The cultural challenge in mathematical cognition[1]

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In their recent paper on "Challenges in mathematical cognition", Alcock and colleagues (Alcock et al. [2016]. Challenges in mathematical cognition: A collaboratively derived research agenda. Journal of Numerical Cognition*, 2, 20–41) defined a research agenda through 26 specific research questions. An important dimension of mathematical cognition almost completely absent from their discussion is the cultural constitution of mathematical cognition. Spanning work from a broad range of disciplines— including anthropology, archaeology, cognitive science, history of science, linguistics, philosophy, and psychology—we argue that for any research agenda on mathematical cognition the cultural dimension is indispensable, and we propose a set of exemplary research questions related to it.*

Keywords: mathematical cognition, numerical cognition, culture, research agenda

1 Introduction

In a recent paper on "Challenges in mathematical cognition" (Alcock et al. [2016]. Challenges in mathematical cognition: A collaboratively-derived research agenda. *Journal of Numerical Cognition*, *2*, 20–41), 16 researchers from the fields of neuroscience, psychology, and mathematical education presented 26 specific research questions "designed to generate a coherent agenda for research on mathematical cognition" (p. 20), commented on in three additional contributions (Berch, 2016; Chinn, 2016; Lee, 2016). Questions were grouped into six broad topics, most of which revolve around competence development. While the authors state that their list of questions "reflects a broad approach to understanding human mathematical cognition" (Alcock et al., 2016, p. 33), they also note the possible limitation of the exercise due to their specific experiences and knowledge, and they explicitly express the hope that their paper will stimulate debate. We take this up and argue here that an important dimension of mathematical cognition is almost completely absent from their discussion, namely its cultural constitution, together with the perspectives of several disciplines that deal with culture, including "anthropological, sociological, linguistic, semiotic, historical, and political viewpoints" and "situated, contextual, or ethnomathematical considerations" (Berch,

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[1] Authors are listed in alphabetical order. This does by no means imply any order with regard to relevance. In fact, the manuscript is joint work, and all authors have contributed equally to its formation and current form.

2016, p. 43). While Alcock and colleagues (2016) envisage that future interdisciplinary communication will result in further refinements of their considerations, we propose a corrective, rather than a refinement, of their list of research areas through the inclusion of the cultural dimension of mathematical cognition, and a set of exemplary research questions related to it.

The question of whether to include cultural aspects is not simply a matter of diverging research interests or a trade-off between comprehensiveness on the one hand and focus on the other. Rather than conceiving of cognition as taking place in the head of individuals and as being separate from the wider human context—and so rendering context seemingly optional for consideration—the two need to be understood as being intrinsically intertwined (Saxe & de Kirby, 2014). Culture is an integral aspect of mathematical cognition, a *conditio sine qua non* (Núñez, 2009, 2017). If the field aims at making substantial progress, its research agenda cannot afford to ignore the basic fact that, without culture, there is no mathematical cognition.

This is not to say that every single aspect of mathematical cognition is culturally mediated. It is generally accepted that humans share with other species two phylogenetically ancestral cognitive systems that may serve as preconditions for the development of numbers and hence for mathematical cognition in general: one for parallel individuation of small quantities in the subitizing range (i.e., for numbers smaller than 4) and one for magnitude approximation (e.g., Feigenson, Dehaene, & Spelke, 2004). However, as soon as we turn to specifically human competences in dealing with numbers and other mathematical concepts such as chance, logic, or graph theory, then individual cognitive processes are intrinsically tied to cultural practices (Núñez, 2017; for examples see Ascher, 1991, 2002; Crump, 1990; Saxe, Guberman, & Gearhart, 1987), and hence also changing over time, which is why mathematics is so different now than it was in the past (Dantzig, 1954).

Higher cognitive functions involving number presuppose cognitive tools; one fundamental instance is a conventionalized counting sequence for the exact assessment of quantities. Such tools emerged in human communities to serve specific purposes, they are socially transmitted, and have been developed over cultural evolutionary time in a continuous and iterative process of reproduction and alteration, involving microgenetic, ontogenetic, and sociogenetic changes (for a case study and a theoretical framework, see Saxe, 2012). During this process, these cognitive tools both were modified by adapting them to new mathematical problems, and they helped to modify the conceptual understanding of the underlying ideas. A strong influence of culture on mathematical cognition is also attested to by the extensive cultural diversity exhibited in which numerical tools are developed and used, how they are valued, taught, and culturally transmitted, and for which practical purposes they are regarded as relevant.

In this paper, we review lines of research that, while taking widely different starting points, all converge on the essential role of culture for mathematical thinking. The review includes perspectives on conventional systems of number representations, on the contribution of individual cognitive processes for the reproduction and alteration of these systems, and on their interaction with mathematical cognition. We then identify challenges to the field arising from a neglect of the cultural dimension before formulating a set of questions for future research. As we largely focus on tools for representing and dealing with natural numbers, we use the term *numerical cognition* instead of *mathematical cognition* unless more general questions are at stake.

2 Culture, the missing dimension

The most fundamental cultural tool for numerical cognition are numeration systems, which come in different modalities: in a verbal modality as number words, in an embodied modality as finger counting or other body-based representations, in a written modality as notational systems, and in other external modalities such as tally sticks, quipus, and abaci. Each system has structural properties that affect its learning and use. For instance, the system of number words in the verbal modality does not provide a durable and manipulable external representation due to the ephemeral nature of vocal utterances, whereas numeration systems in other modalities are more or less durable and open up possibilities for external interactions. Of the numerous ways in which numerical cognition is saturated with culture, we focus on three aspects. We begin by reviewing the origin and variability of numeration systems. Then, we address properties of such systems and their interplay with numerical thinking. Finally, we consider processes of enculturation and the cultural context of numerical cognition.

2.1 Origin and variability of numeration systems

The early archaeological evidence for numerical cognition is limited and therefore interpretations are hotly debated. The first unambiguous numbers—numerical notations in Mesopotamia and Egypt—do not appear until about 5,000 years ago. Earlier devices possibly used to accumulate, represent, and store numerical information include marine shell beads (Blombos Cave, South Africa, about 75,000 years old), notched bones (Border Cave, South Africa, about 42,000 years old), and hand stencils (Cosquer Cave, France, about 27,000 years old). Determining whether such devices represented numerical information has been challenging, because they might serve social purposes other than dealing with numbers (e.g., beads can be personal ornaments, notches decorations, and handprints part of rituals), and many of the more specific claims about their possible functions (e.g., as lunar calendars) have been discounted as unproven (d'Errico, 1989, 1995; Marshack, 1972). Still, such devices provide suggestive evidence that behaviourally modern *Homo sapiens* either possessed concepts like *more*, *next in the sequence* and (one-to-one) *correspondence*, or was at least engaged in behavioural patterns with the potential to generate such concepts (Overmann, 2016c). Moreover, since emerging numeration systems (which often involve finger-counting or devices made of perishable materials) tend to leave little or no material trace, the archaeological record likely underestimates the time depth for numerical emergence (Overmann, 2017).

Two open questions are related to the role of language and communication: First, what was the interactional context for early symbol formation for number? Did notches or other semiotic forms emerge primarily in non-social settings in which individuals were trying to solve a problem for themselves, or in social settings in which individuals were trying to communicate an intended meaning, or both? A focus on primacy of communication would be consistent with Vygotskian and neo-Vygotskian perspectives (Vygotsky, 1986; Sfard, 2008) as well as with perspectives that focus on a bootstrapping of semiotic activities with numerical operations in communicative acts (Saxe, 2012; but also: Werner & Kaplan, 1963). Second, does the emergence of material forms that involve number presuppose a language for numbers or vice versa (cf. Overmann, 2016b)? While it is hard to imagine that sequences of notches on tallies beyond the limits of subitizing would have been used in the absence of number words, or used efficiently without modifications like grouping, devices such as the modern rosary hint at the possibility that some numerical problems could be solved without numerical language: By virtue of encoding numerical content, organization, and structure, material representations like rosaries may have supported an emerging understanding of

number concepts (Overmann, 2016b, 2016c), which then may have been further moulded and elaborated by linguistic and cultural changes over time.

Processes whereby new systems of numeration emerge in contemporary groups can provide some insights on origins. For instance, when studying an urban community of unschooled candy sellers, Saxe (1991) found that child candy sellers in Brazil who were largely non-literate developed new forms of representation. They made use of the colours of bills rather than the digits printed on the notes (which they could not read) to both identify values of bills (with number words) and to engage in arithmetic computations with large bill values. Similarly, Oksapmin people in Papua New Guinea traditionally use a 27-body-part counting system in which they generate one-to-one correspondences between conventional positions on the body and countable objects. But the organization of the system is being altered as people participate in two new kinds of collective practices (Saxe, 2012). In one case, the Oksapmin's formerly 27-part system became a 20-part system as people used it in collective practices of economic exchange with a Western currency system (counting Australian shillings to the 20th position which then signals an Australian pound). In another case, the Oksapmin's unary 27-body part system became constituted as a 10-base-structured system as Oksapmin teachers made efforts to bridge indigenous numerical knowledge forms to Western numeration in collective practices of classroom life, which supports a base-10 written numeration system. Studies like these indicate that the origins of numeration systems are unlikely to be the product of an "immaculate conception". Rather, they emerge from prior cognitive and material resources as individuals engage with emergent problems, structuring those resources to serve new functions.

Given that numerical abilities and practices may date back ten-thousands of years, it will be hardly surprising that present-day numeration systems differ tremendously across languages on a range of dimensions (detailed in the next subsection). While this diversity is a hallmark of cultural forces and hence significant in its own right, it also inspires more profound questions on the nature of numerical cognition. Importantly, it puts into perspective the observation that there are also striking commonalities among numeration systems, as attested to by a well-described set of regularities in the world's languages (Greenberg, 1978; and see Zhou & Bowern, 2015). A parallel but non-overlapping set of regularities has also been documented for written numerical notations (Chrisomalis, 2010). For instance, both the verbal and the written modality make frequent use of ordering from highest to lowest powers of the base, but differ in their use of fundamental operations to structure number representations. Whereas it is common to express addition through iteration in numerical notations (e.g., 300 = CCC), no known language expresses 300 as *hundred hundred hundred*. The existence of such regularities indicates that variability is not limitless, but constrained by a number of cognitive factors deserving further investigation (Chrisomalis, 2013).

Taking into account the extent of cross-cultural diversity in numeration systems will not only help us to delineate those aspects that are universal from those that are variable; mustering this information to reconstruct the changes in numeration systems over time, both on a small scale (e.g., Saxe, 2012) and a large scale (e.g., Calude & Verkerk, 2016; Kirby et al., 2016), will also allow us to identify the driving forces behind these changes. For a more comprehensive understanding of numerical cognition, uncovering the cognitive and cultural affordances and constraints involved in the emergence of cultural variability is therefore an essential step.

2.2 Properties of numeration systems and their interplay with numerical thinking

As semiotic tools, numeration systems have properties that vary across languages and cultures (Bender & Beller, 2012; Chrisomalis, 2004; Widom & Schlimm, 2012; Zhang &

Norman, 1995). These properties include the extent of the system (defined by the largest possible number symbol), its structure and regularity, and the modality in which it is realised. These specific properties interact with how numbers are represented and processed, and may facilitate some numerical understandings and operations, but may also hinder others (e.g., Beller & Bender, 2011; Bender & Beller, 2017; Bender, Schlimm, & Beller, 2015; Chrisomalis, 2017; Nickerson, 1988; Schlimm & Neth, 2008).

One tremendously varying property is the extent of the system. Several languages contain number words that do not reach beyond the limit of subitizing, and few languages lack number words and numerical expressions altogether (Everett, 2005; Hammarström, 2010), preventing their speakers from verbalizing even those quantities directly perceivable without counting (Hurford, 1987) and affecting their numerical cognition (Dehaene, Izard, Spelke, & Pica, 2008; Everett & Madora, 2012; Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004). Other languages possess number words far beyond a million, crucially, developed even in the absence of numerical notations (e.g., Bender & Beller, 2006, 2014). Typically, the extent of numeration systems is expanded over time due to an increasing need to deal with higher numbers; if no such need emerges, words for higher numbers can also be dropped, resulting in reduced extent (Beller & Bender, 2008). Such changes in the extent of numeration systems point to the crucial role of the cultural context for nurturing numerical practices. Correlations between a system's extent and markers of socio-political complexity (Beller & Bender, 2008; Divale, 1999; Epps, Bowerin, Hansen, Hill, & Zentz, 2012) allude to some of the sociogenetic processes involved in numerical cognition (cp. Saxe, 2012).

An even more important property concerns the representation of numerical information, which may give rise to *representational effects* (Zhang & Norman, 1995; Zhang & Wang, 2005). For instance, when represented in a *cumulative* manner (as in "III"), magnitude (or ordinal) information on the value is explicit: that "III" refers to three entities and that it is more than "II" can be perceived directly. By contrast, when represented in a *symbolic* manner (as in "3"), the only information directly perceivable is category (or nominal) information on whether or not the number sign is equal to another one: that "3" is different from "2". Which value it refers to or whether it is smaller or larger than "2" is something that remains implicit in the sense that it needs to be retrieved from memory, thereby increasing cognitive load during number processing (Zhang & Norman, 1995). Notation systems differ in the extent to which they employ cumulative or symbolic representations (Chrisomalis, 2010): for example, the Indo-Arabic digits belong to those symbolic systems in which almost all numerical information is implicit, coded in the value and the place of a digit, whereas Roman numerals combine symbolic and cumulative representations.

However, while such differences in representational format may affect cognitive processing (Schlimm & Neth, 2008), it should be kept in mind that notational systems are not used for one purpose only, but are embedded in a rich set of cultural practices and may alternatively or exclusively be recruited for calculating a numerical value (as in arithmetic), for indicating a quantity (as in measures and prices), or for simply distinguishing entities (as in phone numbers or bus routes). The Roman numerals, for instance, were used as a representational system only and not as a computational system; for arithmetic, the Romans and later medieval Europeans used the abacus as the principal material support for computation (Taisbak, 1965), complemented by finger math (Williams & Williams, 1995).

A third property particularly relevant for children's acquisition of the numeration system is the regularity in composition. For instance, in various East Asian languages, number word construction mirrors the regular structure of written Indo-Arabic number notation, whereas word construction in Indo-European languages such as English, German, or

French comes with several irregularities (e.g., Bender et al., 2015; Brysbaert, Fias, & Noël, 1998; Calude & Verkerk, 2016; Miura, 1987). These irregularities include specific number words such as "eleven" and "twelve" (instead of regular constructions like "ten-and-one" and "ten-and-two"), the inversion of summands (as in number words from "thirteen" through "nineteen" in English, and those up to "neunundneunzig" [99] in German), or irregular multiples of the base (as in French "quatre-vingt-dix" [= $4 \times 20 + 10$]). Several studies suggest that such irregularities contribute to delays in the acquisition of the verbal system and in the comprehension of the place-value notational system, and impede the learning of algorithms for addition and subtraction that are based on the place-value notational system and take advantage of the decimal structure (Fuson & Kwon, 1991; Klein et al., 2013; Miller, Smith, Zhu, & Zhang, 1995; Miura, Okamoto, Kim, Steere, & Fayol, 1993).

Taking seriously the specific properties of numeration systems is therefore indispensable when investigating the processes involved in numerical cognition, and even more so when seeking ways to support and improve mathematical performance in education. But the system-specific properties are by no means the only way in which numerical cognition is intrinsically linked to culture; arguably even more relevant is the cultural context in which numerical cognition takes place.

2.3 Processes of enculturation and the cultural context of numerical cognition

Acquisition of the numeration systems themselves (such as the sequence of number words or a conventionalized finger counting pattern) as well as an understanding of their precise numerical meaning requires participation in enculturating practices of collective life, whether in counting games or other activities with older siblings or adults (and later on also in schooling). Such collective practices vary markedly within and across cultural groups. For example, in a study of working and middle class families and their 2.5- and 4-year-old children, Saxe, Guberman, and Gearhart (1987) identified a wide range of practices in which numerical problems emerged in the everyday lives of children. Some were invented by the children and their parents themselves, like jointly counting steps up to an apartment, pressing numbered elevator buttons, or setting a table, while others were rooted in store-bought materials, like reading of counting books or playing commercial games involving number. In these collective practices, four principal numerical functions emerged for number words: *nominal functions* (using number words without cardinal meanings), *cardinal/ordinal* functions (using number words to represent the sum of a count or ordinal positions in a count), *comparative/reproductive functions* (using number words as an intermediary to compare numerical values of two or more groups or to reproduce the number of a group through the creation of a second group), and *arithmetical functions* (composing/decomposing numerical values in arithmetical transformations). In the 2.5-year-old group, children were largely engaged with nominal and cardinal/ordinal functions, and in the 4-year-old group, the functions expanded to include comparative/reproductive and arithmetical functions. Importantly, children themselves played a role as agents in the kinds of numerical environments that emerged during social interactions, and mothers adjusted the numerical functions that were elaborated in relation to the actions of the child.

The range of number word functions is also attested to by other studies, some of which point to the progressive arithmetization of number words to serve cardinality functions in early development (e.g., Le Corre & Carey, 2007; Sarnecka & Carey, 2008; Wiese, 2003), while others focus on numerical comparison and reproduction functions (Saxe, 1977, 1979). The active and agentive role of both child and parent in enculturating practices is also observed in more challenging contexts. For example, deaf children to hearing parents invent their own home sign language, including gestures vaguely referring to quantities, but none

with precise cardinal meaning (Spaepen, Coppola, Flaherty, Spelke, & Goldin-Meadow, 2013; Spaepen, Coppola, Spelke, Carey, & Goldin-Meadow, 2011), indicating that appropriate cultural input is necessary to develop the respective numerical abilities. Likewise—and in contrast to a widely shared opinion that finger counting is a universal basis for learning numbers—congenitally blind children can develop an understanding of number words without support from finger-based representations (e.g., Crollen, Mahe, Collignon, & Seron, 2011; Crollen et al., 2014).

Learning to count and what to do with this ability also depends on how numbers are valued in a given community (Núñez, Cooperrider, & Wassmann, 2012; Saxe, 2012; Wassmann & Dasen, 1994) and on the properties of the cultural systems for number. For example, in traditional Oksapmin communities in 1978, young children knew many of the 27 body parts in their numeration system, but were engaged with numerical activities infrequently compared to the levels documented in working and middle class populations in the US (see Saxe, 2012). Correspondingly, age norms for children's performance on numerical comparison and numerical reproduction tasks among the Oksapmin differed markedly from those documented for Western children. Further, unlike their Western counterparts who acquire verbal numeration systems, Oksapmin children face a challenge linked to the spatial properties of their 27-body part numeration system, specifically from the fact that most body parts occur in symmetric pairs. When comparing the numerical value of symmetrical body parts, Oksapmin children therefore tended to regard them as indicating the same number (Saxe, 1981). Such challenges are not present for children in groups using only a verbal system.

In sum, we suggest that a careful study of the processes of enculturation and the cultural context in which numerical cognition takes place are important. This work will provide us with crucial insights on the active role that the child plays in the learning process itself, and in shaping the practices of collective life that guide them, rather than being a simple recipient of math education.

3 Challenges to the field: Missing research questions

Even though Alcock and colleagues (2016) point at the possibility of multiple pathways to mathematical success (p. 32), they tend to take as reference the skills, achievements, and learning trajectories of what they label "typical populations" (p. 33). Populations regarded as typical—that is, in most cases, members of Western, Educated, Industrialized, Rich, and Democratic (abbreviated as WEIRD) societies—are known to be outliers along various psychological dimensions (Henrich, Heine, & Norenzayan, 2010). The same is true for the children in these populations, raised in respective countries and influenced by the prevailing cultural patterns, including a specific way of schooling, from early on. Not even infants are truly unaffected by cultural input, as attested to by the fact that they recognize their mother tongue already at birth (Byers-Heinlein, Burns, & Werker, 2010; Moon, Cooper, & Fifer, 1993).

Taking such WEIRD populations as reference may seem justified if one's agenda is aimed at the education systems in North American and Western European countries, with "success in mathematics" being defined as mastery of the respective school curriculum. To the extent, however, that it is also intended as a research agenda on mathematical cognition more generally, neglecting the cultural nature of the phenomena investigated is problematic. If one's goal is "a broad approach to understanding human mathematical cognition" (Alcock et al., 2016, p. 33), then such highly specific populations do not provide an appropriate basis

from which to draw general conclusions, nor should they be the only target for one's efforts to facilitate mathematical achievements—and even more so as, globally, populations become increasingly heterogeneous in terms of their cultural and linguistic composition.

Taking culture more seriously entails important implications. Most generally, it helps us elucidate the nature of mathematical thinking (the first of the six broad topics identified by Alcock et al., 2016). It does this by not only providing new insights to fundamental questions about mathematical thinking, but also by alerting us to such questions in the first place. The most fundamental questions would revolve around the forms of representation that have emerged in different cultures and the functions they serve in collective practices: How do form and function affect each other, and how do they become reproduced and altered in the activities of children and adults (Saxe, 2012)? More specific questions arise from the three aspects we focused on in this commentary, pertaining to (i) the origin and variability of numeration systems, (ii) the properties of such systems and their interplay with numerical thinking, and (iii) processes of enculturation and the cultural context of numerical cognition.

3.1 Origin and variability of numeration systems

What are the regulative processes that constrain and enable the emergence, reproduction, and alteration of numeration systems in human communities, and are these processes similar across different communities? For example, in his studies of economic exchange in Oksapmin communities, Saxe (2012) proposed that, as interlocutors communicate with one another about number-related issues in collective practices like economic exchange, they strive to make their communicative intents clear by using word forms that they assume are known by their audience, while at the same time adapting their communications to newly emerging communicative problems. In this way, participants in a community reproduce prior representational forms, but also alter them as they adjust their communications to the problems of daily life. In striving for both numerical coherence (that what is said in communication is rooted in mathematical sense) and communicative coherence (that what is asserted should be tailored so that the speaker's communicative intent should be clear to the hearer), such a regulative process could well be a fundamental source of both continuity and discontinuity in representations in communities.

If these regulations are fundamental to the reproduction and alteration of numerical forms, how might they be manifest across situations when authority and power are similar or different across interlocutors, whether in adult-child interactions, peer interactions, or interactions between adult members of similar social positions? There is such a diversity of community-specific collective practices that the possibilities for detailed analysis are extraordinary. Examples include deaf children of hearing parents communicating with one another about number and new forms of representation that emerge as communication (e.g., Spaepen et al., 2011, 2013) and groups moving from pidgin languages to creoles in which numerical representations might become regularized (see Hammarström, 2008, for some analyses on this issue).

3.2 Properties of numeration systems and their interplay with numerical thinking

Language appears to have a profound effect on the thinking of individuals (e.g., Boroditsky & Gaby, 2010; Dolscheid, Shayan, Majid, & Casasanto, 2013), with number being a case in point. But more work is needed to understand the interplay between the representational forms that individuals come to use and the functions for which they use them.

A particularly illuminative example is the bootstrapping process in which individuals deploy problems that involve multiple conceptual resources supporting their transcendence of particular cultural forms of representation. For instance, more than half of the world's

population speaks more than one language, with numeration systems in these languages most likely differing from each other in at least subtle, if not fundamental ways. Moreover, even if monolingual, most people nowadays still use several numeration systems in parallel, such as the sequence of number words in one's mother tongue and one or more notational systems (e.g., Indo-Arabic digits and Roman numerals). While serving partly different functions, these systems differ significantly in how they represent the same type of information. What then is the interplay between the different forms of numerical representation and the thinking of (developing) individuals? Do different ways of representing numbers interfere with one another during learning and when manipulating quantity, and do they interact with computational technologies such as counting-boards in the performance of arithmetical tasks? Which role do irregularities in numeration systems play? What were the impacts of straightening out the irregular systems in several French-speaking countries (except France), in Wales, and in Norway? And would the same be advisable for other Indo-European languages as well?

A related question concerns the treatment of external tools. When solving numerical problems, the work is distributed over the material tools with which one engages and cognitive (including cortical) processes recruited for numerical activities. When solving an arithmetic problem, in which ways then do these processes differ depending on whether one deploys a representational artefact like a digital calculator, paper and pencil, or mental arithmetic? Such variation in the way in which number is used in everyday life are commonplace, but understanding how cognition becomes distributed over the artefacts that we use and the cognitive processes involved with number (Hutchins, 1995; Larkin & Simon, 1987; Zhang & Wang, 2005) poses difficult empirical challenges for the cognitive sciences and neurosciences. This is made even more challenging by the substantial restructuring of the brain through literacy (van Atteveldt & Ansari, 2014; Dehaene, 2013; Menary, 2015; Overmann, 2016a), which implies that patterns observed in Western participants cannot simply be generalized to non-literate (yet numeric) people.

3.3 Processes of enculturation and the cultural context of numerical cognition

What is the nature of and the variation in enculturating practices across communities? Saxe and colleagues (1987) documented a variety of collective practices in which young children were engaged in working and middle class families in New York City (as explained in the previous section), including the forms and functions of representation that emerged in the context of interactions and differential numerical cognitive developments associated with participation in those environments. But more such work is in dire need. What are similarities and differences that mark the emergent environments with regard to enculturating practices involving number across human communities? In what way do children personalize the conventions of their community such that the respective cultural forms become the child's own forms and come to serve quantitative functions? In what way does participation in collective practices involving enculturation lead to the socialization of children's personal, idiosyncratic approaches, leading to new ways of thinking and communicating with others? We do know that young children in the Western world are engaged in play with digital technologies like tablets, cell phones, and computers, but what are the characters of the environments that emerge in such play with computational media? How do these environments become interwoven with the number development of children as children develop fluency with digital technologies involving numerical problems with and without adult and/or peer support? Importantly, outside of such WEIRD communities, what are the collective practices that children in other parts of the world are engaged in (cp. Lancy, 2014)? What are the conventional systems of numeration that they are acquiring, and how is their understanding interwoven with the numerical ideas and representations that they form?

Answers to questions like these will allow us to modify and improve our models of numerical cognition such as the triple-code model described by Dehaene (1992). To date, model building has been unnecessarily constrained by nearly exclusively relying on research with literate participants familiar with a decimal place-value system based on the Indo-Arabic digits, and with individuals, rather than communities, as the units that are engaged in recurring numerical problems. Broadening the samples would open the way for incorporating arithmetic abilities and strategies of people using different notational systems or no notational system at all. After all, the numerical cognition developed and used by humans, for most of its existence, was *not* based on a decimal place-value notational system (which, in historic terms, is a very recent invention).

And finally, conceiving of cultural diversity in numeration systems and other ethnomathematical patterns (for examples see Ascher, 1991, 2002; Ascher & Ascher, 1981; Crump, 1990; D'Ambrósio, 1985; Powell & Frankenstein, 1997) as information rather than noise and as equivalent rather than inferior also enriches our understanding in a more general sense by uncovering the range of possibilities in human cognition (Levinson, 2012; Levinson & Gray, 2012). Ultimately, this enables us to address questions on the malleability of the cognitive architecture underlying mathematical cognition more generally, questions on which factors (if any) constrain the extent of diversity in mathematical cognition, or whether the latter is simply a product of arbitrary processes. Respective answers will then help delineate what the human mind is able to learn and process, and how using numeration systems shapes human cognition and culture more generally.

4 Conclusion

Culture is not only "out there", in some exotic corners of the world, but everywhere around us and inherent in the material and conceptual systems we use and the practices in which they are embedded. We *all* are brought up in a structured environment, a social context, and a set of cultural practices and routines in which we acquire cultural tools, learn to use them in culturally agreed upon ways, and modify or alter them. Coming in closer contact with people having a different upbringing and speaking another language increases the exposure to, and hence awareness of, diversity in numeration systems (both verbal and notational) and number-related practices. While this situation may pose additional challenges for instance in mathematical education, it also opens new windows and provides opportunities for developing a deeper comprehension of mathematical concepts by comparing the different cultural instantiations. The same strategy would benefit the scientific approaches to the topic. Only if we recognize it as the quintessentially cultural phenomenon it is, and only if this insight is reflected in research agendas, will we be able to make substantial progress in understanding numerical cognition.

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