With Iterative and Bosonized Coupling towards Fundamental Particle Properties

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Previous results have shown that the linear topological potential-to-phase relationship (well known from Josephson junctions) is the key to iterative coupling and non-perturbative bosonization of the 2 two-spinor Dirac equation. In this paper those results are combined to approach the nature of proton, neutron, and electron via extrapolations from the Planck scale to the System of Units (SI). The electron acts as a bosonizing bridge between opposite parity topological currents. The resulting potentials and masses are based on a fundamental soliton mass limit and two iteratively obtained coupling constants, where one is the fine structure constant. The simple non-perturbative and relativistic results are within measurement uncertainty and show a very high significance. The deviation for the proton and electron masses are approximately 1 ppb (10^{-9}), for the neutron 4 ppb.

Introduction. In the context of the sine-Gordon equation (SG) [1] mass and coupling constants constitute phenomenological dimensions without any direct relation to the topological processes. If topological phases evolve (stepwise) proportional to the potential (as often observed) a possible iterative approach to coupling constants is already included [2] and provides i.e. for an iterative approach to the fine–structure constant. Additionally, a simple bosonization is possible and based on a Dirac Hamiltonian that carries pairs of standard vector and scalar potentials [2] and requires the standard hydrogen Hamiltonian to arrive at the SG that represent bosonized spin-orbit coupling which requires to compare at least two interacting states carrying spin and opposite currents.

Bosonizing a radial symmetric Dirac equation. We start with interaction described by massless vector particles (photons) and massless scalar mesons within a relativistic local quantum field theory. The Dirac equation $\hat{H}_D\Psi = M_\gamma\Psi$ based on a Dirac - Hamiltonian $\hat{H}_D$ for a mixed potential consisting of a scalar potential $V_s(r)$ and a vector potential $V_v(r)$ is given by

$$\hat{H}_D = c\hat{\alpha}\cdot\hat{p} + \hat{\gamma}_0\left[V_0 + V_s\right] + V_v\,, \tag{1}$$

where $\hat{\alpha}$ and $\hat{\gamma}_0$ are the Dirac matrices [3, 4]. The vector and scalar potentials will provide for spin–spin and spin–orbit coupling and be helpful to bosonize the Dirac equation with two opposite parity two–spinors. In the same way the vector Coulomb potential part of V_v corresponds to the exchange of massless photons between a nucleus and leptons orbiting around it, the scalar Lorentz potential potential part of V_s with $-1/r$ characteristics corresponds to the exchange of massless scalar mesons. The resulting energy eigenvalue V_γ corresponds to opposite parity two-spinors components with total mass V_0. The two-spinor wave–function in spherical symmetry is given by

$$\Psi \propto \begin{pmatrix} \psi_R(r)y^m_{jl_R} \\ i\psi_L(r)y^m_{jl_L}\end{pmatrix}, \tag{2}$$

where y^m_{jl} are the normalized spin-angular functions constructed with Pauli spinors and spherical harmonics of order l. The spinors ψ_R and ψ_L are eigenfunctions with eigenvalues $l_R(l_R+1)$, and $l_L(l_L+1)$, respectively. A new parameter κ can be interpreted as orbital (spin) excitation between the two–spinor components, where $2\kappa = l_R(l_R+1) - l_L(l_L+1) > 0$ characterizes a left/right spin-asymmetry or difference. The radial functions have to obey the asymmetry relation

$$\begin{aligned}\frac{d^2\theta_R}{dr^2} &= -\frac{\kappa}{r}\frac{d\theta_R}{dr} + \left[\frac{V_0 - V_\gamma + V_s - V_v}{\hbar c}\right]\frac{d\theta_L}{dr}, \\ \frac{d^2\theta_L}{dr^2} &= +\frac{\kappa}{r}\frac{d\theta_L}{dr} + \left[\frac{V_0 + V_\gamma + V_s + V_v}{\hbar c}\right]\frac{d\theta_R}{dr}.\end{aligned} \tag{3}$$

Topological currents. The $1/r$-terms carry dimensional information and can be interpreted as fractional dimension shifts, κ by spin–asymmetry (a fractional parity property) and α by vector potentials. For SG–solitons a radial topological current can be introduced according to

$$q(r) = \frac{\partial_r\theta}{r}, \quad \frac{2V}{\mu} = (\partial_r\theta)^2 = q(r)^2 r^2, \tag{4}$$

where $q(r)$ can be interpreted as a (fractional) radial dimension shift induced by vector and/or scalar currents [5]. $q(r)$ simply represents the electric charge density. Let the Dirac equation describe topological charge and current density, the 2-dim. case allows to introduce orthogonal topological currents or phase gradients by

$$\psi_L(r) = \frac{\partial_r\theta_L}{r}, \quad \psi_R(r) = \frac{\partial_r\theta_R}{r}\,, \tag{5}$$

where

$$|\Psi|^2 = q(r)^2 = \psi_L(r)^2 + \psi_R(r)^2. \tag{6}$$

Constant q eq.(4) immediately provides for a harmonic oscillator coupling potential $V \propto r^2$ with proportionality between potential and phase

$$V(r) = \frac{\mu}{2}\left(qr\right)^2, \quad V(\theta) = \frac{\mu}{2}q\theta, \quad E_\mu = \mu c^2\,, \tag{7}$$

and unit condition

$$V_0 = V(r=1) = V(\theta = q) = \frac{\mu}{2}q^2\,. \tag{8}$$

Stationary boson exchange. The balancing condition is given by

$$\frac{\psi_R(r)}{\psi_L(r)} = \frac{\alpha}{\kappa} > 1\,, \tag{9}$$

this couples opposite parity radial functions, and provides for a wide variety of possible Lorentz scalar $V_s(r)$ and vector Coulombic potential $V_v(r)$ functions [6, 7]. With eq.(5) both types of $1/r$–coupling, coulombic coupling and spin–asymmetry correspond to relative topological phase evolutions

$$\theta_L = \pi\kappa f(r), \quad \theta_R + \theta_0 = \pi\alpha f(r), \tag{10}$$

where $f(r)$ will be a special function of r and θ_0 a phase offset. This couples opposite parity radial functions, and provides for a wide variety of possible Lorentz scalar $V_s(r)$ and vector Coulombic potential $V_v(r)$ functions that have to obey the asymmetry relation

$$\frac{V_\gamma}{V_0} = \frac{V_v(r) + \frac{\hbar c\alpha}{r}}{V_s(r) + \frac{\hbar c\alpha}{r}} = \frac{\alpha^2 - \kappa^2}{\alpha^2 + \kappa^2}\,. \tag{11}$$

To couple opposite parity components both type of potentials have to include Coulombic $-\hbar c\alpha/r$ terms. But finally those two components will merge after bosonization to one $-\hbar c\alpha/r$ potential and give the SG equation. The two first order ODE can now be combined to a Schrödinger/Klein–Gordon type relativistic wave equation that have bosonic solutions.

Coupling by sine–Gordon. With eq.(9) it is sufficient to consider only one part, therefore we omit the index $\theta_R \to \beta\phi$

$$\frac{d^2\phi}{dr^2} = \frac{\kappa}{r}\frac{d\phi}{dr} + \left[\frac{V_0 + V_\gamma + V_s + V_v}{\hbar c}\right]\frac{\kappa}{\alpha}\frac{d\phi}{dr}\,. \tag{12}$$

With the SG potential

$$V(\phi) = \frac{\mu}{2\beta^2}[1 - \cos(\beta\phi], \tag{13}$$

and SG equation

$$\frac{d^2\phi}{dr^2} - \beta^{-1}\sin(\beta\phi) = 0\,, \tag{14}$$

the vector and scalar potentials necessary to bosonize the Dirac equation with two opposite parity two–spinors merge to one Coulomb-type potential

$$V_s + V_v = -\frac{\hbar c\alpha}{r}, \tag{15}$$

where

$$\frac{(V_0 + V_\gamma)}{\hbar c}\frac{\kappa}{\alpha} = \cos(\beta\phi/2) = \frac{(V_0 - V_\gamma)}{\hbar c}\frac{\alpha}{\kappa}\,. \tag{16}$$

Background-soliton and soliton-soliton coupling. According to [8] basic background fluctuations mediated are by linear waves with mean potential $2\overline{V} = \hbar c/\lambda_1$ that couple to solitons in a Compton–type permanent scattering process. The soliton Compton wavelength λ_μ is related to the effective background (cut–off) wavelength by $\lambda_1 = q^2\lambda_\mu$, q^2 characterizes localization or the reduction of fluctuation amplitude between background and soliton. Another q^2 factor corresponds to the fluctuation reduction due to soliton–soliton coupling. This means, that soliton–soliton interaction and correspondent topological currents scale with q^4 with respect to the linear and massless background reference, where the coupling cascade is given by

$$2V_0 = q^2 E_\mu = q^2\frac{hc}{\lambda_\mu} = 2q^4\overline{V}. \tag{17}$$

More can be found in [2].

Iterative coupling. The linear relationship between potential and phase eq.(8) provides for an iterative solution, where the optimum phase shift θ_M is given by

$$\theta_M = \pi\alpha = \beta\phi\,, \tag{18}$$

$$M\theta_M = -2\pi J\cos\theta_M\,. \tag{19}$$

The Sommerfeld fine structure constant can be well approached by $\alpha_{137} = 1/137.00360094...$ [9, 10] and fits well to the interpretation, that the topological phase gradient or charge/current q provides for a dimensional shift.
In the permanent radial symmetric background–soliton Compton scattering process a radial phase gradient will be induced that corresponds to a radial coupling potential $V_r(\theta)$ driven by $\overline{V}$

$$1 - \cos(\theta) = \frac{V_r(\theta)}{\overline{V}}\,. \tag{20}$$

And again, the linear relation between phase and potential leads quickly with $E_\mu = V(2\pi J)$ to the iterative condition for the optimum phase shift

$$1 - \cos(\theta) = \frac{\theta}{\pi}\frac{q^2}{2\pi J} = \frac{1}{N}, \quad \theta = \pi\kappa\,. \tag{21}$$

Eq.(21), $1/q^2 = 12\pi^2$, and $J = \frac{1}{2}$ provides for $\kappa = 1836.11766....$ The value of κ controls the topological phase gradient induced by radial spin–asymmetry.

TABLE I:
The results of the α-κ model compared to values in [12].

Name	*Not.*	*≈ meas. value*	*dev.*	*calculation*	*comment*
fine structure const.	α^{-1}	137.03601144(498) [13]	$\approx 1.5\cdot 10^{-8}$	137.036009411..	eq.(19), from iteration, [9]
$N = 3805.5$	$\alpha^{-1}_{3805.5}$	137.03599976(50)	$\approx 6\cdot 10^{-9}$	137.036000525..	modal shifts in [9]
spin asymmetry	κ^{-1}	1836.118	$< 10^{-7}$	1836.1176608..	$\approx 6\pi^5$ eq.(21), from iteration
$N = 683174$	κ^{-1}_{683174}			1836.11748207..	eq.(44), incl. modal shift
current w.n.	k_μ, q_μ	$4.76804431085...\cdot 10^{15}\mathrm{m}^{-1}$	exact	$2\pi q^2\Xi^2\mathrm{m}^{-1}$	eq.(24), energy 940.8637..MeV
Vacuum permittivity constant	ϵ_0	$8.95419..\cdot 10^{-12}(\mathrm{s/m})^2$	$\approx 1.35\%$	$8.97413..\cdot 10^{-12}(\mathrm{s/m})^2$	eq.(41)-eq.(43), the fixed SI value is $\epsilon_0 = 10^7/(4\pi c^2)$.
electron	ξ_{e-}	$5.431168277\cdot 10^{-4}$	$-9.8\cdot 10^{-8}$	$5.4311688808\cdot 10^{-4}$	eq.(36)
electron + correction	ξ_{e-}	$5.431168277\cdot 10^{-4}$	$< 10^{-9}$	$5.4311682794\cdot 10^{-4}$	eq.(36), eq.(44), including modal shift $(-9.7\cdot 10^{-8})$
proton	ξ_{p+}	0.9972454102	$+9.6\cdot 10^{-8}$	0.99724531409	eq.(37)
proton + correction	ξ_{p+}	0.9972454102	$< 10^{-9}$	0.99724541114	eq.(37), eq.(44), including modal shift $(+9.7\cdot 10^{-8})$
neutron	ξ_n	0.9986200355	$4\cdot 10^{-9}$	0.99862003154	eq.(38)
? small neutral	ξ_m	$q_m \approx q_\pi$?	?	0.2728144..	eq.(39), 256.68 MeV

Origin of the basic soliton energy. With Planck units assigned to the background fluctuation level we get $\lambda_1 = q^2\lambda_\mu = 1$, $q^{-2}E_\mu = 1$. To get the correct baryon mass scale in our SI system, we have to shift the Planck scale velocity units to human artificial velocity units based on SI length and arbitrary mass units. Planck velocity units demand that the light velocity equals the unit velocity $c = u = 1$ such, that the mean background energy $\overline{V}$ scales with the square of the wave velocity and the SI unit energy scales with the square of the unit velocity u (in SI E_u =1J = 1kg m^2/s^2)

$$\frac{2\overline{V}}{E_u} = \frac{c^2}{u^2} = \Xi^2, \quad \Xi = 299792458\,. \tag{22}$$

Practical applicability of the SI system motivates to expect a unit velocity $0 < u \ll c$ with $\Xi = c/u \gg 1$. Particle and photon energies can be compared via Compton and photon wavelengths that refer to the light velocity. Planck length units demand that the 1-dimensional quantum energy of waves coupling to particles E_μ is inversely proportional to the wavelength, especially to the Compton wavelength with

$$\frac{E_\mu}{E_u} = \frac{\lambda_u}{\lambda_\mu}\,. \tag{23}$$

As a result, the characteristic soliton wavelength of one-dimension coupling is with eq.(17), eq.(22), and eq.(23) exactly given by [2]

$$\lambda_\mu = \frac{\lambda_u}{q^2\Xi^2} \approx 1,31777...\cdot 10^{-15}\mathrm{m}\,. \tag{24}$$

Eq.(24) provides for the basic soliton mass μ via Compton relation $\mu = h/(c\lambda_\mu) = q^2\Xi^2 h/(c\lambda_u)$. Realized in SI units the value is

$$\mu = \frac{\hbar}{c}\frac{\Xi^2}{6\pi\mathrm{m}} \approx 1.67724...\cdot 10^{-27}\mathrm{kg}\,, \tag{25}$$

where

$$\lambda_1 = q^2\lambda_\mu = \hbar c/(2\overline{V}) = q^2\lambda_\mu = |c^{-2}|/(2\pi)\mathrm{m}\,. \tag{26}$$

Currents and bridges. Supported by the Dirac equation, the following steps provide probably for the most effective strategy to obtain very accurate fundamental particle energies. The basic soliton energy will split [see eq.(4) and eq.(6)] into left- and right–handed parts E_L, E_R

$$E_\mu = E_L + E_R = 2q^2\overline{V}\,, \tag{27}$$

where Planck units provide for a common base. The correspondent left, right, and basic mass currents q_L, q_R, q_μ, respectively, can be defined with eq.(9) according to

$$E_\mu = q^2 = q_L^2 + q_R^2 = q_\mu^2(\alpha^2 + \kappa^2), \qquad 2\overline{V} = 1\,. \tag{28}$$

A (self-)bound state energy $V_\gamma < V_0$ will be assigned to the left–right current asymmetry that scales like eq.(17) and defines the ratio

$$\frac{V_\gamma}{V_0} = \left(\frac{q_\gamma}{q_\mu}\right)^4 = \frac{q_L^2 - q_R^2}{q_L^2 + q_R^2} = \frac{q_L^4 - q_R^4}{q_L^4 + q_R^4 + 2q_L^2 q_R^2}\,. \tag{29}$$

According to the soliton-soliton coupling relation eq.(17) coupling currents scale with the fourth power in q. But not the extra term $2q_L^2q_R^2$ in the normalizing denominator: it will be identified as the low-dimensional internal left/right bosonization bridge and leads to the one and only possible definition of a symmetric bridge in accordance with eq.(9) and eq.(29)

$$q_{e-} = \frac{q_L q_R}{q_\mu} \tag{30}$$

that can be normalized with eq.(28) to

$$\xi_{e-} = \frac{q_{e-}}{q} = \alpha\frac{q_R}{q} = \kappa\frac{q_L}{q}, \quad q_L = \alpha q_\mu, \quad q_R = \kappa q_\mu\,, \tag{31}$$

see appendix. Acting as a kind of direct topological bridge q_{e-} connects and balances left– and right–handed currents in accordance with the bosonization relation eq.(9). If the smaller current q_R becomes reciprocal to the bigger current q_L (i.e. by inversion) the coupling bridge $q_{e-} = q_L q_R/q_\mu$ can be removed by setting

$$q_L^4 = q_n^4 - q_m^4, \quad q_\pi = \frac{q_m}{q_n}q_L = \frac{q_n}{q_m}q_R, \tag{32}$$

providing for a neutral and symmetric decoupling bridge q_π with big neutral current q_n and small neutral current q_m, see appendix. The transition generates a decomposition without a bridge or mixed term given by

$$\left(\frac{q_\gamma}{q_\mu}\right)^4 = \frac{q_n^4 - q_m^4}{q_n^4 + q_m^4}\,, \tag{33}$$

where the neutral currents q_n and q_m scale with the fourth power like q_μ and split into two (“orthogonal”) parts as desired

$$E_\mu^2 = E_n^2 + E_m^2 = 4q^4\overline{V}^2\,. \tag{34}$$

Identification of particles Any (self-)bound stable particle should be lighter than μ of eq.(25). With q as the current, q_{e_-}, q_γ, and q_n can now be identified as electron, proton, and neutron currents, respectively. It is proper to assign the coupling currents and correspondent energy eigenvalues to dimensionless $\xi_x = q_x/q = k_x/k_\mu$ values that correspond to the ratio of measured particle Compton wavenumber given by

$$E_x = \hbar\omega_x = \hbar c k_x = \hbar\xi_x\omega_\mu = \hbar\xi_x c k_\mu\,, \tag{35}$$

with respect to the reference soliton Compton wavenumber k_μ, see table 1 and appendix. The bosonizing bridge q_{e-} based on the bridge in eq.(31) is with eq.(35) given by

$$q_{e-} = \frac{q}{\sqrt{\kappa^{-2}+\alpha^{-2}}} = \xi_{e-}q \tag{36}$$

and will represent the electron, the proton coupling and stabilizing partner. In equilibrium the coupling can be assumed to happen in rotary motion where the orbital phase evolution given by α provides for a velocity $v = \alpha c$. The correspondent relativistic factor $\gamma = 1/\sqrt{1-\alpha^2}$ increases the soliton background current of the co-rotating body-fixed frame q_γ and provides for the effective current

$$q_{p+} = \frac{q_\gamma}{\sqrt{1-\alpha^2}} = \left(\frac{\kappa^{-2}-\alpha^{-2}}{\kappa^{-2}+\alpha^{-2}}\right)^{1/4}\frac{q}{\sqrt{1-\alpha^2}} = \xi_{p+}q, \tag{37}$$

that represents the (proton) current in the laboratory frame. The big neutral current eq.(32) can be assigned to the neutron,

$$q_n = \left(\frac{\kappa^{-2}}{\kappa^{-2}+\alpha^{-2}}\right)^{3/4}\left(\frac{\kappa^{-2}-\alpha^{-2}}{\kappa^{-2}+\alpha^{-2}}\right)^{-1/4} q = \xi_n q\,, \tag{38}$$

the small neutral current to an unknown

$$q_m = \sqrt{\frac{\kappa}{\alpha}}q_n = \xi_m q\,. \tag{39}$$

Using the fine structure value $\alpha^{-1} \approx 137.036$ according to the Dirac equation with Coulomb coupling it turns out that for a special $\kappa^{-1} \approx 1836.118$ the system above can describe fundamental particle energies and mass ratios. Possible numbers are shown in table 1, κ is given similar to α as an iterative orbital coupling condition.

3-dim. charge and permittivity. The coupling currents or fluxes “flow” usually 3-dimensional. The Gauss relation connects the 1-d coupling topology to a spherical symmetry such, that the fine structure constant can be defined by

$$\alpha = \frac{q^2}{4\pi\epsilon_{00}\hbar c}\,. \tag{40}$$

This provides for

$$\epsilon_{00} = \frac{\lambda_1 q}{\hbar c}, \alpha_q = \frac{q}{4\pi},\ M = \left[\frac{1}{\alpha_q}\right] = 137\,, \tag{41}$$

where [] means next higher integral value [11]. Leaving Planck units and introducing the Coulomb charge unit by replacing q with the SI elemental charge e, $\epsilon_{00} = \lambda_1 e/(\hbar c) \approx 8.974129 \cdot 10^{-12}(\mathrm{s/m})^2$ is slightly above ($\approx 1.35\%$) the SI vacuum permittivity constant ϵ_0, see eq.(41) and [2]. If we assume, that eq.(41) is correct, the unit of charge is with the fixed SI value $\epsilon_0 = 10^7/(4\pi c^2)$ directly coupled to the unit of mass part of $\hbar$. As a consequence, the dimension of charge in eq.(41) becomes kilogram and we have determined the most likely charge–to–mass ratio of a fundamental baryon, where the reduction ϵ_0/ϵ_{00} could be due to nuclear effects and binding energies (the charge–to–mass ratio of protons depends always on the context) [2].

Local screening? But there is another interesting approach: if we take into account that the ratio $e/\hbar$ has been determined with the reduced V_γ in the bound state and not with the "free" V_0 value, the reduction of V_γ with respect to V_0 due to the radial phase gradient depends only on κ and α and is with eq.(11) $\approx 1.12\%$. The electric field surrounding an electron polarizes the QED vacuum. In QED this phenomenon leads to charge screening for the electron $q_e < q$ or $a = q/q_e - 1 > 0$ that can be found as a g-factor anomaly $g_e = 2(1 + a) > 2$. The screening can be translated a reduction of the topological phase gradient due to the bound state potential reduction $V_\gamma < V_0$ and dimensional shift. Splitting the shift in a q-dependent and V-dependent part (includes λ_μ, c, $e/\hbar$) where

$$\epsilon_{00} = q^2 \frac{\lambda_\mu}{c} \frac{e}{\hbar}, \quad \epsilon_e = \frac{q_e^2 V_\gamma}{q^2 V_0} \epsilon_{00} \,, \tag{42}$$

it turns out that $\epsilon_e = \epsilon_0$ requires a first order correction

$$1 + a = q/q_e \approx 1.00115810.. \tag{43}$$

that is quite near to the electron g-factor $g_e/2 = 1.00115962..$. The overall reduction $V_0 - V_\gamma$ by spin–asymmetry without screening effect for a given λ_μ is about 10.42MeV and could be indeed a source for strong interaction as a form of cooperative phase shift and dimensional reduction.

It should be noted, that the iterative method to determine the fine-structure extended by a variable charge Z in $M\theta_M = -2\pi J \cos(Z\theta_M)$ in eq.(19) is a chaotic system. As an iterative system it shows asymptotic stable and converging regimes but also bifurcations and unstable regimes for special feedback coupling strengths Z, the chaotic dynamics should be found in charged nuclei where $Z > 114$ and $J = 1$ reaches the critical value. A simulation can be found at [14].

The small difference. To fulfill the requirements of single-valuedness, κ coupling provides for both, orbital radial and standing waves, see fig.1. The correspondent adjustment requires to increase $\kappa' \to \kappa$ and angle $\theta' \to \theta$ to obtain an integer N in eq.(21). This process stretches the wave and reduces the orbital wavenumber. With integer $N = 683174$ there is a very small increase in $\kappa' \to \kappa$ given by

$$\delta_\kappa = \frac{1}{\kappa' N} \frac{\pi}{q^2} - 1 \approx 9.7 \cdot 10^{-8}, \kappa N = \frac{\pi}{q^2}, \quad N = 683174,\ N' = 683174.06648.. \tag{44}$$

In table 1 a similar shift can be found for both, the proton (same direction) and electron (opposite direction), while the neutron value is quite exact within measurement uncertainty. Neglecting the orbital effect of α, the orbital wavelength stretch in fig.1 and wavenumber reduction can be assigned to the electron since $E_{e-} \approx \overline{V} 2\pi/N'$. In electron–to–proton mass ratio measurements the proton mass now appears to be heavier. This could be the reason why $E_{p+} = \xi_{p+} E_\mu$ is increased by the relative difference δ_κ.

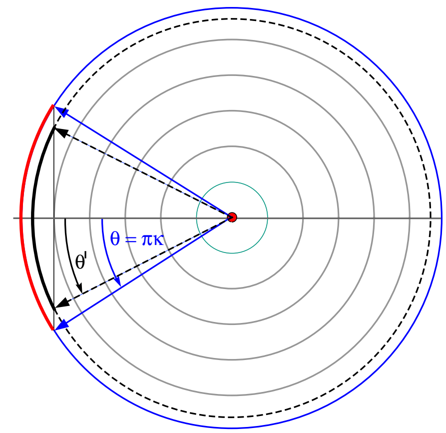

FIG. 1: Integer N in eq.(21) corresponds to both, the orbital and radial electron wavenumber. Increasing the angle $\kappa' \to \kappa$, $\theta' \to \theta$ to adjust to the integer N, stretches the orbital standing wave and reduces the orbital wavenumber, see also eq.(44).

Conclusion. Increasing the angle $\kappa' \to \kappa$, $\theta' \to \theta$ fits to the shifts tabulated in table 1 and provides for an overall deviation for the proton and electron mass of $< 10^{-9}$. Regarding weak interaction, the small neutral partner current should somehow be related to the neutrino, but 256.7 MeV would probably be too heavy for a neutrino. One could ask if it is an unmeasurable massless fluctuation current related to the massless background current. The local current and phase reduction including phase gradient and dimensional reduction (charge screening) provides for a possible understanding of $\epsilon_{00} > \epsilon_0$ connected to strong interaction. Regarding the currents, two neutrons provide approximately for the same reduction like one proton, see table 1. This fits to the well known proton–to–neutron number ratio found in heavy nuclei. Generally, these mass differences could be relevant for approaching (nuclear) binding energies with many–soliton models [8]. The corresponding energies are in the MeV–range. The neutron is energetically lower than μ, this fits with the semi–stability of a free neutron.

Since the vector potential V_v in eq.(15) already includes the negative coulomb term, $V_v = -V_s - \hbar c\alpha/r$ also includes the negative scalar potential. If we would make the scalar component very small we would find simply the relativistic hydrogen formalism providing for the famous result of Dirac [3, 4] including relativistic spin-orbit coupling. The two central coupling constants can be determined iteratively from the a linear SG potential–to–phase relationship - a general key that also provides for balanced and bosonized Dirac currents with a very high significance.

[1] M. J. Ablowitz and H. Segur,*Solitons and the Inverse Scattering Transform*, SIAM, Philadelphia, (1981).
[2] B. Binder,*Bosonization and Iterative Relations Beyond Field Theories* (2002); PITT-PHIL-SCI00000944.
[3] P. A. M. Dirac, Proc. R. Soc. Lond. A **117**, 610 (1928).
[4] W. Greiner, *Relativistic Quantum Mechanics: Wave Equations*, Springer-Verlag Berlin Heidelberg, 1990.
[5] B. Binder,*Higher-Dimensional Solitons Stabilized by Opposite Charge*; PITT-PHIL-SCI00000918, UniTexas mp-arc 02-499.
[6] J. Franklin, Mod. Phys. Lett. A **14**, 2409 (1999); hep-ph/9812464.
[7] A. S. De Castro and J. Franklin, Int. J. Mod. Phys. A **15**, 4355 (2000);hep-ph/0002093.
[8] B. Binder,*Soliton Compton Mass from Auto–parametric Wave–Soliton Coupling*; UniTexas mp-arc 02-475.
[9] B. Binder, 'Berry's Phase and Fine Structure' (2002); PITT-PHIL-SCI00000682.
[10] B. Binder, alpha simulation; http://www.quanics.com/spinorbit.html
[11] B. Binder,*Soliton Coupling Driven by Phase Fluctuations in Auto-Parametric Resonance*; UniTexas mp-arc 02-473.
[12] Groom et al., "The Review of Particle Physics", The European Physical Journal **C15**,1 (2000).
[13] E. Krüger, W. Nistler, W. Weirauch, Metrologia **36**, 147 (1999).
[14] B. Binder, Geometric Phase Bifurcation; http://www.quanics.com/bifurcation137.html

Appendix

Some additional algebra regarding coupled and decoupled partner currents:

$$q^2 = q_R^2 + q_L^2 \tag{45}$$
$$q^2(q_R^2 - q_L^2) = q_R^4 - q_L^4 \tag{46}$$
$$\alpha q_L = \kappa q_R \tag{47}$$
$$q_R = q\kappa/\sqrt{\kappa^2+\alpha^2} \tag{48}$$
$$q_L = q\alpha/\sqrt{\kappa^2+\alpha^2} \tag{49}$$
$$q_\mu = q/\sqrt{\kappa^2+\alpha^2} \tag{50}$$
$$q_{e-} = q_R q_L/q_\mu \tag{51}$$
$$q_{p+}^4 = q^2(q_L^2 - q_R^2)\gamma^4 \tag{52}$$
$$q_n^4 = q_L^4/(1-\kappa^2/\alpha^2) \tag{53}$$
$$q_m^4 = q_n^4(\kappa^2/\alpha^2) \tag{54}$$

$$q_n^4 = (q_L^4 - q_R^4)(1+\kappa^2/\alpha^2)/(1-\kappa^4/\alpha^4)^2 \tag{55}$$
$$q_n^4 = (q_L^4 - q_R^4)/(1-\kappa^2/\alpha^2)/(1-\kappa^4/\alpha^4) \tag{56}$$
$$q_n^4 = (q_L^4 - q_R^4)/(1-\kappa^2/\alpha^2)^2/(1+\kappa^2/\alpha^2) \tag{57}$$
$$q_n^4 = q^4(q_L^4 - q_R^4)/(q_L^2 - q_R^2)^2/(1+\kappa^2/\alpha^2)^3 \tag{58}$$
$$q_n^4 = q^4(q_L^2 + q_R^2)/(q_L^2 - q_R^2)/(1+\kappa^2/\alpha^2)^3 \tag{59}$$
$$q_n^4 = q^6/(q_L^2 - q_R^2)/(1+\kappa^2/\alpha^2)^3 \tag{60}$$
$$q_n^4 = (q_L^4 - q_R^4)/(1-\kappa^4/\alpha^4)^2(1+\kappa^2/\alpha^2) \tag{61}$$
$$q_n^4 = q^2(q_L^2 - q_R^2)/(1-\kappa^4/\alpha^4)^2(1+\kappa^2/\alpha^2) \tag{62}$$
$$q_n^4 = q^2(q_L^2 - q_R^2)/(1-\kappa^4/\alpha^4)/(1-\kappa^2/\alpha^2) \tag{63}$$
$$q_n^4 = q^4/(1-\kappa^4/\alpha^4)/(1+\kappa^2/\alpha^2) \tag{64}$$
$$q_n^4 = q^4/(1-\kappa^2/\alpha^2)/(1+\kappa^2/\alpha^2)^2 \tag{65}$$