Two moves take Newtonian determinism to branching space-times*

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Abstract

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"Branching space-times" (BST) is intended as a representation of objective, event-based indeterminism. As such, BST exhibits both a spatio-temporal aspect and an indeterministic "modal" aspect of alternative possible historical courses of events. An essential feature of BST is that it can also represent spatial or space-like relationships as part of its (more or less) relativistic theory of spatio-temporal relations; this ability is essential for the representation of *local* (in contrast with "global") indeterminism. This essay indicates how BST might be seen to grow out of Newton's deterministic and non-relativistic theory by two independent moves: (1) Taking account of indeterminism, and (2) attending to spatio-temporal relationships in a spirit derived from Einstein's theory of special relativity. Since (1) and (2) are independent, one can see that there is room for four theories: Newtonian determinism, branching time indeterminism, relativistic determinism, and (finally) branching space-times indeterminism.

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1 Overview

Recent work has suggested rigorous but simple notions of indeterminism and free will based on the idea of "branching histories."[1] Philosophy has always contained separate scientific and humanistic pictures of humans in their world (Sellars's well-known scientific and manifest images), and many philosophical enterprises can be described as either focusing wholly within one while either ignoring or being contemptuous of the other, or as trying in some sense to reduce one to the other. The branching-histories enterprise can be seen as an effort to find ways in which the two images fit together, without diminution of either. Our particular strategy is to look for very general quasi-geometrical structures that underlie both. In this sense our theory could be described as equally proto-scientific and proto-humanistic. The enterprise is not itself either scientific or humanistic, but it does try to provide some ways of thinking that are intended as useful for each in just the way that plain old Euclidean geometry helps us to know our way around some aspects of physics and some aspects of perception.

2 Branching histories

The "branching histories" framework offers a theory of possibility, or, much better, *possibilities*. Every philosophy must somehow or other take account of the category of possibilities, a foundation on which many of our most fundamental concepts rest. Here there there are deep divisions.

(1) For some applications one needs only *unreal* possibilities. Perhaps they are given in the mind, as imaginary or fancied alternatives to our actual situation. Or perhaps the possibilities are constructed out of concepts or language or social structures such as conversations. For example, in making sense out of fiction or belief or justification or good reasoning, the alternatives

[1]Many of these ideas related to agency were first developed by von Kutschera thirty years ago against a background of "branching times." For book-length reports, based on branching times, see, Horty 2001, and Belnap et al. 2001; the latter, which contains numerous references, is cited below as "FF." The refinement of branching times to "branching space-times" was introduced in Belnap 1992, just twenty years ago. A variety of important articles on branching space-times has appeared since the initial presentation of the theory; see §7.

one brings into play need only be plausible. It is perhaps heavy-handed of me to put Lewis 1986 in this camp; certainly Lewis himself nominates his possible worlds and their contents as "real." What I have in mind are his views such as "anything can follow anything," which can only make sense to you if you give free reign to your imagination, unconstrained by what can *really* happen. Such possibilities also play a certain role in science. For some purposes, scientific possibilities need to have only epistemic, which is to say, mental status, in someone's mind, or perhaps social status in a family of practices by scientists. For these limited, chiefly heuristic, purposes, the time-worn phrase, "consistency with the laws," has some limited utility as an account of possibility. This remains true even though, as is obvious, "the laws" are just slippery pieces of language, made by man; for heuristic and practical purposes, there is often no need for anything more.

(2) Of fundamental importance in science are possibilities that, although they cannot really happen, serve as alternatives tied mathematically to reality. Historically the first invocation of such mathematical alternatives was by Maurpertuis in his optical Principle of Least Time: To calculate the path of a light ray passing through different media on its way from point a to point b, you consider all the mathematically possible paths, and find the path in which the time of passage is minimized. The alternative paths are on the one hand not mere imaginative concoctions, but on the other hand, they are possibilities that cannot really happen. It is a deterministic fact that only the least-time path will be taken, in spite of the usefulness of considering the mathematical—not real—alternatives in order to calculate the real path.

(3) But for certain concepts, one must insist on—in a phrase of Xu—"possibilities based in reality." In the course of investigating ideas of either indeterminism or free will, to settle for some kind of Humean "compatibilism" that would combine "scientific" or "objective" determinism with slippery subjective or linguistic or merely mathematical notions of possibility is, we think, to lose one's grip. After Leibniz, however, much philosophy has either neglected to take real possibilities seriously, or, having taken up the challenge that they present, has declared them null and void. Because many of us participate in the strict-deterministic attitudes engendered by this philosophical history, it is worth pausing a moment in order to ask whence this mindset. Certainly it goes back to the Stoics of the ancient world, but amid the contending philosophers, it was hardly common coin. Then, after centuries-long preparation by theological meditation on the meanings of omnipotence and

omniscience, presumably a fresh cause was the marvelous visible success of deterministic mathematical physics. Laplace 1820 awards his famous demon total knowledge of a single pre-ordained future course of events ("nothing would be uncertain"). Hume opined that "the same motives always produce the same actions; the same events follow from the same causes." Kant, sharing his century's conviction in the absolute accuracy of the deterministic vision, says that there is no possibility beyond actuality, and indeed, that there is no actuality that goes beyond ironclad necessity. In Kant's effort to make sense out of strict determinism, possibility, actuality, and necessity are the same thing. With the spread of lockstep clocks and machines, and with so much genius philosophizing in behalf of strict determinism, it is hardly surprising that many of our friends fail to take seriously the idea of objective possibilities; we are the legitimate children of our history. Without urging this explanation of why much philosophy has tended largely (but certainly not entirely) to avoid the idea of real possibility, I pass on to the central ideas of branching histories relevant to the problem of fitting objective possibilities together into a single world. In this essay I take one of several beckoning approaches: I start with Newtonian determinism, and indicate as best I can exactly what parts of the description of our world have to be changed in order to accommodate real possibilities for the spatiotemporal future.[2] I am going to sketch a quasi-historical route from Newton to branching space-times. Along the route I will be looking at causal structures of four kinds. They have in common the applicability of the following conventions:

- *Our World* is a nonempty set of events. *Our World* is a representation of our (only) world, a representation that is intended to focus attention on the causal order among its events. No properties are involved.
- *OW*$=_{df}$*Our World*. Often, however, in order to make some technical point, I use "*OW*" as a variable temporarily ranging over world-like abstract structures.
- e ranges over *Our World*. In all structures you should think of e as an *atomic event*; what counts as an "atomic" event, however, will vary with context until we come to branching space-times, at which point it will remain fixed.

[2]It is to be observed that the structures of Lewis 1986 won't do for two reasons. First, his possibilities aren't really real, as we discussed in (1) above; and second, his possible events are all other-worldly, never residing in our world.

- $<$, a binary relation, is meant to be the *causal ordering* on the "atomic" events of *Our World*. The following reading is apt for all four structures: $e_1 < e_2$ iff e_1 is in the causal past of e_2.

These conventions apply equally to four quite different kinds of causal structure. In no case is there any reference to other possible worlds. The use of "*Our World*" is intended to emphasize this.

The structure-kinds discussed will be strict partial orders in the usual sense:

Strict partial order. $<$ is a *strict partial order* of $OW \leftrightarrow_{df} \forall e_1,e_2,e_3 \in OW$: *Irreflexivity*, $e_1 \not< e_1$. *Asymmetry*, $e_1 < e_2 \rightarrow e_2 \not< e_1$. *Transitivity*, $(e_1 < e_2$ and $e_2 < e_3) \rightarrow e_1 < e_3$.

We may express exactly the same idea in terms of the companion relation $\leqslant$.

Partial order. $\leqslant$ partially orders $OW \leftrightarrow_{df} \forall e_1,e_2,e_3 \in$ *Our World*: *Reflexivity*, $e_1 \leqslant e_1$. *Antisymmetry*, $(e_1 \leqslant e_2$ and $e_2 \leqslant e_1) \rightarrow e_1 = e_2$. *Transitivity*, $(e_1 \leqslant e_2$ and $e_2 \leqslant e_3) \rightarrow e_1 \leqslant e_3$.

The ideas of strict partial order and partial order are of course interdefinable.

Now to specifics. As advertised, we begin with a Newtonian account of the world of events.

3 Newtonian world. Non-relativistic and deterministic: World = Line

The causal ordering of the Newtonian world has, as I see it, two features that are so fundamental that they can be described without advanced mathematics. First, the "atomic" events that are related by the causal order are momentary (= instantaneous) super-events: Newtonian physics needs total world-wide information concerning what is going on at time t. Let us call such a super-event (Thomson 1977) a *moment*. In Newton's world there is a one-one correspondence between moments and *times*, but you should nevertheless keep the two at least notionally distict: A moment is a kind of event,

Figure 1: Newtonian world

whereas a time is, ontologically, a real number. Second, the causal order, $<$, of Newton's world is not only a strict partial order, but satisfies the additional constraint of linearity: For any two (distinct) momentary events e_1 and e_2, either e_1 lies in the causal past of e_2, or vice versa: e_2 lies in the causal past of e_1.

Linearity. $<$ is *linear* on $OW \leftrightarrow_{df} \forall e_1,e_2 \in OW[e_1 \neq e_2 \rightarrow (e_1 < e_2 \text{ or } e_2 < e_1)]$. Equivalently, $\forall e_1,e_2 \in OW[e_1 \leqslant e_2 \text{ or } e_2 \leqslant e_1]$.

Newton's version of *Our World* standardly involves structural elements additional to the causal order, but they are not currently part of our story. It is the *linearity* of the causal order that answers to *determinism*, and it is the separable conception of *world-wide* moments falling into a linear causal order that answers to *non-relativistic* "action at a distance": An adjustment in the state of a piece of the world here-now can *immediately* call for an adjustment over there in the furthest galaxy. The picture of the causal order in a Newtonian world is therefore a simple line, with each point representing a moment or world-wide "simultaneity slice," all nature at a certain time t, as illustrated in Figure 1.

From now on we shall think of a "history" as a possible course of "atomic" events, which in the Newtonian case are the moments. When there is no difference between possible and actual, as in the Newtonian scheme, the idea of histories is of small importance since there are not possible histories (plural), but only a single History, so that we might as well say that World = History; this makes the Newtonian world *deterministic* in the most pro-

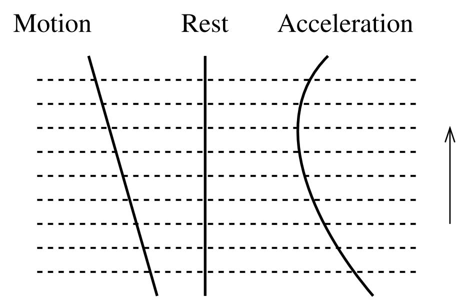

Figure 2: Newton's world with kinematics

found sense.[3] (We shall have later use for the idea of possible histories; but given the presuppositions of this study, the whole idea of "possible *worlds*" is irrelevant, for they would be external to *Our World*.) Furthermore, and independently, the relata of the causal ordering are momentary super-events (Thomson 1977); this makes the Newtonian world *non-relativistic*. With this in mind, we may say that on the Newtonian view, World = Line, as in Figure 1.

It is more common to represent Newton's world as in Figure 2, indicating space by a horizontal line. In such a diagram one can represent some elementary kinematics, such as the difference between motion, rest, and acceleration. One needs to note, however, that as far as the causal order itself goes, there is not the smallest difference between drawing the Newtonian universe as a single line and drawing it as a serial ordering of spatial configurations. The point is that because of the absence of instantaneous "action at a distance," there is no *causal* significance to the "horizontal" separation between points: Each spatial slice, each moment, enters into the *causal* order as an indissoluble whole. An effective statement of this principle is that familiarly called "Laplace's demon"; I have italicized the critical words by which the idea of the demon embodies "action at a distance."

[3]This notion of determinism is "most profound" because simplest. It involves only the causal ordering of momentary events, prescinding entirely from the internal constitution of moments. Most especially, it does not involve any concept of "laws," nor even of properties.

> Given *for one instant* an intelligence which could comprehend all the forces by which nature is animated and the respective positions of the beings which compose it, if moreover this intelligence were vast enough to submit these data to analysis, it would embrace in the same formula both the movements of the largest bodies in the universe and those of the lightest atom; to it nothing would be uncertain, and the future as the past would be present to its eyes (Laplace 1820).

The line of Figure 1 is therefore a better—since less distracting—representation of the Newtonian causal order. That line emphasizes that on this scheme *every* pair of moments is causally ordered, one way or another; the order is "total."

So much for deterministic and non-relativistic "World = Line." As already indicated, branching space-times is to be both indeterministic and relativistic. To make the transition from the Newtonian world to branching space-times, one therefore requires two independent moves.

4 Branching-times world. Non-relativistic but indeterministic: World = Many Lines.

In the first move away from Newton, we *keep* the relata of the causal ordering on *Our World* as momentary super-events—still called *moments*—so that we remain non-relativistic, and we *keep* $<$ as a strict partial ordering of *OW*. In order to represent indeterminism, however, we *abandon* linearity in favor of a treelike order, as sketched in Figure 3. The result of this first transition from the Newtonian world, when taken alone, is exactly what the literature discusses under the rubric "branching time," which we sometimes abbreviate to BT.[4] In branching time there is indeed a single world, *Our World*, but instead of the equation World = Line, the world of branching

[4]The earliest trace of the idea of branching time seems to be in a letter from Kripke to Prior dated September 3, 1958 (see Øhrstrøm and Hasle 1995). There is then a brief discussion in Prior 1967 and a proper working out of the idea in Thomason 1970. Belnap et al. 2001 contains an extended treatment of the concept, chiefly in relation to agency.

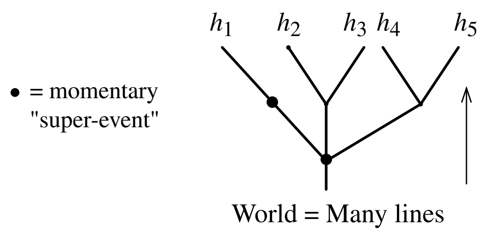

Figure 3: Branching-time world

time involves many line-like histories, that is, many possible courses of events: Branching time is indeterministic. Since, however, we have kept the causal relata as momentary super-events, branching time remains non-relativistic: Splitting between histories in branching time has to be a world-wide matter of "action at a distance" since the consequences of the split are felt instantaneously throughout the farthest reaches of space. We may therefore say that according to branching time, World = Many Lines that split at world-wide momentary super-events called "moments," as in Figure 3. Technically we capture the causal structure of branching time by adding "no backward branching" to the partial order constraint:

No backward branching. $<$ satisfies *no backward branching* on $OW \leftrightarrow_{df}$ $\forall e_1, e_2 \in OW[(e_1 \nleqslant e_2 \text{ and } e_2 \nleqslant e_1) \rightarrow \sim\exists e_3 \in OW(e_1 \leqslant e_3 \text{ and } e_2 \leqslant e_3)]$. Or contrapositively, $\forall e_1, e_2 \in OW[\exists e_3(e_1 \leqslant e_3 \text{ and } e_2 \leqslant e_3) \rightarrow (e_1 \leqslant e_2 \text{ or } e_2 \leqslant e_1)]$.

Figure 4 portrays the self-same splitting, but with the spatial dimension of moments explicitly indicated by the horizontal lines. The subject of this portrait combines in its nature both indeterminism and "action at a distance." Since, however, in exact analogy to Figure 2, a horizontal difference in position has no causal significance, the fundamental causal ordering remains no more complicated than that illustrated by the tree of lines of Figure 3. We might be able to represent some sort of indeterministic kinematics with Figure 4, but the purely causal order remains just a tree. The "demon" corresponding to this picture, given instantaneous knowledge of "all the forces by which nature is animated and the respective positions of the beings which

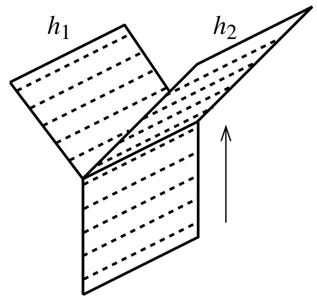

Figure 4: World-wide splitting

compose it," would be able to postdict the entire settled past, and to predict in detail the patterned system of the objectively real *possibilities* for the future. Metaphors aside, however, I repeat that for branching time one drops linearity in favor the principles of partial order and no backward branching.

5 Einstein-Minkowski-space-time world. Relativistic but deterministic: World = Space-time.

The other move away from Newton is that made by Einstein in principle, and more explicitly by Minkowski (see Einstein et al. 1924), as pictured in Figure 5. [**MSnote: Need to add e0 to picture.**] To obtain the Einstein-Minkowski causal order from that of Newton, we *retain* determinism from the Newtonian world; there is no trace of alternative possible futures. The change is rather that now the terms of the causal relation are no longer simultaneity slices, momentary super-events called "moments," that stretch throughout the universe. Instead, the fundamental causal relata are *local* events, events that are limited in both time-like and space-like dimensions. When fully idealized, the causal relata are *point events* in space-time. This, to my mind, is the non-mathematical and pre-physical heart and soul of Einstein-Minkowski

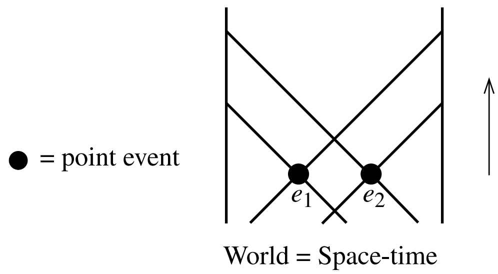

Figure 5: Einstein-Minkowski-space-time world

causal relativity. The move to local events is made necessary by Einstein's argument that there is simply no objective meaning for a simultaneity slice running from one edge of the universe to the other, so that the relata of the causal order cannot be world-wide atomic events. There is no "action at a distance": Adjustments at e_1 influence only events such as e_0, pictured in Figure 5, that are in "the forward light cone" of e_1, or, as we will say, in the *causal future* of e_1. I wish to urge that not only fancy Einstein physics, but even our ordinary experience (when uncorrupted by uncritical adherence to Newton or mechanical addiction to clocks and watches, or to theories known to conflict with relativity) shows us that events are not strung out one after the other. Take an event of our being here now at e_1. Indeed some events lie in our causal future, so that there are causal chains from e_1 to them, and others lie in our causal past, so that the causal chains run from them to e_1. But once we take local events as the relata of the causal order, there is a third category, always intuitive, and now scientifically respectable, since we have learned to be suspicious of the idea of (immediate) action at a distance. In this third category are local events e_2 that neither lie ahead of e_1 nor do they lie behind e_1 in the causal order. Letting $<$ be the causal order relation, I am speaking of a pair of point events e_1 and e_2, as in Figure 5, such that neither $e_1 \leqslant e_2$ nor $e_2 \leqslant e_1$. Instead, e_1 and e_2 have a *space-like* relation to each other. Neither later nor earlier (nor frozen into absolute simultaneity by a mythical world-spanning clock), they are "over there" with respect to each other. Einstein makes us painfully aware that space-like relatedness is non-transitive, which is precisely the bar to the objective reality of momentary super-events capable of being the terms of a linear causal order. Events in their causal relation are not really ordered like a line. Our modern reverence for various parts of Newtonian physics and our related love of clock time delude us.

Since the Einstein-Minkowski relativistic picture is just as deterministic as the Newtonian picture, there are no histories (plural), but only History, so that we have the determinist equation World = History. The difference from the Newtonian picture is with respect to an independent feature: A causally ordered historical course of events can no longer be conceived as a linear chain of momentary super-events. Instead, a history is a relativistic space-time that consists in a manifold of point events bound together by a Minkowski-style causal ordering that allows that some pairs of point events are space-like related. Therefore, if we make the single transition from the Newtonian world to that of Einstein-Minkowski, the result is that World = Space-time as in Figure 5. Evidently from ordering principles so far enunciated, we keep only the idea of partial order, dropping both linearity and no backward branching as having no place. One knows of course that in addition to partial order, the causal ordering of Minkowski space-time has many intricate properties. These are completely laid out in Mundy 1986, which also describes the 1914–36 results of Robb. It turns out, however, that few of these additional features need to play an explicit role in understanding determinism vs. indeterminism, so that for present purposes we may pass over them in silence.

It is a common feeling, and one that I formerly shared, that the very idea of Minkowskian "space-time" implies that indeterminism is incoherent. When one imagines the four-dimensional world described by Minkowski, one somehow seems to be pushed into thinking of our world as like William James's oft-quoted image of determinism:

> [Determinism] professes that those parts of the universe already laid down absolutely appoint and decree what the other parts shall be. The future has no ambiguous possibilities hidden in its womb … . The whole is in each and every part, and welds it with the rest into an absolute unity, an iron block, in which there can be no equivocation or shadow of turning (James 1884).

Although chiefly resting on armchair metaphors, some thinkers have also given detailed arguments that an indeterminism-relativity combination is in principle impossible. Stein 1991 both refutes those arguments and accounts

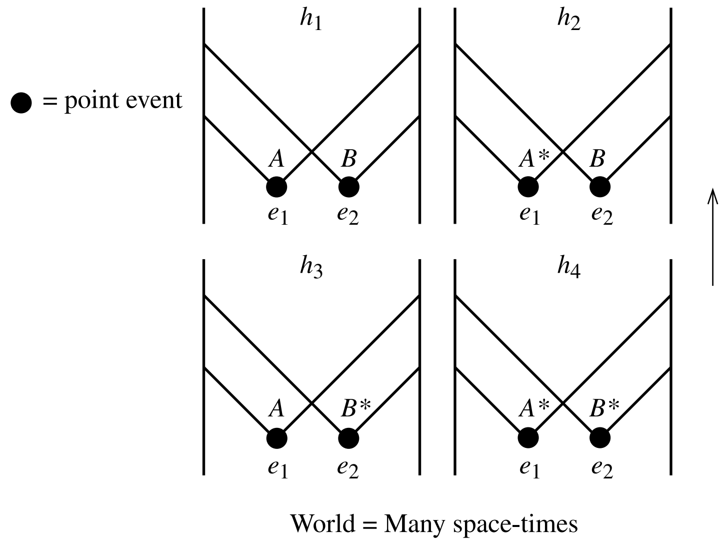

Figure 6: Branching space-times world

for their apparent force. Our discussion sidesteps this conversation by describing a simple theory of a single causal ordering that is at once relativistic and indeterministic. It is a theory of "branching space-times," BST theory for short.

6 Branching-space-times world. Relativistic and indeterministic: World = Many Space-times.

BST now arises by suggesting that the causal structure of our world involves both indeterminism and relativistic space-times; we are therefore to combine two independent transitions from the Newtonian world, with Figure 6 counting as a picture of the result. We can already make a certain amount of capital out of that suggestion. For *indeterminism*, we shall expect not World

= History, but instead World = Many Histories. And for *relativistic* considerations, we shall expect that each singly possible history is not a line, but instead a space-time of point events in something like the Einstein-Minkowski sense. So, once we introduce branching histories, each of which is a space-time, we should expect that World = Many Space-times. This is pictured as best we can in Figure 6. Furthermore, just as histories in branching time (each of which is like a line) split at a world-wide momentary super-event, so in branching space-times we should expect that histories (each of which is like a space-time) should split at one or more point events. To express this complex thought, Figure 6 is an *annotated* diagram, requiring explanations of what the annotations mean.

- Each of the four histories, h_1, h_2, h_3, h_4, is a Minkowski space-time.
- e_1 and e_2 are (distinct) space-like related point events, each of which occurs in all four histories.
- $A, A*, B, B*$ are causal futures. Regions with identical labels contain exactly the same point events. Regions with distinct labels have no point events in common. In addition to e_1 and e_2, all four histories share all point events that are below the zig-zag line passing through e_1 and e_2.
- Given any two histories, h_i, h_j, if any point event, e_0, is in h_i but is not in h_j, then it is above a "splitting point" for h_i and h_j; that is, a point event that is maximal in the intersection $h_i \cap h_j$. Such a point is called a *causal locus*.

  The last item suggests a rough beginning of a theory of causation in branching space-times.

Technically, we represent our world as *Our World*, which is a set of (possible) point events, and we let e range over *Our World*.

I summarize the relation between the four theory schemata of §3–§6 in a proportion:

$$\frac{\text{linear time}}{\text{space-time}} :: \frac{\text{branching time}}{\textit{branching space-time}}$$

And here is a table listing causal relata and relations in an organized way. (Recall that "moment" is jargon for "momentary super-event.")

Structure	*Relata*	*Relation*
linear time	moments	global deterministic causal order
space-time	point events	local deterministic causal order
branching time	moments	global indeterministic causal order
branching space-time	*point events*	*local indeterministic causal order*

We need, however, more information about how the various histories (= space-times) fit together. What is analogous here to the indeterministic way in which branching-time theory structures individual Newtonian line-like histories into a tree? We impose a crucial desideratum: BST theory should preserve our instinct that such indeterminism as there is in our world can be a local matter, a chance event or a choice event here-now that need have no "action-at-a-distance" effect on the immediate future of astronomically distant regions of the universe. We shall satisfy this desire by way of the idea that two histories may "split" from each other at a point event. It needs zero training in mathematics, and but a glance at Figure 6, to see that a theory of how "splitting" lets BST histories fit together into a single world will describe structures that are much more complicated than the branching-time structures that arrange many lines into a single tree. On the other hand, the postulates by which BST theory characterizes these structures form a relatively uncomplicated whole, as we now see.

7 Fundamentals of BST: grammar, definitions, postulates

This section gathers in one place all the postulates of BST theory, with two purposes in mind. One purpose is to buttress my claim that BST axiomatic theory is on the simple side and can be easily surveyed. The other purpose is to give a person interested in BST an easy place to which to turn for the basic definitions and postulates. There is, however, a cost: In this framework we shall be unable to say much about these postulates. Belnap 1992, however, discusses them at some length, and of course the various BST essays that are listed among the references deepen the discussion and exhibit their use.

A number of these postulates are rendered more comprehensible if given in terms of a certain defined notions. Accordingly we begin with an indication of the primitive grammar of BST, then list and comment briefly on those definitions required for smooth statement of postulates, and finally list all of the postulates of BST.

Grammar of BST. There are two primitives in BST theory: *Our World* and $<$. The theory uses the first to name a set and the second to name a two-place relation on that set. There is no additional primitive grammar, except notation for probabilities, which is not employed in this essay (see Weiner and Belnap 2006 and Müller 2005).

Our World (*OW*) is to be taken to name a set-theoretical representation of the one and only world in which we find ourselves, headed from a settled past into a future replete with alternative possibilities, sprawling out all around us, and held together by the causal order. In a definition that will soon be made official, a *point event* is a member of *OW*. Because of indeterminism, point events must be thought of as *possible* point events, with "actuality" relativized to some particular standpoint in *Our World*. Each possible point event, whether representing a possibility for our future or perhaps some might-have-been, is understood to be as fully concrete as any point event in our causal past. It will not do to think of a possible point event as some pale shadow-like imitation of an actual point event.

Read "$e_0 < e_1$" as "e_0 lies in the causal past of e_1" or "e_1 lies in the future of possibilities of e_0" or "e_0 is causally earlier than e_1" or "e_1 is causally later than e_0."

Fundamental definitions. It will help if we enter a number of definitions before stating the postulates. The first batch are merely conventions concerning members and subsets of *Our World*.

- $OW =_{df}$ *Our World*. "*OW*" is pronounced "*Our World*."
- e is a *point event* $\leftrightarrow_{df}$ $e \in OW$. Let e range over point events.
- $e_1 \leqslant e_2 \leftrightarrow_{df} (e_1 < e_2 \vee e_1 = e_2)$.
- Let E range over subsets of *OW*.
- We are careless about the use of "e" vs. "$\{e\}$".

- To put a set-name on the left or right of either $<$ or $\leqslant$ is to make a universal statement, for example, $e < E \leftrightarrow_{df} \forall e_1[e_1 \in E \rightarrow e < e_1]$.

The next pair are intended as revelatory definitions in the (objective) theory of events.

- E is *directed* $\leftrightarrow_{df} \forall e_0 \forall e_1[e_0,\ e_1 \in E \rightarrow \exists e_2[e_2 \in E \,\&\, e_0 \leqslant e_2 \,\&\, e_1 \leqslant e_2]]$.

  Thus a directed set contains a common upper bound for each pair of its members, and indeed for each finite subset. A directed set might be described as "ontologically consistent," or, if that seems too pretentious, merely "event consistent." The intuition is that two possible events are consistent if and only if they both lie in the causal past of some event. Negatively put, two possible events are inconsistent (can't both happen) if and only if there is no event in the past of which they both lie.

- h is a *history* $\leftrightarrow_{df}$ h is a maximal directed subset of OW. That is, where "directed" is as just defined, h is a history $\leftrightarrow_{df}$ $h \subseteq OW$ and h is directed and $\sim\exists E[E \subseteq OW \,\&\, h \subset E \,\&\, E$ is directed]. Let h range over histories.

  Though "history" has a thoroughly technical definition, I associate it with the idea of a possible complete course of events. Each history may be pictured as a space-time, except that it is not intended as a purely geometrical structure. Each history is a possible course of *concrete* events that is represented by a set of point events. Given the definition of "event consistency" suggested in the previous paragraph, a history could be defined as a maximal consistent set of point events, which may help the intuitions of those brought up on classical modal logic. On the other hand, it might be difficult for those so brought up to make the (essential) shift from consistency of propositions to consistency of point events. Evidently two point events are consistent if an only if there is a history that contains them both.

Then come the definitions of chains, infima, and suprema, which, although standard in deterministic space-time, are nevertheless essential to understanding branching space-times.

- E is a *chain* $\leftrightarrow_{df}$ E is a connected subset of OW. That is, E is a chain iff $E \subseteq OW$ and $\forall e_0 \forall e_1[e_0,\ e_1 \in E \rightarrow (e_0 \leqslant e_1 \vee e_1 \leqslant e_0)]$.

  A chain is rather like a portion of a "world line" in the sense of special relativity. A chain may be imagined to be a possible "track" through branching space-times of some enduring entity. If Dorothy is, at e, undecided whether or not to eat a tomato, then there is at least one track through e that represents her eating a tomato, and at least one track that represents her not doing so.

- I is an *initial chain* $\leftrightarrow_{df}$ I is a chain that is nonempty and upper bounded in OW. Let I range over initial chains.

  "Initial" is to remind you of "initial conditions"–but keep in mind that an initial chain is an event. An initial comes to completion in any history that contains it. Hence, it always makes sense to think of an initial as giving rise to one or more possible outcomes.

- O is an *outcome chain* $\leftrightarrow_{df}$ O is a chain that is nonempty and lower bounded in OW. Let O range over outcome chains.

  An outcome chain is a particularly simple sort of outcome event. BST theory relies heavily on *transitions* from initial events to outcome events. O has a beginning in any history that it overlaps, so that provided an initial chain, I, entirely precedes an outcome chain, O, it makes sense to think of $\langle I, O \rangle$ as a *transition* from I to O, and of O as a possible outcome of I. Of considerable technical importance are *immediate transitions*, where there are no point events properly between I and O.

- $inf(O) =_{df}$ the infimum, that is, the greatest lower bound (in the sense of $\leqslant$) of O, if there is one. When $inf(O)$ exists, it is uniquely characterized by the following: $inf(O) \leqslant O$, and $\forall e[e \leqslant O \rightarrow e \leqslant inf(O)]$.

  Infima play a substantial role in the more technical parts of BST theory. When O has no first member, $inf(O)$ is the last point event at which you can truly say "O hasn't occurred yet, but it will."

- $sup_h(I)$ is the supremum of I in h, that is, the least upper bound (in the sense of $\leqslant$) among those upper bounds of I that belong to h, if there is one. When $sup_h(I)$ exists, it is uniquely characterized by the following: $I \leqslant sup_h(I)$ and $sup_h(I) \in h$, and $\forall e[(I \leqslant e \,\&\, e \in h) \rightarrow sup_h(I) \leqslant e]$.

  Suprema are a more delicate matter than infima because of their being relative to histories. The point is that if you are in the middle of a chain without a last member, headed upwards, there may be multiple candidates to fill the role of a point event at which you can first truly say "That's over."

  The next definition, splitting of histories, which has no analogy in a deterministic account of *Our World*; it is the key definition of BST.

- $h_1 \perp_e h_2$, read "h_1 divides or splits or separates from h_2 at e," $\leftrightarrow_{df}$ $e \in h_1 \cap h_2$ and $\sim\exists e_1[e < e_1$ and $e_1 \in h_1 \cap h_2]$.

  That is, e is maximal in the intersection of the two histories, hence, a last point at which h_1 vs. h_2 is not yet decided. Such a point event is a causal locus. Observe that there may be many such, as long as they are pairwise space-like related.

Postulates of BST. The elementary theory of branching space-times restricts itself to the following nine postulates, each of which plays a critical role .

1. **Structural postulate.** OW is a set, and $<$ is a binary relation on OW.

2. **Nontriviality postulate.** $OW \neq \varnothing$.

3. **Nontermination postulate.** There are no terminal elements in OW: $\forall e_0[e_0 \in OW \rightarrow \exists e_1[e_1 \in OW \& e_0 < e_1]]$

4. **Strict partial order postulate.** $<$ is a strict partial order of OW; that is, $<$ is irreflexive ($e \not< e$), transitive ($(e_0 < e_1 \& e_1 < e_2) \rightarrow e_0 < e_2$), and asymmetric ($(e_0 < e_1 \rightarrow e_1 \not< e_0)$. Equivalently, $\leqslant$ is reflexive, transitive, and antisymmetric, and thus partially orders OW.

5. **Density postulate.** $<$ is dense in OW; that is, $e_0 < e_2 \rightarrow \exists e_1[e_0 < e_1 \& e_1 < e_2]$.

6. **Infimum postulate.** For each outcome chain O, $inf(O)$ exists.

7. **Supremum postulate.** For each initial chain I and each history h such that $I \subseteq h$, $sup_h(I)$ exists.

8. **Order preservation postulate.** Given two initial chains, and two histories, the order of the respective suprema is preserved as the histories are varied: If $(I_1 \cup I_2) \subseteq (h_1 \cap h_2)$, then $sup_{h_1}(I_1) < sup_{h_1}(I_2)$ iff $sup_{h_2}(I_1) < sup_{h_2}(I_2)$, and $sup_{h_1}(I_1) = sup_{h_1}(I_2)$ iff $sup_{h_2}(I_1) = sup_{h_2}(I_2)$.

9. **Prior choice postulate.** If an outcome chain O lies in one history h_1 but is excluded from another h_2, then there is a point event e in the proper past of O at which h_1 and h_2 split (or divide or separate). That is, $O \subseteq (h_1 - h_2) \rightarrow \exists e[e < O \ \& \ h_1 \perp_e h_2]$.

   The prior choice postulate gives rise to a theory of causation "in the events" that is devoid of any reference to "laws" or properties. It is the key postulate of BST.

As you can see, the basic definitions used in stating the postulates are straightforward, and the postulates themselves are few and simple.

8 Applications of BST theory

We have finished our appointed task of showing how there are two natural moves that if made in combination take us from Newton to BST, with the first move taking us either to branching time or to special relativity, and the second move taking us to branching space-times. Here we mention, in a strictly bibliographic tone of voice, many of ways in which BST has been applied.

Metaphysics. BST theory itself can be called "metaphysical" in the low-key and negative sense intended to suggest that BST does not employ epistemological or other mentalistic or linguistic concepts, nor even properties. BST is a pure event theory. Since twenty years have elapsed since its introduction, it appears possible that BST may be the only such theory capable of combining special relativity and indeterminism in a simple and rigorous fashion.

Objective possibilities. Being perhaps the essential core of BST theory, it cannot be a surprise that objective possibilities are discussed in many publications; see Belnap 1992 and and the subsequent BST literature as detailed among the references.

Objective probabilities and propensities. To find out how BST can contribute to the difficult task of understanding how objective probabilities might fit into *Our World*, see Müller 2005, Weiner and Belnap 2006, and Belnap 2007b.

"Funny business." The Bell phenomena of quantum mechanics, when redescribed modally instead of probabilistically, are paradigms of a kind of funny business. Take two space-like related initials in BST, each with multiple possible immediate outcomes. Perhaps one is in Texas and the other is on the moon. If (in spite of the space-like relation between them suggesting separate existence and causal independence) a certain immediate outcome of one initial is incompatible with a certain immediate outcome of the other initial, that's funny business. See Szabo and Belnap 1996, Belnap 2002b, Belnap 2003, Placek 2004, Mueller et al. 2006, Müller 2007, and Placek and Wroński 2009.

Quantum mechanics. Müller 2007 shows that the BST approach has consequences for the "consistent histories" approach to quantum mechanics. See also Müller and Placek 2001.

Non-application to quantum mechanics. On the other hand, we do not claim that BST clarifies or contributes much to quantum mechanics: There is a great deal of quantum mechanics, including its most basic parts, on which BST appears (as yet) to throw no light—and if those who argue that quantum mechanics is deterministic are right, then it never will. We find solace in the thought that there seems (as yet) to be no accepted rigorous and thorough-going combination of general relativity and quantum mechanics.

Causae causantes. BST theory leads us to a legitimating theory of *causae causantes* (originating causes), a theory that *requires* indeterminism; see Belnap 2005b. It is seems almost magical that the family of all *causae causantes* of a given effect form a set of inus conditions for the effect in the sense of Mackie 1974: conditions that are individually **i**nsufficient but **n**on-redundant parts of complex (conjunctive) conditions that are each **u**nnecessary but **s**ufficient. Xu 1997 develops an account of causation in branching time.

Agency. Belnap 2005a suggests how the spatio-temporal dimensions of BST theory illuminates agency.

Speech acts. BT and BST each does service as a foundation for a fresh account of speech acts and how they fit into the world; see Belnap 2002a.

Counterfactuals. Placek and Müller 2007 is an investigation of counterfactuals based upon BST, as is Müller 2002. There is also a brief discussion in Belnap 2005b. Nevertheless, I subscribe to R. K. Meyer's judgment that counterfactuals should be left to sports announcers and military historians.

Indeterminism: fitting space-times together. "Fitting" space-times together is just one of several topics of Placek 2000 that interact conceptually with BST. Nevertheless, the next item gives a much-improved solution to the problem of fitting.

True Minkowski branching space-times (MBSTs). The listed postulates for BST suggest that histories are MBSTs, but do not compel that interpretation. Placek and Belnap 2010 show how the MBST interpretation can be smoothly derived by adding some independently attractive postulates involving properties of point events.

An alternative to BST. Placek 2009 works out an alternative to BST in "Possibilities without possible worlds/histories."

To be put somewhere. Müller 2009.

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