The Fine-structure Constant and Some Relationships Between the Electromagnetic Wave Constants

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The four primary wave constants are brought together in one place. The often overlooked wave impedance is proposed to be exactly as fundamental as the speed of light. The little-used relationship between these constants highlights a central confusion in conventional electrodynamics units and theory. The fine-structure constant is reformed in terms of the wave impedance of the vacuum.

Keywords: TEM wave, Z_o, wave impedance, Ivor Catt, fine structure constant

Introduction

The transverse electromagnetic wave ("TEM Wave"), propagating at $c \sim 3\text{x}10^8$ m/sec, through the constant, aethereal, "wave impedance" of $Z_o \sim 376.73$ ohms appears to be the fundamental entity that transmits most, if not all, energy and information [1, 2]. This entity has four known, related properties.

The four crucial relationships below, between the four fundamental electromagnetic constants, have not been found together

on one page in any book or paper yet surveyed by the author. As Catt emphasizes, the wave impedance itself, Z_o, is rarely mentioned. It is treated briefly in a textbook from 1947 [2, p94], and also as a problem in one college-freshman physics textbook [3, p868]. It is not found, for example, in Jackson [4], Griffiths [5], Feynman [6], or in thirty or so other surveyed electromagnetics or electrodynamics textbooks and treatises published over the past two centuries, outside of radio engineering. However, see Note below.

Properties of the Vacuum and its TEM Waves

Z_o = impedance of the TEM wave (wave impedance)
c = speed of the TEM wave (speed of light)
ε_o = electric permittivity of the vacuum
μ_o = magnetic permeability of the vacuum

The two fundamental physical constants, Z_o and c, are 'reciprocal' to the permittivity and permeability of the vacuum (variously called the ether, the quantum foam, free space, and so on):

$$\varepsilon_o = \frac{1}{Z_o c} \tag{1}$$

And

$$\mu_o = \frac{Z_o}{c} \tag{2}$$

These properties are in turn manifest in the orthogonal, transverse axes of the TEM wave, and back. Z_o and c are uniquely determined by ε_o and μ_o, including the necessity of having the same sign, as:

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}} \tag{3}$$

and

$$Z_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} \tag{4}$$

Either of these two pairs of physical constants can be considered *separately* valid as fundamental *sets* of units. The 'reciprocal' set of relations, (1) and (2) above, for ε_o and μ_o, are also uniquely determined, and also have the automatic constraint of same sign.

Recasting the Fine-structure Constant

The Zeeman level-splitting of the electronic emission-bands of atoms in an external magnetic field is associated with the fine-structure constant, α. This was 'classically' derived from a balance of forces for the putative orbital speed, v, of the electron about a single proton in atomic hydrogen- $\alpha = v/c$ [7].

The fine-structure constant is often presumed to contain the speed of light, the electron's charge, the permittivity of the vacuum, and Planck's constant all together. This is not so, for the reasons shown above as Eq. (1) and Eq. (3). With Planck's quantum of action, h, then $\hbar = h/2\pi$, and e = electric charge of the electron, the fine-structure constant, α, is expressed, with four constants, as [7, p151]:

$$\alpha = \frac{e^2}{4\pi\varepsilon_o(\hbar)c} = \frac{e^2}{2\varepsilon_o hc} \cong 1/137 \tag{5}$$

Substituting Eq. (1) into Eq. (5) and cancelling yields:

$$\alpha = \frac{Z_o e^2}{2h} \tag{6}$$

This is a more compact representation in which the speed of light, c, plays no role. Its mutually exclusive, 'conjugate' property Z_o appears

instead. A constant should logically be defined with the fewest number of parameters. The speed-of-light does not necessarily have anything to do with the rhs of the fine-structure constant, contrary to an often-seen opinion, e.g in Tipler.

This expression can be rearranged as:

$$\alpha \frac{h}{Z_o} = \frac{e^2}{2} \tag{7}$$

which provides a different perspective on the nature of these ratios. A similar replacement applies to any electrodynamics relationship in which the three other, commonly used constants implicitly appear together, with two constants that form one of the exclusive sets.

Conclusion

It is unfortunate that members of these two exclusive sets of constants are mixed together in the same units systems and equations. c, μ_o, and ε_o are not independent of one another. They can be made to appear to be algebraically independent, but they are not physically independent: each shows up in the definition of the other. When they are mixed up together in what becomes a self-referential mathematical relationship, so too is the interpretation of what the equation is supposed to represent.

The reformed electrodynamics theory treats the separated relationships above as primary, in its conception and in its units. A units system has been developed in which impedance, Z, replaces electric charge, q, ohm for coulomb [8]. New insight into the physics behind the constants springs from this kind of reformation.

Note

Ivor Catt has brought to the author's attention that the four equations (1) - (4) above are indeed to be found together in his *Electromagnetic Theory*, p241, a book that is most unfortunately out of print [9]. Catt makes a slightly stronger claim, that "it is more correct to start with [Z_o and c], the directly measurable parameters of a region of space [10, p57]." Catt also states, "If the reader believes that [ε_o and μ_o] are fundamental, then he is impelled to equate the importance of [Z_o] with c, because both are simple, similar manipulations of [ε_o and μ_o] [11]."

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