Nominal quantification as top-level anaphora

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Abstract

So far, we have focused on discourse reference to atomic individuals and specific times, events, and states. The basic point of the argument was that all types of discourse reference involve attention-guided anaphora (in the sense of Bittner 2012: Ch. 2). In particular, when grammatical verbal categories, such as *person* (PRN), *tense* (TNS), and/or *aspect* (ASP), are realized in a language, they involve discourse anaphora to a *top-ranked event* (speech act, $\top\varepsilon$, or last-mentioned event, $\bot\varepsilon$) and/or to a top-ranked discourse referent of a category-specific type—to wit, *individual*, $\top\delta$ or $\bot\delta$, for PRN; *time*, $\top\tau$ or $\bot\tau$, for TNS; or *state*, $\top\sigma$ or $\bot\sigma$, for ASP.

We now turn to discourses involving anaphora to and by quantificational expressions of various types. Today, we focus on quantification over individuals, but the analysis we develop will directly generalize to temporal quantification (over times, events, and/or states, see March 22) as well as modal quantification (over worlds, see March 29–April 5). The basic idea is to build on the classical Fregean theory of quantificational determiners as logical relations between sets of individuals, but to factor in attention-guided anaphora to an by quantificational expressions and to extend this approach to quantification to discourse referents of other types (including times, events, states, and ultimately, worlds).

To implement this research program we build on prior work on update with *plural information states* by van den Berg (1994, 1996) and Brasoveanu (2007). However, we revise these earlier update systems to improve intuitive transparency as well as generality. For example, we eliminate the 'dummy individual' (★) from model structures. Maximization and distributivity are built into the type-neutral architecture instead of typed object-language operators. Generalization across types then follows automatically, without stipulating additional operators. Last but not least, we replace variable-based anaphora of van den Berg and Brasoveanu with attention-guided anaphora, implemented in extensions of UC_0 and UC_τ with plural discourse reference ($UC_{\delta+}$ and $UC_{\tau+}$, respectively). Quantification then emerges as one more species of top-level anaphora—to wit, anaphora to top-ranked sets.

Outline

1. Quantification as plural reference (vdBerg 1994, 1996, Brasoveanu 2007)
2. Quantification in *Update with Centering* (Bittner 2011, 2012)
3. "Quantifier subordination" as top-level anaphora
4. Quantifier scope: Topic over background

1 QUANTIFICATION AS PLURAL REFERENCE

(1) i. *Al invited some friends.*
ii. *Most people came and they had a good time together.*

• ***Dynamic Plural Logic*** (DPlL, vdBerg 1994, 1996, see Appendix A)
i. contexts as *sets of assignments*, e.g. $J_0 \ldots J_2$ below, w. *non-deterministic* update
ii. predicates relate *sets of entities*, e.g. $\langle \{f_1\}, \{a\} \rangle \in [\![F^2]\!]$, if f_1 is a friend of a
iii. *distr. pred's*, e.g. C^1 for 'came', distribute over sets, e.g. $\{f_1\}, \{f_2\}, \{f_3\} \in [\![C^1]\!]$
coll. pred's, including *quantifiers*, are predicated of sets, e.g. $\{f_1, f_2, f_3\} \in [\![\mathbf{2}^+]\!]$

(2) J_0

x_1	x_2	x_3	x_4	…
★	★	★	★	

Minimal info-state $J_0 = \{g_\star\}$, where '★' denotes the *dummy individual*

i. *Al …*

$\varepsilon_{x1} \wedge \Delta_{x1}(x_1 = al) \ldots$

J_{1a}

x_1	x_2	x_3	x_4	…
a	★	★	★	

Sample output for model $\mathcal{M} = \langle \mathcal{D}, [\![\cdot]\!] \rangle$ s.t. $[\![al]\!] = a$

… invited some friends.

$\ldots \wedge \mathbf{M}_{x2}(\varepsilon_{x2} \wedge \Delta_{x2}(F^2\, x_2x_1)) \wedge \mathbf{M}_{x3}(\varepsilon_{x3} \wedge \Delta_{x3}(x_3 = x_2 \wedge I^2\, x_1x_3)) \wedge \mathbf{2}^+ x_3$

J_{1b}

x_1	x_2	x_3	x_4	…
a	f_1	f_1	★	
a	f_2	f_2	★	
a	f_3	f_3	★	
a	f_4	f_4	★	
a	f_5	★	★	

Assume further:
$\cup\{X \subseteq \mathcal{D} \mid \langle X, \{a\} \rangle \in [\![F^2]\!]\} = \{f_1, \ldots, f_5\} =: F_{1\text{-}5}$
$\cup\{X \subseteq F_{1\text{-}5} \mid \langle \{a\}, X \rangle \in [\![I^2]\!]\} = \{f_1, \ldots, f_4\} =: F_{1\text{-}4}$

ii. *Most people came and they had a good time together.*

$\Delta_{x3}(P^1 x_3) \wedge \mathbf{M}_{x4}(\varepsilon_{x4} \wedge \Delta_{x4}(x_4 = x_3 \wedge C^1\, x_4)) \wedge \boldsymbol{most}\, x_3x_4) \wedge G^1\, x_4$

J_2

x_1	x_2	x_3	x_4	…
a	f_1	f_1	f_1	
a	f_2	f_2	f_2	
a	f_3	f_3	f_3	
a	f_4	f_4	★	
a	f_5	★	★	

Assume further:
$\cup\{X \subseteq F_{1\text{-}4} \mid X \in [\![C^1]\!]\} = \{f_1, \ldots, f_3\} =: F_{1\text{-}3}$
$\cup\{X \subseteq F_{1\text{-}3} \mid X \in [\![G^1]\!]\} = F_{1\text{-}3}$

2 QUANTIFICATION IN UPDATE WITH CENTERING

• ***UC_0 with plural reference*** ($UC_{\delta+}$, Bittner 2011, 2012, see Appendix B)

i. contexts as *sets of sequence pairs*, e.g. c_1, ..., c_8 below, w. *deterministic update*

ii. predicates relate *entities*, e.g. $\langle f_1, a\rangle \in [\![F^2]\!]$, and $f_1, f_2, f_3 \in [\![C^1]\!]$

iii. *collective pred's* are predicated of sums, e.g. $f_{1\text{-}3} := \sqcup\{f_1, f_2, f_3\} \in [\![G^1]\!]$

card. pred's & *quantifiers* are predicated of sets, e.g. $F_{1\text{-}3} := \{f_1, f_2, f_3\} \in [\![\mathbf{2}^+]\!]$

Table 2. New Drt-abbreviations (see Appendix C)

• <u>*static conditions*</u> (type *t*)

$A \bigcirc B$	:=	$\exists u(u \sqsubseteq A \wedge u \sqsubseteq B)$
$A_a = \boldsymbol{max}(B_{at})$	:=	$A \in B \wedge \forall u_a(u \in B \rightarrow u \sqsubseteq A)$
$^{\varnothing}A_{at}$	:=	$A \neq \varnothing \wedge \forall u_a \forall v_a(u \in A \wedge v \in A \wedge u \bigcirc v \rightarrow u = v)$

• <u>*global projections*</u> (type $s(st)a$, for $a \in \mathrm{DR}(\Theta)$)

$A_{sa}\|\|^*$	:=	$\lambda i_s \lambda I_{st} \lambda u_a(\exists j_s(Ij \wedge u = Aj))$
$(A\|\|_{B1\ldots Bn})^*$	:=	$\lambda i_s \lambda I_{st} \lambda u_a(\exists j_s(Ij \wedge B_1 i = B_1 j \wedge \ldots \wedge B_n i = B_n j \wedge u = Aj))$

⋮

(3) i. *Al*$^{\top}$ [*invited*] *some friends*$^{\perp}$.

$^{\top}[x|\ x =_i al]$; $(([x|\ friend\langle x, \top\delta\rangle]\ ^{\perp}; [invite\langle\top\delta, \perp\delta\rangle]); [\mathbf{2}^+\{\perp\delta\|\|\}])$

c_1	c_2	c_3	✓c_3
⟨⟨a⟩, ⟨⟩⟩	⟨⟨a⟩, ⟨f_1⟩⟩	⟨⟨a⟩, ⟨f_1⟩⟩	
	⟨⟨a⟩, ⟨f_2⟩⟩	⟨⟨a⟩, ⟨f_2⟩⟩	
	⟨⟨a⟩, ⟨f_3⟩⟩	⟨⟨a⟩, ⟨f_3⟩⟩	
	⟨⟨a⟩, ⟨f_4⟩⟩	⟨⟨a⟩, ⟨f_4⟩⟩	
	⟨⟨a⟩, ⟨f_5⟩⟩		

ii. *Most people*$^{\top}$ [*came*]$_{most}$

$([pers\langle\perp\delta\rangle]; {}^{\top}[X|\ X =_I \perp\delta\|\|]; [^{\varnothing}\langle\top\delta t\rangle])\ ^{\top}$; $([cm\langle\perp\delta\rangle]; [\boldsymbol{most}\{\top\delta t, \perp\delta\|\|\}])$

✓c_3	c_4	✓c_4	c_5	✓c_5
	⟨⟨$F_{1\text{-}4}$, a⟩, ⟨f_1⟩⟩		⟨⟨$F_{1\text{-}4}$, a⟩, ⟨f_1⟩⟩	
	⟨⟨$F_{1\text{-}4}$, a⟩, ⟨f_2⟩⟩		⟨⟨$F_{1\text{-}4}$, a⟩, ⟨f_2⟩⟩	
	⟨⟨$F_{1\text{-}4}$, a⟩, ⟨f_3⟩⟩		⟨⟨$F_{1\text{-}4}$, a⟩, ⟨f_3⟩⟩	
	⟨⟨$F_{1\text{-}4}$, a⟩, ⟨f_4⟩⟩			

and they$_{\perp}^{\top}$ *had a good time together*

$^{\top}[x|\ x =_I \sqcup\{\perp\delta\|\|\}]$; $[\mathbf{2}^+\langle^{\mathcal{A}}\top\delta\rangle]$; $[hv.good.time.tg\langle\top\delta\rangle]$

c_6	✓c_6	✓c_6
⟨⟨$f_{1\text{-}3}$, $F_{1\text{-}4}$, a⟩, ⟨f_1⟩⟩		
⟨⟨$f_{1\text{-}3}$, $F_{1\text{-}4}$, a⟩, ⟨f_2⟩⟩		
⟨⟨$f_{1\text{-}3}$, $F_{1\text{-}4}$, a⟩, ⟨f_3⟩⟩		

iii. *One girl*$_{\top}{}^{\top}$ [*had a prior engagement*]$_{one}$

$((^{\top}[x|\ girl\langle x\rangle, x \in_i \top\delta t]^{\top};\ [hv.prior.eng\langle\top\delta\rangle]);\ [\mathbf{1}\{\top\delta||\}])$

c$_7$ c$_8$ ✓c$_8$

$\langle\langle f_3, f_{1\text{-}3}, F_{1\text{-}4}, a\rangle, \langle f_1\rangle\rangle$
$\langle\langle f_3, f_{1\text{-}3}, F_{1\text{-}4}, a\rangle, \langle f_2\rangle\rangle$
$\langle\langle f_3, f_{1\text{-}3}, F_{1\text{-}4}, a\rangle, \langle f_3\rangle\rangle$
$\langle\langle f_4, f_{1\text{-}3}, F_{1\text{-}4}, a\rangle, \langle f_1\rangle\rangle$ $\langle\langle f_4, f_{1\text{-}3}, F_{1\text{-}4}, a\rangle, \langle f_1\rangle\rangle$
$\langle\langle f_4, f_{1\text{-}3}, F_{1\text{-}4}, a\rangle, \langle f_2\rangle\rangle$ $\langle\langle f_4, f_{1\text{-}3}, F_{1\text{-}4}, a\rangle, \langle f_2\rangle\rangle$
$\langle\langle f_4, f_{1\text{-}3}, F_{1\text{-}4}, a\rangle, \langle f_3\rangle\rangle$ $\langle\langle f_4, f_{1\text{-}3}, F_{1\text{-}4}, a\rangle, \langle f_3\rangle\rangle$

3 "QUANTIFIER SUBORDINATION" AS TOP-LEVEL ANAPHORA

Observation: An indefinite introduced in the scope of a distributive quantifier can antecede a singular anaphoric pronoun in the next clause only if the pronoun, too, is in the scope of a distributive operator. (Karttunen 1976, Roberts 1987)

(4) i. *Most*$^{\top}$ *chess-sets*$^{\top}$

$^{\top}[x|\ chess.set\langle x\rangle]^{\top};\ ((^{\top}[X|\ X =_I \top\delta||];\ [^{\varnothing}\langle\top\delta t\rangle])^{\top}; \ldots$

c$_1$ c$_2$

$\langle\langle x_1\rangle, \langle\rangle\rangle$ $\langle\langle X_{1\text{-}4}, x_1\rangle, \langle\rangle\rangle$
$\langle\langle x_2\rangle, \langle\rangle\rangle$ $\langle\langle X_{1\text{-}4}, x_2\rangle, \langle\rangle\rangle$
$\langle\langle x_3\rangle, \langle\rangle\rangle$ $\langle\langle X_{1\text{-}4}, x_3\rangle, \langle\rangle\rangle$
$\langle\langle x_4\rangle, \langle\rangle\rangle$ $\langle\langle X_{1\text{-}4}, x_4\rangle, \langle\rangle\rangle$

[*came with a*$^{\perp}$ *spare pawn*]$_{most}$

$([x|\ spare.pawn\langle x\rangle, come.with\langle\top\delta, x\rangle];\ [\boldsymbol{most}\{\top\delta t, \top\delta||\}]))$

c$_3$ ✓c$_3$

$\langle\langle X_{1\text{-}4}, x_1\rangle, \langle y_1\rangle\rangle$
$\langle\langle X_{1\text{-}4}, x_2\rangle, \langle y_2\rangle\rangle$
$\langle\langle X_{1\text{-}4}, x_3\rangle, \langle y_3\rangle\rangle$

ii. DIS

$(^{\top}[X|\ X =_I \top\delta||];\ [^{\varnothing}\langle\top\delta t\rangle])^{\top}; \ldots$

c$_4$

$\langle\langle X_{1\text{-}3}, X_{1\text{-}4}, x_1\rangle, \langle y_1\rangle\rangle$
$\langle\langle X_{1\text{-}3}, X_{1\text{-}4}, x_2\rangle, \langle y_2\rangle\rangle$
$\langle\langle X_{1\text{-}3}, X_{1\text{-}4}, x_3\rangle, \langle y_3\rangle\rangle$

[*It*$_{\perp\top}$ *was taped to the box*$_{\top}$.]$_{DIS}$

$([\perp\delta =_I \boldsymbol{max}\{\perp\delta||_{\top\delta}\}];\ [\mathbf{1}\langle^{\mathcal{A}}\perp\delta\rangle];\ [taped.to.box\langle\perp\delta,\top\delta\rangle];\ [\top\delta t =_I \top\delta||])$

✓c$_4$ ✓c$_4$ ✓c$_4$ ✓c$_4$

4 QUANTIFIER SCOPE: TOPIC OVER BACKGROUND

Model: Al's students $S := \{s_1, s_2, s_3\}$ had test, t, with questions $Q := \{q_1, q_2, q_3\}$
Student s_1 answered q_1, q_2, q_3; s_2 answered q_1, q_2; and s_3 didn't answer anything.

(5)i. *Al's students*$^\top$ *had a test*$^\perp$ (*today*).

$[x|\ x =_i al];\ {}^\top[x|\ x =_i \sqcup std^{of}\langle\perp\delta\rangle, \mathbf{2}^+\langle^{\mathcal{A}}x\rangle];\ [x|\ test\langle x\rangle, take\langle\top\delta,\ x\rangle]$

c_1	c_2	c_3
$\langle\langle\rangle, \langle a\rangle\rangle$	$\langle\langle s_{1-3}\rangle, \langle a\rangle\rangle$	$\langle\langle s_{1-3}\rangle, \langle t, a\rangle\rangle$

ii. *Most*$^\top$ *kids*$^\top$ [*did well*]$_{most}$.

$^\top[x|\ kid\langle x\rangle, x \in_i^{\mathcal{A}} \top\delta]\ ^\top;\ ((^\top[X|\ X =_I \top\delta\|];[^\varnothing\langle\top\delta t\rangle])\ ^\top;\ ([do.well.on\langle\top\delta,\perp\delta\rangle])$
$[\boldsymbol{most}\{\top\delta t,\ \top\delta\|_{\perp\delta}\}]))$

c_4	c_5	
$\langle\langle s_1, s_{1-3}\rangle, \langle t, a\rangle\rangle$	$\langle\langle S_{1-3}, s_1, s_{1-3}\rangle, \langle t, a\rangle\rangle$	$\langle\langle S_{1-3}, s_1, s_{1-3}\rangle, \langle t, a\rangle\rangle$
$\langle\langle s_2, s_{1-3}\rangle, \langle t, a\rangle\rangle$	$\langle\langle S_{1-3}, s_2, s_{1-3}\rangle, \langle t, a\rangle\rangle$	$\langle\langle S_{1-3}, s_2, s_{1-3}\rangle, \langle t, a\rangle\rangle$
$\langle\langle s_3, s_{1-3}\rangle, \langle t, a\rangle\rangle$	$\langle\langle S_{1-3}, s_3, s_{1-3}\rangle, \langle t, a\rangle\rangle$	

iii. *At least one*$^\top$ *student*$_\top$ …

$([std\langle\top\delta\rangle];\ ^\top[X|\ X =_i \top\delta\|_{\perp\delta}];\ [^\varnothing\langle\top\delta t\rangle])\ ^\top;\ \ldots$

✓c_6	c_7
	$\langle\langle S_{1-2}, S_{1-3}, s_1, s_{1-3}\rangle, \langle t, a\rangle\rangle$
	$\langle\langle S_{1-2}, S_{1-3}, s_2, s_{1-3}\rangle, \langle t, a\rangle\rangle$

[*answered every*$^\perp$ *question*$^\perp$]$_{at.least.one}$

$(([x|\ que^{on}\langle x, \perp\delta\rangle];$ $\quad [X|\ X =_I \perp\delta\|])\ ^\perp;\ \ldots$

c_8	c_9
$\langle\langle S_{1-2}, S_{1-3}, s_1, s_{1-3}\rangle, \langle q_1, t, a\rangle\rangle$	$\langle\langle S_{1-2}, S_{1-3}, s_1, s_{1-3}\rangle, \langle Q_{1-3}, q_1, t, a\rangle\rangle$
$\langle\langle S_{1-2}, S_{1-3}, s_1, s_{1-3}\rangle, \langle q_2, t, a\rangle\rangle$	$\langle\langle S_{1-2}, S_{1-3}, s_1, s_{1-3}\rangle, \langle Q_{1-3}, q_2, t, a\rangle\rangle$
$\langle\langle S_{1-2}, S_{1-3}, s_1, s_{1-3}\rangle, \langle q_3, t, a\rangle\rangle$	$\langle\langle S_{1-2}, S_{1-3}, s_1, s_{1-3}\rangle, \langle Q_{1-3}, q_3, t, a\rangle\rangle$
$\langle\langle S_{1-2}, S_{1-3}, s_2, s_{1-3}\rangle, \langle q_1, t, a\rangle\rangle$	$\langle\langle S_{1-2}, S_{1-3}, s_2, s_{1-3}\rangle, \langle Q_{1-3}, q_1, t, a\rangle\rangle$
$\langle\langle S_{1-2}, S_{1-3}, s_2, s_{1-3}\rangle, \langle q_3, t, a\rangle\rangle$	$\langle\langle S_{1-2}, S_{1-3}, s_2, s_{1-3}\rangle, \langle Q_{1-3}, q_2, t, a\rangle\rangle$
$\langle\langle S_{1-2}, S_{1-3}, s_2, s_{1-3}\rangle, \langle q_3, t, a\rangle\rangle$	$\langle\langle S_{1-2}, S_{1-3}, s_2, s_{1-3}\rangle, \langle Q_{1-3}, q_3, t, a\rangle\rangle$

	$]_{every\perp\top}$	$]_{at.least.one\top}$
$([ans\langle\top\delta,\ \perp\delta\rangle];$	$[\perp\delta t =_I \perp\delta\|_{\top\delta}]));$	$[\mathbf{1}^+\{\top\delta\|\}]$
c_{10}	c_{11}	✓c_{11}
$\langle\langle S_{1-2}, S_{1-3}, s_1, \ldots\rangle, \langle Q_{1-3}, q_1, \ldots\rangle\rangle$	$\langle\langle S_{1-2}, S_{1-3}, s_1, \ldots\rangle, \langle Q_{1-3}, q_1, \ldots\rangle\rangle$	
$\langle\langle S_{1-2}, S_{1-3}, s_1, \ldots\rangle, \langle Q_{1-3}, q_2, \ldots\rangle\rangle$	$\langle\langle S_{1-2}, S_{1-3}, s_1, \ldots\rangle, \langle Q_{1-3}, q_2, \ldots\rangle\rangle$	
$\langle\langle S_{1-2}, S_{1-3}, s_1, \ldots\rangle, \langle Q_{1-3}, q_3, \ldots\rangle\rangle$	$\langle\langle S_{1-2}, S_{1-3}, s_1, \ldots\rangle, \langle Q_{1-3}, q_3, \ldots\rangle\rangle$	
$\langle\langle S_{1-2}, S_{1-3}, s_2, \ldots\rangle, \langle Q_{1-3}, q_1, \ldots\rangle\rangle$		
$\langle\langle S_{1-2}, S_{1-3}, s_2, \ldots\rangle, \langle Q_{1-3}, q_2, \ldots\rangle\rangle$		

Model: Al's students $S := \{s_1, s_2, s_3\}$ had test, t, with questions $Q := \{q_1, q_2, q_3\}$
Student s_1 answered q_1 and q_2; s_2 answered q_2; and s_3 answered q_3.

(6)i. *Al's students*$^{\top}$ *had a test*$^{\perp}$ (*today*).

$[x|\ x =_i al];\ {}^{\top}[x|\ x =_i \sqcup std^{of}\langle \perp\delta\rangle, \mathbf{2}^{+}\langle^{\mathcal{A}}x\rangle];\ [x|\ test\langle x\rangle, take\langle \top\delta,\ x\rangle]$

c_1	c_2	c_3
$\langle\langle\rangle, \langle a\rangle\rangle$	$\langle\langle s_{1\text{-}3}\rangle, \langle a\rangle\rangle$	$\langle\langle s_{1\text{-}3}\rangle, \langle t, a\rangle\rangle$

ii. *It*$_{\perp}{}^{\top}$ *was hard but doable.*

${}^{\top}[x|\ x =_I \boldsymbol{max}\{\perp\delta\|_{\top\delta}\}];\ [\mathbf{1}\langle^{\mathcal{A}}\top\delta\rangle];\ [hard\langle\top\delta\rangle, doable\langle\top\delta\rangle];$

c_4	$\checkmark c_4$	$\checkmark c_4$
$\langle\langle t, s_{1\text{-}3}\rangle, \langle t, a\rangle\rangle$		

iii. $[_{every}$

${}^{\top}[x|\ que^{on}\langle x,\ \top\delta\rangle]\ ^{\top};\ ((^{\top}[X|\ X =_I \top\delta\|];\ [^{\varnothing}\langle\top\delta t\rangle])\ ^{\top};\ \ldots$

c_5	c_7
$\langle\langle q_1, t, s_{1\text{-}3}\rangle, \langle t, a\rangle\rangle$	$\langle\langle Q_{1\text{-}3}, q_1, t, s_{1\text{-}3}\rangle, \langle t, a\rangle\rangle$
$\langle\langle q_2, t, s_{1\text{-}3}\rangle, \langle t, a\rangle\rangle$	$\langle\langle Q_{1\text{-}3}, q_2, t, s_{1\text{-}3}\rangle, \langle t, a\rangle\rangle$
$\langle\langle q_3, t, s_{1\text{-}3}\rangle, \langle t, a\rangle\rangle$	$\langle\langle Q_{1\text{-}3}, q_3, t, s_{1\text{-}3}\rangle, \langle t, a\rangle\rangle$

[*at least one student* …

$([x|\ std\langle x\rangle, x \in_i {}^{\mathcal{A}}\top'\delta\rangle]\ ^{\perp};\qquad ([X|\ X =_I \perp\delta\|];\ [^{\varnothing}\langle\perp\delta t\rangle])\ ^{\perp};$

c_8	c_9
$\langle\langle Q_{1\text{-}3}, q_1, t, s_{1\text{-}3}\rangle, \langle s_1, t, a\rangle\rangle$	$\langle\langle Q_{1\text{-}3}, q_1, t, s_{1\text{-}3}\rangle, \langle S_{1\text{-}3}, s_1, t, a\rangle\rangle$
$\langle\langle Q_{1\text{-}3}, q_1, t, s_{1\text{-}3}\rangle, \langle s_2, t, a\rangle\rangle$	$\langle\langle Q_{1\text{-}3}, q_1, t, s_{1\text{-}3}\rangle, \langle S_{1\text{-}3}, s_2, t, a\rangle\rangle$
$\langle\langle Q_{1\text{-}3}, q_1, t, s_{1\text{-}3}\rangle, \langle s_3, t, a\rangle\rangle$	$\langle\langle Q_{1\text{-}3}, q_1, t, s_{1\text{-}3}\rangle, \langle S_{1\text{-}3}, s_3, t, a\rangle\rangle$
$\langle\langle Q_{1\text{-}3}, q_2, t, s_{1\text{-}3}\rangle, \langle s_1, t, a\rangle\rangle$	$\langle\langle Q_{1\text{-}3}, q_2, t, s_{1\text{-}3}\rangle, \langle S_{1\text{-}3}, s_1, t, a\rangle\rangle$
$\langle\langle Q_{1\text{-}3}, q_2, t, s_{1\text{-}3}\rangle, \langle s_2, t, a\rangle\rangle$	$\langle\langle Q_{1\text{-}3}, q_2, t, s_{1\text{-}3}\rangle, \langle S_{1\text{-}3}, s_2, t, a\rangle\rangle$
$\langle\langle Q_{1\text{-}3}, q_2, t, s_{1\text{-}3}\rangle, \langle s_3, t, a\rangle\rangle$	$\langle\langle Q_{1\text{-}3}, q_2, t, s_{1\text{-}3}\rangle, \langle S_{1\text{-}3}, s_3, t, a\rangle\rangle$
$\langle\langle Q_{1\text{-}3}, q_3, t, s_{1\text{-}3}\rangle, \langle s_1, t, a\rangle\rangle$	$\langle\langle Q_{1\text{-}3}, q_3, t, s_{1\text{-}3}\rangle, \langle S_{1\text{-}3}, s_1, t, a\rangle\rangle$
$\langle\langle Q_{1\text{-}3}, q_3, t, s_{1\text{-}3}\rangle, \langle s_2, t, a\rangle\rangle$	$\langle\langle Q_{1\text{-}3}, q_3, t, s_{1\text{-}3}\rangle, \langle S_{1\text{-}3}, s_2, t, a\rangle\rangle$
$\langle\langle Q_{1\text{-}3}, q_3, t, s_{1\text{-}3}\rangle, \langle s_3, t, a\rangle\rangle$	$\langle\langle Q_{1\text{-}3}, q_3, t, s_{1\text{-}3}\rangle, \langle S_{1\text{-}3}, s_3, t, a\rangle\rangle$

answered $]_{at.least.one\perp\top}$ *every question*$_{\top}$]

$[ans\langle\perp\delta,\ \top\delta\rangle];\quad [\mathbf{1}^{+}\{\perp\delta\|_{\top\delta}\}];\quad [\top\delta t =_I \top\delta\|\}]$

c_{10}	$\checkmark c_{10}$	$\checkmark c_{10}$
$\langle\langle Q_{1\text{-}3}, q_1, t, s_{1\text{-}3}\rangle, \langle S_{1\text{-}3}, s_1, t, a\rangle\rangle$		
$\langle\langle Q_{1\text{-}3}, q_2, t, s_{1\text{-}3}\rangle, \langle S_{1\text{-}3}, s_1, t, a\rangle\rangle$		
$\langle\langle Q_{1\text{-}3}, q_2, t, s_{1\text{-}3}\rangle, \langle S_{1\text{-}3}, s_2, t, a\rangle\rangle$		
$\langle\langle Q_{1\text{-}3}, q_3, t, s_{1\text{-}3}\rangle, \langle S_{1\text{-}3}, s_3, t, a\rangle\rangle$		

APPENDIX A: DYNAMIC PLURAL LOGIC (DPlL, vdBerg 1996, unless noted)

D1 (DPlL basic expressions)

Var = $\{x, y, z, \ldots\}$

Prd^1 = $\{A^1, B^1, C^1, \ldots, Z^1, \mathbf{1}, \mathbf{1}^+, \mathbf{2}, \mathbf{2}^+, \ldots\}$
Prd^2 = $\{A^2, B^2, C^2, \ldots, Z^2, \textit{\textbf{most}}, \textit{\textbf{all}}, \textit{\textbf{no}}\}$
Prd^3 = $\{A^3, B^3, C^3, \ldots, Z^3\}$

D2 (DPlL syntax)

i. $Ru_1 \ldots u_n \in$ **Wff** , if $R \in$ **Prd**^{n} & $u_1, \ldots u_n \in$ **Var**
ii. $u_1 = u_2 \in$ **Wff** , if $u_1, u_2 \in$ **Var**
iii. $\neg\varphi \in$ **Wff** , if $\varphi \in$ **Wff**
iv. $(\varphi \wedge \psi) \in$ **Wff** , if $\varphi, \psi \in$ **Wff**
v. $\varepsilon_u \in$ **Wff** , if $u \in$ **Var**
vi. $\Delta_u(\varphi) \in$ **Wff** , if $u \in$ **Var** & $\varphi \in$ **Wff**
vii. $\mathbf{M}_u(\varphi) \in$ **Wff** , if $u \in$ **Var** & $\varphi \in$ **Wff**

D3 (DPlL models, assignments, info-states)

$\mathcal{M}$. A DPlL-*model* is a structure $\mathcal{M} = \langle \mathcal{D}_\mathcal{M}, \star, [\![\cdot]\!]_\mathcal{M} \rangle$ such that:

i. $\mathcal{D}_\mathcal{M}$ is a non-empty set (of $\mathcal{M}$-*entities*) and $\star \notin \mathcal{D}_\mathcal{M}$ (*dummy*)
ii. For $B \in$ **Prd**^{n}, $[\![B]\!]_\mathcal{M} \subseteq \{\langle X_1, \ldots, X_n\rangle |\ \varnothing \subset X_i \subseteq \mathcal{D}_\mathcal{M}\}$
iii. $[\![\boldsymbol{n}]\!]_\mathcal{M} = \{X: |X| = n\}$ $\quad [\![\textit{\textbf{most}}]\!]_\mathcal{M} = \{\langle X, Y\rangle: |X \cap Y| > |X \backslash Y|\}$
$[\![\boldsymbol{n}^+]\!]_\mathcal{M} = \{X: |X| \geq n\}$ $\quad \vdots$ (other quantifiers)

g. $\mathcal{G}_\mathcal{M} := \{g|\ g: \mathbf{Var} \to (\mathcal{D}_\mathcal{M} \cup \{\star\})\}$ (set of $\mathcal{M}$-assignments)
$g_\star \in \{g|\ g: \mathbf{Var} \to \{\star\}\}$ (default $\mathcal{M}$-assignment)
$g[u/d] := (g \backslash \{\langle u, g(u)\rangle\}) \cup \{\langle u, d\rangle\}$ (u-to-d alternative to g)

$\mathcal{G}$. An $\mathcal{M}$-*info state* is a set of $\mathcal{M}$-assignments. For any info-states $G, H \subseteq \mathcal{G}_\mathcal{M}$:

$G(u) := \{g(u): g \in G\ \&\ g(u) \neq \star\}$ $\quad u \in$ **Var**
$G|_{u=d} := \{g \in G|\ g(u) = d\}$ $\quad u \in$ **Var**, $d \in \mathcal{D}_\mathcal{M} \cup \{\star\}$
$G|_{u \neq d} := \{g \in G|\ g(u) \neq d\}$ $\quad u \in$ **Var**, $d \in \mathcal{D}_\mathcal{M} \cup \{\star\}$
$G[u/d] := \{g[u/d]: g \in G\}$ $\quad u \in$ **Var**, $d \in \mathcal{D}_\mathcal{M} \cup \{\star\}$
$G[u/X] := \{g[u/d]: g \in G\ \&\ d \in X\}$ $\quad u \in$ **Var**, $X \subseteq \mathcal{D}_\mathcal{M} \cup \{\star\}$
$G[u_1 \ldots u_n]H$ iff $G(u_1) = \ldots = G(u_n) = \varnothing$
$\&\ \forall d_1, \ldots, d_n \in \mathcal{D}_\mathcal{M}: G[u_1/d_1]\ldots[u_n/d_n] = H[u_1/d_1]\ldots[u_n/d_n]$

D4 (DPlL semantics). For any DPlL-model $\mathcal{M} = \langle \mathcal{D}_\mathcal{M}, \star, [\![\cdot]\!]_\mathcal{M} \rangle$ and info-states $G, H \subseteq \mathcal{G}_\mathcal{M}$:

i. $[\![Ru_1 \ldots u_n]\!]^\mathcal{M} = \{\langle G, G\rangle: \langle G(u_1), \ldots, G(u_n)\rangle \in [\![R]\!]_\mathcal{M}\}$
ii. $[\![u_1 = u_2]\!]^\mathcal{M} = \{\langle G, G\rangle: G(u_1) = G(u_2)\}$ Murray 08
iii. $[\![\neg\varphi]\!]^\mathcal{M} = \{\langle G, G\rangle: G \neq \varnothing\ \&\ \neg\exists H, K: \varnothing \subset H \subseteq G\ \&\ \langle H, K\rangle \in [\![\varphi]\!]^\mathcal{M}\}$ Brsvn07
iv. $[\![(\varphi \wedge \psi)]\!]^\mathcal{M} = \{\langle G, H\rangle: \exists K(\langle G, K\rangle \in [\![\varphi]\!]^\mathcal{M}\ \&\ \langle K, H\rangle \in [\![\psi]\!]^\mathcal{M})\}$
v. $[\![\varepsilon_u]\!]^\mathcal{M} = \{\langle G, H\rangle: G[u]H\}$ vdB diss+96
vi. $[\![\Delta_u(\varphi)]\!]^\mathcal{M} = \{\langle G, H\rangle: G|_{u=\star} = H|_{u=\star}\ \&\ \forall d \in G(u): \langle G|_{u=d}, H|_{u=d}\rangle \in [\![\varphi]\!]^\mathcal{M}\}$
vii. $[\![\mathbf{M}_u(\varphi)]\!]^\mathcal{M} = \{\langle G, H\rangle: \langle G, H\rangle \in [\![\varphi]\!]^\mathcal{M}\ \&\ \forall K: \langle G, K\rangle \in [\![\varphi]\!]^\mathcal{M} \Rightarrow K(u) \subseteq H(u)\}$ Brsvn07

D5 (DPlL-truth).

φ is *true* in $\mathcal{M}$, written $\mathcal{M} \models \varphi$, iff $\exists H: \langle \{g_\star\}, H\rangle \in [\![\phi]\!]^\mathcal{M}$

APPENDIX B:
UC_0 WITH PLURAL REFERENCE ($UC_{\delta+}$, Bittner 2012: Ch. 5)

D1 The set of $UC_{\delta+}$ *types* is the smallest set Θ s.t.: (i) $t, \delta, s \in \Theta$, and (ii) $(ab) \in \Theta$ if $a, b \in \Theta$. The subset $\text{DR}(\Theta) = \{\delta, \delta t\}$ is the set of *discourse referent types.*

D2.1 A $UC_{\delta+}$ *frame* is a set $\mathcal{F} = \{\mathcal{D}_a |\ a \in \Theta\}$ such that:

i. $\mathcal{D}_t = \{0, 1\}$ and $\mathcal{D}_\delta$ are non-empty disjoint sets

ii. $\mathcal{D}_s = \cup_{n \geq 0,\, m \geq 0}\{\langle\langle \mathsf{d}_1, \ldots, \mathsf{d}_n\rangle, \langle \mathsf{d}'_1, \ldots, \mathsf{d}'_m\rangle\rangle: \mathsf{d}_i, \mathsf{d}'_j \in \mathcal{D}_{dr}\}$,
with $\mathcal{D}_{dr} = \cup\{\mathcal{D}_a: a \in \text{DR}(\Theta)\}$

iii. $\mathcal{D}_{ab} = \{f |\ \varnothing \subset \text{Dom } f \subseteq \mathcal{D}_a \;\&\; \text{Ran } f \subseteq \mathcal{D}_b\}$

D2.2 An $\mathcal{F}$-*mereology* is a structure $\mathbf{M} = \langle \mathcal{D}_\delta, \sqsubseteq_\delta, {}^{\mathcal{A}}\rangle$ such that $\langle \mathcal{D}_\delta, \sqsubseteq_\delta\rangle$ is a join-semilattice and $\mathsf{y} \in^{\mathcal{A}} \mathsf{x} \Leftrightarrow \mathsf{y} \sqsubseteq_\delta \mathsf{x} \;\&\; \forall \mathsf{z}: \mathsf{z} \sqsubseteq_\delta \mathsf{y} \rightarrow \mathsf{z} = \mathsf{y}$

D3 A $UC_{\delta+}$ *model* is a tuple $\mathcal{M} = \langle \mathcal{F}, \mathbf{M}, [\![\cdot]\!]\rangle$ s.t. $\mathcal{F}$ is a $UC_{\delta+}$ frame, $\mathbf{M}$ is an $\mathcal{F}$-mereology, and $[\![\cdot]\!]$ maps $A \in Con_a$ to $[\![A]\!] \in \mathcal{D}_a$. Moreover, $\forall a \in \text{DR}(\Theta)$, $\mathsf{i} \in \mathcal{D}_s$:

$[\![\top a]\!](\mathsf{i}) \doteq ((①\mathsf{i})_a)_1$	$[\![\top' a]\!](\mathsf{i}) \doteq ((①\mathsf{i})_a)_2$	$[\![\top^{\Rightarrow} a]\!](\mathsf{i}) \doteq \{((①\mathsf{i})_a)_n: n \geq 1\}$
$[\![\bot a]\!](\mathsf{i}) \doteq ((②\mathsf{i})_a)_1$	$[\![\bot' a]\!](\mathsf{i}) \doteq ((②\mathsf{i})_a)_2$	$[\![\bot^{\Rightarrow} a]\!](\mathsf{i}) \doteq \{((②\mathsf{i})_a)_n: n \geq 1\}$

D4 ($UC_{\delta+}$ syntax)

i.	$Con_a \cup Var_a \subseteq Term_a$	
ii.	$(A = B) \in Term_t$	, if $A, B \in Term_a$
	$(A \precsim B) \in Term_t$	, if $A, B \in Term_s$
	$(A \sqsubseteq B) \in Term_t$	, if $A, B \in Term_\delta$
iii.	$\boldsymbol{n}(A), \boldsymbol{n}^{+}(A) \in Term_t$	, if $A \in Term_{at}$ and $\boldsymbol{n} \in \{\mathbf{1}, \mathbf{2}, \ldots\}$
	$\boldsymbol{most}(A, B) \in Term_t$	, if $A, B \in Term_{at}$
iv.	$\neg\varphi, (\varphi \wedge \psi) \in Term_t$	, if $\varphi, \psi \in Term_t$
v.	$\exists u_a \varphi \in Term_t$	, if $u_a \in Var_a$ and $\varphi \in Term_t$
vi.	$\lambda u_a(B) \in Term_{ab}$	, if $u_a \in Var_a$ and $B \in Term_b$
vii.	$BA \in Term_b$	, if $B \in Term_{ab}$ and $A \in Term_a$
viii.	${}^{\mathcal{A}}A \in Term_{\delta t}$	, if $A \in Term_\delta$
ix.	$(A_a \,{}^{\top}\!\bullet B), (A_a \,{}^{\bot}\!\bullet B) \in Term_s$	, if $a \in \text{DR}(\Theta)$, $A_a \in Trm_a$ and $B \in Term_s$
x.	$(A\,{}^{\top}; B), (A\,{}^{\bot}; B) \in Term_{(st)st}$	, if $A, B \in Trm_{(st)st}$

D5 ($UC_{\delta+}$ semantics)

i.	$[\![A]\!]^g$	$= [\![A]\!]$	, if $A \in Con_a$
	$[\![A]\!]^g$	$= g(A)$	, if $A \in Var_a$
ii.	$[\![(A = B)]\!]^g$	$= 1$	, if $[\![A]\!]^g = [\![B]\!]^g$; else, 0
	$[\![(A \precsim B)]\!]^g$	$= 1$	, if $[\![A]\!]^g \precsim_s [\![B]\!]^g$; else, 0
	$[\![(A \sqsubseteq B)]\!]^g$	$= 1$	, if $[\![A]\!]^g \sqsubseteq_\delta [\![B_\tau]\!]^g$; else, 0

iii. ABBR: If $A \in Trm_{at}$, ${}^{\{\}}[\![A]\!]^g := \{\mathsf{d} \in \mathcal{D}_a |\ [\![A]\!]^g(\mathsf{d}) = 1\}$ (*set characterized by A*)

$[\![\boldsymbol{n}(A)]\!]^g$	$= 1$	, if $\lvert{}^{\{\}}[\![A]\!]^g\rvert = n$; else, 0
$[\![\boldsymbol{n}^{+}(A)]\!]^g$	$= 1$	, if $\lvert{}^{\{\}}[\![A]\!]^g\rvert \geq n$; else, 0
$[\![\boldsymbol{most}(A, B)]\!]^g$	$= 1$	, if $\lvert{}^{\{\}}[\![A]\!]^g \cap {}^{\{\}}[\![B]\!]\rvert \geq \lvert{}^{\{\}}[\![A]\!]^g \backslash {}^{\{\}}[\![B]\!]\rvert$; else, 0

iv. $[\![\neg\varphi]\!]^g = 1$, if $[\![\varphi]\!]^g = 0$; else, 0

$[\![(\varphi \wedge \psi)]\!]^g = 1$, if $[\![\varphi]\!]^g = 1$ and $[\![\psi]\!]^g = 1$; else, 0

v. $[\![\exists u_a\varphi]\!]^g = 1$, if $\{\mathsf{d} \in \mathcal{D}_a |\ [\![\varphi]\!]^{g[u/\mathsf{d}]} = 1\} \neq \varnothing$; else, 0

vi. $[\![\lambda u_a(B)]\!]^g(\mathsf{d}) \doteq [\![B]\!]^{g[u/\mathsf{d}]}$, if $\mathsf{d} \in \mathcal{D}_a$

vii. $[\![BA]\!]^g \doteq [\![B]\!]^g([\![A]\!]^g)$

viii. $[\![{}^{\mathcal{A}}A]\!]^g \doteq {}^{\mathcal{A}}([\![A]\!]^g)$

ix. $[\![(A\ {}^{\top}\!\bullet\ B)]\!]^g \doteq \langle([\![A]\!]^g \cdot \textcircled{1}[\![B]\!]^g), \textcircled{2}[\![B]\!]^g\rangle$

$[\![(A\ {}^{\perp}\!\bullet\ B)]\!]^g \doteq \langle\textcircled{1}[\![B]\!]^g, ([\![A]\!]^g \cdot \textcircled{2}[\![B]\!]^g)\rangle$

x. $\mathsf{c}[\![(A\ {}^{\top}; B)]\!]^g = \{\mathsf{k} \in \mathsf{c}[\![A]\!]^g[\![B]\!]^g |\ \exists \mathsf{i} \in \mathsf{c} \exists \mathsf{j} \in \mathsf{c}[\![A]\!]^g \exists a \in \mathrm{DR}(\Theta)\text{: } (\textcircled{1}\mathsf{j})_1 \in \mathcal{D}_a$
$\quad \&\ \textcircled{1}\mathsf{i} \prec \textcircled{1}\mathsf{j}\ \&\ (\textcircled{1}\mathsf{j})_a = (\textcircled{1}\mathsf{k})_a\ \&\ [\![B]\!]^g \neq [\![B[\top a/\bot a]]\!]^g\}$

$\mathsf{c}[\![(A\ {}^{\perp}; B)]\!]^g = \{\mathsf{k} \in \mathsf{c}[\![A]\!]^g[\![B]\!]^g |\ \exists \mathsf{i} \in \mathsf{c} \exists \mathsf{j} \in \mathsf{c}[\![A]\!]^g \exists a \in \mathrm{DR}(\Theta)\text{: } (\textcircled{2}\mathsf{j})_1 \in \mathcal{D}_a$
$\quad \&\ \textcircled{2}\mathsf{i} \prec \textcircled{2}\mathsf{j}\ \&\ (\textcircled{2}\mathsf{j})_a = (\textcircled{2}\mathsf{k})_a\ \&\ [\![B]\!]^g \neq [\![B[\bot a/\top a]]\!]^g\}$

APPENDIX C: DRT-ABBREVIATIONS FOR $\mathrm{UC}_{\delta+}$

Table 1. Old Drt-abbreviations (as for UC_τ)

• <u>*static terms* ($a \in \mathrm{DR}(\Theta)$)</u>

$(A \neq B) := \neg(A = B)$

$A = B \sqcup B' := B \sqsubseteq A \wedge B' \sqsubseteq A \wedge \forall u(B \sqsubseteq u \wedge B' \sqsubseteq u \rightarrow A \sqsubseteq u)$

• <u>*(local) projections*</u> (type *sa* for $a \in \mathrm{DR}(\Theta)$)

$(A_a)^\circ := \lambda i(A)$

$(A_{sa})^\circ := \lambda i(Ai)$

$(B_{ab}\, A_{sa})^\circ := \lambda i(B\ (A)^\circ i)$

• <u>*(local) conditions*</u> (type *st*)

$B\ \mathsf{R}_i\ A := \lambda i(B^\circ i\ \mathsf{R}\ A^\circ i)$ for $\mathsf{R} \in \{=, \in, \ldots\}$

$B\langle A_1, \ldots, A_n\rangle := \lambda i(B(A_1{}^\circ i, \ldots, A_n{}^\circ i))$

$(C_1, C_2) := \lambda i(C_1 i \wedge C_2 i)$

• <u>*updates*</u> (type *(st)st*)

$[C] := \lambda I\lambda i(Ii \wedge Ci)$

${}^{\top}[u_1 \ldots u_n] := \lambda I\lambda j(\exists u_1 \ldots \exists u_n \exists i(Ii \wedge j = (u_1\ {}^{\top}\!\bullet \ldots (u_n\ {}^{\top}\!\bullet\ i)\ldots)))$

$[u_1 \ldots u_n] := \lambda I\lambda j(\exists u_1 \ldots \exists u_n \exists i(Ii \wedge j = (u_1\ {}^{\perp}\!\bullet \ldots (u_n\ {}^{\perp}\!\bullet\ i)\ldots)))$

${}^{\top}[u_1 \ldots u_n|\ C] := \lambda I\lambda j(\exists u_1 \ldots \exists u_n \exists i(Ii \wedge Ci \wedge j = (u_1\ {}^{\top}\!\bullet \ldots (u_n\ {}^{\top}\!\bullet\ i)\ldots)))$

$[u_1 \ldots u_n|\ C] := \lambda I\lambda j(\exists u_1 \ldots \exists u_n \exists i(Ii \wedge Ci \wedge j = (u_1\ {}^{\perp}\!\bullet \ldots (u_n\ {}^{\perp}\!\bullet\ i)\ldots)))$

$(D_1; D_2) := \lambda I\lambda j(D_2\, D_1 I\, j)$

$\sim\! D := \lambda I\lambda j(Ij \wedge \neg\exists k(j \precsim k \wedge DI\, k))$

Table 2. New Drt-abbreviations

• <u>*static conditions* (type *t*)</u>

$A \bigcirc B := \exists u(u \sqsubseteq A \wedge u \sqsubseteq B)$

$A_a = \boldsymbol{max}(B_{at}) := A \in B \wedge \forall u_a(u \in B \rightarrow u \sqsubseteq A)$

${}^{\varnothing}A_{at} := \mathbf{2}^{+}A \wedge \forall u_a \forall v_a(u \in A \wedge v \in A \wedge u \bigcirc v \rightarrow u = v)$

• <u>*global projections*</u> (type $s(st)a$, for $a \in \mathrm{DR}(\Theta)$)

$(A_a)^*$	:=	$\lambda i_s \lambda I_{st}(A)$
$(A_{sa})^*$	:=	$\lambda i_s \lambda I_{st}(Ai)$
$(A_{sa}\|\|)^*$	:=	$\lambda i_s \lambda I_{st} \lambda u_a(\exists j(Ij \wedge u = Aj))$
$(A\|\|_{B1\ldots Bn})^*$	:=	$\lambda i_s \lambda I_{st} \lambda u_a(\exists j(Ij \wedge B_1 i = B_1 j \wedge \ldots \wedge B_n i = B_n j \wedge u = Aj))$
$(B\{A\|\|_{\ldots}\})^*$	:=	$\lambda i_s \lambda I_{st}(B\,(A\|\|_{\ldots})^* iI)$

• <u>*global conditions*</u> (type $s(st)t$)

$B \;\mathrm{R}_I\; A$	:=	$\lambda i_s \lambda I_{st}(B^* iI \;\mathrm{R}\; A^* iI)$	for $\mathrm{R} \in \{=, \in, \ldots\}$
$B\{A_1, \ldots, A_n\}$	:=	$\lambda i_s \lambda I_{st}(B(A_1{}^* iI, \ldots, A_n{}^* iI))$	

• <u>*global updates*</u> (type $(st)st$)

$[G]$	:=	$\lambda I_{st} \lambda i_s(Ii \wedge GiI)$
${}^{\top}[u_a\| G]$	:=	$\lambda I_{st} \lambda j_s(\exists u_a \exists i_s(Ii \wedge GiI \wedge j = (u \,{}^{\top}\!\bullet i)))$
$[u_a\| G]$	:=	$\lambda I_{st} \lambda j_s(\exists u_a \exists i_s(Ii \wedge GiI \wedge j = (u \,{}^{\perp}\!\bullet i)))$

Examples (C1–12) below illustrate how *Table 1* (T1) and *Table 2* (T2) can be used to turn Drt-abbreviations into 'official' $\mathrm{UC}_{\delta+}$ (Appendix B) for some updates in discourses (3)–(6):

C1 Test whether the input $\perp\delta$-column contains at least two individuals (3i)

$[\mathbf{2}^{+}\{\perp\delta\|\}]$
$:= \lambda I \lambda i(Ii \wedge (\mathbf{2}^{+}\{\perp\delta\|\})iI)$ T2.$[G]$
$:= \lambda I \lambda i(Ii \wedge \mathbf{2}^{+}(\perp\delta\|)^* iI)$ T2.$B\{A_1$
$:= \lambda I \lambda i(Ii \wedge \mathbf{2}^{+}\,\lambda x_\delta(\exists j(Ij \wedge x = \perp\delta j)))$ T2.$(A_{sa}\|)^*$

C2 Introduce the local $\perp\delta$-column as a topical set (3ii)

${}^{\top}[X\| X =_I \perp\delta\|]$
$:= \lambda I \lambda j(\exists X_{\delta t} \exists i(Ii \wedge (X =_I \perp\delta\|)iI \wedge j = (X\,{}^{\top}\!\bullet i)))$ T2.${}^{\top}[u\| G]$
$:= \lambda I \lambda j(\exists X_{\delta t} \exists i(Ii \wedge (X)^* iI = (\perp\delta\|)^* iI \wedge j = (X\,{}^{\top}\!\bullet i)))$ T2.$B \;\mathrm{R}_I\; A$
$:= \lambda I \lambda j(\exists X_{\delta t} \exists i(Ii \wedge X = \lambda x_\delta(\exists j(Ij \wedge x = \perp\delta j)) \wedge j = (X\,{}^{\top}\!\bullet i)))$ T2.$(A_a)^*$, $(A_{sa}\|)^*$

C3 Test whether the local topic set consists of pairwise disjoint elements (3ii)

$[{}^{\varnothing}\langle\top\delta t\rangle]$
$:= \lambda I \lambda i(Ii \wedge ({}^{\varnothing}\langle\top\delta t\rangle)i)$ T1.$[C]$
$:= \lambda I \lambda i(Ii \wedge {}^{\varnothing}(\top\delta t^{\circ} i))$ T1.$B\langle A_1$
$:= \lambda I \lambda i(Ii \wedge {}^{\varnothing}(\top\delta t\; i))$ T1.$(A_{sa})^{\circ}$
$:= \lambda I \lambda i(Ii \wedge \mathbf{2}^{+}\,\top\delta t\; i \wedge \forall x_\delta \forall y_\delta(x \in \top\delta t\; i \wedge y \in \top\delta t\; i \wedge x \bigcirc y \rightarrow x = y))$ T2.${}^{\varnothing}A$

C4 Test whether most of the elements of the local topic set are in the input $\perp\delta$-column (3ii)

$[\boldsymbol{most}\{\top\delta t, \perp\delta\|\}]$
$:= \lambda I \lambda i(Ii \wedge (\boldsymbol{most}\{\top\delta t, \perp\delta\|\})iI)$ T2.$[G]$
$:= \lambda I \lambda i(Ii \wedge \boldsymbol{most}((\top\delta t)^* iI, (\perp\delta\|)^* iI))$ T2.$B\{A_1$
$:= \lambda I \lambda i(Ii \wedge \boldsymbol{most}(\top\delta t\; i, \lambda x_\delta(\exists j(Ij \wedge x = \perp\delta j))))$ T2.$(A_{sa})^*$, $(A_{sa}\|)^*$

C5 Introduce the sum of the input $\perp\delta$-column as a topical individual (3ii)

${}^{\top}[x\| x =_I \sqcup\{\perp\delta\|\}]$
$:= \lambda I \lambda j(\exists x_\delta \exists i(Ii \wedge (x =_I \sqcup\{\perp\delta\|\})iI \wedge j = (x\,{}^{\top}\!\bullet i)))$ T2.${}^{\top}[u\| G]$
$:= \lambda I \lambda j(\exists X_{\delta t} \exists i(Ii \wedge (x)^* iI = (\sqcup\{\perp\delta\|\})^* iI \wedge j = (x\,{}^{\top}\!\bullet i)))$ T2.$B \;\mathrm{R}_I\; A$
$:= \lambda I \lambda j(\exists X_{\delta t} \exists i(Ii \wedge x = \sqcup\lambda x_\delta(\exists j(Ij \wedge x = \perp\delta j)) \wedge j = (x\,{}^{\top}\!\bullet i)))$ T2.$(A_a)^*$, $B\{A_1$

C6 Test whether the local topical individual is a plurality (at least two atomic parts) (3ii)

$[\mathbf{2}^{+}\langle^{\mathcal{A}}\top\delta\rangle]$

$:= \lambda I\lambda i(Ii \wedge (\mathbf{2}^{+}\langle^{\mathcal{A}}\top\delta\rangle)i)$ T1.$[C]$

$:= \lambda I\lambda i(Ii \wedge \mathbf{2}^{+}((^{\mathcal{A}}\bot\delta)^{\circ}i))$ T1.$B\langle A_1$

$:= \lambda I\lambda i(Ii \wedge \mathbf{2}^{+}(^{\mathcal{A}}(\top\delta i)))$ T1.$(B_{ab}\,A_{sa})^{\circ}$

C7 Introduce a girl from the local topic set as a topical individual (3iii)

$^{\top}[x|\ girl\langle x\rangle, x \in_i \top\delta t]$

$:= \lambda I\lambda j(\exists x_\delta \exists i(Ii \wedge (girl\langle x\rangle)i \wedge (x \in_i \top\delta t)i \wedge j = (x\ {}^{\top}\!\bullet\ i)))$ T1.$^{\top}[u|\ C], (C_1, C_2)$

$:= \lambda I\lambda j(\exists x_\delta \exists i(Ii \wedge girl\,(x)^{\circ}i \wedge (x)^{\circ}i \in (\top\delta t)^{\circ}i \wedge j = (x\ {}^{\top}\!\bullet\ i)))$ T1.$B\langle A_1, B \mathsf{R}_i A$

$:= \lambda I\lambda j(\exists x_\delta \exists i(Ii \wedge girl\,x \wedge x \in \top\delta t\ i \wedge j = (x\ {}^{\top}\!\bullet\ i)))$ T1.$(A_a)^{\circ}, (A_{sa})^{\circ}$

C8 Test whether the input $\top\delta$-column contains exactly one individual (3iii)

$[\mathbf{1}\{\top\delta||\}]$

$:= \lambda I\lambda i(Ii \wedge (\mathbf{1}\{\top\delta||\})iI)$ T2.$[G]$

$:= \lambda I\lambda i(Ii \wedge \mathbf{1}\,(\top\delta||)^{*}iI)$ T2.$B\{A_1$

$:= \lambda I\lambda i(Ii \wedge \mathbf{1}\,\lambda x_\delta(\exists j(Ij \wedge x = \bot\delta j)))$ T2.$(A_{sa}||)^{*}$

C9 Test whether the local background individual ($\bot\delta i$) is the $\sqsubseteq_\delta$-maximum of the $\bot\delta$-column for the substate with the given local topic ($\top\delta i$) (4ii)

$[\bot\delta =_I \boldsymbol{max}\{\bot\delta||_{\top\delta}\}]$

$:= \lambda I\lambda i(Ii \wedge (\bot\delta =_I \boldsymbol{max}\{\bot\delta||_{\top\delta}\})iI)$ T2.$[G]$

$:= \lambda I\lambda i(Ii \wedge (\bot\delta)^{*}iI = (\boldsymbol{max}\{\bot\delta||_{\top\delta}\})^{*}iI))$ T2.$B\ \mathsf{R}_I\ A$

$:= \lambda I\lambda i(Ii \wedge (\bot\delta)^{*}iI = \boldsymbol{max}\,((\bot\delta||_{\top\delta})^{*}iI))$ T2.$(B\{A||\ldots\})^{*}$

$:= \lambda I\lambda i(Ii \wedge \bot\delta i = \boldsymbol{max}\ \lambda x_\delta(\exists j(Ij \wedge \top\delta i = \top\delta j \wedge x = \bot\delta j)))$ T2.$(A_{sa})^{*}, (A_{sa}||_B)^{*}$

C10 Test whether the local topic set ($\top\delta t\ i$) is the input $\top\delta$-column (4ii)

$[\top\delta t =_I \top\delta||]$

$:= \lambda I\lambda i(Ii \wedge (\top\delta t =_I \top\delta||)iI)$ T2.$[G]$

$:= \lambda I\lambda i(Ii \wedge (\top\delta t)^{*}iI =_I (\top\delta||)^{*}iI)$ T2. $B\ \mathsf{R}_I\ A$

$:= \lambda I\lambda i(Ii \wedge \top\delta t\ i = \lambda x_\delta(\exists j(Ij \wedge x = \top\delta j)))$ T2.$(\cdot)^{*}$

C11 Introduce the plural sum of students of the local background ($\bot\delta i$) as a topic (5i)

$^{\top}[x|\ x =_i \sqcup std^{of}\langle\bot\delta\rangle, \mathbf{2}^{+}\langle^{\mathcal{A}}x\rangle]$

$:= \lambda I\lambda j(\exists x_\delta \exists i(Ii \wedge (x =_i \sqcup std^{of}\langle\bot\delta\rangle)i \wedge (\mathbf{2}^{+}\langle^{\mathcal{A}}x\rangle)i \wedge j = (x\ {}^{\top}\!\bullet\ i)))$ T1.$^{\top}[u|\ C], (C_1, C_2)$

$:= \lambda I\lambda j(\exists x_\delta \exists i(Ii \wedge x^{\circ}i = (\sqcup std^{of}\langle\bot\delta\rangle)^{\circ}i \wedge \mathbf{2}^{+}(^{\mathcal{A}}(x^{\circ}i)) \wedge j = (x\ {}^{\top}\!\bullet\ i)))$ T1.$B\langle A_1, B \mathsf{R}_i A$

$:= \lambda I\lambda j(\exists x_\delta \exists i(Ii \wedge x^{\circ}i = \sqcup std^{of}\,(\bot\delta)^{\circ}i \wedge \mathbf{2}^{+}(^{\mathcal{A}}(x^{\circ}i)) \wedge j = (x\ {}^{\top}\!\bullet\ i)))$ T1.$(B_{ab}\,A_{sa})^{\circ}$

$:= \lambda I\lambda j(\exists x_\delta \exists i(Ii \wedge x = \sqcup std^{of}(\bot\delta i) \wedge \mathbf{2}^{+}(^{\mathcal{A}}x) \wedge j = (x\ {}^{\top}\!\bullet\ i)))$ T1.$(\cdot)^{\circ}$

C12 Test whether most of the elements of the local topic set are in the input $\top\delta$-column for the substate with the given background (exam, $\bot\delta i$) (5ii)

$[\boldsymbol{most}\{\top\delta t, \top\delta||_{\bot\delta}\}]$

$:= \lambda I\lambda i(Ii \wedge (\boldsymbol{most}\{\top\delta t, \top\delta||_{\bot\delta}\})iI)$ T2.$[G]$

$:= \lambda I\lambda i(Ii \wedge \boldsymbol{most}((\top\delta t)^{*}iI, (\top\delta||_{\bot\delta})^{*}iI))$ T2.$B\{A_1$

$:= \lambda I\lambda i(Ii \wedge \boldsymbol{most}(\top\delta t\ i, \lambda x_\delta(\exists j(Ij \wedge \bot\delta i = \bot\delta j \wedge x = \bot\delta j))))$ T2.$(A_{sa})^{*}, (A_{sa}||_B)^{*}$

C13 Test whether the local background set is the input $\bot\delta$-column for the substate with the given topic (student, $\top\delta i$) (5iii)

$[\bot\delta t =_I \bot\delta\|_{\top\delta}]$

$:= \lambda I\lambda i(Ii \wedge (\bot\delta t =_I \bot\delta\|_{\top\delta})iI)$ T2.$[G]$

$:= \lambda I\lambda i(Ii \wedge (\bot\delta t)^*iI = (\bot\delta\|_{\top\delta})^*iI)$ T2.$B\ \mathsf{R}_I\ A$

$:= \lambda I\lambda i(Ii \wedge \bot\delta t\ i = \lambda x_\delta(\exists j(Ij \wedge \top\delta i = \top\delta j \wedge x = \top\delta j)))$ T2.$(\cdot)^*$

C14 Introduce the $\sqsubseteq_\delta$-maximum of the $\bot\delta$-column for the substate with the given local topic (sum of Al's students, $\top\delta i$) as a topic (6ii)

${}^\top[x|\ x =_I \boldsymbol{max}\{\bot\delta\|_{\top\delta}\}]$

$:= \lambda I\lambda j(\exists x_\delta\exists i(Ii \wedge (x =_I \boldsymbol{max}\{\bot\delta\|_{\top\delta}\})iI \wedge j = (x\ {}^\top\!\bullet\ i)))$ T2.${}^\top[u|\ G]$

$:= \lambda I\lambda i(Ii \wedge (\bot\delta)^*iI = (\boldsymbol{max}\{\bot\delta\|_{\top\delta}\})^*iI))$ T2.$B\ \mathsf{R}_I\ A$

$:= \lambda I\lambda i(Ii \wedge (\bot\delta)^*iI = \boldsymbol{max}\ ((\bot\delta\|_{\top\delta})^*iI))$ T2.$(B\{A\|_{\ldots}\})^*$

$:= \lambda I\lambda i(Ii \wedge \bot\delta i = \boldsymbol{max}\ \lambda x_\delta(\exists j(Ij \wedge \top\delta i = \top\delta j \wedge x = \bot\delta j)))$ T2.$(A_{sa})^*$, $(A_{sa}\|_B)^*$

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