APPARENT SIMULTANEITY

Hanoch Ben-Yami

ABSTRACT

I develop Special Relativity with backward-light-cone simultaneity, which I call, for reasons made clear in the paper, 'Apparent Simultaneity'. In the first section I show some advantages of this approach. I then develop the kinematics in the second section. In the third section I apply the approach to the Twins Paradox: I show how it removes the paradox, and I explain why the paradox was a result of an artificial symmetry introduced to the description of the process by Einstein's simultaneity definition. In the fourth section I discuss some aspects of dynamics. I conclude, in a fifth section, with a discussion of the nature of light, according to which transmission of light energy is a form of action at a distance.

1 Considerations Supporting Backward-Light-Cone Simultaneity

In an earlier paper (Ben-Yami [2006]) I argued, among other things, that in Special Relativity we can define the events simultaneous with a given event as those on its backward light cone. In this paper I shall examine some implications of this definition for our understanding of various physical and philosophical issues.

I start with a brief summary of the relevant points made in my earlier paper. First, following, with minor modifications, Reichenbach ([1928], § 22), I maintained that an effect cannot precede its cause; this was supposed to be an analytic truth expressing a conceptual relation between temporal and causal concepts.[1] Next, this time following Malament ([1977]) and Rynasiewicz ([2000]), I examined which rules explicitly defining simultaneity relative to an observer are possible, given some minimal constraints on simultaneity, and demanding that our rule give acceptable results independently of the future history of the observer. If we limit ourselves to inertial observers, then three such relations are possible: any event in an inertial observer's history can be defined as simultaneous either with all events on its backward light cone, or with all events on its forward light cone, or with all events simultaneous with it according to the standard simultaneity introduced by Einstein in his 1905 Relativity paper (pp. 893-4).[2] If we admit, however, non-inertial segments to the history of an otherwise inertial observer, then Einstein simultaneity relative to that observer is no longer an acceptable simultaneity relation. By contrast, both backward and forward light cone simultaneity are definable not only for such observers, but for any non-inertial observer as well.

1.1 Temporal order

We shall first demonstrate a conceptual problem involved in Einstein simultaneity. Suppose A and B are inertial bodies at rest relative to each other, a distance d apart; an astronaut is traveling from A to B with velocity v relative to A, and upon arrival at B immediately turns back and returns to A with velocity $-v$. When the astronaut arrives at B, then, relative to the astronaut, a star C explodes, C being on the line connecting A and B, and, relative to A, AB = BC = d.

When did C explode relative to the astronaut? It seems that I have just said just that: when he arrived at B. But the answer is not that simple. The astronaut arrived at B at time d/v relative to A. Applying Lorenz transformation, where the primed frame is the astronaut's:

$$t' = (t - vx/c^2)/(1 - v^2/c^2)^{1/2},$$

we see that the explosion happened at $t' = (d/v)(1 - v^2/c^2)^{1/2}$ according to the astronaut's clock. Since the explosion happened at $x = 2d$ relative to A, then, using the same formula again, we see that it happened at $t = (d/v)(1 + v^2/c^2)$ relative to A. However, on his way back, the astronaut's time, t'', is related to A's by the formula:

$$t'' = (t + vx/c^2 - 2dv/c^2)/(1 - v^2/c^2)^{1/2}$$

The constant $-2dv/c^2/(1 - v^2/c^2)^{1/2}$ is the result of the fact that the origin of the astronaut's present reference frame did not coincide with A's at $x = 0$, $t = 0$. If we now calculate when did C explode relative to the astronaut's present reference frame, we get the result:

$$t'' = (d/v)(1 + v^2/c^2)/(1 - v^2/c^2)^{1/2}$$

[1] In the meantime I have published a paper against Dummett's argument supporting the possibility of such 'backwards' causality (Ben-Yami, [forthcoming]).

[2] Standard or Einstein simultaneity is that according to which the speed of light in vacuum is constant between any two points relative to any observer. Since the term 'standard' lends it some linguistic pride-of-place over other simultaneity schemes, I shall use the phrase 'Einstein simultaneity' below.

That is, sometime on his way back from B to A (assuming $v < c/\sqrt{3}$, otherwise the astronaut first arrives at A).

So when did C explode relative to the astronaut? It seems that according to Einstein simultaneity that single explosion is simultaneous with two different events in the astronaut's history: first, with his arrival at B, and later with some event—α, say—on his way back to A. So after his arrival at B but before α, has C exploded or hasn't it relative to him? Well, it both has exploded and hasn't yet exploded. And at α, it is exploding although that explosion has already occurred. Similarly, things that happened after the explosion happened before that same explosion as well (and we can add more absurdities). Nothing much is left of our concepts of temporal order.

These are results we get if we try to standardly define simultaneity and temporal order relative to inertial observers with non-inertial segments in their history. They are a consequence of the fact that any such observer has simultaneity plans that are tilted relative to each other. Of course, we can still introduce this coordinate *t* and make all sorts of calculations with it—this is, in fact, what I have just done. But if we wished this coordinate to express what we understand by time and temporal order, we have to give up much of that idea, more than is usually realized.

By contrast, if every event in the history of an observer is defined as simultaneous with all events on its backward light cone, no such absurdities follow, and what we understand by temporal order is, in this respect, preserved. I think this is a conceptual advantage of backward light cone simultaneity over Einstein's.

1.2 The meaning of coordinates

Let us linger a little longer on the meaning of coordinates. There are of course many ways in which coordinates can be introduced. We can, for instance, measure distances in the east-west direction in meters, and those in the north-south direction in feet. As a result, turning a table ninety degrees would usually change its coordinate length. Still, we would like to say that its length did not really change due to this turn—the change is merely nominal, so to say; in this case the coordinates no longer express length properly. We define length by means of rigid objects, which are those that preserve their relative lengths, up to the desired accuracy, when transformed in space and time; and our coordinates no longer express this feature. So if we wish our coordinates to express some physical reality, not every measurement convention that enables the scientifically necessary mathematical representation of facts is equally acceptable.

These considerations also apply to the representation of time by our coordinates. If all we desire is the ascription of a number to every event in history, then there are of course many ways of doing *that*. We can even have 'time' flowing backwards, if we wish. Brown, for instance, in his recent defense of the conventionality of distant simultaneity, notes ([2006], p. 97): 'Let me testify, having flown from New Zealand to both North and South America, that arriving before you left is survivable!' I hope that Brown would agree, though, that crossing the International Date Line eastward is not a way of becoming a day younger. Revolving your clock's hands 24 hours backwards is a legitimate way of maintaining your coordination with society, but by doing that your clock has stopped representing correctly the time you have lived.

Suppose we chose our coordinates so that none represented what we understand by length or as the time that has passed on the observer whose coordinates they are. This is of course mathematically possible. However, it will constitute a deviation from what we ordinarily understand by coordinates. Moreover, since the physical information captured by coordinates as ordinarily understood is significant, we should in that case find ways to extract that information from our coordinates. They would, in a sense, *conceal* the information. I therefore think that to the extent that coordinates can represent physical reality, we better have coordinates that do that. If we *can* introduce coordinates that express length, duration and the relation of causality to time, we should.

This might be impossible, or impossible globally, in some circumstances—e.g. curved space-time; but in this paper we consider flat space-time, with only occasional excursions to how curvature, or some kinds of curvature, would affect our results.

Let us return, then, to our discussion of coordinates in Special Relativity. Our t-coordinate was originally meant, in Newtonian space-time, to express what we understand by time. If we want to preserve as much of that as we can within the framework of Special Relativity, then we should not allow for an effect to precede its cause, or to a single instantaneous event to be simultaneous with more (or less) than one moment in an observer's history. This last constraint, as I have shown above, makes backward-light-cone simultaneity preferable over Einstein simultaneity.[3]

1.3 Dependence of simultaneity on location and relative motion

We shall now consider another respect in which the former synchronization is preferable over the latter. As I noted in my [2006] paper, and as was previously noted by Sarkar and Stachel ([1999], abstract and p. 218), while according to Einstein simultaneity, simultaneity is independent of an observer's place, according to backward-light-cone simultaneity it is independent of the observer's velocity. That is, if two inertial observers are at rest relative to each other, events that the one would consider Einstein-simultaneous would be considered such by the other as well; while two inertial observers at the same place, one moving relative to the other, would not consider the same events Einstein-simultaneous. By contrast, two remote observers would not consider the same events backward-light-cone simultaneous, while two observers at the same place—whether inertial or not—would.

In this respect, two remote inertial observers at rest relative to each other, form, according to Einstein simultaneity, a system—a reference frame, at least from a temporal point of view. I think this does not reflect any physical reality. These observers are influenced by different causes and in different order, and the information exchange between them may take time of several orders of magnitudes greater than intervals already significant for each observer. If a system is primarily understood as 'an organized or connected group of objects' (OED), then these observers do not constitute any.

On the other hand, suppose we are in the same room, doing various things together and frequently moving relative to each other. I think it is safe to say that by virtue of our interactions we constitute a system. However, due to our relative motion, not the same events are Einstein-simultaneous relative to each of us. If I am moving relative to you in velocity v, then what is happening now at x relative to you happens at

$$-vx/c^2/(1 - v^2/c^2)^{1/2}$$

relative to me. Even if v is very small relative to c, for sufficiently great distances this term can be made arbitrarily large. Yet this disagreement in simultaneity judgments,

[3] My discussion in this section was influenced by the work of Meir Buzaglo on the logic of concept expansion and its relation to rationality; see Buzaglo [2002].

the fact that a distant explosion of a supernova that is happening now relative to you will happen in a day's time relative to me, reflects no relevant difference between us. Einstein simultaneity creates a nominal distinction and does not reflect the fact that due to the instantaneous interaction between us we form a system.

By contrast, according to backward-light-cone simultaneity, observers in relative motion but in the same place judge the same events simultaneous. Since relative to both of them the fastest possible signal travels with the speed of light, the events they judge simultaneous with a certain event in their common history are those events that can then influence them by signals moving with the speed of light. Their common place in the nexus of causes is thus reflected in their common simultaneity judgments. On the other hand, two distant observers, whether in relative rest or motion, disagree in their simultaneity judgments, again reflecting the fact that events of different sets simultaneously influence each of them.

In this respect, backward-light-cone simultaneity represents the causal order and what we understand by a physical system better than does Einstein simultaneity. By contrast with the latter, it considers as frames of reference only physically real systems. In view of our previous discussion of coordinates, that too is an advantage it has over Einstein simultaneity.

1.4 Appearance as Reality

We continue with a different kind of advantage backward-light-cone simultaneity has over Einstein simultaneity.

According to our pre-scientific approach, what we are now seeing is happening now. This view presupposes that the speed of light from source to observer is infinite; and indeed, most natural philosophers from Aristotle to Descartes argued that that is light's speed.

With the advent of modern science, however, with its claim that light's speed is finite, this pre-scientific view had to be given up. Our improved knowledge of the world (science) seemed to establish that the world is not as it appears to be. The things we now see are not only not happening now, but they did not even happen together at all: we now see the sun where and as it was eight minutes ago, but the moon where and as it was just over a second ago. In this way a gap opened between how the world appears to us and how we think it really is. The scientific image of the world became radically different from its manifest or apparent image.[4] And, since in our non-reflective moments, which constitute the great majority of our active life, we act on the basis of the world's apparent image, we have to concede that our everyday attitude to the world involves an illusion.

However, if one adopts backward-light-cone simultaneity, this gap partly closes (yet there are others, of course). According to backward-light-cone simultaneity, what we now see through vacuum is what is happening now. The world is as it appears to be, in this respect. Science does not force us to reject our everyday view of the world.

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[4] I shall use below the phrase *apparent image*, and not *manifest image*, for several reasons. First, the latter might imply commitment to the specific description of that image supplied by Sellars, who coined the phrase, and I'd rather avoid these implications. Secondly, *manifest* means clearly revealed or obvious (OED), which implies truth, an implication that would prejudge the status of that image; *apparent*, by contrast, *can* mean obvious, but can also be used to contrast something with what is real, as in 'it is merely apparent'. No prejudging of the status of the apparent image is therefore implied.

Since according to backward-light-cone simultaneity what appears to be happening now, that is, what we now see, is indeed happening now relative to us, this simultaneity can also be called *Apparent Simultaneity*. This is the phrase I shall use below. (The phrase 'backward-light-cone' should be rejected also because once these coordinates are accepted, that surface is no longer a cone but a plane.)

A reservation should be noted at this place. According to Apparent Simultaneity what we see is occurring now relative to us only if we see through vacuum. In case any medium in which light's speed is slower than in vacuum is between observer and light source, the event seen is *inside* the lightcone of the event of seeing it, and therefore earlier, relative to the observer, than the seeing event. The effect of this is perhaps negligible in actual cases: taking the refractive index of air as 1.0002926, and giving the atmosphere a generous 100 km height above the Earth's surface (the speed of light in vacuum defined as 299,792,458 meters per second), light reaches the Earth less than 10^{-7} of a second later than it would through a vacuum. But such possibilities are of course of theoretical significance.

The fundamental idea of Apparent Simultaneity can be generalized to such cases as well, but I shall not do that systematically or in detail in this paper. I consider this paper as a first step towards a more general theory; its world is that of Special Relativity, with electromagnetic radiation propagating in flat and empty space-time. Although this is obviously an idealization, I think it is methodologically justified. Moreover, I believe its results are sufficiently interesting even if limited to that domain. I shall raise the question of the generalization of these results to full or curved space-time only at a few points, mainly towards the end of this paper.

1.5 The Time Lag Argument

Apparent Simultaneity has significant applications in the theory of perception as well. In this subsection we shall apply it to the Time Lag Argument. This argument first appeared in Leibniz's *New Essays on Human Understanding* ([1996], Bk. II, Chap. ix, § 8, p. 135), written during the first years of the 18th century (but published only posthumously, in 1765). The argument, which relies on the hypothesis that light has a finite speed, was probably developed in the wake of Ole Römer's observations, published in 1675, which seemed to establish that hypothesis. It reappeared in Russell's *The Problems of Philosophy* ([1912], chap. 3, pp. 16-7), perhaps derived from Leibniz, with whose work Russell was well acquainted. From there it spread to become a commonplace of contemporary philosophy of perception (see the abundant references in Suchting [1969]). Here it is as it appears in Russell's later *Human Knowledge* ([1948], p. 219):

> [T]hough you see the sun now, the physical object to be inferred from your seeing existed eight minutes ago; if, in the intervening minutes, the sun had gone out, you would still be seeing exactly what you are seeing. We cannot therefore identify the physical sun with what we see…

As some would like to distinguish, unlike Russell, between what we see and the qualities of which we are directly aware but which we should not be said to be seeing (the 'qualia'), we shall formulate this argument a little differently:

1. Light reaches the seeing subject after it has left the seen object.
2. During that time lag the seen object may have ceased to exist.

3. Necessarily, what we are directly aware of when we see an object, exists at that moment.
4. Therefore, what we are directly aware of when we see an object is not the object seen.

Some have tried to criticize the Time Lag Argument by claiming that what we are directly aware of need not exist when we are aware of it. But I don't find this plausible. When we see anything, whether near by or far away, the qualities of which we are directly aware are there for us to inspect and study, and we often can, if we wish, pay them more or less attention. It seems preposterous to claim that they may not be instantiated while these cognitive processes, focusing on them, are taking place. If the argument is unsound, its fault lies somewhere else.

Apparent Simultaneity suggests a different way out: according to it, the argument's first premise is false. Relative to the observer, the event of seeing something is simultaneous with the event being seen. Accordingly, what we are directly aware of when we see an object may be the seen object. Direct realism is unharmed by the Time Lag Argument.

To show that the Time Lag Argument is unsound, we do not have to accept Apparent Simultaneity as the *true* simultaneity; it is enough that it is an *optional* simultaneity definition. If that is so, then the argument's first premise in not true *simpliciter*; its truth is a matter of convention. Yet the argument assumes that its first premise is simply true.

Some might try to save the argument by claiming that even if light cannot be said to leave the seen object before reaching the seeing subject, it still takes the nervous system some time to transfer the signal from retina to brain, and during *that* time the seen object may have ceased to exist. But this response presupposes a Cartesian model of perception, as if the perceiving subject is a homunculus located somewhere deep in our brain. If, following an approach developed by Dennett [1991], we maintain that the time we see anything—we, the embodied human beings—cannot be determined more accurately than the vague interval between the activation of our retinas by the light and the subsequent activation of the relevant parts of our brain, then this line of response is no longer available.

Lastly, the Time Lag Argument is ineffective as regards other senses. We do hear people and smell flowers by means of 'signals' they emit, and the events of emitting these signals *are* earlier than the events of receiving them. Thus, the heard person and the smelled flower may have ceased to exist by the time they are heard and smelled. This might make the Time Lag Argument seem applicable to hearing and smell. However, there is a significant difference between the way we ordinarily assume we see things and the way we assume we hear and smell them. We do not take ourselves to see objects by means of seeing or perceiving the light they emit—we immediately see the objects themselves; but we do take ourselves to hear people by hearing their voices and smell flowers by smelling the scent, odor or smell they emit. We also say that sounds and smells reach us, while we do not say that colour—the corresponding primary object of sight—reaches us. That is, according to the apparent image, perception in the case of hearing and smell is indeed indirect. Thus, the applicability of the Time Lag Argument to these senses does not create any difficulty: it *should* indeed apply to them, since in their case perception is *not* direct. I presume this is the reason we do not find it applied to them in the literature.

My refutation of the Time Lag Argument assumed that we see things through vacuum. This, however, is hardly ever the case: the distant heavenly bodies of our examples are always seen through the atmosphere. Although, given the calculations in the previous subsection, this makes practically next to no observable difference, possible cases in which the difference is huge can easily be imagined. And of course, 10^{-7} seconds and 10^{+7} seconds may be the same from a theoretical point of view. So the general acceptability of my refutation of the argument depends on the applicability of these ideas to full and curved space-time. I shall return to these issues towards the end of my paper, where we shall see that my refutation requires a kind of generalization in order to be effective in the general case. I still thought it is worthwhile to bring it here in its more specific from, since it is generally advisable to commit oneself to fewer hypotheses where possible. With this caveat entered, I think that the acceptability of Apparent Simultaneity supplies us with a refutation of the Time Lag Argument.

1.6 The speed of light as the greatest possible speed

Why is the speed of light the greatest possible speed? To most natural philosophers up to and including Descartes, the answer was obvious, even analytic: because light's speed is infinite. Greater speeds were inconceivable. With Ole Römer's observations, however, light's speed lost its special place, and since then and until 1905 physicists did not consider it as the greatest possible speed. Only with Einstein's work did this change, when Einstein noted that 'the speed of light physically plays in our theory the role of infinitely great speeds.' ([1905], p. 903) Many found this result a kind of unsatisfactory brute fact, as is witnessed by the recurring discussion of the possibility of tachyons.

Apparent Simultaneity provides a different perspective on the issue. From its point of view, if light is emitted by A and arrives at B, then relative to B the event of light's emission at A is simultaneous with the event of its arrival at B. That is, relative to B, light travels in an infinite speed from A to B. So relative to B, nothing could travel towards it with greater speed. In fact, if anything did travel from A to B with a greater speed, then relative to B it would arrive before it left; that is, relative to B it would be traveling from B to A. The explanation of light's approaching speed being the ultimate speed is again analytic, and no brute fact is involved.

But this does not give us all we want. It is an empirical fact, the Light Postulate, that the time it takes light in vacuum to travel from A to B and back depends only on the distance D between A and B. It is $2D/\mathrm{c}$, where c is the constant designating light's average speed on any such journey. Since according to Apparent Simultaneity, the time, relative to A, in which the light is reflected at B is the time of its arrival back at A (light's speed towards A being infinite), it follows that relative to A light moves away from it with speed c/2. So although we have seen why according to Apparent Simultaneity the ultimate *approaching* speed is light's, namely infinity, we still need an argument for why relative to any body of reference, the ultimate speed between *any* two points is light's. Why cannot anything travel *away* from A, say, in a speed greater than c/2 relative to A?

Suppose that were possible, and that some thing traveled from A to B with a speed greater than c/2 relative to A. We shall assume that a signal *S*, carrying energy E_A relative to A, is sent from A to B with $v > c/2$ relative to A. So relative to A, the energy at A after the transmission was reduced by E_A, and the energy at B after the reception was increased by E_A. Assume also that simultaneously with *S*, a light signal is sent from A to B. We saw above that relative to B, since *S* 'arrives' at B before the light emitted at A simultaneously with *S* does, *S* is actually traveling from B to A relative to B. So relative to B, the energy at B after the event of *S* being at B is reduced by E_B, and the energy at A is correspondingly increased by E_B. To have energy preservation, we must have $E_B = -E_A$. So if, relative to A, A transferred energy to B by means of *S*, then relative to B, B increased its energy content by means of emitting *S*. We could increase the energy content of bodies by having them emit *S*-like signals. I don't think this makes much sense. Moreover, if we think of A and B as agents—and there seems to be nothing in the description of the situation to make this impossible—then either A sent *S* or B did, but not both, as this scenario demands. For all these reasons, the idea of super-luminal signals seems to be incoherent.

This explanation might suspiciously look as sort of hocus-pocus: after all, couldn't we introduce the analogous kind of simultaneity definition for *any* ultimate speed, and in this way turn it into 'analytically' ultimate?

The acceptability of Apparent Simultaneity depends, like that of Einstein's, on the Light Postulate. And we had to rely on the latter to prove that light's speed is the greatest possible speed between any two points relative to any body of reference. A given speed being a *de facto* ultimate speed would not therefore suffice to prove any higher speed to be incoherent. So not *any* ultimate speed would do.

Apparent Simultaneity can also explain why the speed of light is independent of the speed of its source: we see things when they happen, never mind whether or not they move relative to us.

Lastly, if light arrives at B, relative to B, the moment it was emitted from A, we can also understand why no body can be accelerated to the speed of light. If a body moved in a straight line from A to B in the speed of light, then it would be, relative to B, simultaneously in all places between A and B. But no body can be in more than one place at the same time. So no body can be accelerated to the speed of light. (But wouldn't light itself then be in more than one place at the same time? And is *this* acceptable? We shall discuss this issue in the paper's last section.)

1.7 More on the Apparent Simultaneity approach

The literature abounds with papers on simultaneity definitions different from Einstein's. Einstein himself suggested one already in his first 1905 Relativity paper (p. 893), and the discussion has been flourishing ever since. The discussion in this paper, however, differs from previous ones in an important respect, I think. Earlier discussions were interested in one or more of these three things: either in showing why Einstein simultaneity is the only acceptable one, or in showing that other definitions of simultaneity are also acceptable, or in distinguishing by means of alternative acceptable simultaneity definitions the factual from the conventional in Relativity Theory. By contrast, I attempt to show here that in several respects Apparent Simultaneity reflects reality better than does Einstein simultaneity. In this way it can help us understand various issues better than does Einstein's. These issues, as I have tried to demonstrate above, involve both questions belonging to physics proper and questions pertaining to our interpretation of nature in a wider sense.

An important advantage of Apparent Simultaneity over other possible simultaneity definitions, Einstein's included, is that Relativity theory should include, whichever definition of simultaneity it uses, the description of things according to Apparent Simultaneity as well. The way Apparent Simultaneity describes the phenomena is not conventional: this is the way things *really appear*. Even if we use Einstein simultaneity, we may ask: how would things *look* to observers? Would moving rigid rods *appear* contracted? Would time in a moving system *appear* dilated? The replies to these and related questions are not given by the length contraction and time dilation resulting from the Einstein simultaneity definition, but by those resulting from Apparent Simultaneity. These latter results should therefore be derived in any presentation of Special Relativity. All the results of Apparent Simultaneity in which it differs from other simultaneity definitions represent real facts: the way things look. No analogous different result of any other definition represents an analogous matter of fact. In this sense, all other simultaneity definitions add, to the facts of Relativity, cancelable middle terms, mediating between one observation and another, but representing no fact themselves. Their experiential predictions, by contrast, are also derivable if the theory is developed on the basis of Apparent Simultaneity.

This is true with respect of *forward* light cone simultaneity as well. The advantages mentioned above of apparent or backward light cone simultaneity over Einstein simultaneity with respect of temporal order and the concept of a physical system are shared by it and forward light cone simultaneity. The one just mentioned, as well as those mentioned in the subsections 'Appearance as Reality' and the one on the Time Lag Argument, are distinctive of Apparent Simultaneity. That is the reason this paper discusses it, and not forward light cone simultaneity.

The factuality of reality's representation by means of Apparent Simultaneity, the non-factuality of all other representations by means of alternative simultaneity definitions, and the other advantages of Apparent Simultaneity discussed in previous subsections, support it as the preferable simultaneity definition. It is still possible, however, that Apparent Simultaneity would lead to a mathematically more complex representation of reality (and it might have some additional drawbacks, of course.) If that is indeed the case, then Einstein simultaneity is preferable over Apparent Simultaneity for pragmatic reasons. There would be a clash, then, between different values a representation of the theory may have.

In the next section I develop the kinematics of Special Relativity with Apparent Simultaneity. After applying the results obtained there to the Twins Paradox in Section 3, I continue in Section 4 to discuss and present some of the corresponding dynamics. For reasons explained there, further developments, which are beyond the scope of this paper, are necessary in order to judge the full acceptability of Relativity with Apparent Simultaneity. Thus, despite the elegance of the results obtained below, the verdict on the mathematical practicability of this approach should await further work.

But before proceeding to the next section, I shall mention here (for lack of any better place) another aspect in which Apparent Simultaneity is comparable to Einstein's. For reasons that were never clear to me, the fact that Einstein simultaneity coincides with simultaneity by slow clock transport was claimed by some, mainly in the past, to show that Einstein simultaneity is not conventional (see summary in Janis [2006]). I cannot see why, if A and B are at rest relative to each other, a clock that moves as slowly as possible from A to B should show which event in A is 'really' simultaneous with which one at B relative to their frame (the exact claim concerns of course taking limits etc.). Be that as it may, one can propose an analogous method for Apparent Simultaneity: *fast* clock transport. Suppose a clock is moved from A to B; suppose also that the clock is synchronized so that the time it shows on arrival at B is the same as that shown then by a clock stationary at B; let us denote the arrival event e_{arrival} and its time relative to B, t_{B}; let us also denote by e_{LPD} (for 'latest possible departure') the event at A which is the earliest one later than which no clock can be transported from A and arrive at B at e_{arrival}; then the time of e_{LPD} relative to B is t_{B}.

Now I don't think this procedure shows that Apparent Simultaneity is non-conventional or superior to any other simultaneity definition; but it does show that a kind of logic, whatever its merits, that was thought by some to support Einstein simultaneity, can be employed in support of Apparent Simultaneity as well. (That is, these considerations are intended as an *ad hominem* against an approach some thought confers Einstein simultaneity with some advantage over other simultaneity definitions.)

2 Length and Time Change According to Apparent Simultaneity

2.1 Purpose and framework

This section develops the mathematical presentation of the kinematics of Special Relativity with Apparent Simultaneity. Since I am mainly interested in showing how Apparent Simultaneity throws new light on various phenomena, I shall derive in this section the formulas for length and time change (not always contraction and dilation!) only for a rigid body moving towards or away from the observer: this would simplify the mathematics, suffice for our purposes here and make the paper accessible to a wider audience. I shall, however, supply without proof the general results at the end of this section. The proofs for the general case are given in an appendix.

The coordinates appropriate to Apparent Simultaneity are spherical, and not Cartesian. This is because the way an observer sees a body is dependent on the angle between its direction of motion and the line connecting it to the observer. For example, a body *A* moving towards an observer and an otherwise identical body *B* moving away from the observer with the same speed would not appear the same to the observer. By contrast, two otherwise identical bodies moving, say, towards an observer with the same speed but not on the same line of motion would appear the same.[5]

As was mentioned above, it is an empirical fact, the Light Postulate, that the time it takes light in vacuum to go from A to B and back depends only on the distance D between A and B. It is $2D/c$, where c is light's constant average speed on such two-way journeys (it is also of course the speed of light in vacuum according to Einstein simultaneity). If we define c as light's average speed, then it is independent of synchronization method, since we measure the interval between light's departure and return *at the same point*. Since according to Apparent Simultaneity, the time, relative to A, in which the light is reflected at B is the time of its arrival back at A (light's speed towards A being infinite), it follows that according to A light moves away from it with speed c/2.

2.2 Description of the thought-experiment

A rigid rod AB is lying along the straight line connecting two material points, C and D, at rest relative to each other, point C being to its left and point D to its right. The rod AB is moving leftward toward point C with velocity v relative to C. The rest length of rod AB is L. Since we use the same method of synchronization according to each point (A, B, C and D), reciprocity holds, and point C is moving rightward

[5] My results are also not a particular case of Winnie's [1970] general results for any synchronization, since Apparent Simultaneity is not a particular case of his Reichenbachian ε-method of synchronization.

towards AB with velocity v relative to A and to B. (I am interested only in the velocity's absolute value here, and I therefore do not distinguish v from $-v$.) See figure 1.

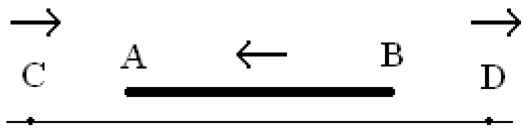

Figure 1

We shall distinguish the speed of approaching from the speed of receding: we should not assume a priori that they are the same. Following the German, we shall call the former v_{her} and the latter v_{hin}. The velocity in which rod AB approaches C, relative to C, and in which C approaches AB, relative to A and to B, is v_{her}. The velocity in which AB recedes from D, relative to D, and in which D recedes from AB, relative to A and to B, is v_{hin}.

2.3 Coordination of inertial clocks, at rest relative to each other

We shall start by assuming that C is inertial. Since AB and D have constant velocity relative to C (in D's case, zero), they are inertial as well. However, according to Apparent Simultaneity, if body β is at a certain moment in its history at the same place as the inertial body α, what is then simultaneous with α is simultaneous with β as well, whether or not β is inertial. Moreover, if body β is at momentary rest relative to α, then even if it is accelerating, the same bodies will be at rest relative to both, having their rest lengths, and therefore distances relative to α will then be equal to distances relative to β. Length change relative to α will therefore equal that relative to β. Moreover, since time change relative to α will be shown to depend only on velocity relative to it, the same (differential) time change would then occur relative to β as well. Our formulas below for length and time change are therefore applicable relative to *any* body, whether inertial or not. Consequently, inertial bodies have no privileged place in the kinematics of Special Relativity developed below. According to the kinematics of Special Relativity with Apparent simultaneity, acceleration is also relative. We shall see in Section 3 how this result is applied to the Twins Paradox.

Returning to our thought-experiment, although A's clock and B's clock are not mutually synchronized, each sees the other's clock as ticking in the same pace as its own. This can be shown as follows. Suppose at $t_A = 0$, A sends a light signal to B. When this light signal reaches it, B sets its clock to $t_B = 0$. A sees this zeroing event at $t_A = 2L/c$, the time it takes the light signal to travel from A to B and back according to A. Now the event $t_A = 2L/c$ is seen at B at $t_B = 2L/c$, the time it takes a light signal to travel from B to A and back relative to B. And so on. So we see that a time interval at A equals the same time interval at B. According to the relativity principle, the contrary also holds, as can be shown directly as well. And this can be generalized to any time interval.

More generally, any inertial clock sees any clock stationary relative to itself as ticking in the same pace as it. Moreover, since we can imagine a clock at any point in space, and relying on transitivity, it follows that for any two events, if two inertial clocks, stationary relative to each other, see these two events as occurring at the same place, they will measure the same time interval between the two events.

2.4 Relation of approaching to receding velocities

Let us first determine the relation of v_{her} to v_{hin}. Suppose that, according to B's clock, C arrives at A at $t_{\text{B}} = 0$. Then C arrives at B, according to B's clock, at $t = \text{L}/v_{\text{her}}$. Suppose further that according to A's clock, C arrives at A at time $t_{\text{A}} = 0$. Then according to A, C arrives at B at $t = \text{L}/v_{\text{hin}}$. Moreover, since B's clock shows zero when B sees C arrive at A, A will see B's clock showing zero at $t_{\text{A}} = 2\text{L/c}$: the time it takes a light signal to travel from A to B and back, relative to A. Following the previous subsection, let our two events occurring at the same place relative to both A and B be B's zeroing its clock and C's arrival at B. As we saw in the previous subsection, A and B measure the same interval between these two events. B of course measures the interval L/v_{her} between them. Since according to A, B zeroed its clock 2L/c after C's arrival at A, the time between the two events at B is the time it takes C to get to B minus 2L/c. Accordingly:

$$\text{L}/v_{\text{hin}} - 2\text{L/c} = \text{L}/v_{\text{her}}$$

or

1 $$1/v_{\text{hin}} - 1/v_{\text{her}} = 2/\text{c}$$

and for future use:

$$v_{\text{hin}} = \text{c}v_{\text{her}}/(\text{c} + 2v_{\text{her}}) = v_{\text{her}}/(1 + 2v_{\text{her}}/\text{c})$$
$$v_{\text{her}} = \text{c}v_{\text{hin}}/(\text{c} - 2v_{\text{hin}}) = v_{\text{hin}}/(1 - 2v_{\text{hin}}/\text{c})$$

2.5 Relation of approaching to receding lengths

Secondly, let us determine the relations between the length of an approaching rod and that of a receding one. Following our former conventions, we shall call the former L_{her} and the latter L_{hin}. Now D, being to the right of AB, sees its length as L_{hin} and its velocity as v_{hin}. It therefore measures the time between A's arrival at C (B then being a distance L_{hin} to the right of C according to D) and B's arrival at C as $\text{L}_{\text{hin}}/v_{\text{hin}}$. C, by contrast, sees the length of AB as L_{her}, and it sees B approaching in velocity v_{her}. It will therefore measure the time $\text{L}_{\text{her}}/v_{\text{her}}$ between A's arrival and B's arrival. Since C and D are at rest relative to each other, then according to Section 2.3 they measure the same interval between the two events occurring at the same place relative to each of them. Hence:

2 $$\text{L}_{\text{hin}}/v_{\text{hin}} = \text{L}_{\text{her}}/v_{\text{her}}$$

2.6 Length change as a function of velocity

We shall now determine the relation between the observed length of a moving rod and its rest length as a function of its velocity, both for an approaching and for a receding rod. We shall do that by considering different ways of measuring the interval between the meeting of A and C and the meeting of B and C.

Relative to C, B moves towards it in velocity v_{her}, and, from the moment A reaches it (C), crosses a distance L_{her}. So the interval C measures between the two events is L_{her}/v_{her}.

Relative to B, C moves with velocity v_{her} towards it, and the distance between A to itself (B) is L. So B measures the interval L/v_{her} between the events.

As we saw in Section 2.4, since B zeroed its clock when it saw C reach A, A saw B's clock as showing the time $t_B = –2L/c$ when C reached it (A). So at that moment, C being at the same place as A, C also saw B's clock as showing the time $t_B = –2L/c$. And we saw in the previous paragraph that when C and B meet, B's clock shows the time L/v_{her}. Therefore, relative to C, the time that passes on B between the two events is $L/v_{her} + 2L/c$.

Accordingly, the ratio between the rate time passes on an approaching body and on a stationary body, relative to C, is:

$$(L/v_{her} + 2L/c) / L_{her}/v_{her}$$

Now according to B, C did zeroed its clock when it arrived at A, so the time C's clock shows when they meet is indeed the time that passed on C between the two events. Therefore, the ratio between the rate time passes on an approaching body and on a stationary body, relative to B, is:

$$L_{her}/v_{her} / L/v_{her} = L_{her}/L$$

Since B and C use the same synchronization method, and since their relative velocities are the same, then, *assuming linearity* (i.e., the aforementioned ratio is independent of the distance crossed), the two ratios are the same:

$$(L/v_{her} + 2L/c) / L_{her}/v_{her} = L_{her}/L$$

With a little algebra we now get the formula for the length-change of an approaching body:

3a $$L_{her} = L\ (1 + 2v_{her}/c)^{1/2}$$

And by formulas (1) and (2) and a little more algebra we get the formula for the length-change of a receding body:

3b $$L_{hin} = L\ (1 – 2v_{hin}/c)^{1/2}$$

If we use spherical coordinates, with the approaching velocity considered negative, we get only one formula for length change, formula (3b), with v replacing v_{hin}.

The length changes are different from each other, and from the length-contraction according to Einstein simultaneity. Notice also that an approaching rod expands, while a receding one contracts. A rod approaching in a velocity approaching that of approaching light—i.e., infinite velocity—will expand to infinity; one receding in a velocity approaching that of receding light—i.e., c/2—will contract to zero. *These are the lengths approaching and receding rods will actually look to observers*. In this sense they are not conventional, a sense that is not paralleled by anything characterizing length contraction according to Einstein simultaneity.

2.7 Time change as a function of velocity

Next, we shall determine the relation between the time that passes on a moving clock and that of a clock at rest, relative to the clock at rest. In Section 2.6 we saw, when considering B's point of view, that the ratio between the time that passes on C between the two events we considered and that that passes on B is:

$$\Delta t_{\text{her}}/\Delta t = \mathrm{L}_{\text{her}}/\mathrm{L}$$

And by (3a) we now get:

4a $$\Delta t_{\text{her}} = \Delta t(1 + 2v_{\text{her}}/\mathrm{c})^{1/2}$$

Time on an approaching body is seen to pass *faster*.

Similarly, according to A's clock the time that passed between the two events is $\mathrm{L}/v_{\text{hin}}$, while according to A the time that passed on C between these events is the time its clock shows, $\mathrm{L}_{\text{her}}/v_{\text{her}}$, which, by (2), equals $\mathrm{L}_{\text{hin}}/v_{\text{hin}}$. We accordingly get:

$$\Delta t_{\text{hin}}/\Delta t = \mathrm{L}_{\text{hin}}/v_{\text{hin}} \,/\, \mathrm{L}/v_{\text{hin}}$$

And from (3b) we then get:

4b $$\Delta t_{\text{hin}} = \Delta t(1 - 2v_{\text{hin}}/\mathrm{c})^{1/2}$$

In a receding body the time moves slower, time dilation. These are the changes in the speed of clocks, and of processes generally, that observers will actually *see*.

Again, if we use spherical coordinates and consider the velocity of an approaching body negative, we get the same formula for time change of both approaching and receding bodies, namely (4b), with v replacing v_{hin}.

To the second approximation, the formula for time change in a moving body then becomes:

$$\Delta t_v = \Delta t(1 - v/\mathrm{c} - v^2/2\mathrm{c}^2)$$

By contrast to Einstein synchronization, we see that according to apparent synchronization there is a time dilation factor of the first order. (Again, this is the time dilation that will actually be observed, independent of simultaneity choice.) Since $v\Delta t = D$, the distance traveled by the moving body relative to the observer, we get a time dilation factor proportional to the distance traveled; to the second approximation, the time dilation formula becomes:

$$\Delta t_v = \Delta t(1 - v^2/\mathrm{c}^2) - D/\mathrm{c}$$

Over great distances, time dilation of –*D*/c will be observed even for non-relativistic velocities. This explains Ole Römer's observations: when Jupiter traveled away from the Earth, a time dilation proportional to the distance traveled was observed, Jupiter's moons reappearing later than expected.

2.8 Length perpendicular to the direction of motion

We shall now show that the length of a rigid rod in a direction perpendicular to that of its motion does not change. Suppose two observers, Alice and Bob, are moving towards each other with relative velocity v, each carrying a thin ring of rest radius r around herself or himself, perpendicular to the line of motion. Suppose length perpendicular to the direction of approaching motion contracts. Then, relative to Alice, Bob's ring contracts, and on meeting it should pass through her ring. This could be empirically determined: Bob's ring could cut a thread connecting Alice to her ring, while Bob's corresponding thread would remain uncut.

However, given the Relativity Principle, Bob should see Alice's ring contract, and relative to him it should pass through *his* ring: again something that could be empirically determined as above: Bob's thread, and not Alice's, will be cut. But this is impossible, so there is no length contraction—and for similar reasons, no length expansion—in a direction perpendicular to their relative motion.

These considerations apply only to observers moving towards each other and meeting, but they can be generalized as follows. Suppose Bob is moving with his ring a distance d from Alice, at an angle θ to the line connecting them. Now put Canis, their dog, at the place where Bob is, facing Bob, with a ring identical to Alice's and Bob's, and at rest relative to Alice. Canis and Bob are in the same relation to each other in which Alice and Bob were in the previous thought-experiment, so Bob's ring does not change its radius relative to Canis. But since Canis is at rest relative to Alice, its ring does not change its radius relative to her. So Bob's ring does not change its radius relative to Alice either. (Bob's ring may still look contorted to Alice, its surface not perpendicular to Bob's direction of motion.)

We thus get the general result: the length of a moving body perpendicular to its direction of motion remains unchanged.

2.9 Generalization of the former results

I shall now give without proof the general results for length and time change for a body moving relative to an observer. The proofs are supplied in an appendix.

Suppose a body B is moving with velocity v relative to an observer A, in an angle θ to the straight line AB (see Figure 2).

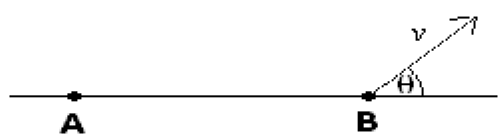

Figure 2

The length of the body relative to the observer is then given by:

3 $$L_v = L_0(1 - 2v\cos\theta/c - v^2\sin^2\theta/c^2)^{1/2}.$$

For the formula for time change, we should take into account that the angle between AB and *v* changes as the body is moving. We should therefore give the differential, relating dt', the time change observed by A on B's clock, to dt, the time change according to A:

4 $$dt' = dt(1 - 2v\cos\theta/c - v^2\sin^2\theta/c^2)^{1/2}.$$

For $\theta = 0$ or π, these formulas reduce to formulas 3b and 4b, or 3a and 4a, respectively, for a body moving away from or approaching the observer. As mentioned above, these formulas are valid for non-inertial bodies as well.

3 The Twins Paradox

In this section I shall try to show that Apparent Simultaneity yields a better understanding of the Twins Paradox than does Einstein simultaneity. More specifically, while according to Einstein simultaneity the situation is sufficiently symmetrical to generate a paradox, or at least a puzzle, this symmetry does not exist according to Apparent Simultaneity and consequently the paradox or puzzle does not arise. Moreover, according to Apparent Simultaneity there is no need to assume that one of the twins was not inertial in any absolute sense during some part of the journey: acceleration is also observer relative. Initially, when describing the paradox according to Einstein simultaneity, I *shall* assume that only one system is inertial; but this assumption will later be eliminated.

3.1 Einstein simultaneity: a paradox

Let us begin with a description of the situation according to Einstein simultaneity. I describe the paradox in its simplest form. An astronaut in a spaceship travels with a constant velocity *v* relative to the Earth in a straight line from the Earth to a distant star, turns back, and returns in the same way with the same velocity. Let us further suppose that the Earth is an inertial system, and that the star is at rest relative to the Earth, a distance L away from it. On Earth, the time that passes between the astronaut's departure and his return is $2L/v$. Since relative to the Earth the astronaut undergoes time dilation, the time that passes according to Earth on the astronaut is $2L/v(1 - v^2/c^2)^{1/2}$. On his return, the astronaut's clock is retarded relative to the Earth's.

The problem of course arises since the astronaut considers the Earth as first moving away from him with velocity *v*, and then returning with the same velocity. Since the astronaut considers the whole stellar system, which is at rest relative to the Earth, as moving with the same velocity as the Earth relative to him, the whole Earth-star distance undergoes contraction relative to him. The Earth-star distance relative to the astronaut would thus be $L(1 - v^2/c^2)^{1/2}$, and the journey would last relative to him $2L/v(1 - v^2/c^2)^{1/2}$. We see that the astronaut agrees with the Earth on the time his clock shows at the journey's end, and that the acceleration he has undergone when turning back on arrival at the star—the only moment in which he was not inertial—does not affect his clock.

The disagreement appears when we calculate Earth's time relative to the astronaut. The Earth's clock should slow down relative to him, and thus the Earth's clock should show, when they meet:

$$2L/v(1 - v^2/c^2)^{1/2}\cdot(1 - v^2/c^2)^{1/2} = 2L/v(1 - v^2/c^2) = 2L/v - 2Lv/c^2$$

This last term, $-2Lv/c^2$, is of course redundant. However, it is agreed that the above result *is* the time that passes on Earth relative to the astronaut on his way to the star and back. The conclusion is that the astronaut's moment of non-inertiality should be responsible for the Earth's clock jumping relative to him $2Lv/c^2$ moments forward.

But this is problematic. First, why should the astronaut's brief non-inertiality influence what happens to the Earth's clock relative to him? If at all, it should influence *his* clock, which we saw it does not. Secondly, this brief non-inertiality period can be identical for different journeys, to stars with very different distances away from the Earth. And it *prima facie* seems that identical accelerations should have identical effects. But of course they do not: the effect is proportional to the distance from Earth, the further away the astronaut is form the Earth, the greater the jump of the Earth's clock. This seems bizarre.

The situation can be visualized as follows. To turn the astronaut back towards the Earth we give him a jolt. This jolt does not affect his clock, but it affects what happens to the Earth's clock relative to him, and that in proportion to the Earth's distance from him. And this happens whether the jolt is given by means of electromagnetic forces, gravitational fields or what have you. How could that be?

Moreover, the astronaut need not pretend that he is being inertial during all that period and that some force acted on the Earth. By contrast to how the situation has often been presented, the astronaut can reason on the basis of the Relativity Principle as follows: 'First the Earth was moving with velocity $-v$ relative to me and its clock underwent dilation; than a force acted on *me* for a very short time (I activated my engines), this force having next to no effect on my clock and of course none on the Earth's; and then the Earth was moving toward me with velocity v, its clock again undergoing dilation.' Its seems his conclusion *must* be that on meeting again, the Earth's clock should be retarded relative to his.[6]

I therefore think that if we consider the Twins Paradox from the point of view of Einstein simultaneity, a paradox, or at least a puzzle, still remains: the situation is by and large symmetrical, and it is implausible that the astronaut's brief non-inertiality period should have the effects it is supposed to have. As Einstein himself put it ([1918], p. 698; translation p. 68), the usual solutions to the puzzle leave the skeptic feeling 'more convicted than really convinced' ('mehr überführt als überzeuget').

3.2 Apparent Simultaneity: symmetry and paradox removed.

When we come to describe the Twins Paradox from the point of view of Apparent Simultaneity, we should remember the distinction between approaching and receding

[6] I am not the first one, of course, to claim that it is far from clear whether the astronaut's non-inertiallity can resolve the paradox. See Debs and Redhead ([1996]) and Brown ([2005], p. 105). Einstein himself thought that, because of the astronaut's acceleration, the resolution of the paradox cannot be given by means of Special Relativity, but that considerations pertaining to General Relativity should be introduced: gravitational fields etc. (Einstein [1918], pp. 698-700; translation pp. 68-72). I shall show below how the paradox is completely resolved *within* Special Relativity.

velocities. What is the same velocity (in absolute value) according to Einstein simultaneity would not be such according to Apparent Simultaneity. The situation described in the former subsection by means of Einstein simultaneity is such that, from the Earth's point of view, the apparent velocity of the spaceship on its way to the star is smaller than its apparent velocity on its way back. If we want to characterize the situation from Apparent Simultaneity's viewpoint, we should say that the spaceship's apparent velocity relative to the star on its way to the star is the same as its apparent velocity relative to the Earth on its way back. We shall call this velocity v_{her}.

Formula (1) in the previous section established the following relation between v_{hin} and v_{her}:

1 $$1/v_{\text{hin}} - 1/v_{\text{her}} = 2/c$$

Accordingly, the time that the journey to the star and back takes relative to Earth is:

$$\Delta t_{\text{Earth relative to Earth}} = L/v_{\text{hin}} + L/v_{\text{her}} = 2L(1/v_{\text{her}} + 1/c)$$

However, relative to the Earth, the astronaut's clock will slow down on the way to the star, while accelerating on his way back. If we use our formulas (4a) and (4b) from the previous section, then relative to the Earth, the time that passes on the astronaut on his journey is:

$$(L/v_{\text{hin}})(1 - 2v_{\text{hin}}/c)^{1/2} + (L/v_{\text{her}})(1 + 2v_{\text{her}}/c)^{1/2}$$

We now use the formulas we extracted from formula (1) in the previous section:

$$v_{\text{hin}} = v_{\text{her}}/(1 + 2v_{\text{her}}/c) \ , \ (1 - 2v_{\text{hin}}/c) = v_{\text{hin}}/v_{\text{her}}$$

And get:

$$\Delta t_{\text{Astronaut relative to Earth}} = (2L/v_{\text{her}})(1 + 2v_{\text{her}}/c)^{1/2}$$

(By means of formula (5) of the Appendix it can be shown that the two results are equal to those obtained by means of Einstein simultaneity.)

What does the astronaut see? First, he sees the star approaching him in velocity v_{her} over the expanded distance $L(1 + 2v_{\text{her}}/c)^{1/2}$, and then he sees the Earth approaching him over the same expanded distance and with the same velocity. So relative to himself, his journey lasts:

$$\Delta t_{\text{Astronaut relative to Astronaut}} = (2L/v_{\text{her}})(1 + 2v_{\text{her}}/c)^{1/2}$$

The same result that we just got for the time that passes on him relative to the Earth.

How much time passes on the Earth relative to the astronaut? Notice that first the Earth recedes from him with velocity v_{hin} and then approaches him with velocity v_{her}. Accordingly, the distance that the Earth first travels is L_{hin}, and this part of its journey lasts $L_{\text{hin}}/v_{\text{hin}}$; it then travels L_{her}, this journey lasting $L_{\text{her}}/v_{\text{her}}$. Taking into account the time change as expressed by formulas (4), then substituting the formulas (3) for length change, and lastly expressing v_{hin} by v_{her} according to (1), we get:

$$\Delta t_{\text{Earth relative to Astronaut}} = (L_{hin}/v_{hin})(1 - 2v_{hin}/c)^{1/2} + (L_{her}/v_{her})(1 + 2v_{her}/c)^{1/2} =$$

$$L(1/v_{hin} - 2/c) + L(1/v_{her} + 2/c) = L(1/v_{hin} + 1/v_{her}) = 2L(1/v_{her} + 1/c)$$

Again the same result that we got above, when we calculated the time that passes on Earth between departure and return relative to the Earth itself.

(When the astronaut turns back upon his arrival at the distant star, he sees the distance between him and the Earth as rapidly growing, from $L(1 - 2v_{hin}/c)^{1/2}$ to L and then to $L(1 + 2v_{her}/c)^{1/2}$. This is because receding bodies contract, while approaching ones extend. Since we unrealistically assumed that the turnabout is instantaneous, we did not have to calculate the time that passes on Earth or on the astronaut during that period. This length-change is not peculiar to Apparent Simultaneity. According to Einstein simultaneity as well, when the astronaut slows down his distance to Earth increases relative to himself, and then decreases again when he starts moving back towards it.)

We see that straightforward calculations give the same results for the two points of view. The paradox does not arise. By contrast to Einstein's own solution of the paradox, which made use of considerations pertaining to General Relativity, considerations pertaining to Special Relativity are sufficient from Apparent Simultaneity's point of view. Moreover, when the astronaut turns back upon arrival at the star, there is no jump on the Earth's clock relative to him. From his point of view, the process that takes place on Earth is continuous, with the Earth's clock first going slower and then faster than his.

Notice also that we did not have to specify that the Earth, but not the astronaut, is inertial. It is irrelevant to the situation whether any of the two participants is inertial, and we do not have to assume that any of the two accelerates in any absolute sense. Relative to the Earth, the astronaut accelerated; relative to the astronaut, the Earth did: acceleration is observer-relative. In this respect the Relativity Principle is applicable to *any* observer or body, and there is no need to assume a preferred set of bodies, inertial bodies, that play a special role in the theory.

The fact that we do not need to distinguish inertial from non-inertial bodies enables us to maintain a relationist view of space-time within Special Relativity. No 'bent' in an observer-independent space-time on the astronaut's turning point is necessary in order to account for the Twins Effect.

Why isn't there symmetry between the points of view? This is because the Earth and the astronaut *see* different things. The Earth *sees* the voyage to the star as lasting longer than the voyage back: the *same* distance, but different velocities and *different* times. The astronaut, by contrast, *sees* the Earth's voyage away as lasting the same time as its voyage back: *different* distances and different velocities, but the *same* time. Remember that we proved above (formula (2)) that $L_{hin}/v_{hin} = L_{her}/v_{her}$. There is no apparent symmetry.

This apparent *a*-symmetry should of course be acknowledged on *any* conception of simultaneity in Special Relativity. This is the fact of the matter about how things *look* to Earth and to the astronaut.[7] But while according to Einstein simultaneity this asymmetry is by-and-large *merely* apparent, the objective situation

[7] The first to have carried in any detail calculations on how things would look to the Earth and to the astronaut was, to the best of my knowledge, David Bohm ([1964], pp. 168-71). He did that by means of the formula of the Doppler shift. Bohm too, however, thought that the acceleration of the astronaut is the key for resolving the paradox, and that considerations pertaining to general relativity are necessary for doing that (ibid., pp. 166-7).

being more symmetrical, according to Apparent Simultaneity there is no additional fact over and above the observed asymmetry. Einstein simultaneity introduced into the description of the process an unobservable symmetrical layer, which we should deem a physically imaginary artifact of the simultaneity definition, and which generated the paradox.

4 Dynamics

In Section 2 we proved and formulated several formulas expressing the kinematics of Special Relativity with Apparent Simultaneity. I believe the reader would agree that these were in general not much more cumbersome than the parallel formulas of the Einstein simultaneity formulation. Moreover, in Sections 1 and 3 we saw several advantages of the Apparent Simultaneity approach to Relativity over the standard one.

Dynamics, however, poses a problem—perhaps solvable—to the Apparent Simultaneity approach. This problem can be demonstrated by the following observation. Suppose A is an inertial observer, and body B is moving along a straight line relative to A, with a constant velocity according to Einstein simultaneity. Suppose further that A is not on B's line of motion. Then according to Apparent Simultaneity, *B's motion is not uniform*. If B first approaches A and then moves away from it, B's speed continuously decreases relative to A. (If A is *on* B's line of motion, then B's motion is uniform apart from at point A, which is a singularity point in its motion: its velocity changes there from v_{her} to v_{hin}.)

Accordingly, Apparent Simultaneity might force us to change the laws of dynamics: since the sum of forces acting on B is zero (according to the accepted way of defining forces), it follows that according to Apparent Simultaneity Newton's First Law, the Law of Inertia, does not hold.[8]

With a few exceptions, previous discussions of alternative simultaneity definitions have concentrated on kinematics. One such recent exception is Ohanian [2004]. 'The fundamental error of Reichenbach and his conventionalist followers', Ohanian claimed, 'was their neglect of dynamics.' (p. 147) This is because nonstandard 'synchronization introduces pseudoforces into the equation of motion, and these pseudoforces are fingerprints of the nonstandard synchronization, just as the centrifugal and Coriolis pseudoforces are fingerprints of a rotating reference frame.' (Abstract) Accordingly, 'when we adopt an inertial reference frame, the requirement of absence of pseudoforces determines the synchronization uniquely, and no further synchronization convention can be imposed.' (p. 147)

But I think Ohanian is mistaken. As Macdonald [2005] argued, 'Ohanian has only shown that *if* Newton's laws must take their standard form, then nonstandard synchronizations are ruled out. But he has given no reason that the laws *must* take their standard form.' (p. 455) That is, instead of introducing pseudoforces we can change the laws of dynamics, and in this way admit nonstandard synchronizations. In his reply to Macdonald, Ohanian has indeed retreated to simplicity arguments in favour of standard simultaneity ([2005], third paragraph). So in this way one can still admit nonstandard synchronizations.

However, especially in light of our previous discussion of the meaning of coordinates (Section 1.2), this response seems insufficient. If Newton's First Law articulates an important fact about interactions in nature, then modified coordinates

[8] The essentials of this problem were already noted by Torretti ([1999], p. 70).

might stop expressing this fact; they would in effect conceal it, and we would have to extract it from them by various mathematical manipulations. We could still use them, of course, but at the cost of part of what we expect of coordinates. So this response leaves us with a strong—albeit not conclusive—argument against the representation of nature by means of Apparent Simultaneity.

I think the way out of this difficulty is different, and it does enable us to preserve what is physically essential, in a sense, in Newton's First Law. This law, in its common formulation, is the only conservation law in which the conserved kinetic or dynamic magnitude is velocity. In all the other related conservation laws—namely, in those in which bodies collide with each other, or interact in any other fashion—the conserved magnitudes are momentum and energy. To achieve greater generality, we should therefore reformulate Newton's First Law with respect to momentum, energy, or both. In fact, Newton's First Law can be considered a limiting case of laws of conservation applying to collisions and other interactions between bodies: it describes the situation in which a body is involved in a 'zero' interaction, that is, the case in which a given body does not interact with any other body or physical entity (field, say). In such a case, a particular case of the general conservation law, the body's momentum and energy do not change with time. This should be the general formulation of Newton's First Law. And we have justified this general formulation without recourse to any specific idea of Apparent Simultaneity.

Since according to Newtonian mechanics, as well as according to Relativity Theory with Einstein simultaneity, a body's momentum and kinetic energy do not change if and only if its velocity does not, we can *derive* Newton's First Law in its ordinary formulation from the law's general formulation. But the general formulation is the one of more fundamental theoretical significance. And this general law can still be valid according to Special Relativity with Apparent Simultaneity. The only difference would be that this time, the conservation of momentum and energy would not entail the conservation of velocity.

In order to have this conservation law, as well as the general conservation law of momentum and energy, all we have to do is to define momentum and energy according to Apparent Simultaneity as equal in value to their corresponding magnitudes according to Einstein simultaneity, which we know are conserved. I derive the resulting formulas in the Appendix. Here I shall simply state the results.

Suppose body B with rest mass m is moving with velocity $\boldsymbol{v}$ relative to body A, in an angle θ to the straight-line AB. B's momentum and energy relative to A are:

$$\mathbf{P} = m\boldsymbol{v}/(1 - 2v\cos\theta/c - v^2\sin^2\theta/c^2)^{1/2}$$
$$E = mc^2(1 - v\cos\theta/c)/(1 - 2v\cos\theta/c - v^2\sin^2\theta/c^2)^{1/2}$$

We may consider the relativistic mass of a body as dependent on its velocity, this time according to the following formula:

$$m_v = m/(1 - 2v\cos\theta/c - v^2\sin^2\theta/c^2)^{1/2}.$$

We then get the simplified results:

$$\mathbf{P} = m_v\boldsymbol{v}$$
$$E = m_vc^2(1 - v\cos\theta/c).$$

As on Einstein simultaneity, the energy of a body at rest equals E = mc^2, m being its rest mass. Again, since **P** and E according to Einstein simultaneity are conserved, and since we have defined **P** and E according to Apparent Simultaneity as equal in value to their corresponding Einstein magnitudes, both momentum and Energy are conserved according to Apparent Simultaneity as well. In particular, Newton's First Law in its general formulation still holds: if a body does not interact with any other body or physical entity, then its momentum and energy do not change with time.

Although I have just shown how Newton's First Law can be maintained in Special Relativity with Apparent Simultaneity, I shall now digress to express some doubts concerning that very law. We opened this section by describing a situation in which body B is moving along a straight line relative to an inertial observer A, with constant velocity according to Einstein simultaneity. Could A and B in our example be the only bodies in the universe? The answer is negative: if they were, then, on Einstein simultaneity as well, body B would not be moving with a uniform speed and along a straight line relative to A: due to their gravitational interaction, they would accelerate towards each other. And adding bodies to this universe would not improve things: we will get more forces, and in general no body would be moving with constant speed along a straight light relative to any other body. Some arrangements in which some body would move for some time in a constant speed along a straight line are possible, but it is doubtful whether these represent any significant physical generalization. And this problem cannot be avoided by making A or other bodies gradually lighter, in order to see what B's acceleration would be at the limit: according to Newtonian mechanics, A's acceleration towards B is independent of A's mass; so B's relative acceleration towards A, the relative acceleration A would ascribe to B, would not approach zero as A's mass approaches zero.

So perhaps, if we wish to articulate the idea behind Newton's First Law, we should abstract, *per impossibile*, from gravitational interactions, and also assume that some of our bodies have no charge, no magnetic moment, no higher magnetic multipole moments, no intrinsic angular momentum, and 'furthermore (nearly) any other physical property imaginable, or for which experimentalists have invented a measuring device, should also be zero'? This is the approach Pfister thought necessary in his careful examination of Newton's First Law in order even to adequately formulate it ([2004], p. 54). If we follow Plato (*Sophist* 247d-e) in taking the ability to act or to be acted upon as a criterion for being, it seems Pfister ended up with nonentities.

One might think that the problem could be resolved if we made our body B the only body in the universe. We could then ascribe to B any physical properties whatsoever: any mass, charge, magnetic moment, and so on. B would not be involved in any interaction, and it would still be moving in a straight line with constant velocity or momentum relative to inertial reference frames. But of course, in such a universe, the only physically real point of view is that of body B itself; there isn't any other thing to which we can relate B's motion. And relative to itself, any body is always in the same place, irrespective of the interactions in which it is involved. On the other hand, for any moving body we can always introduce arbitrary reference frames relative to which it is in uniform motion along a straight line. For Newton's First Law to have any real content, it seems we need to describe motion relative to a physically realized point of view. That is, we need to introduce additional bodies and view B's motion relative to them. But then, as we saw above, it is doubtful whether we can describe physically significant configurations in which B's motion is indeed uniform.

Our conflict here is reminiscent of Kant's dove ([1787], B8, Kemp Smith's translation):

> The light dove, cleaving the air in her free flight, and feeling its resistance, might imagine that its flight would be still easier in empty space.

The bottom line of this sketch is that Newton's First Law is conceptually problematic (although perhaps not irreparably so). And of course I am not the first to air such misgivings; here is Einstein on the Law of Inertia ([1920], p. 1010; translation taken from Pfister [2004], p. 56):

> It reads in detailed formulation necessarily as follows: Matter points that are sufficiently separated from each other move uniformly in a straight line—*provided that the motion is related to a suitably moving coordinate system and that the time is suitably defined*. Who does not feel the painfulness of such a formulation? But omitting the postscript would imply dishonesty.

So although Newton's first Law, in its general formulation, holds in Special Relativity with Apparent Simultaneity as well, I am not sure what status this Law should have in physics.

I end this section with a few additional formulas of dynamics. Since we have defined momentum and energy according to Apparent Simultaneity as equal in value to their corresponding magnitudes on Einstein simultaneity, it follows that the following equation still holds:

$$E^2 - P^2c^2 = m^2c^4$$

This result can of course be verified by direct calculation as well.

If we take the partial derivative relative to time of this equation we get:

$$2E\ \partial E/\partial t - 2\mathbf{P}c^2\ \partial \mathbf{P}/\partial t = 0,$$

or

$$E\ \partial E/\partial t = \mathbf{P}c^2\ \partial \mathbf{P}/\partial t.$$

After inserting the expressions for energy and momentum, and a little algebra, we get the basic dynamical law:

$$(1 - v\cos\theta/c)\ \partial E/\partial t = \boldsymbol{v}\ \partial \mathbf{P}/\partial t.$$

This expression differs from the parallel expression for Einstein simultaneity by the factor $(1 - v\cos\theta/c)$ on the left.

This concludes my discussion of dynamics. We haven't seen, I think, any conceptual disadvantage of Apparent Simultaneity compared with Einstein simultaneity as far as dynamic considerations were involved, and the formulas we derived were not much more complex than Einstein's. Still, to assess the acceptability of this formulation of the theory, we should also see how it represents electromagnetic phenomena. It would also be interesting to try and see how it adapts to circumstances involving gravity. This is, however, beyond the scope of this paper.

5 Of Light

The Apparent Simultaneity approach suggests a new interpretation of the nature of light or of electromagnetic radiation. This interpretation and some of its implications will be developed in this last section.

The idea that light is a kind of wave propagating in space, or that it is a stream of minute particles—photons—moving from one place to another, suggests itself if we think that its propagation speed is finite. It is then most natural to construct models of its nature assuming resemblance with other things that move from one place to another with finite speed. This analogy breaks down, however, if we think of light as propagating with infinite speed. Indeed, Aristotle and Descartes (among others), who thought its speed is infinite, developed different accounts of the way light energy is transmitted from source to absorber. (Although it is anachronistic to talk of energy in its modern sense when describing their theories, the relevant aspects of the concept can easily be adapted to the appropriate factors there.)

While it would be inaccurate to say that light's speed is infinite according to Apparent Simultaneity, it is correct to say that according to it, if light is transmitted in vacuum from B to A, then relative to A its speed is infinite. Assuming that nothing can be in more than one place at the same time, particles or waves cannot constitute the transmitted light.

Aristotle and Descartes, who also thought that light propagates with infinite speed, thought that what light transmits is transmitted by means of the transparent medium between source and absorber. For Descartes, for instance, light was a kind of pressure exerted by the light source on the absorber by means of the rigid bodies filling the space between them (*Le Monde*, Chap. 13). However, if we maintain that light can propagate in vacuum as well, this approach is not open to us: light transmission can occur in the absence of any medium.

The option that then suggests itself is that the electromagnetic interaction between source and absorber is *immediate*. Electromagnetic transmission of energy is an immediate interaction at a distance between source and absorber.

Although the idea of action at a distance is not an integral part of modern physics, it has surfaced time and again, from Newton on. Today it is discussed mainly in the context of EPR and related phenomena. So it is not foreign to contemporary physics thought. Einstein objected to it, but his only argument against it with which I am familiar is, I think, invalid. He claimed that if we allowed for action at a distance, then 'the idea of the existence of (quasi-) closed systems would be made impossible, and with it the specification [Aufstellung] of empirically verifiable laws in the ordinary sense' ([1948], p. 322; my translation). But I cannot see why that should be so: why cannot we isolate a system from distant influences, and not only from proximate ones? Moreover, this objection, even if it were valid, would only show that action at a distance poses difficulties to the researcher, and not that it is incoherent. And I am not familiar with any other good argument against action at a distance. (Notice that we are *not* contemplating the problematic idea of influence that propagates faster than light.) So I cannot see why action at a distance should in principle be rejected. In fact, it seems the idea has always had the appeal of a taboo of which no one knows why exactly it is there.

If the idea that transmission of light energy is an interaction at a distance between source and absorber seems outlandish to us, this is only because we are used to judge ideas by means of our theoretical frameworks. This idea is, in fact, in agreement with how things appear. When we look at the moon, say, it seems that we directly observe that distant object, and that we are aware of it not by virtue of being aware of or being affected by any other thing. The interaction we call sight between observer and object seen *appears* to be interaction at a distance. Thus, the interpretation of the nature of light suggested by Apparent Simultaneity again narrows the gap between the apparent image of the world and its scientific image, between appearance and reality.

If transmission of light energy is done as action at a distance, we can explain why the rest mass of the photon had to turn out equal to zero (what a strange particle!). Since light energy is transmitted directly from source to absorber, there is no energy to be lost to an alleged mediator carrying the transmitted energy—otherwise the system's energy would not be conserved. So if we introduce such a mediator into our models, it has to carry exactly the energy it transmits. It therefore cannot have any energy, and accordingly no mass, of its own.

The direct interaction theory may perhaps help us understand some quantum phenomena as well. (What follows is a sketchy, tentative, attempt.) Let us reconsider the Double-slit Experiment with photons. Assume the light source's intensity is low enough so that there is at any moment, according to the photon model of light, a single photon at most between the source and the screen. We of course still get an interference pattern on the screen. Yet although each photon is supposed to pass through a single slit, so that the presence or absence of the other slit should not influence it, if we close any slit the interference pattern is gone. This is notoriously hard to explain on the particle model.

How may we interpret what is happening in this experiment on the direct interaction approach? We should think of the interaction between source and absorber (i.e., points on the screen) as determined by all spatial relations between them and other bodies. This determination is perhaps best expressed by Feynman's path integral approach (Feynman and Hibbs [1965]). By contrast to Feynman, however, we should not think of each path between source and screen as representing a possible history. I provide a brief description of the calculation method, to give those unfamiliar with it some idea of how it works.[9] Let us consider only those paths from the source to points on the screen that contain no other body along the way, i.e., paths through vacuum. Every such possible path contributes a vector of the form $\rho e^{iS/\hbar}$ to the probability that the source transmit a light-energy quantum to that point on the screen, where S is a function of the path, the *action*, and $\hbar=h/2\pi$, where h is Planck's constant. In our case, if the energy quantum transmitted is E, then $S = -(E/c)x$ and is thus proportional to the path's length. For each point we sum all these vectors, and the square of the length of the final vector is then proportional to the probability of transmission of a light-energy quantum to that point. In the Double-slit Experiment, with both slits open, each point on the screen is on *two* paths leading from the light source to it (actually, due to the finite width of each slit, on two *families* of neighboring paths). The probability that a light quantum be transmitted to it is therefore determined by adding two different vectors, and an interference pattern results. But if one slit is closed, each screen-point is on a single path, and the interference pattern is gone.[10]

[9] My own knowledge of the mathematics involved is also incomplete.

[10] More generally, we should also consider paths that contain additional bodies on them; in that case we should add the contribution of the other bodies to the probability of interaction. This does not require any modification to the ideas considered in this paper, but is rather a natural consequence of them.

But doesn't this talk of paths make sense only if we assume that something *travels* along the paths, in which case the suggested model of direct interaction at a distance is inapplicable? This is not how we should think of the process. Rather, we should think of the spatial relations between bodies as determining the bodies' possibilities of interactions. The interaction between source and screen is direct, but it is determined by the distribution of all bodies in space. There is therefore no sense in asking, through which slit did the photon pass, since there is no photon. In this way, if we adopt the direct interaction approach to the transmission of light energy, we may perhaps get a beginning of an intelligible interpretation of some quantum puzzles. (But there are additional puzzles, of course; primarily those related to the origin of the uncertainty principle, the collapse of the wave-packet and to entanglement.)

The interpretation of the Double-slit Experiment by means of the direct interaction approach to light suggests the application of this interpretation to similar experiments with electrons and other dual wave-particle phenomena. However, two difficulties with this further application immediately suggest themselves. First, since in such cases the particle carrying the energy is supposed to travel slower than light, its emission from the source should be in the past relative to its detection, and not simultaneous with it as in the case of light. It therefore seems that the application of this interpretation to such cases would commit us to direct interaction with the past. Secondly, by contrast to photons, we should not deny the reality of electrons, not to mention heavier particles that display similar behavior in the Double-slit Experiment. Consequently, it seems that between the emission of such particles from the source and their later arrival at the screen they must be somewhere, and more specifically, it seems they *must* follow a definite path. But the logic that was supposed to resolve the puzzle for photons involved assuming that nothing is moving through any specific path. It thus seems that this approach to the Double-slit Experiment cannot apply to particles. I shall conclude this paper with a discussion of these difficulties.

I start with the second issue: I think we have good independent conceptual reasons for maintaining that the position of the electron, in the Double-slit Experiment, between emission and arrival at the screen, is indeterminate; that it follows no path. The electron does not interact with anything along the way between its emission and its arrival at the screen: its energy does not change, and it does not affect the state of any other body.[11] What could it *mean*, then, to claim that it passed in a certain place at a certain moment between its emission and arrival at the screen? When a ball travels through the air, it interacts with the air all along its way; when the Earth travels through space, it continuously absorbs radiation from other sources, affects the trajectory of other celestial bodies, and much more; the electron does nothing of the sort. If we do not wish to imagine it rubbing its shoulders with the celebrated absolute points of absolute space-time, then we cannot ascribe any *meaning* to the claim that it has been in any definite place between its emission and arrival at the screen. The place of a thing is determined only by its interaction with other things; the electron did not interact with anything between emission and arrival at the screen, so its place was indeterminate then. Accordingly, the facts of the Double-slit Experiment, independently of my attempt at interpreting the process, force us not to ascribe any definite position to the electron between its emission and arrival

[11] My description applies to the basic form of the experiment. Other, more elaborate forms allow for some kinds of interaction along the way that affect but do not cancel the interference pattern (see Feynman and Hibbs [1965], pp. 7-9). With appropriate elaborations, the considerations developed here apply to these experiments as well. As I have said, what I intend to supply in these paragraphs is only a first sketch.

at the screen. The logic developed above for the experiment with light is therefore also applicable to the experiment with electrons and other particles. (On the other hand, these considerations make it applicable to light even if one does adopt its photon model. Additionally, these considerations can be applied to entanglement phenomena as well.)

Let us turn to the admissibility of direct interaction with the past. It can now be seen that the Double-slit Experiment with electrons (and other particles) does not commit us to this possibility: the screen directly interacts with the electron, and not with the source. However, as was several times implied in previous sections, the possibility of direct interaction with the past might be suggested by some other phenomena. If interaction between light source and absorber is direct when light propagates through vacuum, it seems it must also be direct when light propagates through some transparent medium. But light's propagation speed through a medium is slower than its speed in vacuum. It follows that the event of light emission occurs earlier than it should have, had it had to get to the absorber at the same time from the same place but through vacuum. But according to Apparent Simultaneity, that *later* event is simultaneous with the event of light reception, relative to the absorber. So the actual event of light emission is earlier, relative to the absorber, than the event of its reception. Accordingly, the absorber directly interacts with something which is in its past. Is this result acceptable, or even coherent?

I think it is both. Notice first that no problematic backward causation is involved: the absorber, which receives light energy from the source, cannot by that means influence the source prior to the emission event in any way, or similarly transfer information to the source. So no causal-loop paradoxes can be generated.

Secondly, accepting the idea of direct interaction with the past brings with it some modification of our ideas of the relation between temporal order and what is real, relative to an observer or, more generally, relative to a physical body. (What is real is always relative to a physical body or entity: physics allows of no external viewpoint that would give us what is 'absolutely' real; the only viewpoints it allows are physically realized ones.) What is *now* real relative to an observer—'*now*' signifying a certain moment in the observer's history—is not only whatever occurs simultaneously, relative to the observer, anywhere in the universe, but also everything with which the observer directly interacts at that moment, although these things might belong to the observer's past. Relative to any moment t in an observer's history, the present signifies, for any object, the *latest* moment in its history in which it could directly influence the observer at t.

In practice, that latest moment is usually, to all intents and purposes, the only moment in which the object can directly influence the observer at t; the past can usually influence us only by means of traces left in the present. But in some cases, direct influence of the past is possible. One such case is transmission of light energy through thick transparent medium. Another could occur in curved space-time, where due to gravitational lenses, for instance, light energy can be transmitted from source to absorber along more than a single path, different paths being of different lengths. And cases of other kinds can also be found.

It might have been nicer had this conceptual change not been a necessary consequence of the direct interaction theory of light. However, if we are to avoid *ad hoc* stipulations, it does seem to me unavoidable. Moreover, if we think of temporal relations as determined by the causal nexus between bodies, it is difficult to see why it should not be possible for two non-simultaneous stages in one body's history to influence directly another body at the same stage of its history (as long as causal loops are impossible). Distant simultaneity would then just indicate the latest moment in the remote body's history of such possible direct influence. So I think this conceptual change does not constitute a strong objection to the direct interaction theory of light. (Of course this conceptual change is far more modest than the extravagance standard common in widely held related physical and philosophical theories: many-worlds and many-minds interpretations, block universes and others.) Moreover, and more significantly, it again agrees with the world's apparent image: turning back to issues discussed in the subsection on the Time Lag Argument, when we observe a star through the atmosphere it does appear to us to be real, existing there in the sky; and the direct interaction theory of light allows us in this case as well to save the phenomena.

The ideas developed in this last section obviously require elaboration, both within physics and within a more general theory of perception. Some possible threads were indicated in the course of this paper; others were passed over in silence. But I hope enough has been said to make the interest in Apparent Simultaneity's view of the world evident.[12]

6 Appendix: Some Derivations

In this appendix we develop the general formulas for length and time change in a moving body. In doing that we shall rely on the results of Section 2. We shall also generalize the empirical Light Postulate formulated there. In its general form it reads:

> Light Postulate: The time it takes light to travel along a closed path in vacuum is a function of the path's length alone.

Light's average speed along the path is denoted c.

Let us now proceed with the derivations. Suppose A, B and C are inertial material points, at rest relative to each other. Let us designate the distances AB = r, BC = d. The angle between AB and BC is θ. A body of rest length L_0 is moving from B to C with a constant velocity v_{her} relative to C. See figure 3.

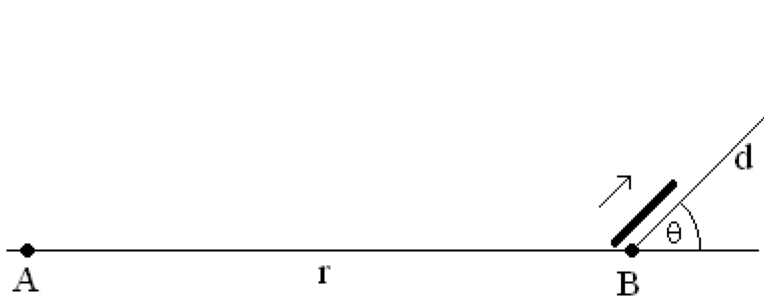

Figure 3

Since we cannot assume that the body's speed is constant relative to A (in fact it is not), we shall designate its average speed, relative to A, v.

[12] I am indebted to Meir Buzaglo and Barry Loewer for comments on earlier versions of some parts of this paper.

Now the experiment is arranged as follows. At time $t_A = 0$, A sends a light signal to B. When the signal reaches B, B sets its clock to $t_B = 0$, and at that moment the body is at B on its journey towards C. C sets its clock to $t_C = 0$ when it sees the body at B.

Since the time it takes light to travel along a closed path of length D is D/c, A sees C zeroing its clock at $t_A = (r + d + AC)/c$. For the same reason, A sees the body at B at $t_A = 2r/c$. The body reaches C when C's clock shows $t_C = d/v_{her}$, which is also the time A sees on C's clock when the body reaches C. So the time it took the body to get from B to C relative to A, which is d/v, also equals:

$$d/v = d/v_{her} + (r + d + AC)/c - 2r/c.$$

According to the Cosine Law, $AC^2 = r^2 + d^2 + 2rd\cos\theta$. We thus get:

$$(r^2 + d^2 + 2rd\cos\theta)^{1/2}/c = d/v - d/v_{her} + r/c - d/c.$$

Square the two sides and cancel identical terms:

$$2rd\cos\theta/c^2 = d^2/v^2 + d^2/v_{her}^2 - 2d^2/vv_{her} + 2rd/cv - 2d^2/cv - 2rd/cv_{her} + 2d^2/cv_{her} - 2rd/c^2$$

We divide by d:

$$2r\cos\theta/c^2 = d/v^2 + d/v_{her}^2 - 2d/vv_{her} + 2r/cv - 2d/cv - 2r/cv_{her} + 2d/cv_{her} - 2r/c^2$$

Let now d approach zero, the limit in which v is the momentary speed of the body at B relative to A:

$$2r\cos\theta/c^2 = 2r/cv - 2r/cv_{her} - 2r/c^2$$

Multiplying by c/2r, and a little more manipulation:

1 $$1/v_{her} = 1/v - (1 + \cos\theta)/c.$$

This is a generalization of formula (1) of Section 2.

Let us now derive the time change in a moving body. Since the body sees BC moving towards it, it will see BC expand to length $d(1 + 2v_{her}/c)^{1/2}$ (see formula (3a)). The time it will take the body to get to C is therefore $\Delta t' = d(1 + 2v_{her}/c)^{1/2}/v_{her}$. On the other hand, the time it takes the body to get to C relative to A is $\Delta t = d/v$. Accordingly:

$$\Delta t'/\Delta t = (d(1 + 2v_{her}/c)^{1/2}/v_{her})/(d/v) = (v/v_{her})(1 + 2v_{her}/c)^{1/2}$$

And in the limit:

$$dt' = dt(v/v_{her})(1 + 2v_{her}/c)^{1/2}$$

We can now use the relation of v_{her} to v, as expressed in formula (1). With a little algebra we eventually get:

4 $$dt' = dt(1 - 2v\cos\theta/c - v^2\sin^2\theta/c^2)^{1/2}.$$

Lastly, according to A and C, it takes body L the same time to cross B, since they are measuring the interval between two events occurring at the same place (see Section 2.3). Accordingly:

2 $$L_{her}/v_{her} = L_v/v.$$

This is a generalization of formula (2) in Section 2.5. Accordingly:

$$L_v = L_{her}(v/v_{her}) = L(v/v_{her})(1 + 2v_{her}/c)^{1/2},$$

3 $$L_v = L_0(1 - 2v\cos\theta/c - v^2\sin^2\theta/c^2)^{1/2}.$$

This concludes the derivation of the general formulas for length and time change, given in Section 2.9.

We move on to the derivation of the formulas for momentum and energy of a moving body of rest mass m, moving with velocity $\boldsymbol{v}$ relative to an observer, in an angle θ to the straight-line connecting it with the observer (see previous figure). These results are most simply obtained if we translate their Einstein expressions to the Apparent Simultaneity ones. To do this, we have to find the functional relation between the Einstein velocity of a body and its apparent velocity.

Suppose a body is moving from the material and inertial point A to the material point B, at rest relative to A and a distance d from it, with velocity v_{ES} according to Einstein simultaneity, and velocity v_{her} relative to B according to Apparent Simultaneity. Suppose further $t_A = 0$ when the body is at A, and that $t_B = 0$ according to Einstein simultaneity when the body is at A. Since a light signal sent from A to B at that moment will reach B at $t_B = d/c$, the body was at A relative to B and according to Apparent Simultaneity at $t_B = d/c$. According to Einstein simultaneity, B's clock should show $t_B = d/v_{ES}$ when the body reached B. According to apparent simultaneity, B's clock should show then $t_B = d/v_{her} + d/c$. We thus get:

$$d/v_{ES} = d/v_{her} + d/c$$
$$1/v_{ES} = 1/v_{her} + 1/c$$

Let us recall equation (1):

1 $$1/v_{her} = 1/v - (1 + \cos\theta)/c.$$

Where v is the velocity according to Apparent Simultaneity. We accordingly get:

5 $$1/v_{ES} = 1/v - \cos\theta/c$$

All we now have to do is substitute these results in the Einstein expressions for momentum and energy:

$$\mathbf{P} = m\boldsymbol{v}/(1 - v^2/c^2)^{1/2},$$
$$E = mc^2/(1 - v^2/c^2)^{1/2}.$$

v in these formulas being the velocity according to Einstein simultaneity, v_{ES}. After the substitution we get the results stated in Section 4, with v now designating Apparent velocity:

$$\mathbf{P} = m\boldsymbol{v}/(1 - 2v\cos\theta/c - v^2\sin^2\theta/c^2)^{1/2}$$
$$E = mc^2(1 - v\cos\theta/c)/(1 - 2v\cos\theta/c - v^2\sin^2\theta/c^2)^{1/2}$$

Philosophy Department
Central European University
Nádor u. 9
1051 Budapest, Hungary

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