

CERN LIBRARIES, GENEVA



CM-P00058496

Ref.TH.1582-CERN

Archives

SUBJECT AND OBJECT

J.S. Bell  
CERN - Geneva

Contribution to the Symposium in honour of  
P. A. M. DIRAC  
Trieste, 18-25 September 1972

Ref.TH.1582-CERN  
8 November 1972

I have been invited to contribute under this heading. The subject-object distinction is indeed the very root of the unease that many people still feel in connection with quantum mechanics. Some such distinction is dictated by the postulates of the theory, but exactly where or when to make it is not prescribed. Thus in the classic treatise <sup>1)</sup> of Professor Dirac we learn the fundamental propositions :

"...any result of a measurement of a real dynamical variable is one of its eigenvalues..."

"...if the measurement of the observable  $\xi$  for the system in the state corresponding to  $|x\rangle$  is made a large number of times, the average of all the results obtained will be  $\langle x|\xi|x\rangle$ ..."

"...a measurement always causes the system to jump into an eigenstate of the dynamical variable that is being measured..."

So the theory is fundamentally about the results of "measurements", and therefore presupposes in addition to the "system" (or object) a "measurer" (or subject). Now must this subject include a person? Or was there already some such subject-object distinction before the appearance of life in the universe? Were some of the natural processes then occurring, or occurring now in distant places, to be identified as "measurements" and subjected to jumps rather than to the Schrödinger equation? Is "measurement" something that occurs all at once? Are the jumps instantaneous? And so on.

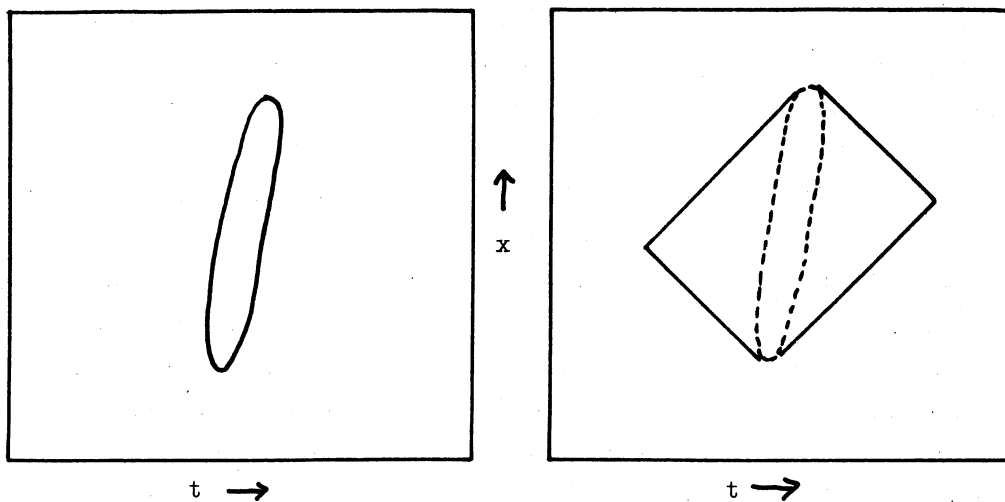
The pioneers of quantum mechanics were not unaware of these questions, but quite rightly did not wait for agreed answers before developing the theory. They were entirely justified by results. The vagueness of the postulates in no way interferes with the miraculous accuracy of the calculations. Whenever necessary a little more of the world can be incorporated into the object. In extremis the subject-object division can be put somewhere at the "macroscopic" level, where the practical adequacy of classical notions makes the precise location quantitatively unimportant. But although quantum mechanics can account for these classical features of the macroscopic world as very (very) good approximations, it cannot do more than that <sup>2)</sup>. The snake cannot completely swallow itself by the tail. This awkward fact remains : the theory is only approximately unambiguous, only approximately self-consistent.

It would be foolish to expect that the next basic development in theoretical physics will yield an accurate and final theory. But it is interesting to speculate on the possibility that a future theory will not be intrinsically ambiguous and approximate. Such a theory could not be fundamentally about "measurements", for that would again imply incompleteness of the system and unanalyzed interventions from outside. Rather it should again become possible to say of a system not that such and such may be observed to be so but that such and such be so. The theory would not be about "observables" but about "beables". These beables need not of course resemble those of, say, classical electron theory; but at least they should, on the macroscopic level, yield an image of the everyday classical world <sup>4)</sup>, for "it is decisive to recognize that, however, far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms" <sup>5)</sup>.

By "classical terms" here Bohr is not of course invoking particular nineteenth century theories, but refers simply to the familiar language of everyday affairs, including laboratory procedures, in which objective properties - beables - are assigned to objects. The idea that quantum mechanics is primarily about "observables" is only tenable when such beables are taken for granted. Observables are made out of beables. We raise the question as to whether the beables can be incorporated into the theory with more precision than has been customary.

Many people must have thought along the following lines. Could one not just promote some of the "observables" of the present quantum theory to the status of beables? The beables would then be represented by linear operators in the state space <sup>6)</sup>. The values which they are allowed to be would be the eigenvalues of those operators. For the general state the probability of an beable being a particular value would be calculated just as was formerly calculated the probability of observing that value. The proposition about the jump of state consequent on measurement could be replaced by: when a particular value is attributed to a beable, the state of the system reduces to a corresponding eigenstate. It is the main object of this note to set down some remarks on this programme. Perhaps it is only because they are quite trivial that I have not seen them set down already.

The state vector (or density matrix) in what follows will always be that of the Heisenberg picture : all time dependence is in the operators and the state refers not to a single time but to a whole history. This permits us, if we wish, to define the "system" under study simply as a limited space-time region. This seems a less intrinsically ambiguous and unrealistic way than any other I can think of to separate off a part of the world from the rest. Of course, one could try to think of the world as a whole, but it is less intimidating to think of only a part. In the approach <sup>8)</sup> known as the "theory of local observables" a Heisenberg state (pure or mixed) can indeed be attributed to any limited region of space-time. It gives, roughly speaking, the expectation value of all functions of the Heisenberg field operators with space-time arguments in that region. If something like a Lorentz-invariant causal connection between field operators is postulated then the region of relevance of the state vector can be extended by including all points whose forward or backward light cones pass entirely through the original region :



It is then the Heisenberg state of the extended region which reduces, whenever a "local beable" in that region is attributed a particular value, to its projection in the subspace with the given eigenvalue. Whatever the particular space-time location of the beable considered, there is no question of any particular space-time location of the associated state reduction, which is coextensive with the whole history of the system under study.

Whereas "measurement" was a dynamical intervention, from somewhere outside, with dynamical consequences, it is clear that "attribution" must be regarded as a purely conceptual intervention. It is made, say, by a theorist rather than an experimenter ; he is quite remote in space and time from the action, and simply shifts his attention from the whole of a statistical ensemble to a sub-ensemble. It follows that attributing a particular value to some beable cannot change particular values already attributed to some other beables. It follows that only those states can be allowed which are simultaneously eigenstates of all beables, or superpositions of such states. Moreover, we need only consider incoherent superpositions, for the beables, unable to induce transitions between different eigenstates, are insensitive to any coherence. Now the beables may not be a complete set, and a list of their eigenvalues may not characterize a state completely. However, the converse is true : when a particular member state of the incoherent superposition is specified, definite values are specified for all beables. Thus the theory is of deterministic hidden-variable type, with the Heisenberg state playing the rôle of hidden variable. When this state, which may originally refer only to the limited region in the figure, is specified, all beables in the extended region are determined.

I suspect that a stronger conclusion would be possible, that one cannot in fact find interesting candidates for beables in interesting quantum mechanical systems. But my own indications in this direction seem to me unnecessarily elaborate and I will not attempt to present them here. The preliminary conclusion is in a way more striking. In the basic propositions quoted from Professor Dirac there was in fact another element, in addition to the vague subjectivity, which could have disturbed a nineteenth century theorist. That is the statistical undeterministic character of the basic notions. In following what seemed to be a minimal programme for restoring objectivity, we were obliged to restore also determinism.

REFERENCES

- 1) P.A.M. Dirac - "The Principles of Quantum Mechanics".
- 2) In this connection there are many very relevant investigations involving considerations which may be roughly identified by the words "ergodicity" or "irreversibility". They tend to show that the effect of wave packet reduction associated with macroscopic observation is macroscopically negligible. (Or it may even be shown that the effect is accurately zero in some hypothetical limit : e.g., K. Hepp 3) takes infinite time.) The relevance of these investigations is of course to the question of the sufficient unambiguity of the theory for practical purposes, and not at all to the question of principle considered here.
- 3) K. Hepp - Helvetica Physica Acta 45, 237 (1972).
- 4) A more extreme position would be that the beables need refer only to mental events.
- 5) N. Bohr.
- 6) Such beables would be related to the "classical observables" of Jauch and Piron (see for example the contributions of these authors in "Foundations of Quantum Mechanics", Proceedings of the International School of Physics "Enrico Fermi", Course II, Academic Press, New York, 1971 ; also H. Primas 7)). However, these authors (loc.cit. and private communications) intended their "classical observables" to refer only to "apparatus" while not in interaction with "quantum systems" and perhaps to be only approximately "classical". Here we wish to avoid any arbitrary division of the world into "systems" and "apparatus", and any arbitrary limitation on the range and duration of interactions, and are concerned with the question of principle and not with that of practical approximation.
- 7) H. Primas - "Advanced Quantum Chemistry of Large Molecules ; Vol. 1 : Concepts and Kinematics of Quantum Mechanics of Large Molecular Systems", Academic Press, New York (1973), and preprint (July 1972).
- 8) See, for example :  
R. Haag - in "Lectures on Elementary Particles and Quantum Field Theory", 1970 Brandeis Lectures, Editors S. Deser, M. Grisaru and H. Pendleton, M.I.T. Press (1970). In this theory the over-all system need not be finite. The idea that the measurement problem might be significantly different in such a context has sometimes been expressed 3),7),9).
- 9) See, for example, the preface to B. d'Espagnat's "Conceptual Foundations of Quantum Mechanics", Benjamin (1971).