

What is the Point of Persistent Disputes?

The meta-analytic answer

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ABSTRACT. Many philosophers regard the persistence of philosophical disputes as symptomatic of overly ambitious, ill-founded intellectual projects. There are indeed strong reasons to believe that persistent disputes in philosophy (and more generally in the discourse at large) are pointless. We call this the pessimistic view of the nature of philosophical disputes. In order to respond to the pessimistic view, we articulate the supporting reasons and provide a precise formulation in terms of the idea that the best explanation of persistent disputes entails that they are pointless. We then show how to answer the pessimistic argument. Taking a well-known mathematical controversy as our paradigm example, we argue that some persistent disputes reflect substantive disagreements at the “meta-analytic” level, i.e., disagreements about the best way, among quite different candidates, to understand the topic at issue, and the best associated cluster of analytic truths one should accept concerning it. Moreover, our concrete example shows that such meta-analytic disagreements can in principle be settled and yield a genuine theoretical (as opposed to merely pragmatic) breakthroughs. We conclude optimistically that persistent disputes can be an important means of fostering epistemic progress.

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Introduction

It is commonplace to observe that people tend to disagree and argue about a multitude of issues, from the most trivial to the most sophisticated. Some disputes last but briefly, others endure for more significant lengths of time (sometimes for decades or even, arguably, for centuries). The history of philosophy is replete with disputes of the latter, long-lasting kind; or, as we call them, “persistent” disputes. To take a few illustrative examples, consider the nominalism-realism debate, the “free will” debate, or the mind-body problem. The persistence of philosophical disputes has often been taken as symptomatic of over-ambitious and wrongheaded intellectual projects; the very ubiquity of such disputes has been used as an argument for the need for an extensive overhaul of the field.³ For instance, Descartes dismissed both the philosophy and the science of his predecessors as dubious and ultimately ill-grounded, “seeing that it has been cultivated for many centuries by the best minds that have ever lived, and that nevertheless no single thing is to be found in it which is not subject of dispute”.

Ironically, Locke subsequently accused Cartesians of breeding “disputes... never coming to any clear Resolution . . . proper to only continue and increase their Doubts, and to confirm them at last in a perfect Scepticism (Locke 1959I,31).”⁴ In his autobiography, Hume explained that he was struck very early by the fact that “Philosophy . . . contain[s] little more than endless Disputes, even in the most fundamental Articles.”⁵ Kant began his first critique with a gloom-ridden reflection on the fact that metaphysics is nothing but “a battle-field of endless controversies.” In the 20th century, Wittgenstein and Schlick, among others, expressed a similar verdict. “Two thousand years of experience, argues Schlick, seem to teach that efforts to put an end to the chaos of systems and to change the fate of philosophy can no longer be taken seriously (Schlick 1959, 53–54).” Wittgenstein famously construed this chaos as a series of “endless misunderstandings.”⁶

Yet these philosophers disagreed both on the exact diagnosis and on the best treatment of persistent disputes. While Descartes thought that philosophy needed a constructive reestablishment that would put an end to its persistent disputes by answering the questions that had given rise to them, other philosophers thought that the revisions needed would turn out to be destructive rather than constructive, appearing to defuse

³ This theme is developed by Rescher (1985, I).

⁴ In a fragment on medicine, he also says that rationalists “lay a foundation for endless disputes (Locke 1959I, xxiv).”

⁵ In the *Enquiry Concerning Human Understanding*, Hume distinguishes two kinds of pointless persistent disputes that we will later review:

It is true; if men attempt the discussion of questions, which lie entirely beyond the reach of human capacity, such as those concerning the origin of worlds, or the œconomy of the intellectual system or region of spirits, they may long beat the air in their fruitless contests, and never arrive at any determinate conclusion. But if the question regard any subject of common life and experience; nothing, one would think, could preserve the dispute so long undecided, but some ambiguous expressions, which keep the antagonists still at a distance, and hinder them from grappling with each other (Hume 1748/2016, VIII-1).

⁶ “Ordinary language leads to endless misunderstandings (Wittgenstein 1929).” For all these references, we have drawn on Rescher’s (1985, I) useful survey of philosophical diversity.

persistent disputes yet without answering the questions that had given rise to them. While Rationalists such as Descartes took it that persistent philosophical disputes could be solved in principle, if only the proper rational steps were taken, Empiricists and Kantians believed that they could only be “dissolved.” For these latter philosophers, the very fact that the enduring disputes had lasted for such a long time meant that they could not be solved at all (thus the endless characterization), and that it was simply pointless for the contending parties to continue to argue over the disputed matters. In what follows, we shall call “pessimistic” the claim that persistent disputes are always pointless and “optimistic” the claim that they are not always so. We shall come back to the question of why Rationalists, but not Empiricists, tend to be optimistic about persistent disputes. Despite its impressive philosophical pedigree and the admittedly strong intuition it embodies, the pessimistic stance on persistent disputes has seldom been adequately defended. Contemporary researchers do often appeal explicitly to pessimistic intuitions, usually in order to dissolve some perennial disputes (think of Hirsch (2016, p138, 2010, p.241) in metaphysics, of Alston (2005, p.21-3) in epistemology, of van Inwagen (2017, 129-31) or Stoljar (2017) in metaphilosophy)⁷. However, they hardly try to justify or deepen that intuition. To our knowledge, there is no direct argument in the literature purporting to show that, in philosophy or elsewhere, persistent disputes must be pointless in virtue of their very persistence. Moreover, no one has explicitly pointed out what is supposed to be wrong with the fact that a dispute persists for a long time. This paper aims at filling this lacuna while delineating the optimistic defense of persistent disputes.

After having defined disputes in section 1 and their persistence in section 2, we survey in section 3 the different ways in which a dispute may be said to be pointless. We then put forward in section 4 what we take to be the strongest pessimistic challenge to the optimistic claim that persistent disputes may in fact have a point. The challenge relies, as we shall see, on the fact that when a dispute persists for a long time, the best explanation for its persistence seems to render it pointless. In section 5, we consider a real-world example of a persistent dispute that has turned out demonstrably to have a point: the “Functions Controversy.” Drawing on this example, we argue that some persistent disputes do have a point, and that their point is meta-analytic, implicitly concerning the best way, among quite different candidates, of understanding the terms and objects at issue. We show that such meta-analytic disputes can be settled and yield genuine theoretical (as opposed to merely pragmatic) progress.

The topic of disagreement has recently come to the fore of the philosophical agenda, yielding a multiplicity of debates about faultless disagreements, peer disagreements, deep disagreements, philosophical disagreements and the a priori, conceptual engineering and metalinguistic negotiations. The question of persistent disputes, as we shall see, cuts across a variety of debates. It is therefore difficult (if not impossible) to do full justice to the precise ways in which these varied approaches interact. In the penultimate section (7), however, we connect our optimistic defense of persistent disputes to some of these recent debates and argue that it can prove fruitful for our

⁷ Some make it clear that persistent disputes may be “interesting” even though they are pointless (more on that soon). Stoljar (2017) suggests that if philosophical disputes were all persistent philosophy would be pointless. He argues however that they are much less persistent than they seem.

understanding of the importance of metalinguistic negotiations and related phenomena in science and philosophy.

Disputes

At first approximation, a dispute over a sentence q is a situation in which different parties

- seem to disagree about q : while Pro asserts q 's truth, Con denies it⁸,
- argue against each other in order to find out which one is correct, and which one is incorrect.

Note that there are countless ways in which one might object to this first approximation, going on consequently to build in complex and precise detail by way of refinement and exactitude. For our present purposes however, a brief characterization should suffice.

Parties. The parties involved in a dispute might be single individuals, or collectively, they might form groups. Moreover, the weight or preponderance of the argument on each side might well be asymmetric. Consider for example the dispute over whether the earth is flat, opposing (in the present day) a negligibly small number of flat-earthers to virtually everyone else. Or, to take a limit case, think about the disputes opposing some delusional patients to their doctors and families (see Hohwy and Rosenberg (2005); Bayne and Pacherie (2004) for a couple of relevant case reports).

Aims. We assume in this paper that the *primary aim* of a dispute is to find out who is right or wrong, that is, whether Pro's assertion is true, and Con's is false or the opposite. Some might object that the aim of a dispute should be construed in terms of knowledge, or of some other norm of assertion, rather than truth (see e.g. Williamson, 2000). This point is well taken. Because it will make things simpler, however, and because we believe it does not affect the main thrust of our arguments, we will neglect alternative, knowledge-based, views of the primary aim of disputes.

Be that as it may, a dispute might be quite useful even when it does not fulfil its primary aim. Pursuing it might for example allow the disputants to attain other valuable cognitive goals, such as finding out that it is impossible to reach the primary aim of the dispute, or that they need further evidence, or again that this dispute is connected in surprising ways to other classical disputes, and so on. In the case of collective or group-based disagreements, the dispute might allow a select few individuals to realize whether or not they are correct, even in the absence of a collective forming of opinion. In such

⁸ We assume throughout the paper the following equivalence schema: the proposition expressed by a given sentence use is equivalent to the proposition asserting the latter's truth. We also assume that assertion and denial are incompatible speech-acts (one cannot coherently assert and deny the same proposition) and exhaustive speech-acts (someone who has settled his mind about a proposition should be disposed to assert it or to deny it). For simplicity, we suppose (against, e.g., dialetheists such as Priest (2006)) that asserting that q is not true amounts to denying that q is true and thus to engaging in a dispute with someone who asserts that q is true.

Two parties disagree when one asserts and the other denies q 's truth).

cases, we might say that the dispute has fulfilled some of its *secondary aims*, and that it is accordingly *interesting*, even if it has no point. Finally, when neither its primary aim nor its secondary aims can be reached, a dispute might still serve what we might call *adventitious aims*: aims, that are not directly related to epistemic values. Participating in a philosophical dispute to which one has skillfully and adeptly devoted time and effort, for example, might help one keep one's job as a philosopher and pay one's rent on time.

Persistence

What about *persistent* disputes, one might ask? "Persistent" is a rather vague and context-sensitive word. In order to make explicit what we mean by it, we need to be clear as to the role we assign to the notion of persistent disputes. This notion is epistemologically useful and significant, we believe, because the persistence of a dispute casts a doubt on its having a point. For the doubt to arise, two things are necessary. First, a persistent dispute must have existed long enough to allow all participating parties to share their evidence, extensively argue, and thoroughly assess the arguments put forward. Although this might depend on the pace of exchanges and on the number of people involved, it might be surmised that several decades should suffice for the process to be completed. However, this condition is neither precise, nor sufficient, for constraining the analysis. To see why, suppose that new and important experimental results for and against q reliably emerge over a short period of time (say, every year), and that as a result, a couple of antagonistic scientists contend over q for decades. The very long time they have spent arguing would not be epistemically challenging, or not quite as much as it would have been, had the relevant empirical evidence remained constant all along. It would indeed be easily explained by the continuous discovery of new empirical data, contributing to each new iteration of their argument. Accordingly, if we do not want to deprive the category of persistent disputes of much of its epistemological usefulness and significance, we should say that a dispute over q is persistent only if, *while the relevant available empirical evidence did not significantly change, it has lasted long enough to allow all parties to share their evidence, extensively argue and thoroughly assess the arguments put forward*. Conversely, we count as persistent a dispute satisfying this condition. The examples from the history of philosophy given above do not all qualify as persistent disputes in this sense, as for some of them (most notably, the "free will" debate) the relevant empirical evidence has in fact significantly changed over the centuries. But one thing the debates we adverted to should have in common is that they all involve a *series* of persistent disputes in our given sense. Thus, one might say that both the discovery of classical mechanics and the discovery of quantum mechanics ended a form of persistent dispute over free will, and at the same time, gave rise to a new variant. Similarly, when Thomas Young made his two-slit experiment, one arguably persistent dispute over the nature of light (wave or particle) ended, and another one took its place.

Varieties of pointless disputes

What is a pointless dispute?

Throwing a rock at the sky is pointless if it is aimed at knocking the moon off orbit, or at causing rainfall in the Sahara. It is not pointless if it is part of a game or play. More

generally, an action has a point if and only if, given one’s capacities and the laws of nature, it allows one to reach the aim we assign to it. A dispute is a kind of action too, albeit a collective action. And just like throwing a rock, it will have a point if and only if it permits the disputants to achieve the aim of the dispute. As we have seen, disputes can be assigned many aims. Previously, we distinguished the primary aim of a dispute (finding out who is right and who is wrong), from its secondary aims (such as finding out whether the primary aim can be attained), and adventitious aims (such as keeping one’s job as a philosopher). No one would be tempted to say that a dispute has a point only because it allows one to reach some of its adventitious aims. The matter is less straightforward when it comes to secondary aims. There is, in any case, an interesting category of disputes that are pointless in the sense that *given their epistemological profile, taking part in these disputes cannot allow the disputants to reach the primary goal of these disputes, that is, cannot allow them to find out who is right and who is wrong about q*. By the “epistemological profile” of a dispute, we mean not only the rationality of the parties, broadly understood (that is, their epistemic virtues and capacities, and the various epistemic vices, motivational influences and cognitive biases that might hinder the exercise of the former) but also the way rationality itself (in terms of capacities, virtues, biases and influences) evolves over time. We should also include in the epistemological profile of a dispute the distribution of the relevant available evidence and its relation to both parties (i.e., how easily accessible it is to both), and other relevant epistemological factors. In what follows, we focus on disputes that are pointless in this primary sense. Importantly, if a dispute is pointless (in that sense), the fact that the parties want to find out who is right and who is wrong gives them no practical reason to keep arguing against each other. If that is the only thing they are hoping to achieve, then the debate is indeed terminally devoid of point, and the disputants would be better off engaged in other pursuits.

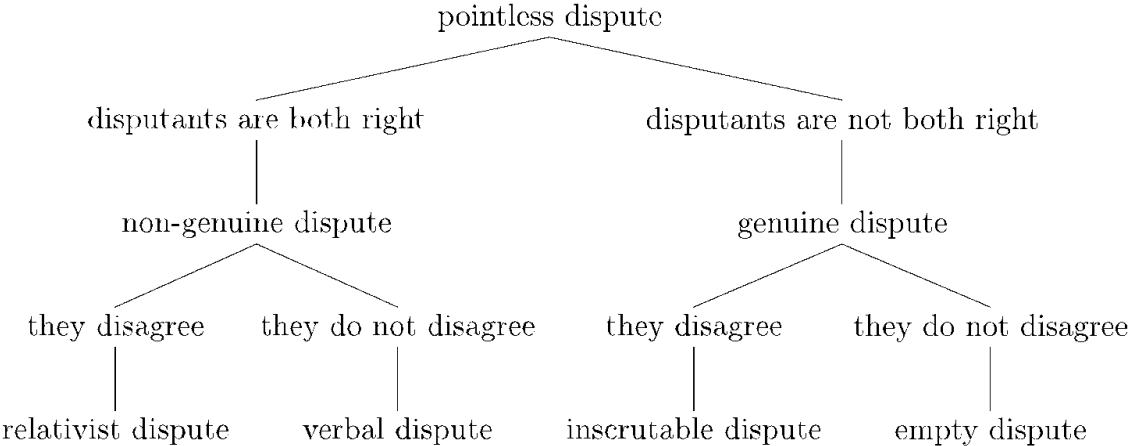


Figure1

A typology of pointless disputes

It is possible to distinguish four types of pointless disputes. Notice that appearances notwithstanding, opposing parties engaged in a dispute might in fact both be right. In such a case, we should say that the dispute is *not genuine*. If a dispute is not genuine, then neither of the disputants is wrong, it is accordingly impossible to find out which of the two is wrong, and *a fortiori* to settle the issue by arguing antagonistically. Non-

genuine disputes are therefore manifestly pointless. There are, however, two different ways for a dispute to be non-genuine, as we shall now explain.

Verbal disputes. Typically, a non-genuine dispute is one in which both parties *do not genuinely disagree*. Yet, one might ask, how can two speakers fail to disagree if one asserts that q is true, while the other denies it? Such an eventuality might easily obtain if the speakers misunderstand each other, for example if q contains ambiguous terms, and the disputants are linguistically at odds over the various intended contents. In such a case, it turns out that if there is disagreement at all, it is about how to use words and their possible meanings, and not matters of deeper substance. Thus the apparent dispute is contrary to first impressions merely *verbal*.⁹ For instance, it is sometimes claimed that in matters of taste disputes are verbal because what 'tasty' means is tantamount to 'tasty for the speaker who utters it', and will on this analysis mean different things as uttered by different speakers. The claim that metaphysical disputes are verbal corresponds to a form of metaphysical pluralism. Carnap (1947) seems to have held such a view about ontology.¹⁰ Hirsch (2009) has recently revived that view, arguing that many (but not all) metaphysical disputes are verbal.

Relativist disputes. There are moreover some non-genuine disputes in which the disputing parties nevertheless genuinely do disagree. That is to say, there is no linguistic misunderstanding of the type above, and yet, intuitively at least, both parties really do put forward conflicting proposal. A similar conundrum arises: how can two persons who are said genuinely to disagree with each other nevertheless both be correct? The answer is that such a predicament might occur if the truth of the disputed sentence is *relative* to certain parameters, be they moral standards or standards of taste, theoretical frameworks or paradigms, and similar. Goodman (1978) argues that even when they are genuinely conflicting and not ambiguous, a sentence q and its negation can be both correct because they are not correct in or relative to the same "world". Goodman calls his view radical relativism, and his relativism is indeed radical in the sense that it is universal. More recently, some philosophers have advocated circumscribed forms of relativism (see MacFarlane, 2014). Some have argued that disputes about taste are not usually verbal because adversaries in matters of taste do not talk past each other; when I say that spinach is tasty and you deny it, our speech acts bear on the same proposition, and our disagreement is tangibly real. Such a disagreement, it has been claimed, might nevertheless be faultless (in the relevant sense that permits both of us to be right) if truth about matters of taste is made relative to latent standards of assessment. Goodman (1978) has held that metaphysical disputes are relativist and endorsed metaphysical relativism. As we understand him, Gallie (1955) argues that many disputes in the

⁹ Some disputes might be verbal and substantive, rather than *merely* verbal, if q is itself about language. For simplicity we will suppose that q is not about language and that verbal disputes are all merely verbal.

¹⁰ More precisely, R. Carnap (1947)'s view was that there are two possible readings of ontological questions: On one 'internal' reading they are verbal, on another 'external' reading, they are empty. It should be reminded, however, that Carnap granted a useful, pragmatic role to certain external ontological questions, namely that of helping us choose and coordinate on a given ontological framework (see Flocke (2018)).

political and social domain are “endless”, because they are relativist. Williams’ (2011VII-X) seems to hold a similar view (which he calls “non-objectivism”) regarding many moral disputes.

Empty disputes. Genuine disputes are disputes in which at least one party is *not* right about q . Yet these types of debate might be pointless too. Starkly, this will obtain when neither party happens to be correct about the matter at issue. In such a case, the primary aim of the dispute — finding out which one of the two parties is right and which is wrong — will as before be impossible to achieve. One might say by way of a stipulative definition that when both opponents are not right their dispute is *empty*. Trivially, if the disputed sentence q is meaningless, the dispute over q is empty. In such a case it is a moot point whether the parties do in fact disagree.¹¹ Expressivists about taste might thus argue that ‘spinach is tasty’ or ‘spinach is tasty’ is true’ are merely expressions of feelings which are neither true nor false and that disputes about such matters are always empty. In metaphysics, the claim that disputes are pointless because they are empty has been maintained by the Logical Empiricists. It expresses a form of metaphysical anti-realism (Rudolf Carnap 1932). Thomason (2009) and Yablo (2009) argue that some metaphysical disputes might indeed be empty.

Empty disputes constitute a central case of the category of pointless genuine disputes. We now come to a third.

Inscrutable disputes. In order to have a point, a dispute must be genuine and non-empty. Let us call “substantive” a dispute in which one party is right while the other party is not. Not all substantive disputes have a point. A substantive dispute will indeed be pointless if it is impossible for the parties to come to an agreement through rational exchange, that is, if the epistemic reasons justifying the assertion or the denial of q are inaccessible to one of the parties. Note that the impossibility and inaccessibility at stake in this context are epistemological. They depend on what we have called the epistemological profile of the dispute, and in particular on the rationality of the disputants. We call those disputes whose epistemological profile makes it impossible to convince by dint of reasons the error-committing party, inscrutable disputes. The claim that traditional metaphysical disputes are pointless because they are inscrutable expresses a form of metaphysical skepticism. This Humean or Kantian view has contemporary advocates. Kriegel (2013) puts forward an argument to the effect that they are always inscrutable. Bennet claims that some of them are.

Verbal, relativist, empty and inscrutable disputes are subcategories of pointless debate. Conversely, if a dispute is neither verbal, nor relativist, empty or inscrutable, it is a substantive dispute in which given their cognitive capacities, the disputants can in principle come to an agreement over q by means of argument and rational persuasion. It will accordingly be a dispute that has a point. Verbal, relativist, empty and inscrutable disputes thus nicely partition the field of pointless disputes (see figure 1).

Note that there is an interesting contrast between verbal, relativist and empty disputes, on the one hand, and inscrutable disputes, on the other hand. Whereas the first three types are pointless for a semantic or an ontological reason, the last type is pointless for an epistemic reason. Importantly, as we have emphasized, a dispute might be pointless

¹¹ We have assumed that the norm of assertion is truth and truth only, and we will suppose that a meaningless sentence cannot be true and should not accordingly be asserted.

but still interesting and accordingly worth having. Bennett (2009) claims that this has been the case of some ontological disputes that are inscrutable, and Sosa (2010, 281) argues that this is the case of many philosophical disputes that are verbal.

The pessimistic challenge to persistent disputes

Our general discussion of the futility of disputes is directly relevant to *persistent* disputes, which may turn out to be pointless in precisely four different ways, on the present analysis: they may be verbal, relativist, empty or inscrutable. Our question is now the following: Is there something in the persistence of a dispute that makes it likely to fall into one of these categories? It may be assumed that *some* persistent disputes are pointless, but why should the very persistence of a dispute always make it pointless? Since we are envisaging an internal connection between persistence and pointlessness, we need to examine *general* arguments for the pointlessness of persistent disputes. We shall see, however, that these general arguments can also be applied on a case-by-case basis, yielding more cogent conclusions for some persistent disputes, as opposed to others.

There is an obvious inductive argument which infers endlessness from persistence: if a dispute has existed for a very long time without having been settled successfully, it will never be. When Empiricists or Kantians say that metaphysical disputes are *endless*, they seem to appeal implicitly to an argument of this kind. The weakness of the inductive argument is easy to see once the latter is made explicit. For instance, a similar argument would have concluded twenty years ago that the perennial search for a demonstration of Fermat's last theorem was pointless, which we know is obviously false.¹²

More significantly, the inductive argument fails to distinguish between persistent *disputes* and persistent *questions*. When a dispute exists for a very long time, the intuitive worry is not so much that a complicated question fails of an answer (persistent questions are legion in mathematics and natural sciences, and few would claim that their persistence means that they are pointless). The intuitive worry is rather that *despite their common knowledge of the unsettled issues, the parties do not relinquish their dispute and continue to hold and argue for (apparently) dissenting views*. In a genuine persistent

¹² The inductive argument is probably stronger in the special case of *philosophical* persistent disputes, as one could contend that none of the philosophical disputes that have lasted a long time have been solved. This last claim, however, is less obvious than it seems. Stoljar (2017) has for example argued, rather convincingly, that if we individuate philosophical disputes properly (and distinguish, for example, the various questions that we loosely put under the heading mind-body problem but that have been raised at very different historical periods and are indeed quite different), the track record of philosophy is similar to that of other fields, and that once philosophical questions are properly individuated, philosophical disputes last much less than is usually assumed (many remain persistent in our sense). Moreover, there are other fields in which disputes tend to last. In a classical paper, Gallie (1955) argued that some concepts are "essentially contested" i.e. are bound to lead to endless disputes, in part because of their evaluative character. These include the central concepts of political science and legal theory. In any case, our question at this point is not limited to philosophy: we are wondering whether the persistence of a dispute per se *generally* gives us reason to deem it pointless.

dispute, one of the parties does not know that she is not right, and that she does not know the answer to the question. But this is not so, in general, with a persistent question (think again of the many conjectures and open problems in mathematics and physics which do not yield persistent disputes). Unlike persistent questions, persistent disputes involve a form of *reflective opacity*. Accordingly, they seem much more worrying from an epistemological point of view than mere persistent questions.¹³

This intuitive worry forms the basis of a serious philosophical challenge, a challenge that is abductive rather than inductive. The challenge is to explain the persistence of a given dispute without assuming that it is pointless. What might account for the fact that parties persist in disputing a sentence's truth if their dispute is not, in one way or another, pointless? Below, we will introduce two important and connected problems that the theorist we have characterized as *optimist* must face in order to answer this challenge. The first one is, roughly, that if a dispute *which has a point* persists, both parties should become competent enough to settle it after a reasonable time. This dispute should not accordingly be persistent. This is the *competence problem*. The second one, which we call the *problem of apt a priori disagreement*, can be stated thus: when a dispute persists and involves sufficiently rational subjects who can share the relevant empirical evidence, it reflects a persisting *a priori* disagreement among rational subjects whose judgments are both apt. But it is hard to see how such a thing could be possible. Taken together, these two problems suggest that the challenge cannot be met and that persistent disputes are pointless.

The Competence Problem

How can a dispute persist if it is not pointless? A successful explanation should first grant that the dispute is substantive: one party must be wrong and the other right, otherwise the dispute would be merely verbal, relativist or empty, and hence pointless. It should accordingly explain the persistence of the dispute in epistemological terms, invoking a bad epistemological profile of the dispute. The epistemological profile must not be too bad, however; that is to say, it must not be incorrigibly bad, for otherwise the dispute would be inscrutable and pointless. In other words, the parties should be competent enough to settle the dispute, but their performance should be impeded by some epistemological obstacles liable to be overcome, albeit extremely slowly.

Let us see how this might happen by singling out the epistemological obstacles that might plausibly explain persistent disagreements — call these persistent disagreement factors¹⁴ — and see whether they can explain a persistent dispute. Persistent disagreement factors all hinge on an asymmetry in the distribution of certain epistemic features that need to be overcome.

Asymmetric access to empirical evidence. Rational agents do not as a rule have equal access to all available empirical evidence relevant to a given question. This fact explains

¹³ There is an additional difference between persistent questions and persistent disputes that we shall not consider here. In a persistent dispute, the parties typically know that someone who is likely to be a peer disagrees with them. This knowledge gives rise to the problem of “peer disagreement” (see Feldman (2003); Elga (2007); Christensen (2007) as well as Kelly (2005; 2010)).

¹⁴ We borrow the term “disagreement factor” to Frances (2014).

many of our persistent disagreements. For instance, I believe that the male rather than the female of the seahorse species carries eggs because I recall coming across this information in a book on marine life roughly thirty years ago. My partner believes the opposite because it seems to him less implausible as a scientific hypothesis. We have disagreed all this time (to be frank, we never much talked about it).

Similarly, I can disagree with my neighbor about the claim that vaccines are on the whole more dangerous than the disease against which they offer immunity at least in part because I happen to have access to far more reliable scientific sources than he does, and because my sources, but not his, inform my opinion correctly in view of the relevant facts. Consequently, the disagreement can rage on unabated for a considerable period.

Some theists likewise explain their disagreement with atheists, as well as with advocates of rival religions, by claiming that *they* have experienced the presence of (their version of) God (among philosophers, see, among others, the influential accounts of Plantinga (2000); Alston (1993)).

Differences in rationality. Psychologists have shown that we are almost without exception affected by cognitive biases, and that consequently, different thinkers display different cognitive “styles”. They have also shown that our motivations can significantly affect our beliefs and their entrenchment. It is safe to suppose that cognitive and motivational biases can account for a range of persistent disagreements.

Take the following puzzle, a paradigm case for attracting disagreement. Suppose Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations. Which is more probable? That Linda is a bank teller, or that Linda is a bank teller and is active in the feminist movement? Kahneman, Slovic, and Tversky (1982) have argued convincingly that many people wrongly believe that the second option is the more probable because they use a misleading representativeness heuristic to assess probabilities.

Moreover, it has been observed that psychological factors can affect real-life persistent philosophical disagreements. Enoch (2011, 192–95) has argued that many moral disagreements are partly grounded on the distorting effects of self-interest. As an illustration, he mentions the view advocated by Peter Singer and Peter Unger to the effect that unless we give almost all our money to the famine relief we are nearly as morally condemnable as murderers. As he says, “refusing to see the (purported) truth of Singer’s and Unger’s claims thus has tremendous psychological payoffs (193)”.

Feltz and Cokely (2013) have likewise argued that some “persistent philosophical disagreements” are predicted by individual differences, in particular by personality traits, which determine certain cognitive biases. They show, for instance, that extroverts tend to endorse the compatibility of free will and determinism.

Different epistemic paths and starting points. Finally, some persistent disagreements can be explained by factors that do not directly depend on differences in rationality or access to the relevant empirical evidence, but only on what we might call the *topography* of the disagreement. That is, on the different starting points, and on the different paths, taken in the course of a disagreement. The idea is to compare the evolution of someone’s opinion on a given topic to climbing a mountain. Even if two people are aiming at the same terminus (by analogy, truth), and even if they are in a perfect physical condition

(by analogy, even if they are perfectly rational and have common access to the relevant empirical evidence) they might end up in different places simply because they had different starting points, took different paths thereafter, and because the landscape itself is full of pitfalls.¹⁵

Arguably, the most notable pitfalls are what philosophers call *vicious epistemic circles*. Often such circles successfully entrap ordinary subjects, altering the form of their beliefs and creating the conditions for long-standing divergence of opinion. Thus, the prevalence of conspiracy theories in some social contexts has been explained in terms of the fact that some people do not trust the accredited experts because they do not trust the institutions bestowing credentials upon them. But they do not trust the institutions accrediting the latter because they believe in conspiracy theories. Conspiracy theorists are trapped in a vicious epistemic circle. Basham (2001, 270ff) has argued that as a result we are not in general in a position to find out whether a claim of conspiracy is correct. We cannot but assume a prior answer to the core question of how conspiracy-prone our society is, in order to derive a well-justified position on the issue (p.274). If Basham's view is correct, those who start by trusting institutions end up rejecting conspiracy theories, and those who distrust them are bound to adopt conspiracy theories. Vicious epistemic circles have also been invoked to explain the fixity of delusional beliefs (Hohwy and Rosenberg 2005; Bayne and Pacherie 2004) and the persistent disagreement between for-vaccine and anti-vaccine factions, and flat-earthers and their opponents (see e.g., Levy (2019)'s account of scientific denialism and Nguyen's (2021) account of echo chambers).

Note that these disagreement factors can explain persistent *disagreements*. Can they explain persistent *disputes*, and persistent disputes that have a point, and are hence substantive and "scrutable"? In a dispute (as opposed to in a mere disagreement) the parties argue to correct and convince each other.¹⁶ In a scrutable dispute, moreover, the epistemological profile must be good enough to allow the parties ultimately to settle the dispute by means of rational argument. The parties must be sufficiently rational (the cognitive biases and the motivational influences on beliefs affecting them must be benign and corrigible), vicious circles must be eschewed, and the relevant available evidence must be equally accessible to both. In such a scrutable dispute, time will accordingly have a beneficial effect. It will progressively cancel not only performance errors, but also the impact of differences in rationality (due for example in cognitive biases and motivational influences) as well as the asymmetries in the access to the empirical evidence. For a scrutable dispute to persist, this beneficial effect of time must be real, but extremely slow. The gist of the competence argument is that in most cases, such a very slow effect is simply implausible : either the disputants are competent enough to settle the dispute, and it should be settled in a reasonable amount of time, or they are not competent, and the dispute is pointless.

Let us see how this works on the above examples. It is reasonable to suppose that the disagreement about seahorses' eggs and the disagreement about probabilities in the "Linda" example will not yield anything like a persistent dispute, or at least not one that is scrutable. If the dispute is scrutable, both parties have the capacity to acknowledge without further ado the decisive evidence to the effect that male seahorses carry eggs or

¹⁵ We borrow the term "starting point" from Feldman (2006).

¹⁶ Connection with active sense of disagreement

that it is more likely that Linda is a bank teller rather than a bank teller and something else. It is hard to see what could prevent them, then, to quickly come to an agreement.

In the theistic example, on the other hand, it seems that the disagreement could indeed yield a persistent dispute, but it is dubious that the asymmetries in the access to the relevant empirical evidence can be redressed by means of simple debate. As James (1902, 371) emphasized, religious experiences are usually very difficult to communicate. They seem to provide what is sometimes called subjective or private or first-person evidence (Schellenberg 2007VIII). Accordingly, if the religious disagreement case yields a persistent dispute, this is likely to be merely of the pointless kind.

The moral disagreement case, the free will case, and the anti-vaccine case are less straightforward to analyze. Historically, disagreements of their type have given rise to genuine disputes, both at the factional (group) level, and at the level of individual thinkers. There are reasons, however, to believe that such group-level disputes are pointless. Take the moral disagreement for example, and suppose, for the sake of the argument, that Singer and Unger are right and that their opponents are simply self-deceived. For the dispute to have a point it must be possible, through rational exchange to correct the distorting influence of their self-interest on their beliefs and have them change their mind. But even if we could do that, it would not suffice to settle the debate, as there would always be new, self-deceived comers joining the ranks of Singer's and Unger's opponents, who have not benefited yet from the virtues of rational redress. The ensuing dispute would arguably be pointless. A similar analysis might deal with the anti-vaccine and the free will cases. The problem in such cases is that new members of one group seem to be selected by their epistemic limitations (more precisely, by how they fare on some disagreement factor), which prevents the dispute from being settled.

The competence challenge is not a knockdown objection against persistent disputes that have a point. Nothing prevents, theoretically at least, the possibility that a dispute exists which is shaped by cognitive biases, asymmetries in the access to evidence, or differences in starting points and epistemic circles that can be overcome albeit extremely slowly. The competence challenge can however yield a schema of abductive arguments that should be applied on a case-by-case basis, as we have illustrated above. For a given persistent dispute, depending on the precise details of the case, the strategy of appealing to the argument schema implies that the best explanation of why such a dispute persists makes it pointless. It is in fact arguable that many pessimistic views about the debates in metaphysics and elsewhere stem implicitly from the idea that in these cases of interest, disagreement factors are set at rest once by one, making persistence mysterious.

There is moreover a broad category of cases to which the competence challenge can be applied directly, as opposed to on a case-by-case basis, characteristic of our argument schema strategy. It is the category of disputes in which differences in rationality are sufficiently easy to overcome, the epistemic circles sufficiently easy to escape, the starting points sufficiently close, the relevant available evidence sufficiently easy to access or share, and the epistemic profile of the dispute, more broadly, sufficiently auspicious. Call such disputes *virtuous disputes*. In such cases, the disagreement factors we discussed which might explain the dispute without making it pointless will most likely be cancelled after a short period of rational exchanges (perhaps in the space of a couple of years). Virtuous disputes, it should be emphasized, need not have a point. Virtuous disputes are such, however, that their epistemic profile seems incapable of

explaining their persistence without making them pointless. But it is hard to see what else could explain their persistence; accordingly, the argument concludes, they will only persist because they are pointless, i.e., because they are verbal, relative or empty.

The point raised above is interesting, since many disputes seem at least *prima facie* virtuous, and some of these seem persistent too. Think of disputes among researchers on such topics as mereological composition in ontology, or fundamental axioms in mathematics (for example, disputes about the truth of the continuum hypothesis). Or consider, in biology, the disputes over the choice of a classification system based on phylogeny vs interbreeding (LaPorte 2009, 70–76); or, in cognitive neuroscience, the dispute over the neural correlates of visual consciousness, or in cosmology disputes over the status of multiverses. Many people engage in these disputes with the hope of settling them in a reasonable time, and they seem to believe that these disputes are virtuous (the case of ontological debates is perhaps moot). There is no question self-interest sometimes plays a role in them, some researchers being motivated, for example, by the perspective of promotions and social recognition. It is, however, at least *prima facie* plausible that such motivational influence, and other aspects of the epistemic profile, cannot explain the persistence of these disputes. At least this is what many researchers engaged in these disputes seem to believe.

In short, the competence challenge enjoins us to find an explanation why some disputes stubbornly persist, which does not entail pointlessness. In many cases, it is difficult to understand how the dispute may persist for protracted periods of time without being pointless, since, as we have outlined, if the dispute has a point the participating parties must be sufficiently competent to settle it, and the passage of time must bring with it adequate and timely redress. This then is the *Competence Problem*. It might be possible to meet this challenge for some forms of persistent debates. It is difficult, however, to see how this might proceed, especially in the case of virtuous disputes.

The Problem of Apt A Priori Disagreements

The *Competence Problem* is related to a second, cognate difficulty, namely the *Problem of Apt A Priori Disagreements*. Roughly sketched, this says that (i) when a virtuous dispute is persistent, it becomes a priori (ii) however, given that the disputants involved in a virtuous dispute are equally competent to assess a priori claims, it is very hard to see what could explain the persistence of their dispute. We tackle these two premises in turn.

Since the disputants engaged in a persistent virtuous dispute are said to gain quick and easy access to a shared empirical body of *relevant* evidence, one might suppose that their disagreement would at some early point become independent of relevant empirical evidence. Since other pieces of empirical evidence are, *ex hypothesis*, not relevant to this dispute, the disagreement is also independent of these latter. Overall, the dispute thus becomes independent of *all* empirical evidence, relevant as well as irrelevant, and, accordingly, a priori.

If the virtuous dispute over the sentence q is not pointless, the persistent disagreement will in fact be grounded on a (more or less explicit) disagreement over a more fundamental sentence q^* , to the effect that the available empirical evidence provides decisive reasons for q . The sentence q^* will be a priori not only because the difference in attitudes toward it (namely, one party believes that q^* is true, the other one that it is

false) is not grounded on a difference in empirical evidence, but also because, if the parties were to settle the dispute, their correct attitude toward q * would not be similarly grounded either.

There are classical, Platonic and Kantian arguments to the effect that fundamental disagreements in metaphysics and ethics hinge on a priori claims.¹⁷ Our argument is much simpler and much more modest than these. First, our argument relies on a dialectical and quasi operational conception of the a priori (expressed by the necessary condition that to the effect that a disagreement that do not depend on problems of rationality or on empirical evidence must be a priori) that remains neutral on the cognitive mechanisms implied.¹⁸ Moreover, our argument only targets disputes (not just disagreements) with a point, and only those, moreover, that are both persistent and virtuous. To reiterate, for a dispute to have a point, the relevant empirical evidence must be equally accessible. If the dispute is, moreover, virtuous and persistent, this equally accessible evidence must quickly become equally accessed in actual fact. Hence the dispute must quickly become a priori, depending only on a priori claims.¹⁹

Let us illustrate this point with an example. For the last two decades, neuropsychologists have disagreed about the neural correlates of visual consciousness; all the while, the accessible relevant empirical evidence did not change significantly. Roughly, while some (call them Pro) believe that the neural correlate necessarily involves frontoparietal networks, others (call them Con) believe that an activation of primary visual areas in the occiput is sufficient for visual consciousness.²⁰ Strikingly, they all agree on the data collected by both camps, and on their prima facie relevance to the debate. While some have characterized this debate (in this and ancillary areas) as merely verbal (see for

¹⁷ These arguments hinge roughly on the idea that fundamental claims in ethics and metaphysics are necessary, and that necessary claim are a priori. See Wedgwood (2019) for an updated defense of the Kantian argument concerning ethics.

¹⁸ Interestingly, this conception would classify as a priori a dispute that hinges on the weighing of different theoretical “super-empirical” virtues. We side with Hirsch (2009, p.233, fn.3) who takes such disputes to be straightforwardly a priori, and against Hawthorne (2009, p.217) here.

This conception also sidesteps an influential objection raised by Williamson (2007) against the significance of the a priori / a posteriori distinction. On the one hand, his objection relies heavily on what he considers the mechanisms of a priori knowledge should be, an issue on which we remain neutral. On the other hand, our dialectical conception and the pervasiveness of persistent virtuous dispute do suggest that our notion of the a priori is indeed quite natural and philosophically important.

¹⁹ Our thesis here should not be confused with the claim, made by Chalmers (2011), to the effect that a sentence is such that any “disputes over it involving a competent disputant is verbal” iff it is in a sense analytic. Our claim, we shall see, allows for persistent virtuous disputes that have a point (and hence are not verbal) and that are a priori but arguably synthetic rather than analytic.

²⁰ Advocates of the first “Pro” view include Dehaene and Naccache (2001; Sergent and Dehaene 2005; Naccache 2005; Kouider et al. 2006; Kouider, Sackur, and Gardelle 2012). Advocates of the second, “Con” view include Zeki and Ffych (1998); Lamme (2004); Block (2005, 2007).

example Bayne (2007, 100), Rosenthal (2002, 660) and even more specifically Gottlieb (2018)), it is arguable that nevertheless the dispute is substantive, granted that they disagree on the way the universally accepted common data should be weighed and interpreted, and that their disagreement is grounded on a priori claims about scientific methodology and scientific concepts. Pro scientists explicitly suggest, for example, that consciousness is a priori tied to reportability and that the only scientifically tractable concept of consciousness is that of “cognitive access”; while Con scientists argue that consciousness is not tied a priori to reportability but is still scientifically tractable (see for example Block (2007)’s insightful analysis of this debate).²¹

It may already seem mysterious that thinkers disagree on an a priori truth, when being rational they are competent enough to find out that it is indeed true. It gets all the more mysterious when their disagreement persists despite lively rational exchanges, since we can safely assume that they correct each other’s performance errors and that their disagreement does not stem from such errors — it is an apt a priori disagreement. The problem here is not so much that one of the parties persistently fails to assent to a truth (q * or its negation) that is a priori even though he is rational enough to do so and does not commit performance errors. After all, many competent subjects have persistently failed to see that some complex mathematical claims, such as Fermat’s theorem or Poincaré’s conjecture, follow from the relevant axioms. We already know that some a priori *questions* can persist for decades or centuries. The problem is rather that one of the parties wrongly and persistently *dissents* on the matter of the disputed a priori truth, and that both parties accordingly disagree persistently. In the case of Fermat’s theorem, Poincaré’s conjecture and many other classical conjectures, the historical landscape is starkly different — at least if we attend to the categorical assertions published in peer-reviewed journals and backed by tentative proofs, as opposed to hypothetical assertions expressed in conversation and backed by intuitions. Mathematicians may dissent for a couple of years about whether a particular complex proof of a given conjecture is correct (the recent example of the six hundred pages-long proof of the *abc* conjecture is a

²¹ One might concede that a virtuous persistent dispute that has a point quickly becomes independent of the empirical evidence that is *directly* relevant to the dispute, and hinges on background disagreements concerning, say, methodological principles or wide-ranging, philosophical or ethical conceptions (these background disagreements might be considered as coming from differences in what we have called the starting points of the disputants). One might question, however, whether the latter disagreements need to be a priori; one might argue that they often depend on empirical evidence as well, even if the empirical evidence here is only indirectly relevant to the initial dispute. In response, it should be recalled that if the dispute is indeed virtuous, and if the empirical evidence mentioned is indeed relevant to the dispute (even if only indirectly), both camps should come to scrutinize it and share it, and their disagreement should quickly become independent of it. We believe that this answer is valid. It is fair to acknowledge, however, that it puts strain on the real-world relevance of the notion of a virtuous dispute. Someone skeptical of the claim that persistent disputes quickly become a priori can indeed deny that there are many genuinely virtuous disputes. This is probably what someone who believes that persistent disputes in philosophy are “just hard” to settle, but not a priori (maybe Williamson and Hawthorne ?) should do. If she does not want to be accused of mere hand-waiving she should however answer our pessimistic by a detailed analysis of the dynamics of “just hard” disputes showing exactly what kind of cognitive difficulties can make them persist.

particularly eloquent example (Castelvecchi 2020)). When no convincing proof has been published, they may persistently fail to know the truth of the matter, and consequently suspend their (considered) judgments for a long time, but they do not generally disagree persistently about it.²² The problem of non-pointless but persistent virtuous disputes is that being reflectively opaque, they seem to imply the existence of a kind of *deceptive a priori* truths, truths, that is, such that rational enough subjects not only fail to know them, but also wrongly believe them to be false (not knowing that they do not know them). We take it that deceptive a priori truths typically represents a kind of a priori truths whose existence will be granted by Rationalists, but denied by Empiricists, and that the challenge of apt a priori disagreements thus goes some way towards explaining why Rationalists, but not Empiricists, tend to be optimistic about persistent disputes.

Logical Empiricists notably argued that all a priori truths are analytic, and that rational subjects should assent to analytic truths merely in virtue of understanding them (at least if they do not make any performance errors). Assuming that two parties are sufficiently rational, and therefore capable of grasping a prior truth, there should be no room for disagreement about which a priori truths are true. Conversely, if rational subjects disagree about an a priori sentence, it follows that either they understand the disputed sentence differently and the dispute is verbal, or they do not really understand it and it is empty.²³ Logical Empiricists must thus reject the existence of the *deceptive a priori* and deny that persistent virtuous disputes can have a point.

One preliminary conclusion to draw from our discussion is that a theorist who believes that some persistent virtuous disputes have a point is committed to maintaining either that some a priori claims are *synthetic* rather than analytic, or else that some analytic claims are such that understanding them does not suffice to assent to them.

The first option makes ineliminable use of the notion of the synthetic a priori. Plausibly, it entails that persistent virtuous disputes are grounded in a difference in the rational or a priori evidence accessed by both parties. Such a difference would be an additional disagreement factor, one that we have not considered so far but that has the potential in principle to explain persistent virtuous disputes. The second option has an air of oddity about it. It implies that one could, after decades of reflection, completely change his mind about an analytic claim he understood very well all along.²⁴ We believe that

²² We should emphasise that our claim here only bears on classical conjectures such as Fermat's theorem, Poincare's conjecture, Goldbach's conjecture and others. We shall see, with the Functions controversy, that there *are* in fact persistent disputes in mathematics, most notably disputes that, unlike these classical conjectures, concern the best way to understand certain mathematical objects, and so the choice of definitions and axioms (what we call "meta-analytic disputes"). We thank an anonymous referee for pressing us on that point.

²³ Unsurprisingly, Hirsh (2009)'s argument for the neo-Carnapian view that certain metaphysical disputes are verbal hinges on the fact that parties involved in these disputes regard their claims as "a priori and necessary".

²⁴ Williamson (2007,IV) has argued that any purportedly analytic sentence is such that two subjects who understand it could disagree about its truth. His argument does not make it clear, however, that the two subjects could persistently disagree, or even be rational enough to settle their dispute and disagree (see especially Williamson (2007, 91–92)). We shall see,

neither option is completely implausible (we are in fact quite sympathetic to the synthetic a priori option). Yet, unless they are fleshed out in more details, it seems that both strategies can only rename the problem of persistent disputes but not resolve it.

We are now able to sum up the pessimistic challenge to persistent disputes.

First, if persistent disputes have a point, they must involve disputants that are competent enough to settle the dispute. Yet it is difficult to see how such disputes may persist for an inordinately long time since, if they have a point, obstacles hindering the disputants' performances will be gradually overcome. Indeed, it seems that the longer a dispute lasts, the less reasons there are to persist.

Second, since parties in a persistent virtuous dispute swiftly gain access to the same relevant empirical evidence, their disagreement becomes apt and a priori in due course. This means that persistent virtuous disputes involve deceptive a priori truths: a priori truths that sufficiently rational thinkers, who do not err because of performance errors, reject and unknowingly fail to know.

We believe that even perfectly virtuous disputes can persist and have a point, hence that the pessimistic challenge may be answered, and in a rather mundane way. In order to answer this challenge, one need not appeal to any dubious form of rational intuition, nor posit cognitive biases, epistemic circles, or asymmetries in the access to the empirical evidence that can only be overcome at an extremely slow pace. One need just acknowledge the existence of a common type of disputes, that we call meta-analytic, and that, for reasons we will soon explain, can be extremely long to settle. Our argumentative strategy will rely on a real-world example: a well-known persistent mathematical dispute, which uncontroversially proved to have a point.

An example of scientific persistent dispute: the Functions Controversy

Persistent disputes are not specific to philosophy and may occur, as we have seen, within science as well. Showing that a given scientific controversy that seems persistent really is persistent is far from trivial, however, as it requires showing that it not covertly fuelled by new empirical discoveries (recalling that we have individuated persistent disputes by the relevant empirical evidence available).

The simplest way to circumvent this problem and to find an uncontroversial example of a persistent scientific dispute is to opt for an illustration coming from a purely formal science such as pure mathematics. Arguably, in this domain empirical evidence is irrelevant or at least non-decisive, and cannot end a persistent dispute.²⁵

in any case, that our proposed solution to the pessimistic challenge makes room for persistent disputes (those that have a point) concerning analytic sentences. Thereby, it does not threaten what we see as an important connection between analytic sentences and assent to such sentences by subjects who understand them. See §5.4 and especially fn. 29.

²⁵ Note that this means moreover that it is not necessary to show that such a dispute is virtuous in order to show that it is a priori if it persists.

Among disputes that have proved persistent, it is also difficult to find one that has uncontroversially proved to have point. Often enough, *prima facie* persistent disputes do not appear to be clearly or definitively settled. Equally, we believe that the domain of pure mathematics is interesting in virtue of its (approximately) cumulative character (pace Lakatos (1976)). In mathematics, the fact that a dispute has been deemed settled for a very long time seems to be a very strong reason to believe that it is indeed settled.

We understand that using a mathematical example might bring with it some additional complications. The semantics and ontology of mathematics are often deemed less straightforward than those of (say) geology or biology. We believe that these complications are rather light, and largely outweighed by the advantages of mathematics mentioned in the preceding paragraph.

The example we have chosen from pure mathematics is the Functions Controversy. This scientific dispute has the advantage of having been without a doubt both persistent and uncontroversially proved to have had a point.

Between the beginning of the 18th century and the end of the 19th century many controversies arose around different mathematical “results” concerning functions. Some of the controversial results were rather technical, but they included the following two simple claims²⁶:

1. Every function is continuous except possibly at a finite number of points.
2. Every continuous function is differentiable except possibly at a finite number of points (see Hawkins (1975, 43–44)).

Those claims were disputed because mathematicians were seemingly “discovering” some “objects” whose existence appeared inconsistent with their truth. In 1826 Abel, for example, showed that a certain function defined as a convergent series of continuous functions is discontinuous in an infinite number of points, apparently falsifying (1).²⁷ In 1829, Dirichlet discovered the eponymous “monster” function which seemed like a function continuous nowhere and thus to falsify (1).²⁸ In 1872, finally, Weirstrass introduced his own monster, which seemed to be a function that is continuous everywhere but nowhere-differentiable and to falsify (2).²⁹

²⁶ The mathematical layman can construe functions as graphs, discontinuities of a function as gaps in its graph, and the points at which it is non-differentiable as those where its graph does not admit a tangent.

²⁷ That function was $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sin(nx)}{n}$, which is discontinuous for every value $(2m + 1)\pi$ of x where m is an integer.

²⁸ The Dirichlet monster is $\chi_{\mathbb{Q}} = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{otherwise} \end{cases}$

²⁹ The Weirstrass monster is $f(x) = \sum_{n=1}^{\infty} b^n \cos(a^n \pi x)$ (with a an odd integer, b a real number in $]0,1[$ and $ab > 1 + 3\pi/2$).

These disputes involved earnest and rational thinkers, indeed some of the greatest mathematicians of the epoch ranged themselves on both sides of the debate. Yet, the disputes concerning (1-2) were persistent and were not clearly settled until the second decade of the 20th century and the acceptance of modern set theory. The question arises as to how (1-2) could be maintained by many thinkers of quality despite the above counterexamples. It would appear that some proponents of the controversial statements denied that the alleged counterexamples were significant exceptions to the general rule. Others denied that they were genuine *functions* or even that they existed at all.

Was the Functions Controversy verbal?

It is tempting to make the charge that the Functions disputes were at bottom merely verbal. Indeed, not all disputants understood the term “function” in the same way. Neither did they all define it with an equal degree of rigor and precision. Reviewing the best textbooks in analysis, Hankel noticed in 1870 that among them, “one [text] defines function in the Eulerian manner; the other that y should change with x according to a rule, without explaining this mysterious concept; the third defines them as Dirichlet; the fourth does not define them at all; but everyone draws from them conclusions that are not contained therein” (Kleiner 1989, 293).

There are however decisive reasons to think that even if the mathematicians’ understanding of functions and their standards of rigor differed quite significantly, this was not the cause of their disputes. If their dispute had been merely verbal (i) it would have been defused by the introduction of new undisputed names to refer to different kinds of functions (ii) its solution could only have brought about a terminological advance, as opposed to a substantial, genuinely mathematical progress. Neither of these was the case in the event.³⁰ By 1870, it was already clear to many that one could distinguish between the “algebraic” functions, which are defined by an “analytic expression” (i.e., algebraic formula), the “geometric” functions (i.e., whose curve can be drawn freehand) and the “logical” functions (i.e., arbitrary correspondences between two sets of value). Indeed, those who introduced this revisionary and more encompassing, logical definition of function still wondered whether all “logical functions” really existed, and if they did, whether they really were functions. Thus Lakatos (1976, 151) points out that according to Dirichlet himself, the “monster” he had discovered was “an example not of an ‘ordinary’ real function, but of a function which does not really deserve the name.” As late as 1904, Poincaré distinguished between logical functions and analytic functions (locally expandable in power series) and suggested that the former were not legitimate in mathematics (see Poincaré (1952, 125)).³¹ Even more strikingly, in 1905 Lebesgue, whose works permitted the generalization of the theory of integration to some “monstrous” logical functions, still argued that “*true*” functions are analytically representable (i.e., representable by an algebraic formula) (Lebesgue 1905, 139). Hermite essentially shared this sentiment concerning “this lamentable evil of functions without derivatives” (for Hermite’s view, see Kleiner (1989, 294)).

³⁰ The introduction of new names to settle a verbal dispute is what D. J. Chalmers (2011) calls the “method of elimination.”

³¹ Yet Poincaré seemed more open to mere “logical” functions in 1899 (Poincaré 1899).

Moreover, the lack of rigor and precision found in many of the mathematicians' definitions did not result from inattention or neglect. Hence, the disagreement could not have been solved simply by substituting more precise definitions for the imprecise ones. Many mathematicians at that time explicitly rejected our modern standards of rigor. It was common, for instance, to regard theorems as rules and mathematical predicates as not in need of a precise formal definition (see especially Richards (2006, 700–713), and Lakatos (1976, 24)). This seems also to have been the conception of Euler himself (Youschkevitch 1976, 67)). Rigor and precision could only develop, it was thought, at the cost of fruitfulness. As Maloney (2008) puts it, Lebesgue for one “[saw] the more precise and general definition of function, which we essentially use today, as a frivolity at best and a liability at worst.”

Ultimately, the solution to these disputes did not stem from terminological advance, but from a substantial mathematical progress. Modern set theory and distribution theory were developed in response to such controversies. These controversies were laid to rest eventually, but not before the emerging new theories had shown their credentials and become entrenched in mathematical practice.

Was the Functions Controversy empty, relativist or inscrutable?

As we explained, the Functions Controversy was not verbal. It did not hinge on the fact that some mathematicians, but not others, used a definition of functions, or true functions, that excluded the “monsters”. Rather, it rested on the fact that participants in these debates disagreed on which definition was the best, and ought to have been used. At this point, it might be suggested that the dispute was perhaps empty, or relativist. There is, however, a straightforward argument to the effect that the dispute was neither empty nor relativist. If it had been empty or relativist, it could not have been settled, and we could not be said to know that (1-2) are in fact false. The same argument, it should be noted, *ipso facto* shows that the dispute was not inscrutable either.

Before moving forward, it is worth pausing on the decisive claim that the Functions controversy has been settled, and has, accordingly a point. We believe that in the present state of mathematics this claim is uncontroversial. We also believe that it is (almost) uncontroversial that settling this dispute *that way* constituted a mathematical progress (denying this would require developing a revisionary / reactionary view of function that has no serious advocate today). What is less clear, and will be important later, is the normative status of this resolution, this progress and the point of the dispute. A radical conventionalist might argue that the Functions controversy was solved by the mere acceptance of a stipulation (to the effect that functions are logical functions) rather than by the discovery of a fact. He will probably concede that this resolution constituted a progress, but only because this stipulation was useful for us (and more useful than other conflicting ones) and insist that we only have practical reasons to consider (1) and (2) as true, not theoretical ones, and that the point of the Functions Controversy was somehow insubstantial or superficial. On the opposite side, Platonist, Kantians, Intuitionists and even, arguably, Poincaré style conventionalists will consider that mathematic truths do not depend on mere stipulations but on the structure of the world or of our minds, that mathematical progress is genuinely theoretical and substantial rather than merely pragmatic, and that the point of the Functions Controversy was thus deep or substantial. Let us call the first view of mathematical progress deflationist. We do not need to take a stand on this deflationism vs. non-deflationism debate here. What is important,

however, non-deflationism is very plausible and clearly is the majority view, that many philosophers attracted by the claim that progress in philosophy is impossible, scarce or at best pragmatic — and that the point of persistent disputes in philosophy is at best superficial — would be tempted to grant that mathematical progress is common and usually deep and theoretical.

The point of the Functions Controversy

If the Functions Controversy was neither verbal nor empty, and by the same token, neither relativist nor inscrutable, it follows that it must have had a point. What, then, was its point? One thing that our discussion suggests already is that this controversy did not concern the properties of something (namely, functions), of which the participants had a *common subjective understanding*. Neither did it concern the best way to articulate such a common understanding. There were no such common understanding. Rather, disputants understood functions quite differently, and they accordingly defined them quite differently and accepted conflicting clusters of analytic claims about them. And their dispute was (implicitly) about the best among their rival understandings. Some mathematicians thought that the best understanding was the algebraic or geometric one, and they assessed under its light all claims about functions. Others favored the logical construal, and these latter ended up on the right side of the debate, correctly denying (1-2). Granting that one's understanding of something is reflected in the analytic claims one is disposed to accept concerning that thing, we might say that the point of Functions Controversy was not analytic but rather *meta-analytic*. The fact that the Functions Controversy was not verbal shows that a dispute whose parties appeal to very different understandings of the object at issue need not be verbal, provided that it is meta-analytic.

This is not a trivial conclusion. It might even seem problematic. On the standard, neo-Fregean views of concepts (viz. ways of understanding something that determine the reference to that thing in context) different understandings imply different concepts, and if the parties disagree because they use (or preferentially use) different concepts, it seems that their dispute must be verbal after all. Fortunately, recent work in philosophy of language and metaphilosophy focused on related phenomena provide interesting ways out of this problem.

The first line of research in philosophy of language puts forward “relationist” or neo-Gricean semantics that canvass the possibility of successful communication between two subject that do not share the same concepts.³² More germanely still, the second line of research in metaphilosophy explicitly argues that what we call meta-analytic disputes are not verbal. Some philosophers working in the rapidly developing field of metalinguistic negotiations, conceptual ethics, and conceptual engineering understand meta-analytic disputes as meta-conceptual, but argue that the concepts involved, even if different still share a common feature which prevents the dispute from lapsing into the verbal. For instance, they are said to be about the same “topic” (Cappelen 2018, 102–3),

³² See e.g., Recanati (2012 VIII) on the first-person, “relationist” semantics according to which successful communication requires mere “coordination” or “de jure coreference,” (Fine 2007; Taschek 1995; Pryor 2016), and neo-Gricean views that can likewise grant a form of mutual understanding without concept sharing (Buchanan 2013).

or are said to play the same role (Thomasson (2020)). Others claim that meta-analytic disputes need not be verbal because the disputants share a similar meta-analytic aim. For instance, Belleri (2021) writes of a “semantically progressive inquiry” and asserts that the unity of inquiry is at the bottom teleological. Yet others invoke externalist views of concepts to argue that even though disputants understand the object at issue in inconsistent ways, they might still share the same concepts (Schroeter 2014; Schroeter and Schroeter 2016). Notably, Ball (2020) has argued that one should construe what we have called meta-analytic disputes as *metasemantic disputes*, that is, as disputes about the way one should “fix the meaning of words as we have used them before.” In this article, we remain neutral on the best view of concepts and meta-analytic disputes.³³ We observe however that there are many ways to do justice to the non-verbal character of such disputes.

The Functions Controversy and the Pessimistic Challenge

We say that a dispute is *meta-analytic* when it bears on the choice of the best way, among quite different candidates, to understand something, rather than on the attribution of properties to something the disputants understand in the same way, or on the best way to articulate their shared understanding of it.

Interestingly, the meta-analytic reading of the Functions Controversy allows us to provide a simple answer to the pessimistic challenge.

Take the competence problem first. According to the proposed interpretation of the dispute, what prevented disputants from agreeing was that they did not all understand (and hence define) functions in the same way. More deeply, they disagreed about which understanding was the best. But how, one might ask, could they disagree about that if they were all competent enough to find out which understanding is the best, and time cancelled the “usual suspects” for performance errors?

The comparative quality of competing understandings in pure mathematics and elsewhere depends importantly on their consistency and relative fruitfulness. It depends, more broadly, on their inferential profiles, that is, on all the inferences one can draw by their means. For finite minds like ours, however, evaluating such an inferential profile is not instantaneous. Each inference takes a very small amount of time to assess, but the number of inferences that need to be assessed is virtually infinite. Assessing the inferential profile is thus an *open-ended process*, that is, a process to which we cannot assign an a priori upper bound in time, be it in terms of years, or even centuries. Moreover, this process may prove surprising, as apparently consistent understandings may sometimes prove inconsistent (think of the naive understanding of sets, for a classical example), and apparently useless re-construals may sometimes prove fruitful. This means that assessing the relative merits of different ways to understand an object will not only be an open-ended, but also *anon-monotonic* one: a process that may lead from a time when we have most reason to favour one understanding U_1 over the other one U_2 , to a time when we have most reason to favour U_2 over U_1 .

For example, Poincaré, Lebesgue and Borel did not know, and they arguably could not have known without years of inquiries and intricate discussions with peers, that the

³³ We would like to thank an anonymous referee of this journal for pressing us on the multiple possible interpretations of meta-analytic disputes.

logical understanding of a function would find its place in an important and consistent mathematical theory (set theory), that classical analysis would easily accommodate it and that it would prove extremely fruitful in many fields (the popular Fractal theory is precisely a theory of “monstrous,” supposedly merely logical, functions) and help provide many mathematical insights.³⁴ It is in fact arguable that they had good reason, at the very beginning of the 20th century, to dismiss merely logical functions as useless curiosities.³⁵

The open-ended character of the process of assessing competing understandings successfully explains why it took mathematicians so long to answer the *questions* surrounding (1-2), and consequently to find out which understanding of “function” was the best. Conjoined with the non-monotonic character of such a process, it furthermore explains how such persistent questions gave rise to persistent *disputes*. Each time a new aspect of the inferential profile was discovered, its assessment necessarily took some time, allowing for the emergence of dissenting views on the questions under scrutiny. In general, as time passes, new results are made public, cognitive biases and performance errors are removed through fruitful dialogues and debates, and experts become able to fully grasp them. But by the time this process reaches completion, new aspects of the competing inferential profiles may have been discovered, whose assessment may once again give rise to dissenting views through additional performance errors, cognitive biases, or simply ordinary delays and difficulties in communication. If assessing the comparative merits of two understandings were a monotonic process, it could be argued that disputants should have agreed sooner or later, owing to the gradual cancellation of communicative difficulties, biases and performance errors. Arguably, they should have inferred, by monotonicity that the dispute was settled one and for all. Nevertheless, as we explained above, the comparative assessment of two understandings is far from monotonic.

We pointed out at the outset that there is nothing mysterious in a dispute that lasts for a very long time if new relevant empirical evidence arises through continuous discovery. We are now able to make this thought more precise. There is no mystery because the process of assessing a growing body of empirical evidence is open-ended, if the body of evidence grows, and non-monotonic. The Functions Controversy persisted because it a special kind of rational, non-empirical evidence whose assessment is both open-ended and non-monotonic, similar, in that respect, to the assessment of a growing body of empirical evidence, and unlike the assessment of trivial analytic evidence. The relevant a priori evidence was in a sense *accessible* all along to all parties, granted sufficient

³⁴ Commenting on the set theoretic paradoxes Poincaré reportedly prophesied: “later generations will regard Mengenlehre (set theory) as a disease from which one has recovered” (but see Gray (1991)).

³⁵ It should be noted that in pure mathematics the comparative quality of two ways of understanding and defining an object is an a priori matter. Arguably, an understanding of an object is better than another if it is mathematically more fruitful and does not lead to contradictions; that is, roughly, if it can yield better mathematical insights. It is true that the claims that most mathematicians prefer a certain understanding, or that they find it more fruitful, are a posteriori, but that is merely a posteriori evidence of an a priori truth (just like the fact that most mathematicians believe last Fermat’s theorem has been proved is a posteriori evidence of the a priori truth that its purported proof is valid).

rationality. Being, however, open-ended and non-monotonic, its assessment took a very long time.

Pessimists grant — or should grant — that new empirical evidence may fuel ongoing debates in such a way that thinkers continue to disagree over the same issue for decades or even centuries. We suggest that their outright rejection of persistent disputes, in which by our definition the empirical evidence is fixed, reveals an unjustified refusal to acknowledge the existence of a type of evidence that is akin to empirical evidence in that its assessment is open-ended and non-monotonic, but that is, like trivial analytic evidence, a priori. This evidence concerns in particular the assessment of different understandings, which is open-ended, non-monotonic and sometimes a priori. It is meta-analytic.

The meta-analytic reading of the Functions Controversy thus answers the competence problem. It also explains why disputants could disagree on an a priori claim. Thus, it can solve the problem of apt a priori disagreements. Even though the participants in the dispute preferentially resorted to different understandings of the concept of a function, we have seen that they were not talking past each other, and that their disputes were not verbal, because they were meta-analytic. The fact that in a meta-analytic dispute two parties can *without misunderstanding* understand a disputed sentence in a radically different manner should already dispel the suspicion, associated with certain views of the a priori and the analytic, that any apt a priori disagreement must be verbal. The fact that a priori meta-analytic disputes can be solved shows that apt a priori disagreement need not imply that the disputes are either empty or relativist. More broadly, the meta-analytic reading of the Functions Controversy implies that there are some a priori claims that can only be known and understood by rational subjects' appeal to the best kind of understanding of the subject matter. So, for example, the statement to the effect that "monster functions are genuine functions" can only be known to be true by a subject who understands functions in the right way. While a subject interpreting it in the correct manner will endorse it, one who interprets it in another way is likely to deny it, even though she understands it, hence to fail to know that she does not know that monster functions are true functions.

This explains why, despite lively exchanges, some rational subjects might fail to assent to a given a priori truth, or even might dissent from it, unknowingly failing to know that it is true. We have called *deceptive a priori truths* truths on which rational subjects can aptly disagree, and that they can, accordingly, wrongly believe to be false, not knowing that they do not know them. On the meta-analytic reading, the existence of *deceptive a priori truths* is not mysterious. It does not require us to posit unusual or non-standard analytic truths, or a *puzzling form of synthetic a priori*. Rather, it stems from the fact that different subjects associate different understandings, and so different analytic truths, with a given term, even though they both understand the term, and therefore don't misunderstand each other, or talk past each other.³⁶

³⁶ Interestingly, in the course of his argument to the effect that two subjects who understand a purportedly analytic sentence can nevertheless disagree over it, Williamson (2007) considers the hypothesis that subjects might disagree because they associate different concepts to the same words (and different thoughts to the same sentences) but rejects it on the ground that it would undermine Frege's requirement of the publicity of senses (p.114)

The point of persistent disputes

The Functions Controversy allows us to draw the following conclusions. First, the fact that a dispute is persistent, or even persistent and virtuous does not entail that it is pointless. Second, a good explanation as to why some disputes persist is that they are meta-analytic and that meta-analytic evaluations, being open-ended and non-monotonic, can take decades or even centuries. In order to find the best understanding of a term, one might need to assess the full inferential profile of the latter, which requires much time and can always prove surprising. Finally, and given the plausibility of the non-deflationary view of mathematical progress, the point of persistent meta-analytic disputes can arguably be deep, substantial than merely pragmatic.

The mere fact that a dispute is meta-analytic, as the example of the Functions Controversy shows, does not entail that it is pointless. The same could be said about the fact that the dispute is a priori. Even if virtuous persistent disputes become a priori, that does not make them pointless, because some evaluative claims about the comparative quality of different understandings are a priori, yet can yield persistent disputes that have a point.³⁷

When the Pessimist proposed an abductive argument to the effect that all persistent disputes are pointless, she may well have been right to suppose that the disagreement factors in the epistemic profile of a dispute (to recall, asymmetries in rationality, in the access to the empirical evidence, and vicious epistemic circles) cannot explain its persistence. The Pessimist was wrong, however, to draw the conclusion that the best explanation of the persistence of a dispute is always that it is pointless. In some cases, the best explanation is that the dispute is meta-analytic and that meta-analytic disputes can involve the open-ended and non-monotonic assessment of priori evidence. In such cases, a persistent dispute needs not be pointless. The competence challenge is only challenging for someone who neglects, among the disagreement factors, the difficulty of meta-analytic evaluations.

For all we know, there might be persistent disputes that are not pointless even though they are not meta-analytic. Yet we would like to suggest that our diagnosis is quite general, and that many persistent disputes in philosophy, in sciences and in public life (i) are meta-analytic and a priori (ii) persist precisely for this reason (iii) crucially, are not necessarily pointless.

and that it would render the dispute verbal (p.115). We believe that Frege's requirement is already challenged on other grounds — something Frege (1956, 298) himself seems to acknowledge and that can be accommodated rather well (Recanati 2012 VIII) — and that associating different concepts with a word in a disputed claim does not make the dispute verbal if, like in the case of the Functions controversy, the dispute is (implicitly) about the best way to understand the word and its denotation. Cf. fn. 27.

³⁷ It is in fact tempting to dispel the apparent mysteries of the notion of synthetic a priori by claiming that that synthetic a priori claims are simply meta-analytic claims.

Meta-analytic disputes, metalinguistic negotiations and deep disagreements

The view that many persistent disputes are meta-analytic disputes (as we have called them) is not entirely new. Arguably, it has been held under various guises by many philosophers, in relation to certain scientific and philosophical persistent disputes. Carnap's argument against traditional ontology, for example, relied on the thesis that disputes over meta-analytic questions (which he dubbed "external questions") are empty, or perhaps relativist, (see fn. 7). The view that persistent disputes are meta-analytic may well be at the root of Gallie (1955)'s influential take on "essentially contested concepts." It may also be said to inform Williams' (2011VII-X) analysis of ethical disputes and, arguably, Kuhn (2012)'s understanding of (the disputes surrounding) scientific revolutions.

More recently Sider (2009) has construed metaphysical disputes as disputes over the best understanding of quantifiers (and the best quantifiers concepts). Many works in the field of metalinguistic negotiations, conceptual ethics, and the conceptual engineering literature have argued in a similar vein that philosophical disputes are often metaconceptual (and hence meta-analytic) disputes (Plunkett 2015; Burgess and Plunkett 2013; Cappelen 2018).

Likewise Fogelin (1985) noticed that many disputes are "deep" in the sense that they stem from "a clash in underlying principles", can accordingly persist even though "the parties [are] unbiased, free of prejudice, consistent, coherent, precise and rigorous" and "by their nature, are not subject to rational resolution". Godden and Brenner (2010) and Shields (2018) have all argued that deep disagreements are in fact meta-conceptual.

Our view that some meta-analytic disputes are both substantive and scrutable, and can persist without being pointless, is much less widespread, however. Indeed, all these authors, except the most recent (e.g., Sider, Plunkett, Sundell, Burgess, Capellen, Shields (2018)), seem to believe that meta-analytic or "metaconceptual" questions are pointless. To our knowledge, even the latter do not put forward, as we do, an explicit argument to the effect that such disputes can be *persistent* and still have a point.³⁸ More importantly, all of them seem to hold that the point of a meta-conceptual dispute is always somehow pragmatic rather than deep and substantial.³⁹ The plausibility of the non-deflationist view of mathematics strongly suggest that they are wrong.

It is also worth noting that we have hinted at an argument for the pervasive character of meta-analytic disputes, just above, but that this argument --- call it the pervasiveness argument --- is quite different from those typically proposed in the metalinguistic and

³⁸ Plunkett and Sundell (2013, 241–44) do claim that metaconceptual disputes are *worth having*. Plunkett (2015) argues that much philosophy is (at least implicitly) metaconceptual. However, as we have seen, a dispute can be *interesting* and hence worth having without *having a point* (see p.4). It can even be worth having while being non-empty, non-verbal, and non-relativist but pointless (see p.8).

³⁹ This is connected to the claimed Carnapian inheritance of the conceptual engineering literature and to the claimed Wittgensteinian inheritance of the deep disagreement literature. See especially Shields (2018).

conceptual ethics literature. First, in this literature meta-analytic disputes are always construed as metaconceptual or metalinguistic. We saw that there are other construals of meta-analytic disputes. Second, the most thorough arguments for the pervasive character of metaconceptual and metalinguistic disputes essentially rely on the linguistic data surrounding some (potentially pointless) ordinary as well as philosophical disputes. Plunkett (2015)'s important argument in this vein is a case in point, insofar as it is a linguistic argument applied to metaphilosophical questions. Roughly, his argument is that:

- (i) some linguistic data suggest that a given exchange is a dispute whose parties really disagree (i.e., they do not misunderstand each other), but mean different things by the disputed sentence.
- (ii) the claim that their dispute is a metalinguistic negotiation can explain these data, and it can explain them more simply than the claim that the dispute is relativist or empty, which relies on complex non-standard semantic frameworks (such as recent brands of expressivism or relativism) (p.848-9).

Our argument relies partly on linguistic data as well, to wit, the data surrounding the Functions Controversy. It relies mostly, however, on epistemological and historical considerations to the effect that:

- Some persistent meta-analytic disputes have proved to have a point (the Functions Controversy).
- The meta-analytic reading of a persistent virtuous dispute allows us to defuse the best arguments for the charge that it is pointless, in answer to the pessimistic challenge.

Accordingly, the ground for ruling out the rival relativist or expressivist analyses is not the greater complexity, but the implication of pointlessness carried by these alternative interpretations. One might see our pervasiveness argument as contributing the metalinguistic negotiations literature by providing an additional, optimistic reason to believe that many scientific and philosophical disputes are implicitly meta-analytic (and thus maybe metalinguistic and metaconceptual) because they persist and have a point. And of course our main argument strengthens the interest of such disputes as it shows that they can have a point even though they are persistent.

Conclusion

In this article, we examined and rejected the widespread imputation that persistent disputes are pointless. Thus, we characterized pointless disputes, put forward a typology, and reconstructed the strongest pessimistic argument against the claim that persistent disputes might have a point. To defuse the pessimistic argument, we proposed a meta-analytic reading of a concrete example: the illustrious "Functions" controversy. In general, when a dispute is meta-analytic, disputants disagree about which understanding or set of analytical truths among different candidates is the best one. The epistemic difficulty of settling the disagreement at this level is what renders their dispute persistent. Significantly however, it does not render it pointless, as this collective task is achievable in principle.

If this is true, then one should not have unnecessarily sanguine expectations of the time it takes to settle such a dispute. To paraphrase Hegel, who might here be classified as one of the greatest optimists in the history of philosophy, one should trust the “power of the negative,” for, in some instances, the very negativity of a sustained disagreement may strengthen the natural power of reason.

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