be the essence of bodies and is therefore considered to be the firmest foundation of all philosophy. Therefore the axiom on the strength of which we gather this property of bodies must not be repudiated. Of the same kind is the argument for inertia and (mobility and force of resistance) of matter. (McGuire, 1967, p. 237)

It is clear from this draft that Newton is distinguishing a long list of qualities that can be intended and remitted from qualities that cannot. But the important thing to notice is that Newton connects invariance with a part-whole relation. He says that a quality that cannot be remitted cannot be taken away *from the whole*. By which he seems to mean that that given a composite entity, and that various attempts to transform this entity (say, by manipulations of force) are unable to increase or diminish the quality's intensity, implies that it is inseparable from the

whole. The implication of this invariance is that a quality that is invariant in the whole is the direct sum of the qualities that are present in the parts, and so it cannot be diminished when the whole is transformed or when the body's texture is manipulated. Similarly, Newton says that if the quality were separable from the parts, it would be diminished in the whole. Thus, the notion of inseparability from the parts is shown to be intimately tied to the invariance of a quality in the whole.

One should also notice from this quote the distinction Newton draws between finding the essential qualities of the atomic parts and the epistemic role of the intension/remission criterion. Newton argues that "the essential properties of bodies do not become known by the light of nature". That is, unlike Descartes and other speculative philosophers, who inquire after the essence of bodies via some conceptual criterion, the purpose of this criterion is determine the basic qualities of the atomic parts via experience. Because the intension/remission criterion is based on our experience with transformations of texture, the list of primary qualities might grow in tandem with our understanding of the phenomena. Newton is therefore articulating an empiricist criterion for primary qualities.

It is neither Descartes's nor Boyle's expectation that gravity should be anything but a secondary quality, one that is explained by other primary qualities and various configurations, motions and frictions of bodies that move through the ether pervading all of space. But Newton's main scientific discovery, and perhaps his unparalleled achievement, is to uncover the fact that gravity does not behave like a secondary quality whose intensity changes whenever the texture of a body changes. For example, we know that gravitational acceleration is the same for all bodies on the surface of the earth, regardless of their varying textures. We can also appreciate why the mention of essential qualities was dropped in the final formulation of Rule III. While Rule III demonstrates that gravity is a primary

quality, one cannot think of gravity as an essential or inherent quality, given that a body's weight is dependent on the distance to other massive bodies.

The type of invariance described in Rule III is distinct from the kind of invariance involved in finding the same parameter under all changes in relations to other bodies. The invariance of quality Newton is concerned with is invariance under the active reconfiguration of the body's parts. Such invariance does not necessarily presuppose that the quality at stake is *inherent*, since it may be that a quality that is invariant under changes of texture is still a relative or a non-inherent quality (i.e., determined by a set of relations to parts that are *not* part of the body). If the quality does not increase or diminish its intensity while changing the configuration of the parts, all this means is that the quality is *reducible* to the qualities present in the parts, and is not a function of the specific configuration of the parts. Thus the reading we provide here is distinct from the standard reading which assimilates Rule III to a *general* invariance criterion.

We might summarize Newton's inference as follows:

> All observed bodies have a quality Q.
> The quantity describing the intensity of Q is invariant under transformations of texture (i.e., reconfiguration of the parts).
> Therefore, all bodies have a quality Q.

The inference described by Rule III is therefore a particular kind of inductive inference. It is an inference that requires a wider basis than most inductive procedures, because it relies on a quality that is found in *all* observed bodies. But the inference is not a simple inductive inference, as it incorporates two distinct steps. From the fact that all observed bodies have quality Q, and that the quantity describing the intensity of Q is invariant under transformations of texture, Newton concludes that all ultimate parts of observed matter have quality Q. Let us call

this type of inference the "transductive" inference from the observed composites to their unobserved, ultimate parts. The second inductive step derives from the fact that all ultimate parts of *observed* matter have quality Q the claim that *all* parts of matter have quality Q. Let us call this step "universal induction". From the fact that all parts of matter have quality Q, Newton concludes that all bodies have quality Q, assuming here that a primary quality Q of any composite body is a direct sum of the qualities present in the parts.

Thus we see that a proper reading combines all standard readings into one coherent reading, where induction, transduction, and invariance all have a specific role to play. But to do so we must revise the traditional way in which commentators have understood Newton's intension/remission criterion, and think of it as invariance under changes of texture.

5 Newton's list of primary qualities

It remains to be seen how the reading we provide is consistent with Newton's list of universal qualities, namely, extension, hardness, impenetrability, mobility, force of inertia, and gravity. In what sense can we say that each of these qualities has a quantity invariant under transformations of texture?

When considering the quality under consideration, and the claim that the quality's intensity cannot be intended or remitted, we need to consider the quantities that represent those qualities. In the case of extension, we may think of the geometric quantity of volume, which represents the volume or space a body takes up. It is clear that while a body may increase or decrease in volume, as for example, in the case of a gas that expands or contracts, such changes in volume are ordinarily the result of a body's density decreasing or increasing, or the body's empty regions and pores increasing or diminishing. But the body itself, pores

excluded, has a volume that is a mathematical sum of the volumes of the parts. One may consider this additive nature of volume as perhaps the defining characteristic of Euclidean Geometry. A central presupposition for Euclidean Geometry is the notion that when segments of lines are added, the magnitude attributed to the composite line is a sum of the magnitudes attributed to the parts. A similar assumption holds for the entire volume attributed to a composite body as a function of the volumes attributed to the parts. Thus, it seems to be an essential assumption in Euclidean Geometry that the volume of a body will not change as a result of the reconfiguration of the parts, as long as we do not include pores and empty regions in the volume's calculation. This implies that the extension we attribute to composite bodies is analogous to the extensions attributed to the body's ultimate parts, and that extension is therefore a universal quality.

Hardness and impenetrability seem to be associated with a body, and make the body distinct from space itself. Notice that Newton is not mentioning mass, or Quantity of Matter, as the universalized quality, because for him the mass parameter is merely a *quantity*, not the qualitative aspect of bodies that make them impenetrable or hard, i.e., the qualitative aspects that make them material bodies. This point perhaps ought to be emphasized, because commentators often ignore the actual list of qualities mentioned by Newton, which include hardness and impenetrability and simply indicate mass as a universal quality, itself not listed by Newton as a universal quality. The difference between a quantity and a quality is that a quantity may indicate a measure attributed to a quality, but it does necessarily represent the quality itself, given that the quantity attributed to the body depends also on the *number* of bodies present or the *magnitude* of a composite body. Thus, the mass of a composite body does not indicate solely the inherent quality of a body, but is oftentimes amassed from the measures

representing the inherent qualities present in the body's ultimate parts. If we divide a composite body into two, it is often the case that nothing inherently changes in that composite body, we simply separate out one part of the aggregate from another. However, the mass, or quantity of matter of the composite body will be divided into two distinct quantities of matter. Thus, it is a misreading of Newton's words when commentators take the mass parameter itself to be a quality, as the mass parameter is nothing but a measure, a magnitude representing a certain intensity associated with a quality, or a measure associated with an aggregate of bodies that have the quality.

The claim that hardness and impenetrability are universal qualities is implied by the fact that mass parameter is invariant under transformations of texture. We can conclude from this feature of the mass parameter that impenetrability and hardness are universal qualities. Given that the Quantity that represents the measure of bodies behaves like an *extensive* quantity, analogous to temporal duration and extension, it is divisible into the ultimate parts of matter.

Mobility is the power of bodies to move from one place to another, and force of inertia compels a body to move in a straight line. Both of these qualities are reflected in the Quantity associated with moveable bodies, i.e., the quantity of motion (Newton's term for momentum). The quantity of motion of a composite body is always the sum of the quantities of motion attributed to the parts. Thus, there is strong indication that changes in the configuration of the parts do not correlate with changes in the overall quantity of motion (assuming a particular distribution of quantities of motion in the parts). The divisible nature of quantity of motion is also implied by the fact that the quantity of motion of a solid body is proportional to the quantity of matter. This implies that each part of the solid body carries its own quantity of motion, that is, each part of the body has both

mobility and force of inertia.

The fact that quantity of motion is proportional to quantity of matter, renders mobility and force of inertia universal qualities. But this is *not* because mass is an inherent parameter, and it causes the force of inertia. Rather, the fact that quantity of motion is proportional to the mass parameter is mathematical evidence that quantity of motion is a divisible Quantity, and that it arises from the quantities of motion attributed to the ultimate parts of matter, *and not* from the particular configuration of the parts. This is a subtle point, because the traditional invariance view is unable to make sense of the force of inertia being a universal quantity. The force of inertia can be intended or remitted as the result of changes in the body's velocity.[8] If one views the quantity of motion as a simply a vector quantity attributed to a particle, equal to the product of mass and velocity, then one will not be able to understand how the invariance criterion applies to force of inertia. But consider Newton's explication of his definition of quantity of motion:

> The motion of the whole is the sum of the motions of the individual parts, and thus if a body is twice as large as another and has equal velocity, there is twice as much motion, and if it has twice the velocity

[8]To make sense of how the force of inertia is explained by Rule III, Janiak (2010, p. 95) argues that the force of inertia is simply mass, given that Definition III of the *Principia* associates the force of inertia with a body's mass. But this is simply a confused reading of Definition III, where Newton asserts that the force of inertia is *proportional* to the body, and is the *inertia* of the body. The force of inertia is also determined by the body's velocity, since it can be considered as the body's impetus. Moreover, identifying force of inertia with mass is odd, since Newton lists four universal qualities, hardness, impenetrability, mobility, and force of inertia – are all these qualities intended to simply represent mass? It would be quite a redundant and perplexing list.

there si four times as much motion. (Newton, 1999, p. 404)

Newton does not clearly distinguish between the extension of the body and its mass, but he clearly thinks that quantity of motion behaves partly like an extensive quality, given that it is proportional to the "size" of the body. The quantity of motion *can* be intended or remitted, but it *cannot* be intended or remitted as a result of changes in texture, unless those changes in texture involve causal interactions with external objects.

And finally we get to the quality of gravity. It is the purpose of Rule III, I contend, to show that gravity is primary like any other primary quality. It is the surprising conclusion derived from Newton's work, and the main scientific achievement and innovation of his philosophy of nature. Rule III, therefore has the sole role of convincing Newton's readers of the reducible, or divisible nature of gravity, and therefore that such divisible nature implies its universality. His explication of Rule III continues as follows:

> Finally, if it is universally established by experiments and astronomical observations that all bodies on or near the earth gravitate toward the earth, and do so in proportion to the quantity of matter in each body, and that the moon gravitates toward the earth in proportion to the quantity of its matter, and that our sea in turn gravitates towards the moon, and that all planets gravitate toward one another, and that there is a similar gravity of comets toward the Sun, it will have to be concluded by this third rule that all bodies gravitate toward one another. Indeed, the argument from phenomena will be even stronger for universal gravity than for the impenetrability of bodies, for which, of course, we have not a single experiment, and not even an observation, in the case of the heavenly bodies. Yet I am by no means affirming that

> gravity is essential to bodies. By inherent force I mean only the force of inertia. Gravity is diminished as bodies recede from the earth. (Newton, 1999, p. 796)

Notice that Newton highlights two properties of gravity; that gravity is present in all observable bodies, and that it is proportional to quantity of matter. This, according to him, suggests that criteria articulated by Rule III applies to gravity as well. Under the traditional invariance reading of Rule III, this is a baffling claim. It is one thing to claim that quantity of matter is universal, given that it is invariant. But why do we think that a force *proportional* to quantity of matter is a universal quality? Does this imply that any force law that includes an invariant parameter is a universal law?

A more charitable reading connects the claim about gravitation being proportional to quantity of matter to a claim about the divisible nature of the gravitational force. Take for example the empirical fact that all bodies on the surface of the earth experience the same gravitational acceleration. For Newton, this empirical fact implies that the gravitational force is proportional to the quantity of matter. Take a body with a certain mass of 1 Kg. It experiences a gravitational acceleration of $g = 9.8\frac{[m]}{[s]^2}$. If we divide the body into two parts, each of which is 1/2 kg, each part experiences the exact same gravitational acceleration as the composite whole. This implies that the gravitational *force* must be twice as large in the composite body as the gravitational forces operating on the parts, to produce the exact same gravitational acceleration. Divide the body further and further into parts, since each of the divided part has the same gravitational acceleration as the whole, one may conclude that the gravitational *force* is proportional to the quantity of matter that is found in each of the parts, i.e., the gravitational force is exactly divisible in proportion to the masses found in the

parts. If we divide a body further, until we reach the atomic parts, the force of gravity will divide itself until we reach the force of gravity that operates on each atomic part. The important fact about gravity, is that its intensity does not vary as a result of reconfiguring the atomic parts, or changing the body's texture. The force of gravity of a composite body is a simple sum of the forces of gravity present in each of the atomic parts. Thus, the fact that the force is proportional to quantity of matter is essentially equivalent to the claim that a quantity representing the quality cannot be intended or remitted as a result of reconfiguring the body's parts.

The invariance of gravity under changes of texture is in direct conflict with mechanical theories of ether. When we imagine gravity to be a product of the friction created by an ether, we must suppose that the configuration of a body, its particular shape, whether its surface is smooth or coarse, would affect the intensity of gravity. Newton's *Principia* is also a study of the theory of fluids which demonstrates how bodies of varying shapes and volume would move differently through a fluid medium. Thus, Newton's claim is that contrary to any expectations produced by mechanical ether theories of gravity, gravity is a divisible quantity, not produced by any pressure between the ether fluid and surface of bodies. Such pressure would necessarily depend on the body's texture and the relative configuration of the parts.

This reading of Rule III enables us to view gravity as primary *and* non-essential or non-inherent. It is primary, in the sense that it can be universalized to all atomic parts. However, it is not essential, because the force of gravity that operates on each part varies in its intensity as the parts recede from the central gravitating body. Thus Newton's innovation is in articulating a criterion for distinguishing primary from secondary qualities, without this criterion necessarily

demonstrating the essential nature of the quality.

One difficulty with this view is that it seems to suggest that a world containing a single atom might not have the gravitational quality. If the gravitational force is necessarily derived from relations between bodies, it cannot be present in a single atom, if that atom is the only one existing. Newton has two interpretive options to deal with such a case. One option is to argue that Rule III shows gravity to be a universal quality, *except* for the case of a universe containing a single atom, where gravity does not exist. This option takes the universality of gravity to lack the metaphysical grounds that make it a *truly* universal quality. Another interpretive option would argue that the universality of gravity implies that a universe containing a single atom is ruled out as physically impossible. Given that gravity is universal, one may require that each atomic part would experience a gravitational pull, so that any single atom requires at least one other atom to exit. Either of these interpretive options seems counterintuitive, but the best way to accommodate everything that Newton says, especially the claim that gravity is both primary and non-essential, compels us to adopt one of these two options.

6 Conclusion

Recent scholarship regarding Newton's inductive procedures has emphasized his use of DI arguments, and have demonstrated the argumentative strategies that distance Newton from the HD account. However, emphasizing the role of DI arguments requires further analysis of the method by which a phenomenal law can be elevated to a law of nature, without which the DI arguments do not have any hinges on which to turn. Newtonian induction combines DI arguments, with a distinct inductive procedure for unravelling primary, or universal qualities. Newton's methodological atomism is based on the idea that those qualities that are

derived from qualities of the atomic parts have an intensity that does not depend on a body's transformation of texture. Once we have reduced a quality to qualities of the atomic parts, we can generalize to all atoms and to all composite bodies. Laws that describe the nature of primary qualities are taken to be universal laws of nature.

One might argue that methodological atomism is a hypothesis, and a false one at that. Not all parts of matter are the same, and there are good reasons for thinking that the analogy of nature fails. I cannot enter this important debate given the limited scope of the paper, except to say that the universal nature of extension, inertial forces, and gravity is still part of the foundation of contemporary physics, despite the radically different theories that we have about these qualities. So, perhaps in the physical sciences methodological atomism has its modern variations. What was important to uncover in this paper was the distinct nature of Newton's inductive procedure, and how it differs from Baconian induction.

References

Belkind, O. (2012). Newton's scientific method and the universal law of gravitation. In E. Schliesser and A. Janiak (Eds.), *Interpreting Newton: Critical Essays*, Chapter 6, pp. 138–168. Cambridge University Press.

Burtt, E. A. (1954). *The Metaphysical Foundation of Science*. Garden City, NY: Doubleday Anchor Books.

De Pierris, G. (2012). Newton, locke, and hume. In E. Schliesser and A. Janiak (Eds.), *Interpreting Newton: Critical Essays*, pp. 257–279. Cambridge University Press.

Dorling, J. (1973). Demonstrative induction: Its significant role in the history of physics. *Philosophy of Science 40*(3), 360–372.

Ducheyne, S. (2012). *The Main Business of Natural Philosophy*. Springer.

Finocchiaro, M. A. (1974). Newton's third rule of philosophizing: A role for logic in historiography. *Isis 65*(1), 66–73.

Glymour, C. and D. Stalker (1980). *Theory and Evidence*. Princeton University Press.

Hanson, N. R. (1970). Hypotheses fingo. In R. E. Butts and J. W. Davis (Eds.), *The Methodological Heritage of Newton*, pp. 14–33. Toronto University Press.

Harper, W. (1990). Newton's classic deductions from phenomena. *PSA (1990) Volume 2: Symposia and Invited Papers*, 183–196.

Harper, W. (2002). Newton's argument for universal gravitation. In I. B. Cohen and G. E. Smith (Eds.), *The Cambridge Companion to Newton*, pp. 174–201. Cambridge University Press.

Harper, W. (2012). *Isaac Newton's Scientific Method: Turning Data into Evidence about Gravity and Cosmology*. Oxford University Press.

Hooker, C. A. (1991). From phenomena to metaphysics. In D. Prawitz and D. Weterstal (Eds.), *Logic and Philosophy of Science in Uppsala: Papers from the 9th International Congress of Logic, Methodology and Philosophy of Science*, pp. 159–184. Springer.

Janiak, A. (2010). *Newton as Philosopher*. Cambridge University Press.

Koyré, A. (1965). *Newtonian Studies*. London: Chapman & Hall.

Lange, M. (2002). *The Philosophy of Physics, Locality, Fields, Energy and Mass.* Blackwell.

Mandelbaum, M. (1966). *Philosophy, Science and Sense-Perception: Historical and Critical Studies.* John Hopkins University Press.

McGuire, J. E. (1967). Transmutation and immutability: Newton's doctrine of physical qualities. *Ambix: The Journal of the Society for the Study of Alchemy and Early Chemistry 14*(2), 69–95.

McGuire, J. E. (1968). Force, active principles, and newton's invisible realms. *Ambix: The Journal of the Society for the Study of Alchemy and Early Chemistry 15*, 154–208.

McGuire, J. E. (1970). Atoms and the 'analogy of nature': Newton's third rule of philosophizing. *Studies In History and Philosophy of Science Part A 1*(1), 3 – 58. Reprinted in *Tradition and Innovation: Newton's Metaphysics of Nature*, 1995, Kluwer.

McMullin, E. (1978). *Newton on Matter and Activity.* University of Notre Dame.

Newton, I. (1999). *The Principia: Mathematical Principles of Natural Philosophy.* University of California Press. Trans. by I. B. Cohen and A. Whitman.

Okruhlik, K. (1989). The foundation of all philosophy: Newton's third rule. In J. R. Brown and J. Mittelstrass (Eds.), *An Intimate Relation: Studies in the History and Philosophy of Science*, Volume 116 of *Boston Studies in the Philosophy of Science*, pp. 97–113. Kluwer.

Popper, K. (2003). *The Logic of Scientific Discovery.* Routledge. Originally published in 1935 by Springer.

Smith, G. E. (2002a). From the phenomena of the ellipse to an inverse-square force: Why not? In D. Malament (Ed.), *Reading Natural Philosophy: Essays in the History and Philosophy of Science and Mathematics*, pp. 31–70. Open Court.

Smith, G. E. (2002b). The methodology of the *principia*. In I. B. Cohen and G. E. Smith (Eds.), *The Cambridge Companion to Newton*, pp. 138–172. Cambridge University Press.

Stein, H. (1970). On the notion of field in newton, maxwell, and beyond. In R. H. Stuewer (Ed.), *Historical and Philosophical Perspectives of Science*, Volume V, pp. 264–310. University of Minnesota Press, Minneapolis.

Stein, H. (1990). "from the phenomena of motions to the forces of nature": Hypothesis or deduction? *PSA (1990) Volume Two: Symposia and Invited Papers*, 209–222.

Torreti, R. (1999). *The Philosophy of Physics*. Cambridge University Press.

Whewell, W. (1971). *Philosophy of Discovery*. New York: Academic Press.

Worrall, J. (2000). The scope, limits, and distinctiveness of the method of 'deduction from the phenomena': Some lessons from newton's 'demonstrations' in optics. *The British Journal for the Philosophy of Science 51*, 45–80.