What Chance Doesn't Know*

Harjit Bhogal and Michael Townsen Hicks

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Abstract

Humean accounts of chance have a problem with *undermining futures*: they have to accept that some series of events are physically possible and have a nonzero chance but are inconsistent with the chances being what they are. This contradicts basic platitudes about chances (such as those given by Bigelow et al. (1993) and Schaffer (2007)) and leads to inconsistency between plausible constraints on credences. We show how Humeans can avoid these contradictions by drawing on *metaphysically impossible* worlds that are, nevertheless, *scientifically possible*. One major advantage of our approach is that one single move deals with these both problems, whereas previous Humean approaches to undermining (such as that given by Lewis (1994), Thau (1994), and Hall (1994)) have only addressed the connection between credence and chance. Furthermore, our approach connects more closely with the way we employ stochastic scientific theories. And it's part unified solution to other challenges that the Humean faces.

The *Humean* takes the world to be fundamentally non-modal. How then, do they deal with a cluster of concepts that seem to be suffused with modality — causation, counterfactuals, laws of nature, objective chance and so on? The standard Humean approach is to reduce facts about these concepts to the arrangement of local, occurrent facts: the *Humean Mosaic*. Consequently, such facts metaphysically supervene upon the mosaic.

However, this reduction of modal facts to the mosaic leads to a series of difficulties for the Humean. Here, our focus will be on chance, and in particular, a problem that Lewis called the 'Big Bad Bug' for Humeanism — the problem of *undermining*.

The standard Humean approach to chance starts with an account of the laws of nature: the Best System account (BSA) (see Lewis; 1983, p. 42-3). It says, very roughly, that the laws are the propositions that simply and informatively summarize the non-modal facts.

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Slightly more precisely, consider various sets of axioms concerning the Humean mosaic and the systems produced by taking the logical closure of the axioms. The BSA says that these systems can have two virtues. Firstly, a system is better if it is simpler — that is, it has fewer and simpler axioms. Secondly, a system is better if it is more informative about the Humean mosaic. The best system is the one which best balances these virtues. The axioms of the best system are the laws of nature. This description glosses over lots of details and choice points, but it's sufficient for now.

As we will discuss later, the standard Humean approach to chance adapts the BSA so that it outputs chances as well as laws. Just as with laws, we postulate chances to simply and informatively summarize the non-modal facts.

The problem of undermining arises because the chances depend upon the Humean mosaic, specifically on the frequencies of events. So if the frequencies were different the chances would be different. Imagine, for example, that there is actually a 50% chance of a coin landing heads. If the mosaic were such that the coin was tossed exactly 10 million times and it landed heads every time, then saying that the coin has a 50% chance of landing heads would not be an informative summary of the mosaic. Rather, a better, more informative, summary would say that the coin has a chance of 1 of landing heads. Consequently, if the coin were to land heads in all 10 million tosses then the chances would be different. Such an outcome, where if it were to come about the chances would be different, is called an *undermining outcome* or *undermining future*.[1]

However, the actual chances assign a positive chance to the coin landing heads 10 million times in a row — it's extremely unlikely but not impossible. So the Humean has to accept, it seems, that chance assigns a positive probability to situations where the chances are different. And this, as we will discuss, runs into problems with some some intuitive principles, some call them platitudes, concerning chance — the Basic Chance Principle (BCP) and the Principal Principle (PP).

Our aim in this paper is to describe a solution to these undermining problems. The solution is based upon a move that is becoming more popular in the Humean literature as a response to other problems that Humeans face — accepting that there are certain metaphysically impossible worlds that are relevant to scientific practice, and to the analysis of scientific modalities like laws and counterfactuals. In particular, we are going to claim that chance does not, in fact, assign a positive chance to situations where the chances are different. Rather, it assigns a

[1]Our example here is a bit too quick. For there may be other events — die rolls, other sorts of coins, roulette wheels — which, in addition to our coin, are part of the mosaic. A chance theory which fits these non-coin events but departs from the actual frequencies of coin tosses might, on balance, still fit the world well enough to be the laws. When discussing this case, then, we will assume that there are no such similar events at our world.

positive chance to metaphysically impossible situations where the chances are the same. This, we argue, allows us to deal with the undermining problems better than previous Humean approaches, notably, the approach developed by Lewis (1994), Thau (1994), and Hall (1994).

But we need to start with some preliminaries. So next we will introduce and motivate the Humean appeal to impossible worlds. And after, we will describe the Humean approach to chance and how it leads to tension with the BCP and the PP.

1 Impossible Worlds

To introduce the Humean appeal to impossible worlds it will be useful to start by looking at a couple of standard objections to Humeanism.

Perhaps the most common is the *non-supervenience* objection — there appear to be counterexamples to the claim that the modal facts metaphysically supervene upon the mosaic.

For example, it's a consequence of Newtonian physics that, if there were just one particle in a world without forces, it would move in a straight line forever. But Newtonian physics isn't the only set of laws that leads to this motion; for example, the particle's trajectory could be chancy. These seem like distinct possibilities: one in which the particle moves inertially because Newton's laws hold, and nothing exerts a force on it, and another in which the particle moves in a straight line as a result of some chancy laws. If there are these two possibilities with the same Humean mosaic (one particle moving in a straight line) but different laws (Newton's laws or a stochastic physics) then Humeanism seems to be false (see, for example, Tooley (1977, p. 669), Carroll (1994, pp. 57-67) and (Maudlin; 2007, pp. 67-68) for other such cases.)

A different, but related, objection to Humeanism says that Humeanism gets the wrong results about certain counterfactuals, in particular, *nested counterfactuals*.

Imagine that we live in a Newtonian world. Then consider the nested counterfactual 'if there was only one particle in the world, then if there was another particle they would be attracted in accordance with the laws of gravitation'. This counterfactual seems true. If there was only one particle we would be in a one-particle Newtonian world. And in such a world then if there were two particles they would be attracted in accordance with the Newtonian laws.

The problem is that the Humean seems to say that the counterfactual is false. The reason is that, if we were in a one particle world then the BSA would not output the laws of Newtonian mechanics — the Newtonian laws wouldn't provide the best balance of simplicity and informativeness with respect to this mosaic. A simpler system which just states that everything

moves in a straight line would be just as informative about that mosaic. So, a one-particle Newtonian world is metaphysically impossible.

Consequently, if there were only one particle, we would be in a world where the laws are much simpler than those of Newtonian mechanics, and those laws would not imply that if there were another particle they would be attracted in accordance with the laws of gravitation. (Lange (2009, p. 54), Sklar (2014, pp. 79-80) and Hall (2012, pp. 32-33) develop versions of this objection.)

Perhaps the most common response to the non-supervenience problem is to simply deny the relevant intuitions — just denying the intuition that there could be a one-particle world governed by Newton's laws. But this response requires us to provide an error theory for their use in scientific modeling — an error theory which isn't well developed or compelling. When scientists investigate Newtonian mechanics they seriously consider what would happen in possibilities like the one-particle Newtonian world, and similarly sparse worlds. The Humean should make sense of this rather than merely claiming that such worlds are metaphysically impossible (Bhogal; 2021).

The most common response to the nested counterfactual problem is to tweak the semantics of counterfactuals (see Hall (2012)). But such tweaks often look ad hoc, adding unmotivated context-dependent parameters (Carroll (2018)) or sacrificing compositional semantics (Hall; 2012). And they don't touch on the related non-supervenience problem.

The impossible worlds solution seems more promising to us. To see how it works notice that the non-supervenience problem and the nested counterfactual problem are closely related. Both arise from the fact that there are certain worlds, like the one-particle Newtonian world, that are intuitively possible and are taken seriously in scientific practice, but that the Humean claims are metaphysically impossible.

In fact, there are a variety of other problems that arise in the same way. As Bhogal (2021, section 3.4) notes, we can generate almost identical problems by, instead of nesting counterfactuals, nesting other scientific modalities. Just as the Humean misevaluates certain nested counterfactuals they misevaluate certain counterfactuals about explanation, counterfactuals about causation, claims about the physical possibility of counterfactuals, and so on.

For example, imagine again that we live in a Newtonian world. Then consider this counterfactual about explanation: if there were only one particle in the world then the explanation for why that particle moves at a constant velocity would be Newton's first law. This counterfactual is true, but the Humean doesn't seem to get this result. Again, the reason is that, for them, there isn't a one-particle world where Newton's first law is one of the laws, so it can't be the explanation of the constant velocity.

Similarly, because physical possibilities are standardly taken to be a subset of the metaphysical impossibilities, the standard Humean view implies that it is physically impossible for there to be a one particle world where the constant velocity of the particle is explained by Newton's first laws. That's a claim about the physical possibility of an explanation that the Humean misevaluates.

All these problems arise because the Humean has too few worlds to work with when they analyze various scientific modalities. The anti-Humean accepts that there is a one particle Newtonian world, and so can appeal to this world when analyzing counterfactuals and other physical modalities. The Humean, though, says that such a world is not metaphysically possible.

And of course, the problem is not merely with the one particle Newtonian world. Anti-Humeans, such as Tim Maudlin (2007, p.67), often make claims like 'every model of a set of laws is a possible way for a world governed by those laws to be'. Take a possible set of laws L. L has a series of models. Maudlin's thought is that for each model of L, there is a possible world where L are the laws and where that model holds. And this is true for every possible set of laws, not just L. So, for example, given that Newtonian mechanics is a possible set of laws and that a single particle moving intertially is a model of Newtonian mechanics then there is a possible single-particle Newtonian world. He claims that this is a natural consequence of scientific practice — that scientific practice takes seriously every model of every possible set of laws as a possible world.[2] But many of these worlds will be metaphysically impossible, according to the Humean.

Again, Humeans encounter problems because they have too few metaphysically possible worlds to work with. So a natural solution is just to accept that some worlds can be important for the practice of science even when they are metaphysically impossible. Accepting, for example, that a one-particle Newtonian world is important for science, and relevant to the analysis of scientific modalities, like laws and counterfactuals, even though it is metaphysically impossible. This can be understood as an acceptance that certain worlds are *scientifically possible* even though they are metaphysically impossible. More generally, the Humean can accept that the all the worlds described by Maudlin are scientifically possible, even though many are metaphysically impossible.

This move is gaining popularity in the Humean literature. It was gestured at in Bhogal and Perry (2017, footnote 12) and developed further, though in rather different ways, in Bhogal

[2] A slight hedge is in order here. According to one view of symmetries, symmetry-related models represent the same possibility. If this view of symmetries is correct, there is a distinct world for each equivalence class of symmetry-related models, rather than a world for each model (see e.g. Wallace and Greaves (2014), Wallace (2019)).

(2021) and Hicks (forthcoming). And Humean moves in this spirit go back to at least Ward (2007, 2002) and Halpin (2003) who have 'projectivist' conceptions of Humean laws.

The authors cited above differ on the exact nature of these scientifically impossible worlds and how the space of such worlds is generated. But the core idea is that there are scientific possibilities that outrun the space of metaphysical possibilities for the Humean. A one-particle Newtonian world is scientifically possible even though the best system account, as applied to the one particle world, would not generate the laws of Newtonian mechanics.

1.1 Scientifically Possible but Metaphysically Impossible Worlds

We think that this appeal to impossible worlds, and a space of scientific possibility that outruns metaphysical possibility, helps the Humean solve some key problems with their account. But, of course, such a view of scientific possibility will be unfamiliar to many readers. So it would be useful to take a bit more time introducing it.

However, the aim of this paper is not to give a thoroughgoing defense of why it's legitimate for the Humean to appeal to impossible worlds like this. Rather, the aim is to show how this kind of appeal to metaphysically impossible worlds can solve the undermining problem. Of course, this itself provides a motivation for such an appeal to impossible worlds — if appealing to such worlds really does solve the undermining problem then that is a powerful reason for the Humean to take this approach. But again, it is the application and utility of this view for theories of chance that is our focus.

Consequently, the discussion of the rest of this section should be understood as clarifying and introducing this approach to scientific possibility, and specifically the appeal to metaphysically impossible worlds, rather than arguing that anyone is forced to take this approach.

1.1.1 Which Worlds are Scientifically Possible?

Here's a natural first question about the notion of scientific possibility that we are using: Which worlds are scientifically possible? This is a good question, and it's important to note that there can be dispute over this, just as there is dispute over which worlds are metaphysically possible, without that casting doubt on the notion of scientific possibility itself. But, as we saw in the last section, for the Humean to usefully appeal to such scientifically possible worlds as part of a response to the non-supervenience and nested counterfactual problems it needs to be the case that worlds like the one-particle Newtonian world are scientifically possible.

As a generalization of this, Maudlin's claim that there must be a distinct possibility for each model of each possible set of scientific laws gives us a way to characterize the space of scientific possibility. The natural move for the Humean defender of this impossible worlds strategy is to agree that all the worlds Maudlin describes are taken seriously in scientific practice and so are scientifically possible. But not all of them are metaphysically possible — the single-particle Newtonian world, for example, is not metaphysically possible. For our purposes, going forward, we can understand the space of scientifically possible worlds in this way.

Of course, this does not fully characterize the space of scientifically possible worlds — what are the possible scientific laws? — but it provides enough for our purposes.

1.1.2 Further applications of Impossible Worlds

But a more substantial cause for concern might be a resistance to the appeal to impossible worlds, or even to the very idea of impossible worlds. In response, we can start by noting the usefulness of appealing to impossible worlds in the modern literature in metaphysics and the philosophy of science.

Metaphysical impossibilities have recently been fruitfully applied to a number of metaphysical problems, including counterpossible counterfactuals (Nolan (1997), Berto and Jago (2013), Berto et al. (2018), Jago (2013), Hicks (forthcoming)), accounts of mental content (Priest (2016), Jago (2013)) metaphysical or grounding explanations (Wilson (2016), Schaffer (2016)), hyperintensional explanations (Wilhelm (2020)), and reasoning concerning different logical systems (Priest (2016)).

Apart from motivations for appealing to impossibilities which come from metaphysics more generally, Humeans and philosophers of science have plenty of reasons to posit a space of scientific possibility distinct from the space of metaphysical possibility.

(1) Scientific models frequently appeal to simplifying assumptions which are metaphysically impossible. For example, models of fluids like water, which idealize them as continuous (Tan (2019)), or of animal populations which allow for fractional rabbits (McLoone (forthcoming)) appeal to metaphysical impossibilities (since water is metaphysically necessarily a molecular composite of hydrogen and oxygen, and living rabbit populations are necessarily numbered by integers).

(2) One of the goals of scientific modelling is to represent subsystems of the world — the sort of bounded situations with which we interact. It's telling that many of the possibilities that lead to nested modality problems are situations which are not metaphysically problematic when embedded in a larger world: a single stationary particle can, compatibly with

Humeanism, be subject to Newton's laws if it is embedded in a larger world rich in more complicated behavior, and any sequence of coin flips — even 10 million heads — can be part of a larger sequence whose frequency approaches 0.5. Metaphysical possibilities model the way modal notions of lawhood or chance depend on the world as a whole, but scientific possibilities can abstract away from these holistic connections so we can better understand the world piece by piece.

(3) There is reason to think that the nature of explanation in science diverges from the nature of explanation in metaphysics. It's common for Humeans to claim that there is a distinction between these two types of explanation as part of a solution to the problem of *explanatory circularity* (for an overview of the problem, see Lange (2013, 2018), for responses distinguishing types of explanation see Loewer (2012) Hicks and van Elswyk (2015), Marshall (2015), Miller (2015), Dorst (2019), Bhogal (2021)). But if there is a link between explanation and modality then this distinction between two types of explanation generates a corresponding distinction between scientific and metaphysical modality — see (Bhogal; 2021).

So, we think, impossible worlds have a wide application across metaphysics and the philosophy of science.

1.1.3 Against the Very Idea of Impossible Worlds

But still, many will resist an appeal to worlds that are scientifically possible but metaphysically impossible. The thought is that there just aren't such worlds. They don't exist! They are impossible!

This is a reasonable reaction. And certainly we won't be able to make everyone comfortable with the idea of impossible worlds in this space. Again, the aim of this paper is to use the notion of scientific possibility to solve issues about chance. But there are a few things we can say about the background picture we are working with.

Firstly, one might be concerned that worlds like the single-particle Newtonian world are simply contradictory or incoherent since the details of the BSA contradicts the world being Newtonian and only consisting of a single particle moving at constant velocity. But on the approach that we will develop this is not the case — at such single-particle Newtonian worlds the BSA (and Humeanism) doesn't hold — that's what makes them metaphysically impossible. This will be very important when we discuss the problem of undermining.

Secondly, there is a view of metaphysical possibility, prominent in the 1980s, according to which metaphysical possibilities are closely connected to meanings. Worlds are impossible if

they are incompatible with the (externalist) meanings of words. On this view it's metaphysically impossible for water to be XYZ because the *meaning* of water is H_2O. This view is another reason someone might be concerned that the metaphysically impossible worlds that we appeal to are inconsistent or incoherent in some way. In particular, they might worry that worlds at which Humeanism is false are inconsistent with the meanings of terms — what 'law of nature' means rules out anti-Humean worlds. We, on the other hand, do not think these worlds are inconsistent with the meaning of 'chance', because we think that the Humean theory of chance is a theory of metaphysical explanation rather than a theory of meaning. So we think instead that these worlds are impossible because they violate principles of metaphysical explanation, rather than being inconsistent given the meanings of the words.

Thirdly, one might be less concerned about impossible worlds given certain views of the metaphysics of possible worlds. If possible worlds are ersatz — say, they are sets of propositions or other abstracta — then it's natural to think that there are metaphysically impossible worlds. There can be sets of propositions that correspond to metaphysical impossibilities. This is not to say that an appeal to impossible worlds requires such a view of the metaphysics of worlds but just that views of worlds as less concrete go naturally with an acceptance of impossible worlds.

Fourthly and finally, one might be concerned with the idea of impossible worlds if you hold the view that metaphysical possibility is 'absolute' or 'the most unrestricted grade of possibility'. This would imply that there just can't be scientific possibilities that are metaphysically impossible. If you are fully committed to such claims about metaphysical possibility then this will be a reason to reject our usage of scientific possibility as well as the other appeals to impossible worlds in modern metaphysics that we mentioned above.

It's a commitment of our approach, then, that metaphysical possibility is not 'absolute' or 'the most unrestricted' in this way. In fact this commitment is of a piece with an important strand of thinking in modern metaphysics which decentralizes the role of metaphysical possibility, instead claiming that the core metaphysical questions are about things like metaphysical explanation, or essence, or fundamentality and that issues about modality are less basic. Now is not the time to get into the details of such 'post modal' approaches, except to say that theoretical benefits reaped from an appeal to impossible worlds and a corresponding rejection of the idea that metaphysical possibility is 'the most unrestricted' would be a reason to favor such approaches.

Hopefully this discussion has clarified the appeals to impossible worlds that are central to this paper. But again, the focus of our paper is not on motivating this view about impossible worlds and scientific possibility but, rather, in showing how it solves the undermining

problem for the Humean.

But why is this appeal to impossible worlds important for undermining? Well, we will get to that in full detail later. For now, notice two things. Firstly, that the undermining problem is, just like the nested counterfactual problem, a problem that arises from nesting two physical modalities. Where the nested counterfactual problem involves nesting two counterfactuals, the undermining problem involves the Humean making a claim about the chances of chances — that is, that there is a non-zero chance that the chances are different[3].

Secondly, the undermining problem arises because the Humean has too few worlds to work with. The problem comes from the fact that that there is a world which seems possible, and is, in a sense, taken seriously by scientific practice — specifically, the world where the coin is fair, but it still lands heads 10 million times in a row — but that the Humean says is metaphysically impossible — at any metaphysically possible world where the coin lands heads 10 million times in a row, its chance of heads is much higher than 50%. Nonetheless, this world would count as scientifically possible since it's a model of a possible set of laws. As we noted earlier, the non-supervenience problem, and the nested counterfactual problem arise in the same way.

Given the similarities then, we think that the undermining problem should be dealt with in the same way as the nested counterfactual and non-supervenience problem, by appealing to metaphysically impossible worlds.

But before we get to that we need to first describe the Humean approach to chance and the undermining problems in more detail.

2 Fit and Objective Chance

The standard Humean account of chance is an extension of Humean account of laws. According to the BSA, Humean laws are simple, informative generalizations: they tell us as much about the mosaic as possible in as simple a way as possible. Humean chances come in when the generalizations that best balance simplicity and informativeness about the mosaic are probabilistic.

Let's consider, again, a world where coins are tossed 10 million times in history and not much else happens. If the coin lands heads every time then a generalization that says 'coins always land heads' is both simple and informative about the mosaic. So, plausibly, it would be a law

[3]Or, in an alternative formulation which we will also discuss, the claim is that it is physical possible that the chances are different.

for the Humean. Imagine, an alternate situation in which the 10 million coin tosses seem like a random sequence of coin tosses — HHHTTHHTTTHTTTHT… — where, overall, about 5 million land heads and 5 million land tails. In this case trying to give information about how every coin toss lands would be massively complex. The best way to be informative in a simple way would be a generalization like 'coins have a 50% chance of landing heads'. And so, this generalization will plausibly be a law — the best system has introduced chances for coin tosses.

The way this idea is implemented into the BSA is by adding an additional virtue of systems, *fit*, in addition to simplicity and informativeness. Roughly, speaking, a system fits the mosaic better than another if it assigns that mosaic a higher chance of coming about. The best system is the one that best balances these three virtues[4]. Deterministic laws will automatically maximize fit; but if no deterministic system can adequately balance simplicity and informativeness, chancy laws may be called for.

Notice that fit guarantees that the chances can't diverge too far from the frequencies. Consider again the world where coins are tossed 10 million times and land heads every time. And consider a system S which implies that the chance of a coin landing heads is 0.5 and each toss is independent. It's incredibly unlikely that the coins will land heads every time, given the chance is 0.5. Consequently, S would score very badly on fit, and would not be the best system of the world. A better system would be one where the chance of heads is closer to 1. In fact, whenever the chances outputted by a system diverge widely from frequencies (and there are a sufficiently large number of trials) the fit of that system will be extremely low. The BSA, therefore, generates metaphysical limits on how far the chances and frequencies can diverge.

3 The Basic Problem

Unfortunately, the metaphysically necessary connection between chances and frequencies creates serious problems. If the chance of a coin landing heads is 0.5 and each toss is independent then there is a non-zero chance that the coins will land heads in each of the 10 million times they are tossed. But, if all the coins did land heads, then the chance that a coin would land heads would not be 0.5. It would be (close to) 1. Although it seems possible for chance to be 0.5 but every toss to land heads, the Humean says this is metaphysically impossible.

This is a strange situation. It's strange that chance assigns a positive chance to a situation — coins landing heads every time — that is inconsistent with it being the chance. And it's

[4] Though see Elga (2004) for a discussion of the complications that arise for infinite worlds, and a solution to them.

strange that the Humean says that it is impossible for the chance of each toss to be 0.5, for the tosses to be independent, and for every toss to land heads. But it's often pointed out in the literature on undermining (following Lewis (1994, p.483)) that this strangeness isn't, on its own, a substantial objection to the Humean view of chance.

The standard way to generate an objection from this strangeness is to show how this strangeness leads to tensions with norms connecting credence and chance. We will consider this in section 4. But, in the rest of this section we will note how such undermining cases are inconsistent with other platitudes about chance, notably the Basic Chance Principle and the Realization Principle.

The Basic Chance Principle (Bigelow et al.; 1993) provides a link between chance and modality. It says the following:

> BCP: If a proposition P has a chance of greater than zero at time t, then there must be a possible world w where (i) P is true, (ii) w matches the history of the actual world up to t, and (iii) the chance of P at t in w is the same as the chance of P at t in the actual world.

The BCP is meant to capture the intuitive idea that when a event is chancy then it could, so to speak, go both ways. It's possible, for example, for the radioactive atom to decay in the next minute, and it's possible that it does not. If it were not possible that the radioactive atom decays in the next minute, given the chances and history, then it seems like it can't have a positive chance.[5]

The Realization Principle (Schaffer; 2007) is a slight strengthening of the BCP. It replaces (iii) with (iii'):

> REALIZATION: If a proposition P has a chance of greater than zero at time t, then there must be a possible world w where (i) P is true, (ii) w matches the history of the actual world up to t, and (iii') the laws of w are the same as the laws of the actual world.

These are very compelling principles — Schaffer calls them platitudes about chance; Bigelow et al. say that the BCP is, in part, constitutive of our notion of chance. The problem, as Bigelow et al. point out, is that the BCP and the RP seem to be inconsistent with Humeanism.

[5]Cases where there appear to be non-trivial chances in deterministic worlds might seem to give us a reason to reject the BCP. But there are natural variants of the BCP that are consistent with deterministic chance. (Glynn; 2010, Section 4.3)

Consider again the situation where the chance of the coin landing heads is 0.5 but it is tossed 10 million times and lands heads every time. The chance of this is positive. But, given Humeanism, it's metaphysically impossible that the laws, or the chances for each coin toss, are the same and the coin lands heads 10 million times in a row. This impossibility looks to be a violation of the BCP and the RP. Call this apparent inconsistency between Humeanism and the BCP/RP the *Basic Problem*.

Regardless of specific formulations of the BCP and RP, the intuitive force of the problem is that, on the Humean account, the chance function seems to allow for the possibility of a situation where the fair coin lands heads exactly 10 million times in a row, since it assigns the situation a positive probability. But nevertheless, the facts about chance are metaphysically inconsistent with that situation occurring.[6]

Making sense of this is a problem for the Humean. And notably this problem does not involve the link between chance and credence — it's a purely metaphysical, rather than epistemic, issue.

We will discuss how to deal with this problem soon. But before that let's look at how the strangeness of undermining causes a problem when combined with credence-chance principles.

4 Big Bad Bug

Chancy laws are useful because they provide information about outcomes. But how should we process that information? Lewis (1980) argued that we should match our credences to the chances: if the chance of a coin coming up heads is 0.5, and you know that, then you should have a credence of 0.5 that the coin will come up heads. In general:

PRINCIPAL PRINCIPLE (PP): $b(A|E\&T_{Ch}(*)) = Ch(A)$,

where $b(*|*)$ is any rational initial credence function, $T_{Ch}(*)$ is the proposition that the correct chance theory is $Ch(*)$, and E is any admissible information. (The nature and role of this admissibility condition is highly controversial — see, for example, Meacham (2010) for a discussion – but this won't be important in the rest of the paper.)

[6] In fact, one way to understand the argument of Belot (2016), though he doesn't put it this way, is that this tension forces the Humean to accept that the probability of the fair coin landing heads 10 million times in a row is zero. And that implies that the coin tosses, which we would previously have taken to be probabilistically independent, are not so.

Now, let F be the proposition that all of the coins come up heads in a world where coins are tossed 10 million times, and suppose that here in the actual world the chance of heads is 0.5. F is an undermining future: a future such that, if it happened, the chances would be different. Then according to the actual chances, the chance of F is some small but positive number, $y > 0$. Hence, according to the Principal Principle,

$$b(F|T_{Ch}(*)) = Ch(F) = y > 0.$$

But F is metaphysically incompatible with T_{Ch}; because (by stipulation) if all the coins came up heads, the chance of heads would be 1, rather than 0.5, and so the chance of an all heads sequence would be 1 rather than y. This means that the truth of the chance theory implies that F won't happen. But if we accept that implication, we should set our credences as follows:

$$b(F|T_{Ch}) = 0.$$

But these are incompatible constraints on our initial credences: one tells us to have a nonzero credence in some event, conditional on the chance theory; the other tells us to have a zero credence in the same event, conditional on the same theory. We can't do both (see (Bigelow et al.; 1993) (Lewis; 1994, p.483)).

The Big Bad Bug already has a couple of well-known solutions. The first of these, which has attracted more attention, appeared in a series of papers by Lewis (1994), Thau (1994), and Hall (1994), but it is one that leaves many (including us) a bit unsatisfied. The second has been defended by Jenann Ismael in a recent series of papers (Ismael (2008, 2015)). We will discuss both of these, starting with Ismael's proposal:

Ismael (2008) suggests that we form our credences using the following General Recipe:

General Recipe (GR): $b(A) = \sum b(T_{Ch})Ch(A)$

The general recipe allows chances to assign nonzero probability to situations in which they are not the chances. But Ismael (2015) denies that this will yield an internal contradiction, as we will never be in a position where we have rational beliefs about what the true chance theory is. According to (Ismael; 2008, p. 299), we should "allow that the chance-making pattern at any point in time lies partly in the future, and observe that we are precisely as ignorant of the present chances as we are of the remainder of the chance-making pattern (specifically, of which undermining futures won't occur)".

We worry that the claim that we will never be in a position where we rationally believe the true chance theory is a little strong. If we are not inductive skeptics then it seems that there

will be *some* cases, even if non-actual, where have rational beliefs about the chances. Our ignorance of the future should be tempered by our expectation that our observations of the past give us good information about what will happen. We worry that Ismael's solution closes off this legitimate epistemic approach to uncertainty about the future, namely, taking some epistemic risk with the hope of having true beliefs (for a discussion of the epistemic advantage of risky behavoir, see Pettigrew (2016)). That said, Ismael's reasons for rejecting the proposal of Lewis, Thau, and Hall align in some interesting ways with our own (we will discuss these commonalities in a future footnote[7]).

Next to the response of Lewis, Thau, and Hall, which we will hereafter call 'LTH'. This proposal has two parts: first, it proposes a modified chance-credence linking principle. Second, it places constraints on the chance function to dissolve the contradiction. Most discussion of this solution has focused on the new principle (fittingly named the New Principle). But we think that the constraints LTH put on the chance function deserve more attention.

The first leg of the LTH proposal is the New Principle:

New Principle (NP): $b(A|E\&T_{Ch}) = Ch(A|E\&T_{Ch})$, if defined.[8]

where $b(*|*)$ and $Ch(*|*)$ are once again a rational initial credence function and the chance function, and A is once again any proposition, and T_{Ch} is the proposition that the correct chance theory is $Ch(*)$: it's a proposition that states the fact that Ch is the one true chance function. But now E is any evidence whatever (no admissibility required).

The New Principle embodies the way we defer to a certain sort of experts: *analyst experts* (see Hall (2004) and Elga (2007)). These experts are like medical hotlines. They ask you about your local situation ('what does the rash look like?') and then tell you what attitude to have given your evidence ('unless it changes color, don't worry about it')[9]. We first feed these experts our evidence, and then defer to their beliefs conditional on our evidence. Deference to these experts is given by the New Principle: rather than adopting their unconditional credence, we should adopt their credence conditional on our own total evidence[10]. Lewis and Hall take the chances to be primarily this sort of expert. We provide to the chances a lot

[7] footnote 13.

[8] The *if defined* clause is not present in many formulations of the NP. The reason we have it here is because we don't want to assume that the chance function is defined over exactly the same set of propositions as the rational credence function — perhaps we have rational credence in propositions about which there is no objective chance.

[9] This way of describing chances on the New Principle is partially inspired by Dorst (2023)'s illustration of Humean laws

[10] Note that, to be an analyst rather than a database an expert does not, as Briggs (2009) suggests, need to be *completely* ignorant of the situation she is evaluating, or have *no* more evidence than the person appealing to her. Contra Briggs: just as the phone help line might have plenty of information about your computer or car than you do not, the chances — as an analyst — might have some evidence you do not. What's important, instead, is that you have some relevant evidence that the analyst lacks. Any additional evidence that the analyst has allows them to better evaluate the additional evidence you supply them with.

of information about the current physical state of whatever system we want to know more about, and the chance function spits out likelihoods for various ways the system could develop. This is clearest in quantum mechanics: we use the theory to predict the outcomes of various sorts of quantum events ('will this atom decay in the next few hours'), and we supply as inputs to the theory as much information about the quantum system as possible (its current wave function, or a simplified model of the potential function that governs it). The chance function then tells us the probability of decay, conditional on our evidence. Because we think that this procedure matches the ways in which we use the chances, we agree with LTH that the New Principle is the way to go.

To avoid placing contradictory constraints on our credences, LTH need more than just the NP. They also need a claim about conditional chances. If the chance of F conditional on all our evidence — including T_{Ch} — is zero then the NP implies we should have zero credence in F and there are no contradictory constraints on our credences. The LTH idea, then, is that the chance of F conditional on the chance theory is zero.

The LTH proposal has two key parts then:

LTH 1 The correct chance-credence principle is the New Principle.

LTH 2 The chance of an undermining future (like the 10 million heads world), conditional on the chance theory, is zero: $Ch(F|T_{Ch}) = 0$.

The first claim tells us to defer, not to the unconditional chances, but to the chances conditional on our total evidence — which includes the fact that these are the chances [11]. The second tells us that the conditional chance of an undermining future is zero. Taken together, this implies that when we defer to chance appropriately we should have a zero credence in F, given our total evidence. And this fits with the way that F is incompatible with the facts about the chances.

This certainly does avoid the contradiction, but we think that there are reasons to be unhappy with this approach. In particular, we are happy with the first claim. It is the second claim that we disagree with[12].

[11] This isn't quite right. We may be interested in our conditional credence in F given some chance theory even when we don't know which chance theory is correct. In this case, the fact that T_{Ch} is the correct chance theory isn't part of our evidence. But when we consider this conditional credence, we are considering — roughly — what credence we would have in F if we were to learn $T_C h$ with certainty, and nothing else. So when we consider our credence conditional on $T_C h$, we are justified in including $T_C h$ amongst the evidence we feed into the expert function. Thanks to an anonymous reviewer for discussion of this point.

[12] We retain the New Principle because, as we've argued earlier, we think that it reflects the way we employ chances. But we note that the appeal to mere scientific possibilities gives Humeans new avenues for retaining the Principal Principle, if they so choose.

The idea of the second claim is fairly clear. There is no metaphysically possible world where T_{Ch} and F holds — if the coin landed heads every time, then the Humean account implies that the chances would be different. Consequently, LTH say that $Ch(F|T_{Ch}) = 0$. Notice that the thought is that chance 'knows' the metaphysical facts. When we tell chance that T_{Ch} then it knows that F is ruled out, because, presumably, it encodes the fact that Humeanism is true, and so it assigns zero to F. But this is what we think is wrong. We don't think that chance knows the metaphysical facts in this way.[13]

Notice that this is closely related to our discussion of the Basic Problem. Chance assigns a positive chance to F, even though there is no metaphysically possible world where those are the chances and F holds. This seems to violate the BCP/RP, until we recognize that the relevant world can be scientifically possible, even though it's metaphysically impossible. Again, what's going on here is that chance seems to be assigning values in a way that is insensitive to the metaphysical facts. We will come back to this later.

But why do we have this position? What are the reasons for rejecting LTH 2? Here are three.

First, LTH 2 requires the chance function to be defined over propositions about what the chances are. But the chances we find in the sciences don't seem to be like this at all. (Joyce; 2007, p. 202) makes this point clearly:

> [W]e also believe that quantum mechanics spits out the correct probabilities given the information encoded in quantum states, and so there is an 'analyst' element in our deference as well. However, we do not think of these probabilities as universal analyst-experts in the way NP requires. There are many items of potential information (e.g. the statement that quantum mechanics is false) about which quantum mechanics cannot speak.

[13](Ismael; 2015, p. 198) suggests an different reason for rejecting LTH 2. She suggests chance is ignorant, not of the metaphysics, but of a *that's all* fact: the chance might assign any nonzero credence to a long sequence of heads, but not assign the same probability to the same sequence as a *total* history of the world. "Let H be a total history for w and let's call the proposition that H is a total history H^{total}. There is no entailment from H to H^{total}, and that is sufficient to show that a frequentist or reductionist of Lewisian stripe will never be forced by her historical knowledge to assign a probability to a proposition of the form $[T_ch]$". On Ismael's view, the chances don't need to assign any probability at all to H^{total}, and merely assigning a probability H does not generate the contradiction with Humeanism. This is because Humeanism is compatible with a mismatch between the chances and frequencies for a sequence of any length, provided that sequence doesn't encompass the whole world. Ismael motivates this ignorance of the total sequence by the fact that the chances, and agents using them, exist within time: "So long as, for any time t, she [the ideal agent] assigns a positive probability to the world continuing indefinitely after t she will not by her own lights be in a position to form a second-order belief about which ur-chance function is correct."

Our approach agrees with Ismael that the way to block the Bug comes from the evidence available to the chance function. Ismael argues that the chances lack holistic information about the Universe; we hold that the chance theory lacks metaphysical information about the nature of chance. We discuss the differences between our approaches in §6.

Joyce's worry, which is echoed in Ismael (2015), is that the chances we find in science give us chances of outcomes of certain processes; they don't seem to much care what would make them the chances, nor are they 'modest' in the sense of Pettigrew and Titelbaum (2014): they don't explicitly—or in any reasonable sense implicitly—spit out the probability that they are the one true chance function. They are simply not defined over propositions of the form T_{Ch}. Nor are they likely to be, unless physicists start taking a keen interest in philosophy.

But this isn't the only worry Joyce alludes to. For the LTH proposal doesn't just require the chances to care about, or be defined over, propositions about what the chances are. LTH 2 also requires the chance theory to assign quite strange chances conditional on those propositions. This provides the basis for our second reason for rejecting LTH 2. For the LTH solution to work, the chance of some events like F, conditional on the chance of that very event being very low, must be different from the unconditional chance of F. This means that the chance function must violate what Bigelow et al. (1993) call the 'Chance Conditional on Chance' principle:

> Chance conditional on Chance (CCC): $Ch(F|Ch(F) = x) = x$, if defined.[14]

If the Humean solves the Big Bad Bug by arguing that the chance of some event conditional on the truth of the chance theory is different from that event's unconditional chance, then the chance theory must violate CCC. But theories that violate CCC are (as Bigelow et al. (1993) point out) quite peculiar. And, in line with Joyce's criticism, it's very strange to think of such a theory coming out of even an idealized physics. Why would physicists create a theory with this weird and specific feature (and how would they define it in the language of quantum mechanics)?

Finally, the LTH proposal requires the chance theory to deliver different predictions for isolated subsystems of the universe than it does for the universe as a whole. This is because any particular sequence can, according to the standard Humean picture, be embedded in a much larger Humean world, compatible with the chances being what they are — take any (finite) sequence of coins landing heads, and embed it within a world with many more tails, and you have an arbitrary finite heads sequence a world with a 50% chance of heads. But take that sequence to be the whole world, and metaphysical incompatibility results. This means that the LTH chance function must take the chances of outcomes for a physical system to depend not just on the state of that system, but also whether the system is the whole world or just a

[14] This 'if defined' clause is not present in Bigelow, Collins, and Pargetter's original formulation. It is included here to allow for chances which are not defined over propositions about the chance function.

part of it. This creates more headaches for physicists. It requires physical chances to depend, not just on the features of the system they're describing, but also on whether this system is the whole universe or a subsystem. That's a really strange thing to be incorporated into a physical theory.

Physical theories typically care about whether a system is isolated — that is, whether there are any external causal influences on a system. But they typically do not care whether isolation results from the system being distant or shielded from its environment rather than not having an environment.

This looks like a small departure from standard physics, but it has major implications. It might undermine the practice of physics, since physicists use isolated systems to gain information about larger systems, including the universe as a whole. It seems constitutive of inductive reasoning that we take the theory which applies to the universe as a whole to apply in the same way to the smaller-scale systems we use to test and gain evidence for that theory; on the LTH proposal, though, we need the theory to treat these sorts of systems very differently.

5 Solving the Problems

So we have two undermining problems — that undermining leads to inconsistency with the BCP/RP — we called this the *Basic Problem* and that it leads to inconsistency with the PP — we called this the *Big Bad Bug*. These two problems are closely related. And, we argue, they are also closely related to the *nested modality* problems that we discussed in section 1. Remember, that the nested modality problems were cases where the Humean seems to misevaluate propositions that nest two physical modalities — things like nested counterfactual propositions, counterfactuals about explanation, counterfactuals about causation, claims about the physical possibility of counterfactuals and so on.

The Basic Problem is similarly to do with nesting modalities. In this case, nesting possibility with chance and the nesting of chance with chance. It seems like it is possible for both F to be true — that is for the coin to land heads every time out of the 10 million times it is tossed — and there be a 0.5 chance of coins landing heads. After all, there is a nonzero probability of the coin landing heads every time. But the Humean seems to say that this is not possible.

While the Basic Problem arises due to inconsistencies between two modalities (chance and possibility), the Big Bad Bug arises due to inconsistencies in our beliefs about these modalities. The Basic Problem is due to the fact that H has a positive chance but is not possible given the facts about the laws and chances. The Big Bad Bug is due to the fact it seems like

we should have a positive credence in H, because it has a positive chance, but it also seems like we should have a zero credence, because it's not possible given the laws and chances.

As we discussed in section 1, we think that the solution to nested modality problems involves expanding the space of worlds that are relevant to the analysis of physical modalities — accepting a space of scientifically possible worlds, some of which are metaphysically impossible. This motivates a solution to the Basic problem, which, in turn, motivates a novel solution to the Big Bad Bug. Those solutions will be the topic of the next two sections.

5.1 Solving the Basic Problem

Let's consider how to solve the Basic problem — that is, how the Humean can have an account of chance that is consistent with the BCP and RP in light of undermining.

As a reminder, the RP says that if a proposition P has a chance greater than zero at time t, then there must be a possible world w where (i) P is true, (ii) w matches the history of the actual world up to t, and (iii) the laws of w are the same as the laws of the actual world.

According to Humeanism, there is no metaphysical possibility at which a coin's chance of heads is 0.5 and it comes up heads in all of the 10 million tosses. Most discussions infer from this that there is no possibility—full stop—at which the coin has a 0.5 chance of heads and comes up heads 10 million times. But if, as we discussed in section 1 the scientific possibilities are not a subset of the metaphysical possibilities—if some scientifically possible things are metaphysically impossible— that is not a valid inference. In section 1 we discussed some motivations for this position, and we will review these in our conclusion. In this section we will see how this position helps solve the Basic Problem, and the Big Bad Bug. (Though, again, if the view does solve the undermining problem that is powerful motivation for accepting it.)

We can, then, accept that there is a world at which a coin's chance of heads is 0.5 and the coin lands heads in all of the 10 million times it is tossed. Such a world is not *metaphysically possible*. But, we can accept that it is *scientifically possible* since there is a possible set of laws which implies that the chance of the coin toss is 0.5, and the situation in which the coin lands heads in all of the 10 million times it is tossed is a model of those laws. This idea was pointed to by Bhogal (2021, section 3.2).

Consequently, we can accept the RP (and the BCP), at least if those principles are understood correctly. That is, we can accept that since there is a nonzero chance of the coin landing heads in all of the 10 million times it is tossed then there must be a possible world w where that happens, which matches the history of the actual world, and where the laws of w are

the same as the laws of the actual world. On our view such a world is possible — it's just scientifically, not metaphysically possible.

More generally, given what we said about the space of scientifically possible worlds in section 1 — for any possible set of laws L and any model of L there is a scientifically possible world where L are the laws and where that model holds — we can accept that if a proposition P has a chance of greater than zero at time t, then there must be a *scientifically* possible world w where (i) P is true, (ii) w matches the history of the actual world up to t, and (iii) the laws of w are the same as the laws of the actual world. This is just the original formulation of the RP with the clarification that the relevant type of possibility is scientific possibility. So, we think, this appeal to impossible worlds solves the Basic Problem by expanding the range of worlds that can be appealed to. How does it help with the Big Bad Bug? That will take a little longer to explain.

5.2 Undermining and The New Principle

The Big Bad Bug, recall, is the epistemic shadow of the metaphysical Basic Problem. It arises because some possibilities have a nonzero chance of occurring — and so, it seems, we should have a nonzero credence in them — but are metaphysically incompatible with the chance theory being what it is — so, it seems, we should have a zero credence in them conditional on the chance theory. According to LTH, this puts strict requirements on the chance theory.

As a reminder, here are LTH's two key claims:

LTH 1 The correct chance-credence principle is the New Principle.

LTH 2 The chance of an undermining future (like the 10 million heads world), conditional on a chance-making proposition, is zero:
$Ch(F|T_{Ch}) = 0$.

We accept LTH 1, that is, the New Principle. As we have argued, we think LTH 2 is wrong. Our solution involves replacing LTH 2 with BH 2.

BH 1 The correct chance-credence principle is the New Principle.

BH 2 The chance of an undermining future (like the 10 million heads world) conditional on the truth of Humeanism is zero: $Ch(F|Humeanism) = 0$.

The undermining future F, is inconsistent with the conjunction of Humeanism and T_{Ch}. The coin landing heads all 10 million times is inconsistent with the conjunction of Humeanism and it being a fair coin.

LTH say, in effect, that chance doesn't know T_{Ch}, and that's why it assigns a positive value to F. But chance does know the truth of Humeanism, so once we condition chance on T_{Ch} it outputs zero for F.

Our claim is that, in effect, chance doesn't know Humeanism, and that's why it assigns a positive value to F. But chance does know the truth of T_{Ch} — it doesn't lack confidence in it being the chance function — so once we condition chance on Humeanism it outputs zero for F. That's what BH 2 says.

To be a little more precise, what we mean when we say that chance knows the truth of T_{Ch} is not that $Ch(T_{Ch}) = 1$ because, as we discussed, perhaps chance is undefined over such propositions. Rather it's that conditioning chance on facts about the chances that doesn't change anything. Chances conditional on chances remain the same (when defined); chances conditional on Humeanism change. Remember that the LTH view denied this.

Let's go into more detail about our solution. The New Principle tells us to defer to the chance of a proposition conditional on our total evidence. Among our evidence is the truth of Humeanism. (We are not, of course, asserting that there is conclusive evidence for Humeanism. But the Big Bad Bug is about how belief in Humeanism leads to inconsistent requirements on agents' credences. So in trying to resolve this problem it's dialectically appropriate to assume agents (rationally) believe Humeanism.) So we should condition chance on Humeanism.

But what does it mean to condition chance on Humeanism? For simplicity we will present our solution as if the chances have opinions about things like what the correct chance theory is, or whether Humeanism is true, but we do not think these assumptions are strictly necessary, and we will indicate why not at the end of the section.

Humeanism implies a series of metaphysically necessary conditionals like this:

$$Ch(Heads) = 0.5 \supset \neg F$$

That is, Humeanism implies that if coins are fair they won't land heads in all of the world's 10 million tosses. So what happens when we condition chance on these conditionals?

What, in particular, is the value of $Ch(F|[Ch(Heads) = 0.5 \supset \neg F])$? Again, we take it that chance 'knows' the facts about chance, in the sense that conditioning on the facts about chance doesn't change anything. This means that

$Ch(F|[Ch(Heads) = 0.5 \supset \neg F])$
$=Ch(F|[Ch(Heads) = 0.5 \supset \neg F] \ \& \ [Ch(Heads) = 0.5])$

But $Ch(Heads) = 0.5 \supset \neg F$ and $Ch(Heads) = 0.5$ together deductively imply $\neg F$. So

$Ch(F|[Ch(Heads) = 0.5 \supset \neg F] \ \& \ [Ch(Heads) = 0.5])$
$=Ch(F|\neg F) = 0.$
Hence, $Ch(F|[Ch(Heads) = 0.5 \supset \neg F]) = 0.$

And that is the result we want. Our total evidence includes the truth of Humeanism. When we condition chance on our total evidence, and specifically, on these conditionals that follow from Humeanism, chance assigns zero to the fair coin landing heads all 10 million times. So, the NP tells us we should assign zero credence. This fits with the way that F is incompatible with the facts about the chances — there are no inconsistent constraints on our credence.

This view solves the three problems we raised for the LTH proposal. In reverse order:

First, it doesn't constrain the chance function posited by physics. Rather than requiring the function to know the difference between subsystems and the universe as a whole, it allows our physical theory to treat all systems the same way. This solution replaces these specific requirements with the anodyne requirement that the chances are logically consistent (something we can expect from physicists).

Second, it allows that chances obey the Chance Conditional on Chance principle. If the chances are defined over propositions about the chances, or other self-referential propositions, then our solution is compatible with the chance of those events, conditional on the chance theory, matching their unconditional chance.

Finally, it doesn't require the chances to be defined over propositions like T_{Ch}, or propositions about chance or its nature at all. For simplicity, we presented the solution assuming that the chances have opinions conditional on T_{Ch}. But we can kick away that assumption if necessary. We — the humans that defer to the chances via the New Principle — have opinions about the truth of the chance theory and the nature of chance. Conditional on the true chance theory, and the true metaphysical theory of chance (Humeanism), we can infer that undermining futures like F won't occur. We can feed this information to the chances without going through an intermediary involving the chances' own opinion about themselves. If our evidence includes the truth of Humeanism and the theory of chances, then it includes $\neg F$, and the chances are defined over F and $\neg F$. We require merely that $Ch(F|\neg F) = 0$. To avoid the bug, on our view, the chances need only to have zero credence in contradictions.

6 Why Favor our View?

Our view, then, is that the New Principle is correct. It's correct that we know something that chance does not, and so that when we defer to chances we must 'give' chance the extra information we have, so to speak, before we listen to what it says. This, of course, is also what LTH say. The difference is what chance doesn't know. LTH say that chance doesn't know that it is the true chance theory. We disagree. On our view is that chance doesn't know some fact about the metaphysics, specifically, it doesn't know the truth of Humeanism.

But why favor our view? Well, we just noted how our view avoids three problems that the LTH view faces. But there are other reasons too — many of which have come up in the course of the paper but it is important to reemphasize them here.

(1) Our solution to the Big Bad Bug flows naturally from our solution to the Basic Problem. These problems are intuitively very closely connected, as has been noted by, for example, Bigelow et al. (1993) and Belot (2016). But, the LTH approach to the Big Bad Bug is completely unconnected from the Basic Problem. Our solution recognizes the connection between these problems and delivers a unified resolution of both. To solve the basic problem, we appeal to scientifically possible, but metaphysically impossible, worlds. For example, the undermining world, in which a fair coin lands heads in all of 10 million tosses, is metaphysically impossible, but not ruled out by the laws of physics. When the Humean accepts such a world as scientifically possible, they can accept the BCP (when understood as a principle concerning scientific possibility).

This solution to the Basic Problem means that there is a scientifically possible, but metaphysically impossible, world that is assigned a positive chance — the world in which a fair coin lands heads in all of 10 million tosses. Accepting such a world motivates the two key parts of our solution to the Big Bag Bug. Firstly, it motivates our rejection of LTH's assumption 2 — given that such a world is assigned positive chance then the chance that the coin will land heads every time, conditional on it being a fair coin, is not zero.

And secondly, accepting such a world motivates the idea that when we defer to chance we should first 'give' chance the information about the truth of Humeanism — conditioning chance on Humeanism as we described in section 5. This is because if we know that Humeanism is true then we know that then our evidence rules our the world in which a fair coin lands heads in all of 10 million tosses. But, given that this world is assigned a positive chance, then we know something that chance does not — that this world cannot occur.

(2) Furthermore, our solution to the Basic Problem and the Big Bad Bug are part of a unified solution to nested modality problems. As we noted in section 5, undermining problems arise

for exactly the same reasons as other nested modality problems. The Humean misevaluates many claims that nest two scientific modalities — nested counterfactuals, for example. The problems arise because the Humean has too few worlds to work with when they analyze various scientific modalities. They deny that there is a single particle Newtonian world, for example.

But undermining similarly involves nested modalities — in particular, claims that nest possibility and chance. And the Humean seems to get things wrong because they have two few worlds to work with — there's no world where coins are fair but land heads in all of the 10 million times they are tossed.

Our view deals with these problems together, by accepting certain scientifically possible but metaphysically impossible worlds and so expanding the space of worlds that are relevant to analyzing scientific modalities.

The Ismael and LTH approaches don't recognize the deep connection between undermining and these other problems. So, some Humeans give completely unrelated solutions to the other nested modality problems. Hall (2012, pp. 32-33), for example, deals with the nested counterfactual problem by tweaking the semantics for counterfactuals in a way which has nothing to do with his solution to the Big Bad Bug; Ismael Ismael (2017, 2023) sees nested modality problems as a reason to abandon the letter, if not the spirit, of Humean Supervenience. But perhaps the most common Humean reaction is to just brush off the other nested modality problems, suggesting they are question-begging, perhaps, or just denying the relevant intuitions. But we think this is unsatisfying. Perhaps the Humean has to do this if they have nothing better, but it would be preferable for the Humean to develop a view of scientific modality that can capture these intuitions in a clean and unified way. We think it's substantial advantage of our approach to undermining that it fits into this larger view of modality; our approach deals with these closely related scientific modalities in closely related ways.

(3) Finally, and perhaps most importantly, we think that chance not knowing the facts about metaphysics flows naturally from the Humean account of chance, and the broader Humean picture of physical modality. The chances, for the Humean, are deeply rooted in, and generated by, the actual world. The chances are part of a simple, elegant summary of matters of fact. The theory of chance is a theory whose sole theoretical goal is to efficiently summarize the frequencies of the world. It is not meant as a total theory of the world, and so it has no reason to know about the correct metaphysical theory of modality. Chances need not concern themselves with the correct solution to the special composition problem, the metaphysical possibility of gunk, or whether possible worlds themselves are real or ersatz. These metaphys-

ical questions are entirely outside of the purview of chance, on the Humean conception. So too, we think, is the truth of Humeanism. LTH's assumption that chance must assign 1 to metaphysical necessities strikes us as puzzling and in tension with the aims of Humeanism.

7 Conclusion

The Big Bad Bug and the Basic Problem are serious challenges to Humeanism. If adequate responses can't be developed then the whole metaphysical picture of Humeanism seems untenable. Our view is that the challenges can be met, by appealing to worlds that are scientifically, but not metaphysically, possible. This solution avoids problems with Lewis, Thau, and Hall's solution to the Big Bad Bug. What's more it's part of a unified solution to other challenges that the Humean faces.

Maybe the biggest source of resistance to the view we have developed will be the one mentioned right at the start — a discomfort with the appeal to a space of scientific possibility that outruns the metaphysically possible worlds.

As we noted in section 1, the aim of this paper is not to mount a full defence of that move. But now we have seen the whole story about the Big Bad Bug there are a couple more relevant points we can make.

Firstly, our view is that chances don't know the truth of Humeanism. The Humean conception of chance as summarizing the patterns of non-modal facts doesn't suggest that chance should carry information about the metaphysical truths. And we can say similar things about Humean laws — it's not part of their role to be informative about the metaphysical necessities. Consequently, we shouldn't expect metaphysical truths — like the truth of Humeanism — to follow from the content of the laws.

In addition, there is a very natural conception of the space of possibility relevant to scientific practice — the physical possibilities are the situations that are consistent with the actual laws and chances; the scientific possibilities more generally are situations consistent with any possible set of laws and chances.

But if the truth of Humeanism is not something that is built into the content of Humean laws and chances then the scientific possibilities (and the physical possibilities) can contain situations which are inconsistent with Humeanism. For example, the situation where the chance of coins landing heads is 0.5 but where all the 10 million tosses land heads is consistent with the laws and chances (since the laws and chances don't build in the truth of Humeanism), so it's natural to say it's a scientific possibility. But, it's not consistent with the truth of Humeanism (see Bhogal (2021, section 2.3) for related thoughts).

So, the Humean picture of chance motivates an understanding of what chance knows and what it doesn't know — in particular, it doesn't know the metaphysical truths. And this understanding in fact motivates the appeal to a space of scientific possibility that outruns the space of metaphysical possibility.

Secondly, as we noted in section 1.1.2, many scientifically possible but metaphysically impossible worlds are useful to appeal to in our reasoning about the actual world. (Though this usefulness is not a requirement for a world counting as scientifically possible — worlds don't have to qualify as scientifically possible one by one. Rather, as discussed in section 1.1.2 we take every model of a set of possible laws as a scientifically possible way for a world governed by those laws to be.) The distinctive scientifically possible but metaphysically impossible worlds that we appeal to in this paper are cases like the chance of coins landing heads is 0.5 but where all the 10 million tosses land heads. Appealing to such worlds is useful, and, in fact, vital, in our reasoning about coin tosses. If coin tosses are independent, as our standard theory of coin tosses implies, then there is a sense in which we are forced to countenance such worlds, and not rule them out as impossible. This is because the content of the standard theory of coin tosses includes the fact that each toss is probabilistically independent of every other toss. Since each toss can land heads or tails, and (according to the theory) no toss changes the chance of any other toss, the theory is unable to rule these worlds out. Consequently, the content of the chance theory can't rule out worlds where every coin lands heads, or where every coin lands tails.

We think, then, that this appeal to metaphysical impossibilities should be central to the Humean's analysis of physical modalities. As we have shown in this paper doing so yields an elegant solution to the Humean's problems with chance.

References

Belot, G. (2016). Undermined, *Australasian Journal of Philosophy* **94**(4): 781–791.

Berto, F. and Jago, M. (2013). *Impossible Worlds*, Oxford: Oxford University Press.

Berto, F., Ripley, D., Priest, G. and French, R. (2018). Williamson on counterpossibles, *Journal of Philosophical Logic* **47**(4): 693–713.

Bhogal, H. (2021). Nomothetic explanation and humeanism about laws of nature, volume 12, *Oxford Studies in Metaphysics* .

Bhogal, H. and Perry, Z. R. (2017). What the Humean Should Say About Entanglement, *Noûs* **51**(1): 74–94.

Bigelow, J., Collins, J. and Pargetter, R. (1993). The big bad bug: What are the humean's chances?, *British Journal for the Philosophy of Science* **44**(3): 443–462.

Briggs, R. (2009). The anatomy of the big bad bug, *Noûs* **43**(3): 428–449.

Carroll, J. W. (1994). *Laws of Nature*, Cambridge University Press.

Carroll, J. W. (2018). Becoming humean, *in* W. Ott and L. Patton (eds), *Laws of Nature*, Oxford University Press.

Dorst, C. (2019). Humean laws, explanatory circularity, and the aim of scientific explanation, *Philosophical Studies* **176**(10): 2657–2679.

Dorst, C. (2023). Predictive infelicities and the instability of predictive optimality, *in* C. L. Michael Hicks, Seigfreid Jaag (ed.), *Humean Laws for Human Agents*, New York: Oxford University Press.

Elga, A. (2004). Infinitesimal chances and the laws of nature, *Australasian Journal of Philosophy* **82**(1): 67–76.

Elga, A. (2007). Reflection and disagreement, *Noûs* **41**(3): 478–502.

Glynn, L. (2010). Deterministic chance, *British Journal for the Philosophy of Science* **61**(1): 51–80.

Hall, N. (1994). Correcting the guide to objective chance, *Mind* **103**(412): 505–518.

Hall, N. (2004). Two mistakes about credence and chance, *Australasian Journal of Philosophy* **82**(1): 93–111.

Hall, N. (2012). Humean Reductionism About Laws Of Nature, *Unpublished manuscript* . **URL:** *http://philpapers.org/archive/HALHRA.1.pdf*

Halpin, J. F. (2003). Scientific law: A perspectival account, *Erkenntnis* **58**(2): 137–168.

Hicks, M. T. (forthcoming). Counterparts and counterpossibles: Impossibilities without impossible worlds, *Journal of Philosophy* .

Hicks, M. T. and van Elswyk, P. (2015). Humean laws and circular explanation, *Philosophical Studies* **172**(2): 433–443.

Ismael, J. (2008). Raid! dissolving the big, bad bug, *Noûs* **42**(2): 292–307.

Ismael, J. (2017). An empiricist's guide to objective modality, *in* M. Slater and Z. Yudell (eds), *Metaphysics and the Philosophy of Science: New Essays*, New York: Oxford University Press, pp. 109–125.

Ismael, J. (2023). Humean disillusion, *in* C. L. Michael Hicks, Seigfreid Jaag (ed.), *Humean Laws for Human Agents*, New York: Oxford University Press.

Ismael, J. T. (2015). In defense of ip: A response to pettigrew, *Noûs* **49**(1): 197–200.

Jago, M. (2013). Impossible worlds, *Noûs* **47**(3): 713–728.

Joyce, J. (2007). Epistemic deference: The case of chance, *Proceedings of the Aristotelian Society* **107**(2): 187–206.

Lange, M. (2009). *Laws and Lawmakers: Science, Metaphysics, and the Laws of Nature*, Oxford University Press.

Lange, M. (2013). Grounding, scientific explanation, and humean laws, *Philosophical Studies* **164**(1): 255–261.

Lange, M. (2018). Transitivity, self-explanation, and the explanatory circularity argument against humean accounts of natural law, *Synthese* **195**(3): 1337–1353.

Lewis, D. (1980). A subjectivist's guide to objective chance, *Studies in inductive logic and probability* .

Lewis, D. (1983). *Philosophical Papers, Volume 1*, Oxford University Press USA.

Lewis, D. (1994). Humean supervenience debugged, *Mind* **103**(412): 473–490.

Loewer, B. (2012). Two accounts of laws and time, *Philosophical Studies* **160**(1): 115–137.

Marshall, D. G. (2015). Humean laws and explanation, *Philosophical Studies* **172**(12): 3145–3165.

Maudlin, T. (2007). *The Metaphysics Within Physics*, Oxford University Press.

McLoone, B. (forthcoming). Calculus and counterpossibles in science, *Synthese* .

Meacham, C. J. G. (2010). Two mistakes regarding the principal principle, *British Journal for the Philosophy of Science* **61**(2): 407–431.

Miller, E. (2015). Humean scientific explanation, *Philosophical Studies* **172**(5): 1311–1332.

Nolan, D. (1997). Impossible worlds: A modest approach, *Notre Dame Journal of Formal Logic* **38**(4): 535–572.

Pettigrew, R. (2016). Jamesian epistemology formalised: An explication of ?the will to believe?, *Episteme* **13**(3): 253–268.

Pettigrew, R. and Titelbaum, M. G. (2014). Deference done right, *Philosophers' Imprint* **14**: 1–19.

Priest, G. (2016). Thinking the impossible, *Philosophical Studies* **173**(10): 2649–2662.

Schaffer, J. (2007). Deterministic chance?, *British Journal for the Philosophy of Science* **58**(2): 113–140.

Schaffer, J. (2016). Grounding in the image of causation, *Philosophical Studies* **173**(1): 49–100.

Sklar, L. (2014). *Physical Theory: Method and Interpretation*, Oxford Univerity Press.

Tan, P. (2019). Counterpossible non-vacuity in scientific practice, *Journal of Philosophy* **103**(1): 32–60.

Thau, M. (1994). Undermining and admissibility, *Mind* **103**(412): 491–504.

Tooley, M. (1977). The nature of laws, *Canadian Journal of Philosophy* **7**(4): 667–98.

Wallace, D. (2019). Observability, redundancy, and modality for dynamical symmetries, *Philsci Archive* . `http://philsci-archive.pitt.edu/18813/`.

Wallace, D. and Greaves, H. (2014). Empirical consequences of symmetries, *British Journal for the Philosophy of Science* **65**(1): 59–89.

Ward, B. (2002). Humeanism without humean supervenience: A projectivist account of laws and possibilities, *Philosophical Studies* **107**(3): 191–218.

Ward, B. (2007). Laws, Explanation, Governing, and Generation, *Australasian Journal of Philosophy* **85**(4): 537–552.

Wilhelm, I. (2020). The counteridentical account of explanatory identities, *Journal of Philosophy* .

Wilson, A. (2016). Grounding entails counterpossible nontriviality, *Philosophy and Phenomenological Research* **92**(3): 716–728.