Quantum Mechanics as Generalised Theory of Probabilities

Michel Bitbol
Archives Husserl, CNRS/ENS, 45 rue d'Ulm, 75005 Paris, France

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Prologue

The thesis I will defend here comprises two propositions: Firstly, quantum mechanics is not a physical theory that happens to make use of probability calculus; it is *itself* a generalised form of probability calculus, doubled by a procedure of evaluation that is probabilistic by way of its controlled usage of symmetries. Secondly, quantum mechanics does not have merely a predictive function like other physical theories; it *consists* in a formalisation of the conditions of possibility of any prediction bearing upon phenomena whose circumstances of detection are also conditions of *production*.

Probabilities, Signs, and Secondary Qualities

Before developing and justifying the above propositions, I should like to return briefly to the prehistory of probability calculus, between the sixteenth and seventeenth century. This return will help us in overcoming certain prejudices about probability that are the product of an intermediate era (roughly, the eighteenth and nineteenth centuries), and in approaching quantum mechanics with an open mind. I mean in particular the prejudice that consists in conceiving probability merely as the expression of a certain subjective ignorance of underlying processes, processes which play out in and of themselves, according to determinist laws.

So what conditions permitted the collective elaboration, from the seventeenth century onward,[1] of probability calculus? Ian Hacking has furnished an extensive list of such conditions,[2] but he insists upon one in particular. This crucial condition is the development, in the sixteenth century, of sciences of *signs* or of *secondary qualities*.

The distinction between primary qualities and secondary qualities—in other words, between properties that show themselves such as they are intrinsically, and properties imputed to material bodies on the basis of impressions or signs that result from their interactions with the sense organs—is usually attributed to Locke. It can ultimately be traced to Galileo, Descartes and Robert Boyle. But in fact we already find the trace of it earlier, in Jerome Francastor, a doctor in the first half of the sixteenth century.

From the moment when the distinction was recognised, an opposition could develop between the sciences of first causes and exact proofs (such as astronomy, geometry, and mechanics), and the other sciences (such as medicine and chemistry) which were reduced to *prognostics* acting on the basis of signs, phenomena, or sensible secondary qualities. It is in the field of so-called 'inferior' sciences, these sciences of secondary qualities, that the notion would crystallise of an opinion

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[1] One precursor is Jerome Cardan, in the mid sixteenth century. But his work remained isolated, with no real influence, and his main treatise on games of chance was only printed for the first time in 1663, in the era of Pascal and Huyghens.

[2] I. Hacking, *The Emergence of Probability* (Cambridge: Cambridge University Press, 1975).

supported by signs, which in part gave rise to the concept of probability. The clues as to the outbreak of an epidemic, or the symptoms of an illness, which are secondary in relation to the supposed primary causes of the epidemic or the illness, were, for example, called 'signs of probability' by Francastor in his book *On Contagion*.[3]

This close association between the birth of the concept of probability and the elaboration of the concept of secondary qualities bears an implicit lesson for the understanding of the privileged link between quantum mechanics and probability. For, as Heisenberg wrote, quantum physics confronts a situation where *even* spatio-cinematic variables of the position and quantity of movement, which were considered at the time of Descartes and Locke as direct and 'primary', must be taken as indirect manifestations, relative to an instrumental context—in short, as secondary.[4] To the universalization of the notion of secondary quality, or of the relativity of phenomena in regard to a context, responds, in quantum mechanics, the universalization of the pertinent domain of probability.

One can guess, however, from this short summary, the reason why the concept of probability remained embryonic and marginal in the natural science of the first half of the seventeenth century; a reason which also explains, albeit belatedly, the contemporary reticence to take entirely seriously a physical theory built on a probabilistic armature, such as quantum mechanics. This reason is that, from the beginning, probabilities were considered as a predictive lesser evil in a situation where one finds oneself momentarily incapable of offering an descriptive account based upon well-founded principles and truths (truths concerning efficient causes if one is Aristotelian, figures and movements if one is Cartesian). It is no surprise, under these conditions, that all the efforts of the players in the first scientific revolution were focused on elucidating causal links or describing a real universe of primary qualities by way of geometry, rather than seeking to systematise the estimation of the uncertain in the shifting circumscription of secondary qualities.

The Uncertain and the Middle of Things

As Catherine Chevalley quite rightly emphasises,[5] the estimation of the uncertain only began to constitute an entirely separate theme of investigation in the work of an anticartesian thinker, Pascal, for whom 'the end of things and their beginning are hopelessly hidden from [man] in an impenetrable secret'.[6] If man must content himself, according to Pascal, with 'perceiving the appearance of the middle of things, in an eternal despair of knowing either their beginning or their end'[7], he cannot content himself with denigrating the appearances in favour of an ungraspable backworld governed by principles. Man must learn to inhabit his milieu; he must know how to focus his attention upon the play of his experimental manipulations and the phenomena that result from them; he must admit the inconsistency of cutting up the world into separate and intrinsically-existing objects, since phenomena are so tied one to another that it is impossible to know how to grasp one without grasping all; he must understand, also, that no cognition can free itself from the nexus of interrelations, but can only situate itself within it, remaining cognisant of the

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[3] Ibid., 28.

[4] W. Heisenberg, *Philosophical Problems of Quantum Physics* (Woodbridge, Connecticut: Oxbow Press, 1979), 38.

[5] C. Chevalley, *Pascal, contingence et probabilities* (Paris: PUF, 1995).

[6] B. Pascal, *Pensées*, fragment 199, in *Oeuvres complètes* (Paris: Seuil, 1963), 526.

[7] Ibid.

perspective from which it derives. Finally, man must consent to make the effort to domesticate the uncertainty that is his lot, by mathematizing directly the relations between antecedents and expectations, and between expectations and observations.

Of course, probability calculus was able to develop after Pascal by freeing itself of what some would call an epistemological pessimism motivated by the vertigo of the impenetrability of the Divine plan. The tone, in Laplace's 1814 *Philosophical Essay on Probabilities*, is almost antipodeal to the latter, since Laplace here affirms the all-powerfulness of a Principle of Sufficient Reason incarnated by a God whose work is transparent. According to Laplace, 'the curve described by a simple molecule of air or of vapour is governed in a manner just as certain as that of planetary orbits: the only difference between them is that which our ignorance places there.'[8] And it is only in this interval between the in-principle determination of all things and our perhaps provisional ignorance with regard to them, that probability has any place: 'probability', Laplace continues, 'is relative in part to our ignorance, and in part to our cognitions'.[9]

Such a conception perfectly fulfilled its office in the framework of classical physics, and particularly in classical statistical mechanics (leaving to one side the more recent problematic of sensitivity to initial conditions). But, faced with the recurrent question of the essential or nonessential character of probabilities in quantum physics, and the difficulties it presents to the thesis of probability-ignorance, it was worthwhile our returning to Laplace, and recalling that the calculus of probabilities made one of its first appearances upon an entirely other philosophical terrain. It emerged in Pascal, as we saw, on the basis of a recognition of anthropological limits, of an epistemology close to operationalism, of a generalised holism, and of a gnoseological perspectivism. One cannot but be struck in observing that *all* these traits are present in the most current interpretations of quantum mechanics, and that there is no acceptable interpretation that does not include at least one of them. The most frequently encountered trait, including in the most reliable 'hidden variable' interpretation, is *holism*.

Indeterminism and Contextuality

These two historical remarks—one on the association of the concept of probability with the concept of secondary qualities, the other on the conception of probability calculus as an instrument for the predictive mastery of our situation of entanglement in the network of natural relations—will now help us to undo two interpretative knots of quantum physics, both of which relate to *indeterminism*.

The first concerns the notion, very widespread since Heisenberg's foundational works of 1927-30, of an uncontrollable *perturbation* that the agent of measurement is supposed to exert upon the microscopic measured object. It is interesting to note that this 'perturbation' was assigned a twofold role by those who conceived of it.

On one hand, as Bohr emphasises at the end of the 1920s, the uncontrollable perturbation constitutes the reason for the *indivisibility* of quantum phenomena—that is to say, the impossibility of separating in the phenomena that which belongs to the object and that which belongs to the measuring agent. Perturbation would explain, in other words (borrowed this time from Heisenberg), that quantum physics leads to a generalisation of the model of secondary qualities—with their obligatory reference to

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[8] P. S. de Laplace, *Essai philosophique sur les probabilités* (Paris: Courcier, 1814), 4.

[9] Ibid.

the context within which they manifest themselves—to the detriment of that of primary intrinsic qualities.

But on the other hand, according to the 1927 article where Heisenberg presents his so-called ‘uncertainty’ relations for the first time, the perturbation is also that which takes account of the *indeterminism* of quantum physics. The incompressible and uncontrollable perturbation by the agent of measurement is what prevents complete knowledge of the two groups of variables that compose the initial state of a particle. Consequently, concludes Heisenberg, the principle of causality, which links in a constraining fashion an initial state and a final state, is inapplicable in quantum physics.

The model of ‘perturbation’ thus allows us to bring to light a close relation between contextuality and indeterminism, since perturbation has as a consequence the contextuality of phenomena as well as an indeterminism in regard to them. A relation which is, perhaps, historically translated in the confluence of the concepts of secondary quality and probability at the time of their birth. Unfortunately, the image of the perturbation of the object by the measuring agent also has a major inconvenience which did not escape Bohr or Heisenberg, and which Karl Popper later emphasised: Basically, this image begins by bringing into play a universe of objects endowed with primary spatial and kinematic qualities, and then invoking their mutual alteration so as to subsequently justify the putting aside of the concept of primary quality and the generalisation of that of secondary quality.[10] In this image, then, one puts forward the representation of a universe of figures and movements, with the unique aim of demonstrating its inanity, or (what comes down to the same thing, for a verificationist epistemology) its in-principle inaccessibility.

The image of ‘perturbation’ thus represents a metastable moment in the reflection on quantum mechanics. It invites us to surpass it, in two opposite directions: Either we take wholly seriously the *premises*, and we try to construct an empirically-adequate theory of the inaccessible spatio-kinematic processes that are postulated—this is the strategy of the authors of certain hidden variable theories. Or, on the contrary, we take wholly seriously the holistic *consequences* of the image of perturbation, namely the indivisibility of the quantum phenomena, its unsurpassable *relativity* to an experimental context, and we develop a conception of physical theory that no longer appeals to an imagined representation of the supposedly constitutive moments of the phenomena—this is the strategy that Bohr adopted from 1935 onward, not without certain shortcomings.

It is reassuring for those who, like myself, have chosen to push the second strategy to its ultimate consequences, to observe that it is possible to establish a *direct* formal link between indeterminism and contextuality, without need of an intermediary furnished by the image of perturbation. In 1935, Grete Harmann published a small book in which she hinted at such a link.[11] This young German philosopher remarked that the possible causes of a quantum phenomena *cannot be used to foresee it*, because they are only ever defined afterwards, *relatively* to the very circumstances of the production of this phenomena when measured. Later, at the beginning of the 1950s, Paulette Destouches-Fevrier proved in a much more rigorous way a theorem according to which every predictive theory bearing upon phenomena defined relative

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[10] On this point, see M. Bitbol, *Mécanique quantique, une introduction philosophique* (Paris: Flammarion, 1996), Chapter 3.

[11] G. Hermann, *Les fondements philosophiques de la mécanique quantique,* tr. A. Schnell and L. Soler (Paris: Vrin, 1996), 90.

to experimental contexts certain of which are mutually incompatible, is ‘essentially indeterminist’.[12]

Determinist Ideals, Indeterminist Projections

Let us remark now that, through what has been said above, a second interpretative knot concerning the relation between quantum physics and indeterminism has been implicitly undone. It was often asked in the 1930s whether quantum mechanics, with its probabilistic, or even ‘statistical’ (as Einstein had it) character might one day be rendered obsolete by a determinist theory of the individual underlying processes. The response of the last forty years research to this question is somewhat sibylline, but all the more instructive philosophically.

The first lesson to be drawn from this research is that *it is not possible* to formulate theories which dictate the intrinsic properties of individual objects via determinist laws, but which also reproduce exactly the predictions of quantum mechanics.[13] These so-called hidden variable theories simply find themselves subject to certain constraints, principal among which are non-locality[14] (that is to say, the instantaneous mutual influence of the properties of arbitrarily distant objects) and contextualism[15] (that is to say, the influence of the measuring device on the postulated properties). These two conditions, however, do raise problems. The nonlocal concept of instantaneous interactions at a distance[16] introduces a formal conflict (albeit one

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[12] P. Destouches-Février, *La structure des théories physiques* (Paris:PUF, 1951), 260-80. This theorem is perfectly compatible with the existence of hidden variable (or rather, hidden *process*) theories, since it bears only upon the link between effective or possible phenomena, and not the link between processes *in principle* inaccessible to experimentation (see paragraph 5).

[13] D. Bohm & B. Hiley, *The Undivided Universe* (London: Routledge, 1993).

[14] J. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge: Cambridge University Press, 1987).

[15] S. Kochen and E. Specker, ‘The problem of hidden variables in quantum mechanics’, *Journal of Mathematical Mechanics* 17 (1967), 59-87.

[16] At this stage we must avoid confusions of vocabulary and ideas between standard quantum mechanics and hidden-variable theories, which may lead one to believe wrongly that standard quantum mechanics encounters the same problems as hidden-variable theories with regard to nonlocality. Standard quantum mechanics leads us to *foresee correlations* between distant experimental events; but, in itself, it furnishes *nothing that could be taken for an explanation* of these correlations. In particular, it does not imply in itself any idea of *nonlocal* interactions. All one can remark is the *nonfactorisability* of the components of a global state vector which furnishes (correlated) probabilities of the results of two distant events. But the current interpretation of the state vector poses the temptation to a semantic overdetermination of nonfactorisability. For in this interpretation, a state vector represents the ‘state’ of a ‘system’, and not merely a generalised instrument of the probability calculus of experimental phenomena. Whence the nonfactorisability of the state vector was understood as the *nonseparability* of the states of the subsystems that composed the system; and this nonseparability was sometimes confused with the nonlocality that implies instantaneous influence. The difficulties and confusions here come from the mixing up of an operationalist and predictive orientation with implicitly ontological and descriptive elements (the concepts of ‘system’ and ‘state’).

Hidden-variable theories have at least the advantage that they seek to furnish an *explicitly* ontological interpretation of quantum mechanics, bringing in the intrinsic properties of objects. On this basis, they can claim to *explain* correlations. The correlations are explained either in applying the concept of causes common to the postulated intrinsic properties (local hidden variable theories), or by invoking instantaneous interactions at a distance between these properties (nonlocal hidden variable theories). The explanation via common causes having been excluded by Bell’s theorem, it remains to partisans of hidden variable theories to confront the consequences of the explanation via instantaneous interactions at a distance.

Let us just indicate that an attempt to generalise explanation via causes common to *contextual phenomena* rather than to properties, without soliciting any structure other than that of standard quantum mechanics, and consequently without falling victim to Bell’s theorem, can be found in M. Bitbol, *Mécanique quantique, une introduction philosophique* (Paris: Flammarion, 1996), 189-91.

without practical consequences) with the axioms of relativity theory.[17] As for contextualism, it has as a consequence that measurements do not at all allow us to accede, point by point, to continuous and determinist processes which, according to the theory, would take place in nd of themselves in nature *if one had not modified them by seeking to bring them to light*. In other words, the theory itself implies that the 'independent' determinist processes that it describes are inaccessible to experience.

The conclusion to draw from this is certainly not that we must cast anathema on hidden variable theories, but simply that it is indispensible to revise their ambitions downward. We have seen that one of the principal objectives of their advocates was to reopen the question of determinism, against those who affirmed overhastily that this question had already been settled (in a negative sense) by quantum mechanics. Standard quantum mechanics may well have been 'essentially indeterminist' in structure, but if one could reproduce its results in an *other* theory comprising determinist processes, the determinist option would regain all of its credibility. It is true that the *ontological* question of knowing whether the ultimate laws of nature are or are not determinist is *undecidable*, because determinist appearances can result from a statistical regularity, and, inversely, indeterminist appearances can be a translation of deterministic chaotic phenomena.[18] But at least one could still hope for determinism to rediscover its traditional status as a guiding thread for research. But we have been disabused of even this hope. For in hidden variable theories, the determinist stance does indeed seem to have been lost, even at the level of its epistemological fecundity. The determinist stance was only fruitful because it compelled researchers to conceive of networks of univocal bonds underlying phenomena, to design the type of experiment that would allow these bonds to be brought to light, and to thus define often unprecedented classes of phenomena. Unfortunately, this process is blocked from the outset by the in-principle inaccessibility of the bonds that underlie phenomena in contextualist hidden variable theories capable of reproducing quantum predictions. Once the reciprocal current of information between the determinist project and the definition of new domains of experimentation dries up, the attempt to pursue this project formally becomes nothing more than a jeu d'esprit whose principal (if not sole) interest is its serving as an intellectual stimulant for specialists in the foundations of modern physics.

This situation does not justify, for all that, the inverse excess—namely, indeterminist dogmatism. All one is within one's rights to observe is that henceforth, in the physical sciences, the advantage of epistemological fruitfulness will belong to the stance that consists in maximally developing predictive capacity to the detriment

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[17] But, it will be asked, are there not *also* difficulties in adapting standard quantum theories to relativistic theories, in spite of the advances realised by Dirac and the creators of quantum field theory? Doubtless. However, in the light of the remarks made in the preceding note, one might think that these difficulties are *not of the same nature* as those met with by hidden variable theories. Hidden variable theories confront relativistic theories *on their own terrain*—that of the *description* of spatiotemporal events that can be treated *as if they occurred of themselves*. On the contrary, standard quantum theories operate on a completely different plane: that of the prediction of phenomena whose production is suspended in the presence of appropriate contexts that are sometimes mutually incompatible. The problem of the putting into concordance of standard quantum theories and relativistic physics thus very probably pertains to the difficulty in defining an appropriate terrain upon which the two theoretical universes can encounter each other, rather than to a direct conflict.

[18] On this subject, see Jacques Harthong's 'fifth conflict of transcendental ideas', cited by A. Dahan-Dalmedico in A. Dahan-Dalmedico, J.L. Chabert and K. Chemla (eds.), *Chaos et déterminisme* (Paris:Seuil, 1992), 405; and J. Harthong, *Probabilités et statistiques* (Paris: Diderot, 1996).

of descriptive ambition, the calculus of probabilities rather than determinist models of evolution.

It is true that many thinkers do not stop there; they tend to extrapolate the epistemological observation of the fecundity of the indeterminist option into an ontological affirmation of the intrinsically stochastic character of the laws governing the world. But their position is easily acceptable on the methodological plane, without it being necessary to follow them in the metaphysical aspects of their conclusions. As James Logue has shown in his *Projective Probability*,[19] every coherent system of probabilistic evaluation can be interpreted in realist fashion—that is to say, it can be understood as expressing propositions whose truth status is independent of the means of testing them. And this interpretation in turn might lead the authors of a probabilistic evaluation to project it onto the world. We should not be surprised, in these circumstances, that quantum physics' coherent system of probabilistic evaluations, without the counterbalance of a fruitful determinist programme, could have been conceived by researchers as eminent as Popper (and even Heisenberg, in his own way) as translating, in part or as a whole, a 'real' or 'existent' characteristic of the world.[20] Popper, for example, holds that the world is made of capacities, of potentialities or of natural propensities, which manifest themselves experimentally through particular statistical distributions of phenomena, and which are reflected in quantum theory in the form of a probabilistic algorithm.

Incontestably, the partisans of an ontological indeterminism thereby deliver themselves, just as much as the defenders of hidden variable theories, to what Kant would have denounced as an attempt to extend the application of our concepts beyond the limits of experience[21]—the sole advantage accruing to the partisans of hidden variable theories being that they limit themselves to directly hypostasizing the quantum formalism's mode of operation, rather than seeking to develop a new one. But ought we to reproach them for this? Since every coherent system of probabilistic evaluation can be read in a realist mode, since nothing prevents the interpretation of the quantum algorithm of probability calculus as translating an order of natural propensities, why would we prohibit them from *adhering* unhesitatingly to such interpretations? Why would we refuse their *belief*, without ulterior motive, that quantum theory describes a reality made of pure potentialities?

The type of response we shall try to give to this question is of an epistemological rather than metaphysical order. We shall not ask if reality *is* or *is not* made of potentialities that have the structure of the probabilistic algorithm of quantum theory, but only whether or not we lose anything on the plane of understanding if we interpret this algorithm in realist fashion.

Let us say right away—and this is the meaning of James Logue's statement of equivalence—that neither the practitioner of probabilistic evaluation nor the quantum physicist lose anything whatsoever to such a way of seeing things. They may even gain something that is at the heart of every profession of realist faith—namely, the seriousness with which they consider their theoretical entities, and the motivation for

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[19] J. Logue, *Projective probability* (Oxford: Oxford University Press, 1995).

[20] K. Popper, *A Universe of Propensities* (Bristol: Thoemmes, 1992).

[21] This accusation is addressed to someone who takes Popper's declarations on propensity more literally than Popper himself. For Popper recognised that the *metaphysical* question of determinism remains undecidable before, just as after, quantum mechanics. He considered simply that only the abandonment of dogmatic determinism could open the way to nonconventional theories of change that may prove more fruitful than the causal and spatiokinematic theories inherited from the first scientific revolution of the seventeenth century. One of these alternative theories is no other than the theory of propensions. See K. Popper, *The Open Universe* (London: Routledge, 1988), Chapter 4.

research.[22] On the other hand, the philosopher really does have something to lose in allowing himself to be fascinated by the sole relation between theory and world. For this stance does not at all incite him to reflect upon what the theory owes to the *situation* of man in the world, and in particular what it owes to the very practice of experimental investigation. Unlike the scientist in his everyday work, the philosopher cannot content himself with *occupying* the Pascalian situation of the man in the milieu that he explores; he must *think* this situation, and charge himself with enunciating its consequences. The scientific researcher, moreover, also has an interest in adopting the reflexive stance from time to time, when she arrives at periods of reorientation in her work. And everyone knows that she finds herself almost inevitably led to do so when science is going through revolutionary times.

A Generalised Theory of Probability

It is this type of reorientation that we shall now proceed to undertake. We are going to suspend judgment on the subject of a hypothetical partial isomorphism between quantum mechanics and the real in which we experiment, and interest ourselves selectively in what the structure of this theory owes to the form of experimental activity itself.

Let us begin by rapidly recounting, to this end, the architecture of standard quantum mechanics:

1. The formal kernel of this theory consists in a vector space defined on the set of complex numbers, and endowed with a scalar product; in other words, *Hilbert space*.

2. Upon this space are defined specific operators, called 'observables', which furnish, through their eigenvalues, the list of possible results of an operation of measurement.

3. A vector in Hilbert space, called a *state vector*, is associated with each *preparation* (that is to say, with that which, in an experiment, fixes the conditions necessary for measurement).

4. By applying *Born's Rule* to this state vector, we obtain a function assigning *probabilities* to the results of any measurement whatsoever that is made subsequent to this preparation.

5. As variable spatiotemporal intervals and diverse physical circumstances can separate the end of the functioning of the preparation and the operation of measurement, we take account of them by way of an *evolution equation* of the state vectors: Schrödinger's equation in the non-relativistic case, Dirac's in the relativist case.

Here I would like to insist upon the major difference between the probability functions in the classical theory of probabilities and those that are obtained on the basis of the state vectors of quantum mechanics by applying Born's Rule. Classical probability functions associate a number between 0 and 1 with each 'event' in the broad sense, defined by Kolmogorov[23] as a subset of elementary events. The set of these subset-events comprises the empty set and the exhaustive set, and it is endowed with a Boolean algebra structure by the operations of union and intersection. In other words, classical probability functions are defined upon a Boolean algebra. On the contrary, taking account of the properties of Hilbert space, quantum probability functions are not defined upon a Boolean algebra; they are defined upon different and

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[22] See M. Bitbol, *Schrödinger's Philosophy of Quantum Mechanics* (Dordrecht: Kluwer, 1996), paragraphs 5-9.

[23] A. Kolmogorov, *Foundations of the Theory of Probability* (New York: Chelsea, 1950).

richer structures called ‘orthoalgebras’.[24] I will avoid detailing the axioms of orthoalgebra, and content myself with indicating that the concept of orthoalgebra is not unrelated to Boolean algebra. One might even consider that orthoalgebras constitute a generalisation of Boolean algebras, and that, correlatively, quantum probability functions generalise classical probability functions. For an orthoalgebra contains Boolean algebras as its substructures; and, on the other hand, the restriction of a quantum probability function to these Boolean substructures is equivalent to a classical probability function.

This structural disparity between classical and quantum probability functions justifies our refusing to content ourselves with the observation that quantum mechanics ‘uses’ probability theory. *Quantum mechanics itself consists, in part, in a new and broadened form of probability theory.*

A Metacontextual Predictive Formalism

It would, however, be a shame to limit ourselves to this superficial and formalist exposition of the situation. We can easily enough understand the reasons for the emergence of a new sort of theory of probability by showing that it is a practically inevitable response to the characteristics of the class of experimental phenomena that quantum mechanics deals with. Principal among these characteristics, as already pointed out in our reflections on the concept of secondary qualities, is *contextuality*; in other words, the inseparability of the phenomena and the experimental context of its manifestation. It is this that imposes a great many of the structural characteristics of quantum theory.

But to really bring to light the very strong link between contextuality and quantum mechanics, we must firstly analyse what makes the contextuality of quantum phenomena uncircumventable, and differentiate it from other, more benign and easily surmountable forms of the relation of determinations to a context.

In all sciences, as in many ordinary situations, we can say that to each experimental or sensory situation there corresponds a whole gamut of possible phenomena or determinations. For example, to a context represented by the cones of the retina there corresponds a scale of colours, to a context represented by a ruler corresponds a scale of lengths, to a context represented by a thermometer corresponds a scale of temperatures, and so on. But as long as the contexts can be conjoined, or as long as the determinations are indifferent to the order of intervention of contexts, nothing prevents our fusing the scales of possibilities into one sole scale relative to one sole global context, and then passing over this context in silence and treating the elements of the scale as if they translated so many intrinsic determinations. The presupposition that nothing prevents us from retracting the context is automatic when one makes use of the propositions of ordinary language: for the latter allow us to attribute many determinations to the same object as if they belonged to it. It is important to note that this presupposition and this mode of functioning of language are associated with a classical, boolean logic and a classical, Kolmogorovian, theory of probabilities.

But the appearance of obstacles to the conjunction of contexts, or the observing of an lack of independence of phenomena vis-à-vis the order of utilisation of contexts, as is the case in microscopic physics when one tries to measure canonically conjugated variables, renders traditional methods useless. The strategy of

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[24] R.I.G. Hughes, *The Structure and Interpretation of Quantum Mechanics* (Cambridge, MA: Harvard University Press, 1989), 220.

not taking account of the experimental context fails, and it becomes imperative to make explicit the contextuality of determinations.

In this situation that confronts quantum physics, boolean logic and kolmogorovian probability, at first glance, only survive fragmented into many sublogics and many probabilistic structures, each of them associated with a particular context. To *each* experimental context is associated a scale of possible determinations and a scale of attributive propositions which belong to a classical, boolean, sublogic; and to each determination chosen from the set of possible determinations corresponding to *a* given context, can be attached a real number that obeys Kolmogorov's axioms of probability. But these sublogics and these probabilistic substructures cannot be *fused together*, for they depend on distinct contexts that cannot, in general, be conjoined. Under such conditions, we seek to *articulate* them with each other, respectively in the framework of a metalogic and a metacontextual probabilistic formalism. What is remarkable is that when one constructs such a metalogic, in taking account *only* of the impossibility of conjoining the diverse scales of possibles, one arrives at structures isomorphic with the celebrated nondistributive 'quantum logic' of de Birkhoff and von Neumann.[25] And what is more, when one tries to construct a metacontextual probabilistic formalism, in constraining oneself *only* to respect Kolmogorov's axioms separately for *each* scale of possibilities, and utilising *one unique* generative symbol of subfunctions of probabilities for each preparation, one arrives at a class of structures whose vector formalism in Hilbert spaces of quantum mechanics is hardly a peculiar case. The form of the evolution equation of quantum mechanics is itself derivable from the general conditions bearing upon the temporal stability of the status of the tool of probabilistic evaluation of the state vector.[26]

In its function as a theory-framework, quantum mechanics is consequently nothing less than *a metacontextual form of probability theory*. It brings together the conditions of possibility of a *unified* system of probabilistic prediction bearing upon phenomena inseparable from sometimes incompatible contexts. It only remains to complete this theory-framework with various symmetries[27] in order to draw from it various particular varieties of quantum theory.

Decoherence and Probabilities

We have seen that, short of confronting the grave epistemological difficulties of nonlocal hidden variable theories, quantum probabilities cannot be taken as the expression of an *ignorance* on the subject of processes or events that happen *of themselves* within nature. The quantum calculus of probabilities bears upon phenomena whose occurrence is suspended by the intervention of an appropriate context. The problem is that, qua physical theory, quantum mechanics has a vocation to universality. The metacontextual probability calculus which is its principal

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[25] P. Heelan, 'Complementarity, Context-Dependence, and Quantum Logic', *Found. Phys.* 1 (1970), 95-110.

[26] Bitbol, *Mécanique quantique, une introduction philosophique*; M. Bitbol, 'Towards a transcendental deduction of quantum mechanics', 5th UK conference on the conceptual problems of modern physics, Oxford, September 1996. The essential theorems that allow us to arrive at these conclusions are due to J.L. Destouches, P. Destouches-Février, G. Fano, A.M. Gleason, et J. Bub. See P. Destouches-Février, *La structure des théories physiques* (Paris: PUF, 1951), and R.I.G. Hughes, *The Structure and Interpretation of Quantum Mechanics* (on the subject of the theorems of Fano, Gleason and Bub).

[27] 'Quantum mechanics is not itself a dynamical theory. It is an empty stage. You have to add the actors [...] the missing element that has to be added to quantum mechanics is a principle, or several principles, of symmetry.' S. Weinberg, in R.P. Feynman & S. Weinberg, *Elementary Particles and the Laws of Physics* (Cambridge: Cambridge University Press, 1987), 87.

constitutive element must therefore be able to be applied without restriction and on every scale. But, in our familiar environment, isn't the classical (Kolmogorovian) theory of probabilities perfectly utilisable? And doesn't this classical theory, unlike its quantum equivalent, function such that nothing prohibits us from considering it as expressing a partial ignorance as to intrinsic properties and autonomous events? Thus is posed a problem of *compatibility*, between quantum probability calculus (valid in principle on every scale) and the classical calculus of probabilities (valid in practice on our scale).

The principal objective of theories of decoherence is to test this compatibility. For they allow us to prove that, when applied to complex processes involving an object, a measuring apparatus, and a vast environment, quantum probability calculus is reduced, *up to a very weak approximation*, to the classical calculus of probabilities. This is manifest in the quasi-disappearance of the terms of *interference* typical of quantum probability calculus, and isomorphic to those of a wave process, in favour of a quasi-validity of the classical rule of the additivity of probabilities of a disjunction.

However, rare are those physicists who are content with this purely *probabilistic and predictive* formulation of theories of decoherence. Some among them have even cherished the hope of utilising decoherence as a means of explaining the *emergence* of a classical world, on the basis of a quantum world supposedly 'described' by a universal state vector.[28] The major obstacle they find themselves up against is that, in order to be able to derive, on the basis of a purely quantum calculation, the classical laws and behaviours that prevail at the human scale, they have not been able to avoid introducing hypotheses that already contain anthropomorphic elements.[29]

These discomfitures encourage us to demand no more from decoherence theories than a retrospective assurance of a coherence that is *in practice* sufficient between quantum probability calculus and the presupposition, at once fundamental and elementary, that subtends its experimental testimony. This presupposition consists in admitting that macroscopic events (like the deviation of the indicator of an apparatus) themselves arise in the laboratory, that their trace is for all time available for any researcher who desires to repeat the observation, and that the utilisation of probability calculus *with regard to them,* consequently, only expresses a partial ignorance as to what they are.

Quantum Field Theory, Path Integrals, and Metacontextual Predictive Formalism

The reflections above, it is true, hold something surprising for certain contemporary physicists. For, in manipulating a concept of field sometimes insufficiently distinguished from its classical equivalent, and in taking literally the processes that are figured imagistically in Feynman diagrams, a non-negligible number among them ended up behaving as though the conceptual problems that quantum mechanics raised at its birth were but a bad memory. If physics 'describes' the evolution of fundamental fields, and/or if manages to 'describe', equally, the

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[28] M. Gell-Mann, *The Quark and the Jaguar. Adventures in the Simple and the Complex* (New York: Abacus, 1995).

[29] Bitbol, *Mécanique quantique, une introduction philosophique,* 410-18. For a close critique of the ontological claims of theories of decoherence, see B. d'Espagnat, 'Towards a Separable Empirical Reality', *Foundations of Physics* 20 (1990), 1147-72. R. Omnès's response can be found in R. Omnès, *The Interpretation of Quantum Mechanics* (Princeton, NJ: Princeton University Press, 1994).

dynamics of particles (considered as a state of excitation of a field) by way of the procedure of Feynman path integrals, why still preoccupy oneself with that old Bohrian notion of the inseparability of the phenomena and its experimental conditions of manifestation? Why bring to the fore a notion as opaque for the theoretical physicist as that of 'measurement'?[30] Why insist obstinately upon the *predictive* rather than *descriptive* status of quantum theories? Isn't it possible that many of the philosophical perplexities of the creators of quantum mechanics were linked to the use of a formalism (that of vectors in Hilbert space) which, in the most advanced theories, has been rendered obsolete by the formalism of path integrals?

The response to these questions is that, in truth, *none* of the epistemological constraints exerted by the standard quantum mechanics of 1926 have been relaxed by contemporary varieties of quantum theory, and that new constraints of a similar order have even been added to them. Whatever representations they may give rise to, current quantum theories *always* operate as generalised, metacontextual instruments of probabilistic prediction. And this stems from the fact that they are *always* confronted with phenomena inseparable from their context of manifestation. So as to shore up this response, it will be sufficient to evoke rapidly the renovation of philosophical reflections invited by quantum field theory, and then to redefine the relations between the formalism of state vectors in Hilbert space and that of Feynman path integrals.

The central trait of quantum theories, which is that they consist in a metacontextual structure of probabilistic prediction, is rediscovered, not only intact but amplified, in quantum field theory. At the end of a reflection on the formalisms of Fock spaces, Paul Teller concludes: 'states [in Fock space] simply characterise propensities for what will be manifested with what probability under various activating conditions. Among the items for which there can be propensities for manifestation is the occurrence of various numbers of quanta…'[31] In other words, far from having rendered superfluous contextual notions such as those of propensive state, 'observable', and conditions of 'activation', quantum field theories have generalised their application. The concept of quantum field is derived from the classical concept of the field by putting local observables into correspondence with local functions, and through the introduction of relations of commutation (or anti-commutation) for certain couples of observables. As to state vectors in Fock space, they allow not only the calculation of the probability that this or that 'property' of a particle will manifest itself in a given experimental context, but the probability that a certain *number* of particles will be detected under the appropriate instrumental conditions. This number itself is treated as an *observable*, the set of whose possible values *under appropriate conditions of detection* is identified with the set of whole natural numbers. To the contextualisation of the *predicate* of objects typical of standard quantum mechanics, then, quantum field theory adds the contextualisation of the notion of the denumerable *substrates* of predicates.

That one must from now on hold the very concept of 'particles', and not only that of 'properties of a particle', to be relative to a context of manifestation, is rendered particularly evident by the relativistic phenomenon of so-called 'Rindler

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[30] Let us remark in passing that, if it is true that the concept of 'measurement' cannot claim any particular status within the field of validity of physical theories (since it is not distinct in principle from other physical processes), it belongs to the conceptual resources of a background that is *logically prior* to any *experimental, testable* theoretical elaboration. It thus has an uncircumventable *metatheoretical* function.

[31] P. Teller, *An Interpretive Introduction to Quantum Field Theory* (Princeton, NJ: Princeton University Press, 1995), 105.

particles'. This phenomenon is observed when accelerating a detector in the 'void'.[32] The *accelerated* detector responds, in this environment where no detector *at rest* would detect the least particle, as if it were plunged into a thermic bath of particles.[33] It is thus quite clear that one cannot treat particles as objects that 'are' there or 'are not' there, independently of the conditions of their detection. One has only the right to speak of *phenomena of detection* that imply, indissociably, a milieu (say 'the quantum void'), a detector, and the dynamic state of this detector. Quantum field theories now appear as particular elaborations of the metacontextual probabilistic framework of quantum mechanics—elaborations adapted to a broadened class of contextual phenomena, belonging to a relativistic domain, and pertaining to the formal concept of 'support', beyond that of 'property'.[34]

Now we come to Feynman path integral formalisms, which have often supplanted standard formalisms in the modern practice of quantum field theories.[35] Although the functioning of path integrals is illustrated by linear diagrams evoking spatiotemporal trajectories of particles, their role is only to permit the calculation of the *probability* of a final experimental event (at a certain point) under condition of the occurrence of an initial experimental event (at another point). Here, the dependence of the phenomena whose probability is calculated on an instrumental context is only implicit, but it plays a no less capital role in the very principle of the calculation to be carried out. For what does one concretely do when one evaluates a path integral? One sums 'probability amplitudes', then one takes the square of the modulus of the sum thus obtained, to obtain the *probability* one seeks.[36] Now, the distinction between probability amplitudes and probabilities corresponds pretty much to that between virtual experiments and actual experiments. Read in the framework of standard formalism, probability amplitude is nothing other than the scalar product of the state vector and of a vector belonging to an observable corresponding to an experiment that *would have* taken place (but which did not) in the interval that separates the two actual experiments.[37] On the contrary, probability is calculated for the result of an experiment that will *actually* take place, or which already has done. The path integral formalism, just like that of Hilbert space vectors, manifests the metacontextual predictive structure of quantum theories. It consists in evaluating the probability of a contextual phenomenon by summing the terms corresponding to *virtual intermediary contexts* distinct from those in which the phenomenon is effectively manifest.

Let us add to this two other circumstances that suggest close relations between the functioning of quantum theories utilising a vector formalism in Hilbert space, and those making use of path integrals: Firstly, Feynman has demonstrated the equivalence between the formalism of standard quantum mechanics, which puts a Hamiltonian operator in place in Schrodinger's equation, and the path integral

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[32] The formal translation of the concept of the 'quantum void' is a state vector in Fock space identical to those of vectors belonging to the observable 'Number' that is associated with the proper value *Zero*. This state vector associates a null probability with the detection of a number of particles larger than zero by a detector *at rest*.

[33] Teller, *An interpretive introduction,* 110.

[34] The link between the intervention of *relativistic* processes and the mobilisation of the concept of 'support' alongside that of 'property' is made immediately comprehensible by a clarifying remark of B. d'Espagnat: From the fact that the energy-mass equivalence established by the Theory of Special Relativity, d'Espagnat remarks (in *Le réel voilé* [Paris: Fayard, 1994]), it can always happen that a 'property of objects' (kinetic energy) is transformed into 'objects' (one or several particles).

[35] In *superstring* theory, the linear pathways are replaced by *tubes*, and the summation effectuated by an integral bears upon these tubes of spacetime whose section is an annular 'string'.

[36] R.P. Feynman & A.R. Hibbs, *Quantum Mechanics and Path Integrals* (New York: McGraw Hill, 1965); R.P. Feynman, *QED: The Strange Theory of Light and Matter* (Princeton, NJ: Princeton University Press, 1988).

[37] Generally, these experiments consist in making (virtual or actual) measurements of the *spatial position*.

formalism, which uses the corresponding Lagrangian function.[38] What is more, just as certain principles of symmetry determine the Hamiltonian form of Schrodinger's equation, it is the principles of symmetry that allow us to fix the density of the Lagrangian of each interaction, and thus to determine the path integral.[39] The use of such principles of symmetry has as its most concrete consequence the modulation of path integrals (and consequently also that of the probabilistic evaulations that result from them), annulling the amplitude of certain of the diagrams that intervene in the summation.[40]

Epilogue

All of this leads us to conclude with two propositions that are valid independently of the variety of quantum theory or formalism utilised: Every quantum theory combines an invariable element—a metacontextual form of probability theory—and a variable element—a set of symmetries. The association of these two elements makes it a system of probabilistic evaluation suitable for a class of experimental situations whose extension depends upon the symmetries brought into play.

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[38] Feynman & Hibbs, *Quantum Mechanics and Path Integrals*, Chapter 4.

[39] G. Cohen-Tannoudji & M. Spiro, *La matière-espace-temps* (Paris: Gallimard, 1990), 185.

[40] S. Weinberg, *The Quantum Theory of Fields I* (Cambridge: Cambridge University Press, 1995), 428.