A temporal mereology for distinguishing between integral objects and portions of stuff

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Abstract

We develop a formal theory of mereology that includes relations that change over time. We show how this theory formalizes reasoning over domains of material objects, which include not only integral objects (my computer, your liver) but also portions of stuff (the water in your glass, the blood in a vial). In particular, we use different mereological summation relations to distinguish between the ways in which i) integral objects, ii) portions of unstructured, homogenous stuffs (e.g. the water in your glass), and iii) mixtures (the blood in a vial) are linked to their parts over time.

1 Introduction

We present a formal theory for distinguishing between the mereological properties of different kinds of material objects. We take *material objects* to include, not only integral objects (your car, my computer), but also portions of stuff, such as the water in a glass, the gold in a ring, or the blood in a vial. Our theory is intended to serve as a basis for ontologies in fields like medical informatics where parthood relations play a central role in data-structuring and where domains include integral objects (livers, hearts, blood cells), homogenous unstructured stuffs (oxygen, water), and structured stuffs (in particular, mixtures such as blood or urine). Examples of bio-medical ontologies are the Foundational Model of Anatomy [Rosse and Mejino, 2003; FME, 2003] and GALEN [Rogers and Rector, 2000; OpenGALEN, 2003].

Unlike portions of stuff, integral objects may retain their identities through a full-scale change of parts. A human body, for example, is continuously rebuilt on a cellular level and will retain very few of its cellular parts over a period of ten years. By contrast, each portion of stuff necessarily retains certain ”minimal” components for as long as it continues to exist. For example, a specific portion of water is comprised of the same water molecules throughout its duration and a specific portion of blood is comprised of the same red and white blood cells, platelets, and plasma throughout its duration. But unstructured stuffs, like water, and structured stuffs, like blood, differ significantly in how they how they are linked to their minimal components over time. For example, a given portion of water continues to exist even if its molecules are randomly scattered. It may become part of a chemical compound or it may change its physical state (e.g., from liquid to gas) but it is still, strictly, the same portion of water.[1] By contrast, a portion of blood ceases to exist if, e.g., its cells are separated from its plasma.

Most bio-medical ontologies currently use only time-independent parthood relations, which assume a frozen, fixed time-slice view of organisms and their parts. There is general agreement, however, that more complex time-sensitive ontologies needed for data-tracking, and automated reasoning in medical fields concerned with physiological processes, organism development, and diseases. Developers of these ontologies need to make systematic distinctions about the ways in which the different sorts of items making up an organism (organs, blood, water, and so on) are linked to their parts over time. Our theory provides a vocabulary for making these distinctions as well as basis for reasoning about change in mereological relations over time.

The formal theory developed in this paper builds on that of [Simons, 1987, ch. 4]. Our theory differs in that it is formulated in standard predicate logic (Simons uses a free logic), uses a stronger set of axioms for the parthood relation, and, most importantly, introduces the different cross-temporal parthood and summation relations which are used to distinguish between the characteristic spatio-temporal properties of integral objects, unstructured stuffs, and structured stuffs.

We follow Simons in adopting what is known in contemporary metaphysics as the ‘standard account’ of material coincidence. According to this account, distinct material objects, such as a liver and the liver tissue of which it is made, may coincide (i.e.,occupy exactly the same place at the same time). Other proponents of the ‘standard account’ include [Wiggins, 1980; Lowe, 2003; Doepke, 1982; Fine, 2003]. Although the standard account is not universally accepted (see, e.g. [Rea, 1995], for several alternative positions), it is generally adopted by philosophers who treat objects as three-dimensional entities that gain and lose parts

[1]Notice, however, that the portion of water does not survive a scattering of its *atoms*. If its hydrogen atoms are separated from its oxygen atoms, we are left with just oxygen and hydrogen, not water. It is for this reason that the special constituting parts of the portion of water are its molecules, not its atoms.

over time. Since medicine distinguishes between an organ and the tissue of which it is made and treats organs and other body parts as spatial objects that change over time, we think that the standard account fits in better than alternative accounts with the assumptions grounding current work in medical informatics.

2 Examples

Before developing the formal theory, we first lay out some examples of the kinds of mereological relations among material objects that we expect it to handle. The examples illustrate characteristic distinctions in the cross-temporal mereological properties of integral objects, unstructured stuffs, and structured stuffs. They will be used in the later sections of this paper to illustrate the different kinds of parthood and summation relations introduced in our formal theory.

Although our theory is intended to serve as a basis for spatio-temporal reasoning in medical informatics, we use simple common-sense examples, and not medical examples, to illustrate the theory. We do this because the distinction between different summation relations introduced in our theory is somewhat complicated and difficult to grasp. We find that the theory is more accessible when it is illustrated by simpler sorts of items with which most readers are familiar. But the reader should keep in mind that the points made below about statues (integral objects), portions of gold (unstructured stuffs), and portions of lemonade (structured stuffs), apply equally to organs, portions of water (or oxygen, carbon, and so on), and portions of blood (or urine, liver tissue, plasma, and so on).

A. Suppose that a portion of gold (call it GOLD) is formed into a statue of Julius Ceaser (call the statue Julius). Let TJ be a time immediately after Julius is formed. According to the standard account, at TJ, Julius and GOLD coincide, but Julius and GOLD are not identical since Julius is only a few minutes old at TJ, but GOLD is much older.

In sections 3 - 5, we develop a temporal mereology which (like those of [Simons, 1987; Thomson, 1998]), assumes

(*) object x is part of object y at time t if and only if x is spatially included in y at t.

It follows from (*) that x and y occupy the same place at t (i.e. coincide at t) if and only if x and y have the same parts at t. Thus, every part of GOLD is part of Julius at TJ and every part of Julius is part of GOLD at TJ. In particular, GOLD, all sub-portions of gold in GOLD, and all gold atoms in GOLD are part of Julius at TJ. Also, Julius, Julius' head (JHead), Julius' right hand (JHand), and so on are part of GOLD at TJ.[2]

Note, however, that Julius, JHead, and JRHand are *different kinds* of parts of GOLD than are its gold atoms or its gold sub-portions. Let GOLDsAtoms be the collection of gold atoms which are at TJ part of GOLD and let GOLDsSubPortions be the collection of sub-portions of gold which are part of GOLD at TJ. All members of GOLDsAtoms and GOLDsSubPortions, unlike Julius, JHead, and JHand, *must* be parts of GOLD whenever GOLD exists. Julius may cease to be part of GOLD while GOLD continues to exist (if, e.g. GOLD is melted down), but GOLD cannot survive the loss of a single gold atom or sub-portion of gold.

Moreover, whenever all members of GOLDsAtoms (and consequently also all members of GOLDsSubPortions) exist, GOLD must also exist. Thus, even at times when the members of GOLDsAtoms are distributed randomly throughout the world, GOLD must exist (albeit as a scattered entity). By contrast, Julius, JHead, JHand, and so on might all outlive GOLD's demise if, e.g., a single member of GOLDsAtoms were removed from Julius and destroyed. Since GOLD is bound in this way to the members of GOLDsAtoms and GOLDsSubPortions, but not to Julius, JHead, JHand, etc, it is natural that we think of the former, but not the later, as the primary parts of GOLD.

By contrast, it is not clear that Julius has such strong ties to any of its proper parts. By a step-wise replacement analogous to that performed on the ship of Theseus, Julius can survive the loss of any portion of gold (including the loss of GOLD itself, if GOLD is gradually replaced by another portion of metal), JHead, JHand, and perhaps any other structural proper part. Also, JHead, JHand, and other of Julius' structural parts may exist at times when Julius does not exist (e.g., if JHand, JHand, and other structural parts of Julius were constructed before Julius was assembled).

To create a more complex example for illustrating different aspects of our theory, we assume that GOLD and Julius undergo a few changes after time TJ. Suppose that by time TH, JHand has been removed from Julius and subsequently melted down. At TH, JHand is no longer a part of either Julius or GOLD. Indeed, since JHand does not exist at TH, JHand is not a part of anything at TH. Also, the portion of gold (call it GHand) which has been melted down is no longer a part of Julius at TH. But GHand is still a part of GOLD at TH. Thus, at TH, GOLD and Julius no longer coincide. Instead, Julius coincides at TH with a proper part of GOLD– that sub-portion of gold in GOLD which has not been melted down (call it GHand-Minus).

Now suppose that at some time after TH, GHand-Minus is also melted down and at time TB all of GOLD is formed into a statue of Marcus Brutus. Call the second statue Brutus. At TB, Julius, JHead, and so on are no longer parts of GOLD, but GOLD now coincides with a new statue. Thus, GOLD has acquired new parts– at TB, Brutus, Brutus' head, Brutus' right hand, and so on are all parts of GOLD. Notice, though, that throughout these changes GOLD neither gains nor loses parts which are gold atoms or portions of gold.

B. In addition to distinguishing between the mereological properties of integral objects (Julius) and portions of stuff (GOLD), we would also like to use the mereology developed in this paper to clarify distinctions, made informally in [Barnett, 2004], between different types of portions of stuff. Barnett's distinctions can be illustrated by contrasting GOLD with a portion of stuff that is a mixture. Suppose we have some sugar (SUGAR), some water (WATER),

[2] Not all proponents of the standard account accept (*). See, for example, [Lowe, 2003] and [Doepke, 1982].

and some citric acid (ACID) in separate containers on our kitchen counter. When we mix SUGAR, WATER, and ACID together, each of these portions of stuff continues to exist. But we have in addition new portion of stuff: some lemonade (LEMONADE).[3] Just as all members of GOLDsAtoms must be parts of GOLD whenever GOLD exists, so also all members of LEMsMolecules (the collection consisting of SUGAR's sugar molecules, WATER's water molecules, and ACID's acid molecules) must be parts of LEMONADE whenever LEMONADE is made. However, unlike GOLD and GOLDsAtoms, the mere existence of all members of LEMsMolecules is not sufficient to guarantee LEMONADE's existence– all members of LEMsMolecules are present before LEMONADE is made. LEMONADE exists only when members of LEMsMolecules are suitably mixed together: every sugar molecule in LEMsMolecules must be mixed with water and acid molecules in LEMsMolecules, and so on. Nonetheless, LEMONADE, like GOLD and unlike Julius or Brutus, can survive quite a bit of scattering. We could, e.g., divide LEMONADE into a thousand cups. As long as the division is accomplished in such a way that each of the cups contains a portion of lemonade and each member of LEMsMolecules is part of one of these portions, LEMONADE survives the scattering.

Let TL1 be a time immediately after LEMONADE's creation. At TL1, LEMONADE has as parts not only members of LEMsMolecules, but also sub-portions of lemonade. Let LEMsSubPortionsTL1 be the collection of all portions of lemonade which are part of LEMONADE at TL1. (Notice that some portions of stuff which are part of LEMONADE at TL1 are not portions of lemonade. For example, SUGAR, WATER, are ACID are parts of LEMONADE at TL1, but these are not portions of lemonade and thus are not members of LEMsSubPortionsTL1.)

Like GOLD and GOLDsSubportions, LEMONADE must exist whenever all members of LEMsSubPortionsTL1 exist. However, unlike GOLD and GOLDsSubportions, the members of LEMsSubportionsTL1 need not be parts of LEMONADE whenever LEMONADE exists.[4] Suppose that at some time after TL1, LEMONADE is whipped in a blender. Let TL2 be a time after the whipping. We presume that at TL2 all members of LEMsMolecules are still appropriately mixed with other members of LEMsMolecules and thus that LEMONADE still exists at TL2. But some members of LEMsSubportionsTL1 will no longer exist at TL2, since since *their* water, acid, and sugar molecules will have been scattered (within LEMONADE) as a result of the mixing.

To illustrate this, let L-SMALL be some member of LEMsSubPortionsTL1 that is significantly smaller than LEMONADE. The members of only a small portion of LEMsMolecules are molecular parts of L-SMALL. Call this sub-collection L-SMALLsMolecules. Just as LEMONADE persists only so long as members of LEMsMolecules remain appropriately mixed with other members of LEMsMolecules, so L-SMALL persists only so long as members of L-SMALLsMolecules remain appropriately mixed with other members of L-SMALLsMolecules. But given that L-SMALLsMolecules includes only a small portion of LEMsMolecules, it is highly unlikely that all members of L-SMALLsMolecules are still appropriately mixed with one another after the whipping. Given, further, that LEMsSubPortionsTL1 includes very many portions of lemonade that are at least as small as L-SMALL, we can safely assume that not all members of LEMsSubPortionsTL1 exist at TL2 even though LEMONADE exists at TL2. On the other hand, we can also assume that new collections of molecules have been mixed together as a result of the whipping. and thus that LEMONADE has acquired new sub-portions between TL1 and TL2. Thus, while GOLD can neither lose nor gain parts which are portions of gold, LEMONADE can both lose and gain parts which are portions of lemonade.

It is our task in the remainder of this paper to develop an axiomatic theory that allows for clear characterization of examples such as those presented above.

3 Non-extensional temporal mereology

We present a non-extensional temporal mereology in a sorted first-order predicate logic with identity. We distinguish three disjoint sorts. We use w, x, y, z as variables ranging over material objects; p, q as variables ranging over collections of material objects; t, t_1, t_2 as variables ranging over instants of time. All quantification is restricted to a single sort and leading universal quantifiers are generally omitted. Restrictions on quantification will be understood by conventions on variable usage.

3.1 Time-dependent parthood relations among material objects

Material objects are material entities that exists at certain times and have at each moment of their existence a unique spatial location. They include both integral objects (Julius, Brutus) and portions of stuff (LEMONADE, GOLD).

We introduce the primitive ternary relation P which holds between two objects at a time instant where $Pxyt$ is interpreted as: object x is part of object y at time instant t. We then define: x *overlaps* y at t if and only if there is an object z such that z is part of x at t and z is part of y at t; x is a *proper part* of y at t if and only if x is a part of y at t and y is not part of x at t (D_{PP}); x *exists* at t if and only if x is part of itself at t (D_E); x and y are *mereologically equivalent* at t if and only if x is part of y at t and y is part of x at t ($D_{\approx}$). It follows from these definitions that at any fixed time: O is symmetric; *PP* is asymmetric; $\approx$ is symmetric, and transitive.

$$
\begin{array}{ll}
D_O & O\,xyt \equiv (\exists z)(P\,zxt \wedge P\,zyt) \\
D_{PP} & PP\,xyt \equiv P\,xyt \wedge \neg P\,yxt \\
D_E & E\,xt \equiv P\,xxt \\
D_{\approx} & x \approx_t y \equiv P\,xyt \wedge P\,yxt
\end{array}
$$

GOLD is mereologically equivalent to Julius at TJ. When JHand is removed from Julius, GOLD is no longer mereologically equivalent to Julius. Later, at TB, GOLD is mereologically equivalent to the new statue, Brutus.

We add axioms requiring: every object exists at some time (AP1); if x is a part of y at t then x and y exist at t (AP2);

[3] For an alternative account of this situation, see [Burge, 1977].

[4] This point is taken from [Barnett, 2004] where an analogous example involving a portion of crude oil is developed in detail.

at any fixed time parthood is transitive (AP3); if x exists at t and everything that overlaps x at t overlaps y at t then x is a part of y at t (AP4).

$$
\begin{array}{ll}
AP1 & (\exists t) E\, xt \\
AP2 & P\, xyt \rightarrow E\, xt \wedge E\, yt \\
AP3 & P\, xyt \wedge P\, yzt \rightarrow P\, xzt \\
AP4 & E\, xt \wedge (z)(O\, zxt \rightarrow O\, zyt) \rightarrow P\, xyt
\end{array}
$$

Using (AP1 - AP4), we can prove: if x exists at t then x and y are mereologically equivalent at t if and only if x and y have the same parts at t (T1); if x exists at t then x and y are mereologically equivalent at t if and only and they overlap the same objects at t (T2); the following are equivalent: (i) x exists at t, (ii) x overlaps itself at t, (iii) x is mereologically equivalent with itself at t (T3); if x is part of y at t and x and y are not mereologically equivalent at t then x is a proper part of y at t (T4).

$$
\begin{array}{ll}
T1 & E\, xt \rightarrow (x \approx_t y \leftrightarrow (z)(P\, zxt \leftrightarrow P\, zyt)) \\
T2 & E\, xt \rightarrow (x \approx_t y \leftrightarrow (z)(O\, zxt \leftrightarrow O\, zyt)) \\
T3 & E\, xt \leftrightarrow O\, xxt \wedge E\, xt \leftrightarrow x \approx_t x \\
T4 & P\, xyt \wedge \neg x \approx_t y \rightarrow PP\, xyt
\end{array}
$$

Notice that it does NOT follow from our axioms that (i) if two objects have the same parts at a time then they are identical; and (ii) if two objects overlap exactly the same things at a time, then they are identical. For example, GOLD and Julius are not identical but they have exactly the same parts and overlap the same things at time TJ.

3.2 Constant and bound parts

Though our basic mereological relations are time-dependent, we can define useful time-independent parthood relations in terms of the time-dependent relations.

Object x is a *constant* part of object y if and only if whenever y exists, x is a part of y (D_{CP}). Object x is a *constant proper part* of object y if and only if whenever y exists, x is a proper part of y (D_{CPP}).

$$
\begin{array}{ll}
D_{CP} & CP\, xy \equiv (t)(E\, yt \rightarrow P\, xyt) \\
D_{CPP} & CPP\, xy \equiv (t)(E\, yt \rightarrow PP\, xyt)
\end{array}
$$

For example, each atom in GOLDsAtoms is a constant proper part of GOLD and each portion of gold in GOLDsSubPortions is a constant part of GOLD. Also, all members of LEMsMolecules are constant proper parts of LEMONADE. But not all members of LEMsSubPortionsTL1 are constant parts of LEMONADE, since some of these portions of lemonade are destroyed in the whipping.

Statues may also have constant parts. In our example JHead is a constant proper part of Julius. GHand-Minus is a constant part of Julius. But, as pointed out in Section 2, unlike the atoms in GOLD and the molecules in LEMONADE, Julius *could* survive the loss of these parts.

We can prove that constant parthood is reflexive and transitive and that constant proper parthood is asymmetric and transitive. Notice, however, that the logical relations between *CP* and *CPP* are not exactly analogous to those between P and *PP* (see D_{PP}). We can prove that if x is a constant proper part of y then x is a constant part of y and y is not a constant part of x (T5):

$$T5 \quad CPP\, xy \rightarrow CP\, xy \wedge \neg CPyx.$$

But we cannot prove the converse of this theorem. Our theory allows, e.g., that GHand-Minus is a constant part of Julius and Julius is not a constant part of GHand-Minus, but GHand-Minus is not a constant proper part of Julius (since after JHand is removed, GHand-Minus is mereologically equivalent to Julius).

Object x is a *bound part* of object y if and only whenever x exists, x is a part of y (D_{CP}). Object x is a *bound proper part* of object y if and only if whenever x exists, x is a proper part (D_{CPP}).

$$
\begin{array}{ll}
D_{BP} & BP\, xy \equiv (t)(E\, xt \rightarrow P\, xyt) \\
D_{BPP} & BPP\, xy \equiv (t)(E\, xt \rightarrow PP\, xyt)
\end{array}
$$

For example, Julius (as well as JHead and JHand) is a bound part of GOLD. But no member of GOLDsAtoms or GOLDsSubPortions (including GOLD itself) is a bound part of Julius. In general, the parts that are assembled to construct an artifact are not be bound parts of the artifact because they must exist before the assembly. Similarly, members of LEMsMolecules are not bound parts of LEMONADE.

By contrast, organisms typically have many bound proper parts. Any cell which is manufactured and destroyed within my body is a bound, though not necessarily constant, proper part of my body.

We can prove that bound parthood, like constant parthood, is reflexive and transitive and that bound proper parthood is asymmetric and transitive. Also, we can prove that if x is a bound proper part of y then x is a bound part of y and y is not a bound part of x. But we cannot prove the converse of this theorem: for example, Julius is a bound part of GOLD and GOLD is not a bound part of Julius, but Julius is not a bound proper part of GOLD (since at TJ, Julius exists but is not a proper part of GOLD).

4 Collections and time-dependent sums

4.1 Collections

We use $\in$ to stand for the member-of relation between objects and collections of objects. We refer to a finite collection having $x_1, \ldots, x_n$ as members, as: $\{x_1, \ldots, x_n\}$. Since collections and objects are disjoint sorts, $\in$ is irreflexive and asymmetric.

All collections have at least two members (AC1). Consequently there are no empty collections and no singleton collections. We require that two collections are identical if and only if they have the same members (AC2).

$$
\begin{array}{ll}
AC1 & (\exists x)(\exists y)(x \in p \wedge y \in p \wedge x \neq y) \\
AC2 & p = q \leftrightarrow (x)(x \in p \leftrightarrow x \in q)
\end{array}
$$

The collection p is a *sub-collection* of the collection q ($p \sqsubseteq q$) if and only if every member of p is also a member of q ($D_{\sqsubseteq}$). The collection p is a *proper sub-collection* of the collection q ($p \sqsubset q$) if and only if p and q are not identical and p is a sub-collection of q.

$$
\begin{array}{ll}
D_{\sqsubseteq} & p \sqsubseteq q \equiv (x)(x \in p \rightarrow x \in q) \\
D_{\sqsubset} & p \sqsubset q \equiv p \sqsubseteq q \wedge p \neq q
\end{array}
$$

We can prove that $\sqsubseteq$ is reflexive, antisymmetric, and transitive (a partial ordering) and that $\sqsubset$ is asymmetric and transitive (a strict partial ordering).

Note that collections are identified through their members and thus cannot have different members at different times. In particular, collections do not lose members that cease to exist. But we can distinguish collections according to whether or not all of their members exist at a given time. We say that a collection p is *fully present* at t if and only if all of its members exist at t (D_{FP}).

$$D_{FP} \quad \mathit{FP}\; pt \equiv (x)(x \in p \rightarrow E\; xt)$$

Notice that if p is fully present at t then all of its sub-collections are fully present at t. For example, whenever GOLD (the portion of gold) exists, every sub-collection of GOLDsAtoms (the gold atoms in GOLD) is fully present.

A collection p is *discrete* at time t if and only if distinct members of p do not overlap at t (D_D).

$$D_D \quad D\; pt \equiv (x)(y)(x \in p \wedge y \in p \wedge O\; xyt \rightarrow x = y)$$

For example, the collection GOLDsAtoms is at all times discrete. By contrast, the collection of sub-portions of gold in GOLD is never discrete while GOLD exists. Notice that if p is discrete at t and q is a sub-collection of p then q is discrete at t (e.g. all sub-collections of GOLDsAtoms are discrete at all times).

4.2 Time-dependent sums

We say that object z is a *sum* of (the members of) the collection p at time t, $\mathit{SM}\; zpt$, if and only if p is fully present at t and any object overlaps z at t if and only if it overlaps a member of p at t (D_{SM}). In this case, we will also say that p sums to z at t or that z is a p-sum at t.

$$D_{SM} \quad \mathit{SM}\; zpt \equiv \mathit{FP}\; pt \wedge (w)(O\; wzt \leftrightarrow (\exists x)(x \in p \wedge O\; xwt))$$

Thus, at any time t at which it exists, Julius is a sum of the collection of the objects which are part of it at t. Also, GOLD is at TJ a sum of $\{$GOLD, Julius$\}$ and is at TB a sum of $\{$GOLD, Brutus$\}$. A collection p may sum to more than one object at t. For example, both GOLD and Brutus are sums of $\{$GOLD, Brutus$\}$ at TB. Also, an object may be at a given time a sum of more than one collection. For example, GOLD is at TB a sum of $\{$GOLD, Brutus$\}$, a sum of GOLDsAtoms, and a sum of GOLDSSubPortions.

We can prove: if x is a sum of a collection at t, then x exists at t (T7); if z is a sum of p at t then every member of p is part of z at t (T8); if x is a sum of p at t then y is a sum of p at t if and only if x and y are mereologically equivalent at t (T9); if x is a sum of p at t, y is a sum of q at t, and p is a sub-collection of q, then x is part of y at t (T10).

$$\begin{array}{ll} T7 & (\exists p)\mathit{SM}\; xpt \rightarrow E\; xt \\ T8 & x \in p \wedge \mathit{SM}\; zpt \rightarrow P\; xzt \\ T9 & \mathit{SM}\; xpt \rightarrow (\mathit{SM}\; ypt \leftrightarrow x \approx_t y) \\ T10 & \mathit{SM}xpt \wedge \mathit{SM}\; yqt \wedge p \sqsubseteq q \rightarrow P\; xyt \end{array}$$

T10 tells that if GOLDsAtoms* is a sub-collection of GOLDsAtoms and GOLD is a sum of GOLDsAtoms at t, then any sum of GOLDsAtoms* is a part of GOLD at t. For example, all portions of gold made out of sub-collections of GOLDsAtoms (i.e. the members of GOLDsSubPortions) are parts of GOLD at t. Also, any other objects which happen to be made out of (are mereologically equivalent to) sums of sub-collections of GOLDsAtoms at t (e.g. Julius' head, Julius right hand, and so on) are parts of GOLD at t.

We say that x is *partioned* by the collection p at t (or, p *partitions* x at t) if and only if x is a sum of p at t and p is discrete at t.

$$D_{DSM} \quad \mathit{DSM}\; xpt \equiv \mathit{SM}\; xpt \wedge Dpt$$

For example, whenever GOLD exists, GOLDsAtoms partitions GOLD. When Julius exists, GOLDsAtoms also partition Julius. By contrast $\{$GOLD, Julius$\}$ sums to both GOLD and Julius while Julius exists, but $\{$GOLD, Julius$\}$ never partitions either GOLD or Julius. Also, GOLDsSubportions sums to GOLD when ever GOLD exists, but never partitions GOLD.

Since *DSM* xpt implies *SM* xpt, *DSM* counterparts of theorems T7 - T10 are also theorems. In addition, discrete sums have the following useful properties. If x is partitioned by p at t, then x is not a member of p (T11). If x is partitioned by p at t, y is partitioned by q at t, and p is a proper sub-collection of q, then x is a proper part of y at t (T12). If x is partitioned by p at t and y is a member of p, then y is a proper part of x (T13).

$$\begin{array}{ll} T11 & \mathit{DSM}\; xpt \rightarrow x \notin p \\ T12 & \mathit{DSM}\; xpt \wedge \mathit{DSM}\; yqt \wedge p \sqsubset q \rightarrow \mathit{PP}\; xyt \\ T13 & x \in p \wedge \mathit{DSM}\; zpt \rightarrow \mathit{PP}\; xzt \end{array}$$

5 Time-independent sums and partitions

In section 3.2, we used the time-dependent mereological relations to define several time-independent parthood relations. In this section, we use the time-dependent sum and partition relations to define several different time-independent sum and partition relations. Among other things, we will show how these time-independent relations are useful for clarifying important differences between GOLD and more complicated portions of stuff such as LEMONADE.

5.1 Constant sums

Object x is a *constant* sum of collection p (a constant p-sum) if and only if whenever x exists, x is a sum of p (D_{SM_C}).

$$D_{SM_C} \quad \mathit{SM}_C\; xp \equiv (t)(E\; xt \rightarrow \mathit{SM}\; xpt)$$

For example, GOLD and Brutus are both constant sums of GOLDsAtoms. In addition, Brutus is a constant sum of $\{$Brutus, GOLD$\}$ and of the union of GOLDsAtoms and $\{$Brutus, GOLD$\}$. Also, LEMONADE is a constant sum of LEMsMolecules. By contrast, Julius is not a constant sum of GOLDsAtoms– after JHand is removed Julius continues to exist but no longer has some members of GOLDsAtoms as parts. Also, although GOLD is (necessarily) a constant sum of GOLDsSubportions, LEMONADE is not a constant sum of LEMsSubPortionsTL1.

We can prove: if x is a constant sum of p then whenever x exists, p is fully present (T14); if x is a constant sum of p and y is a member of p then y is a constant part of x (T15).

$$\begin{array}{ll} T14 & \mathit{SM}_C\; xp \rightarrow (t)(E\; xt \rightarrow \mathit{FP}\; pt) \\ T15 & \mathit{SM}_C\; xp \wedge y \in p \rightarrow \mathit{CP}\; yx \end{array}$$

Notice that $SM_C\ xp$ and $SM_C\ yp$ may hold even though x and y never overlap. Notice also that if x is a constant p-sum, then the members of p must be constant parts of x but they will not in general be bound parts of x. For example, none of the water, acid, or sugar particles in LEMsMolecules are bound parts of LEMONADE – each of these particles exists at times when they are not part of LEMONADE.

5.2 Bound sums

Object x is a *bound* sum of collection p (a bound p-sum) if and only if p is fully present at some time and at all times at which p is fully present x is a sum of p (D_{SM_B}).

$$D_{SM_B} \quad SM_B\ xp \equiv (\exists t)(FP\ pt) \wedge (t)(FP\ pt \rightarrow SM\ xpt)$$

For example, GOLD is a bound sum of GOLDsAtoms. Whenever all of the atoms in GOLDsAtoms exist, GOLD also exists and is a sum of GOLDsAtoms. By contrast, LEMONADE is not a bound sum of LEM'sMolecules. At times before the sugar, water and acid are mixed together LEM'sMolecules is fully present, but LEMONADE does not yet exist. On the other hand, LEMONADE is a bound sum of LEMsSubportionsTL1, the collection of all sub-portions of lemonade in LEMONADE at time TL1. Whenever all of these portions of lemonade exist, LEMONADE also exists and is a sum of LEMsSubportionsTL1. LEMONADE is also a bound sum of LEMsSubportionsTL2 and GOLD is a bound sum of, as well as a constant sum of, GOLDsSubPortions.

These examples show that x may be a constant p-sum, but not a bound p-sum– LEMONADE is a constant sum of LEMsMolecules, but not a bound sum of LEMsMolecules. Also, x may be a bound p-sum but not a constant p-sum– LEMONADE is a bound sum of LEMsSubportionsTL1, but not a constant sum of LEMsSubportionsTL1.

We have seen that x may be a bound p-sum even if some members of p are not constant parts of x. (Not all members of LEMsSubportionsTL1 are constant parts of LEMONADE.) x may also be a bound p-sum even if some members of p are not bound parts of x. For example, we may assume that at least one of the members of GOLDsAtoms exists at times when GOLDsAtoms is not yet fully present. Call this atom GAFirst. GAFirst is a constant part of GOLD, but not a bound part of GOLD even though GOLD is a bound sum of GOLDsAtoms.

We can prove: if x is a bound p-sum, then whenever p is fully present x exists (T16); if x is a bound p-sum and y is a bound p-sum, then whenever p is fully present, x and y are mereologically equivalent (T17); if x is a bound p-sum and y is a constant q-sum and p is a sub-collection of q then x is a constant part of y (T18).

$$\begin{array}{ll} T16 & SM_B\ xp \rightarrow (t)(FP\ pt \rightarrow E\ xt) \\ T17 & SM_B\ xp \wedge SM_B\ yp \wedge FP\ pt \rightarrow x \approx_t y \\ T18 & SM_B\ xp \wedge SM_C\ yq \wedge p \sqsubseteq q \rightarrow CP\ xy \end{array}$$

As an example of (T18), let WMolecules be the sub-collection of LEMsMolecules consisting of the water molecules in LEMONADE. Then, WATER, the portion of water in LEMONADE, is a constant sum of WMolecules, since, unlike LEMONADE, WATER's existence does not depend on its molecules being appropriately mixed together. (T18) tells us that WATER is a constant part of LEMONADE. For analogous reasons, SUGAR and ACID are also constant parts of LEMONADE.

5.3 Permanent sums

Object x is a *permanent* sum of collection p (a permanent p-sum) if and only if x is both a constant p-sum and a bound p-sum (D_{SM_P}).

$$D_{SM_P} \quad SM_P\ xp \equiv SM_C\ xp \wedge SM_B\ xp$$

For example, GOLD is a permanent sum of both GOLDsAtoms and GOLDsSubportions. But LEMONADE is not a permanent sum of LEMsMolecules, since it is not a bound sum of LEMsMolecules.

We can prove: if x is a constant p-sum and x is itself a member of p, then x is a permanent p-sum (T19); if x is a permanent p-sum then the following are equivalent for all t: p is fully present at t, x is a sum of p at t, x exists at t (T20); if x is a permanent p-sum and y is a permanent p-sum then the following are equivalent for all values of t: x exist at t, y exists at t, x and y are mereologically equivalent at t (T21).

$$\begin{array}{ll} T19 & SM_C\ xp \wedge x \in p \rightarrow SM_P xp \\ T20 & SM_P\ xp \rightarrow (t)(FP\ pt \leftrightarrow SM\ xpt \wedge SM\ xpt \leftrightarrow E\ xt) \\ T21 & SM_P\ xp \wedge SM_P\ yp \rightarrow (t)(E\ xt \leftrightarrow E\ yt \wedge E\ xt \leftrightarrow x \approx_t y) \end{array}$$

5.4 Time-independent partitions

For our purposes, it is useful to have stronger partition counterparts of the time-independent sum relations introduced in the previous section.

Collection p is a *constant partition* of object x if and only if p partitions x whenever x exists (D_{DSM_C}). Collection p is a *bound partition* of object x if and only if p is fully present at some time and p partitions x whenever p is fully present (D_{DSM_B}). Collection p is a *permanent partition* of object x if and only if p is both a constant and a bound partition of x (D_{DSM_P}).

$$\begin{array}{ll} D_{DSM_C} & DSM_C\ xp \equiv (t)(E\ xt \rightarrow DSM\ xpt) \\ D_{DSM_B} & DSM_B\ xp \equiv (\exists t)(FP\ pt) \wedge (t)(FP\ pt \rightarrow DSM\ xpt) \\ D_{DSM_P} & DSM_P\ xp \equiv DSM_C\ xp \wedge DSM_B\ xp \end{array}$$

For example, GOLDsAtoms is a constant partition of both GOLD and Brutus. GOLDsAtoms is also a bound partition (and thus also a permanent partition) of GOLD, but GOLDsAtoms is not a bound partition of Brutus. LEMsMolecules is a constant partition of, but not a bound partition of, LEMONADE.

Clearly, each of the time-independent partition relations entails its sum relation counterpart. In addition, we can derive the following theorems concerning time-independent partitions: if p is a constant partition of x and y is a member of p, then y is a constant proper part of x (T22); if p is a bound partition of x and y is a member of p, then whenever p is fully present, y is a proper part of x (T23); if p is a bound partition of x, q is a constant partition of y, and p is a proper sub-collection of q, then x is a constant proper part of y (T24).

$$\begin{array}{ll} T22 & DSM_C\ xp \wedge y \in p \rightarrow CPP\ yx \\ T23 & DSM_B\ xp \wedge y \in p \wedge FP\ pt \rightarrow PP\ yxt \\ T24 & DSM_B\ xp \wedge DSM_C\ yq \wedge p \sqsubset q \rightarrow CPP\ xy \end{array}$$

6 Conclusions

In the presented theory, we used parthood and summation relations to distinguish key mereological properties of (i) integral objects such as Julius (ii) portions of homogenous unstructured stuff such as GOLD, and iii) structured stuffs such as LEMONADE. Every portion of gold is a permanently partition by the collection of its gold atoms and is a permanent sum of the collection of its gold sub-portions. By contrast, the collection of its molecules is typically only a constant partition, not a bound partition, of a portion of lemonade. Also, the portion of lemonade is typically only a bound sum of, not a constant sum of, the collection consisting of its sub-portions at a given time.

In general, integral objects will have even loser ties to a constituting collection of atoms or molecules than do portions of mixtures. For example, Julius is neither a constant sum nor a bound sum of any collection of atoms or molecules. Also, Julius is neither a constant sum nor a bound sum of any collection consisting of portions of stuff.

The theory presented in this paper is useful for reasoning about parthood and composition relations among integral objects and portions of stuff, particularly in application in, e.g., medicine where changes in objects are tracked over time. Related work in Artificial Intelligence also includes [Hayes, 1985] and [Collins and Forbus, 1987].

One important area for further work is in expanding the theory included in this paper to include modality. With modality, we could distinguish more sharply between integral objects, homogenous stuffs, and mixtures. For example, Brutus, like GOLD and unlike Julius, is a constant sum of GOLDsAtoms. However, unlike GOLD and like Julius, Brutus might not have been a constant sum of GOLDsAtoms. In general, it is possible for all medium-sized integral objects to lose atomic parts. Similarly, it is possible for all mixtures to lose parts which are smaller portions of the same type of stuff.

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