Hindawi
Complexity
Volume 2017, Article ID 6038973, 12 pages
https://doi.org/10.1155/2017/6038973

Research Article

Optimization of Consignment-Store-Based Supply Chain with Black Hole Algorithm

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Received 23 February 2017; Revised 24 May 2017; Accepted 15 August 2017; Published 27 September 2017

Academic Editor: Petri T. Helo

The globalization of economy and market led to increased networking in the field of manufacturing and services. These manufacturing and service processes including supply chain became more and more complex. The supply chain includes in many cases consignment stores. The design and operation of these complex supply chain processes can be described as NP-hard optimization problems. These problems can be solved using sophisticated models and methods based on metaheuristic algorithms. This research proposes an integrated supply model based on consignment stores. After a careful literature review, this paper introduces a mathematical model to formulate the problem of consignment-store-based supply chain optimization. The integrated model includes facility location and assignment problems to be solved. Next, an enhanced black hole algorithm dealing with multiobjective supply chain model is presented. The sensitivity analysis of the heuristic black hole optimization method is also described to check the efficiency of new operators to increase the convergence of the algorithm. Numerical results with different datasets demonstrate how the proposed model supports the efficiency, flexibility, and reliability of the consignment-store-based supply chain.

1. Introduction

In today's economy, the pressure is on to make the operations of supply chain from purchasing to distribution more and more efficient. The competition is characterized as competition between supply chain networks rather than competition between individual production and service companies. The upstream and downstream linkages of involved organizations like production and service companies, suppliers, 3PL or 4PL providers, wholesalers, and retailers increase the complexity of supply chain networks. This increased complexity led to the implementation of new strategies and tools to make supply chain structure more transparent, while the efficiency and flexibility are increased. One of these tools is the consignment inventory concept. Consignment inventory gives advantages for both the suppliers and the customers since the supplier enjoys the advantages of close connection with customers, decrease of own store capacity, and decreased transportation and packaging costs. Meanwhile, customers using consignment-inventory-based supply can enjoy advantages provided by consignment inventory like low supply risk, decreased supply costs, and transparency of inventories.

The design and operation of consignment inventory or consignment-store-based supply chains include a huge number of problems: facility location, routing, scheduling, budgeting, transportation problem, inventory optimization, assignment, and queuing problems.

The model presented in this work not only combines the facility location of consignment stores and the assignment problems of stores, suppliers, and customers but also takes into account capacity of logistic resources. To the best of our knowledge, the facility location of consignment stores in supply chains and its assignment to customers and suppliers has not been considered in the current literature.

The main contributions of this work include (1) an integrated consignment-store-based supply chain model that combines facility location planning and assignment of involved organizations of the supply chain, (2) a black-hole-optimization-based algorithm, which includes new heuristic operators to increase the convergence, (3) a test of the

Table 1: Selected papers related to topics of the research.

SN	Author(s)	Year	Topics
1	Hallikas and Lintukangas	2016	Risk management in purchasing and supply chain
2	Matopoulos et al.	2016	Modelling in purchasing and supply chain
3	Immonen et al.	2016	Supply chain in B2B services
4	Zhang et al.	2017	In-plant supply chain design, production routing
5	Govindan and Soleimani	2017	Supply chain management in reverse logistics
6	Ma et al.	2016	Integrated supply chain design
7	Diabat and Deskoores	2016	Supply chain optimization with hybrid genetic algorithm
8	Pishvaee and Rabbani	2011	Responsive supply chain network design with heuristics
9	Liu and Chen	2011	Inventory routing in a supply chain with heuristics
10	Chávez et al.	2017	Simulation-based supply chain modelling
11	Dorigatti et al.	2016	Agent-based simulation of collaborative supply chain
12	Ge et al.	2016	Simulation and hybrid optimization of supply chain
13	Ben Othman et al.	2017	Resource scheduling with decision support system
14	Dey et al.	2017	Facility location in supply chain
15	Zahran et al.	2016	Consignment stock modelling with delay-in-payment
16	Hackett	1993	Consignment contracting
17	Bylka	2013	Noncooperative consignment stock strategies
18	Bazan et al.	2014	Consignment stock agreements for two-level supply chain
19	Li et al.	2014	Supply diversification
20	Batarfi et al.	2016	Strategy for dual-channel supply chain
21	Ru and Wang	2010	Consignment contracting
22	Fraser	2016	Schwarzschild radius
23	Piotrowski et al.	2014	Black hole optimization versus other heuristics
24	Dorigo and Gambardella	1997	Ant colony optimization
25	Yang	2014	Nature-inspired optimization algorithms
26	Bhargava et al.	2013	Cuckoo search optimization
27	Lozano et al.	2017	Artificial bee colony algorithm
28	Niknam et al.	2013	Bat-inspired heuristics
29	McKendall Jr. et al.	2006	Simulated annealing heuristics
30	Saha et al.	2014	Gravitation search algorithm
31	Srivastava	2015	Intelligent water drop optimization
32	Bányai et al.	2015	Harmony search optimization
33	Zhang et al.	2008	Random black hole particle swarm optimization
34	Yaghoobi and Mojallali	2016	Black hole algorithm with genetic operators
35	Wang et al.	2016	Black hole base optimization
36	Bouchekara	2013	Black-hole-based optimization technique
37	Wang et al.	2014	Parameter optimization based on black hole algorithm
38	Hatamlou	2013	Data clustering with black hole algorithm
39	Azizipanah-Abarghooee et al.	2014	Power system scheduling with black hole optimization
40	Bouchekara	2014	Power flow optimization with black hole algorithm

modified black hole algorithm with different datasets and test functions based on CEC 2005, and (4) computational results of consignment-store-based supply chain problems with different datasets.

This paper is organized as follows. Section 2 presents a literature review, which systematically summarizes the research background of supply chain, consignment stores, and black hole optimization. Section 3 describes the model framework of the consignment-store-based supply chains. Section 4 presents the black hole optimization and supposes some modification to improve its convergence and enhance its efficiency. Section 5 demonstrates the sensitivity analysis of the algorithm based on CEC 2005 functions. For our study, in Section 6, we focus on the optimization results with numerical analysis. Conclusions and future research directions are discussed in Section 7.

2. Literature Review

Since our study embraces several related research streams, namely, supply chain management, consignment stores, and black hole optimization, we provide a brief review on each stream before to elaborate the model, algorithm, and solution. Table 1 enlists the papers published in these areas related to our research.

Only limited attention has been paid to consignment-store-based supply chain optimization with metaheuristic methods in the literature. There has recently been an increased interest in performance analysis of supply chains [1–14] and some recent analysis has been targeted specifically towards the integration of consignment stores into supply chain processes [15–21] and optimization with metaheuristics based on swarming algorithms [22–40], especially black hole optimization. Firstly, the relevant terms were defined and the initial searches were conducted. The keywords used in the search were supply chain management, supply, logistics, consignment store, consignment contract, heuristics, metaheuristics, and black hole optimization. The literature sources were found through scientific databases (ScienceDirect, Scopus, and Web of Science) and regular search engines (Google) and included journal articles. Initially, more than 300 articles were identified. This list was narrowed down to 42 titles by selecting journal articles focusing on our research field. It is worth mentioning that the search was conducted in February 2017; therefore new articles may have been published since then.

2.1. Research on Supply Chain Management. Successful production and service processes provide a significant competitive advantage over other participants of the market. Logistics-related operations such as warehousing, transportation, loading unit building, packaging, customer service, and inventory carrying account for up to 95% of the total cost, depending on the corporate sector. For that reason, it is important to take the logistics-related operations and processes under control and turn logistics into the source of competitive edges. Logistics can be divided into four important parts: purchasing, production or service, distribution, and recycling.

Supply chain management influences the efficiency of purchasing, because supplier orientation, supplier dependency, customer orientation, and purchasing strategy have an effect on performance [1]. Supply chain modelling in purchasing is important not just at the functional and operational level but also at the organizational and strategic level [2]. The relationship between purchasing strategies and e-business solutions like business-to-business services has been studied in the literature and provides new knowledge on complex purchasing systems [3]. In-plant supply chain design includes the problems of facility location, production routing, and scheduling [4].

There is a growing interest in the field of closed-loop supply chain design and green supply chain. Reverse logistic systems play an important role in the end-of-life product recycling and influence consumer's return practices for collecting used products, like wastes of electric and electronic equipment [5]. The functional sequences of supply chain can be taken into consideration as an integrated process; the integration of production and distribution planning is a good solution to avoid conflict in sales [6].

The increased complexity of industrial and service processes led to the application of complex solution methods and procedures to optimize the parameters of related supply chains. Heuristic optimization methods are used to solve NP-hard problems of supply chains: integrated supply chain problems can be solved by hybrid algorithms [7], graph-theory-based heuristic supports responsive supply chain design [8], and inventory routing and pricing problems in a supply chain can be solved with Tabu Search [9]. Simulation-based methods support the optimization of deterministic and stochastic models in the field of transportation [10], collaboration analysis for jointly working members [11], and supply chain optimization to find the core parameters of supply chain strategies to ensure cost efficiency [12]. Decision support systems make it possible to find optimal solutions for resource scheduling in supply chain [13] and group decision-making is an effective tool for facility location problems of supply chains [14].

2.2. Research on Consignment Stores. There is a great body of research dealing with consignment policies, consignment stores, and consignment strategies. Within the frame of this chapter, we give a short overview on this reach literature source related to our research. Production companies usually have four types of inventories: raw materials, work-in-process, finished products, and manufacturing supplies. Holding inventories makes it possible to avoid losses of sales, gain quantity discounts, reduce order costs, and achieve efficient production run. Consignment is a special coordination mechanism of inventories, because the owner keeps ownership of his goods and products until they are sold. Consignment stock can improve the supply chain improvement, because vendor uses its buyer's warehouse capacity to store goods. The operation of consignment-store-based supply chain depends on the policy of its operation; therefore, it is important to optimize its operation strategy [15]. The first work investigated the consignment contracting and defined the role of consignment; it can limit the middleman's commitment and increase the profitability of sales [16]. In a later study, the importance of noncooperative stock strategies was underlined and generalized consignment policies were considered to minimize the average total costs by individual decisions [17]. Another topic that has received significant attention in the literature is the analysis of multilevel supply chain with consignment stores. The result of these researches showed the following advantages of using consignment stores in supply chain [18]: improved customer service through decreased reaction time on customer's demands, levelling of customer's demands, decreased inventory on the side of the supplier [19], and cost reduction [20]. The possible control processes of consignment inventory have been studied in the literature; consignment arrangements from the point of view of suppliers and retailers are discussed [21].

2.3. Research on Black Hole Optimization. Black holes were predicted by Einstein's theory of general relativity. If the mass of a dead star's core is more than three times the solar mass, the force of gravity overwhelms all other forces. In a black hole, gravity pulls so much that nothing, not even particles, light, or radiation, can escape from it. The boundary of a black hole is called event horizon, beyond which events cannot be observed and particles cannot move in any direction but only closer to the core of the black hole. This boundary is called Schwarzschild radius, which is given as

$$r_s = \frac{2gM}{c^2}, \tag{1}$$

where g is the gravitational constant, M is the object mass, and c is the speed of light [22].

The black hole optimization (BHO) is based on this phenomenon. Black hole optimization can be described as simplification of the well-known particle swarm optimization using inertia weight. The black hole optimization belongs to heuristics inspired by the laws of nature, like Newton's three laws of motion, his law of gravitation, the ideal gas law, and so on. The laws of nature may be as relevant sources of inspiration for heuristics as living bodies or human-depending phenomenon [23]. Living-bodies-inspired heuristics are, for example, ant colony optimization [24], firefly optimization [25], cuckoo search [26], artificial bee colony optimization [27], bat algorithm [28], or bacterial algorithm. Simulated annealing [29], gravitation search [30], intelligent water drops [31], and black hole algorithms [23] are inspired by physical laws, while harmony search [32] is based on a strongly human-depending attitude.

BHO can be used for hybrid metaheuristics [33]. Genetic operators can increase the convergence and the optimization results of BHO performing a more diverse search in the search space [34] and the convergence of the basic algorithm can be increased by improved measurement of distances in the search space [35]. The technique can be proposed for the optimization of different scientific problems, like pole face optimization of a magnetizer [36], optimization of parameters of least squares support vector machine [37], data clustering [38], scheduling of thermal power systems [39], or power flow optimization [40].

2.4. Analysis of Recent Papers. More than 80% of the articles were published in the last 4 years. This result indicates the scientific potential of this research field including the problems of supply chain, consignment stores, and heuristic optimization. The articles that addressed the optimization of supply chain processes are focusing on conventional manufacturing and service processes and only a few of them aimed to identify the optimization aspects of consignment-store-based supply chain. Therefore, the heuristic optimization of consignment-store-based supply still needs more attention and research, especially in the case of robust, networking cases. It was found that heuristic algorithms are important support tools for design, since a wide range of models determines an NP-hard optimization problem. According to that, the main focus of this research is on the modelling and optimization of consignment-store-based supply chain.

The aim of this paper is to investigate the effect of location of consignment stores on the performance of the whole supply chain. The contribution of this paper to the literature is twofold: description of a consignment-store-based supply chain model including the optimization problem of facility location and assignment and development of a black-hole-based algorithm to solve an integrated optimization problem.

3. Model Framework

The model framework of the consignment-store-based supply chain is a two-level supply chain including suppliers, consignment stores, and customers (Figure 1). The supply chain has m suppliers that produce the needs of p different customers. The supplier and the customer want to set up a consignment store network to support the just-in-time and just-in-sequence supply. Depending on the number and location of consignment stores and consignment agreements, the suppliers are able to ship their products to different consignment stores and the customers are able to buy their needs from different consignment stores. The decision variables of this model are the following: optimal location of the consignment stores, types of consignment agreements between suppliers and customers, and assignment of objects of supply chain and order quantities. These decision variables include an integrated optimization problem: facility location problem and assignment problem.

The decision variables describe the decisions to be made. In this model, the following must be decided: (a) how many products from suppliers through consignment stores to customers should be transported; (b) location of each consignment store. These two decisions represent the abovementioned assignment and facility location problem. With this in mind, we define q_{ijk} as amount of products transported from ith suppliers through jth consignment store to the kth customer and (x_i^W, y_i^W) as coordinates of the jth consignment store.

The objective function of the problem describes the minimization of the costs of both the suppliers and the customers.

$$\min \quad C = \sum_{i=1}^{m} C_i^S + \sum_{k=1}^{p} C_k^C, \tag{2}$$

where C_i^S represents the costs of suppliers and C_k^C represents the costs of customers.

The first part of the cost function (2) includes the sum of transportation costs among suppliers and consignment stores, the warehousing costs, and the manufacturing costs of all suppliers.

$$C_i^S = \sum_{j=1}^{n} \sum_{k=1}^{p} q_{ijk} \left[c_j^W + c_{ij}^T l_{ij} \left(x_j^W, y_j^W \right) \right] + C_i^M, \tag{3}$$

where c_j^W is the specific warehousing cost in the jth consignment store, c_{ij}^T is the specific transportation cost between the ith supplier and jth consignment store, l_{ij} is the length of the transportation route between the ith supplier and jth consignment store, x_j^W and y_j^W are the coordinates of the jth consignment store, and C_i^M is the manufacturing cost of the ith supplier.

The second part of the cost function (2) includes the transportation costs from consignment stores to customers and the purchasing costs of products.

$$C_k^C = \sum_{i=1}^{m} \sum_{j=1}^{n} q_{ijk} \left(c_i^P + c_{jk}^T l_{jk} \left(x_j^W, y_j^W \right) \right), \tag{4}$$

where c_i^P is the specific purchasing cost from the ith supplier, c_{jk}^T is the specific transportation cost between the jth consignment store and kth customer, and l_{jk} is the length of the

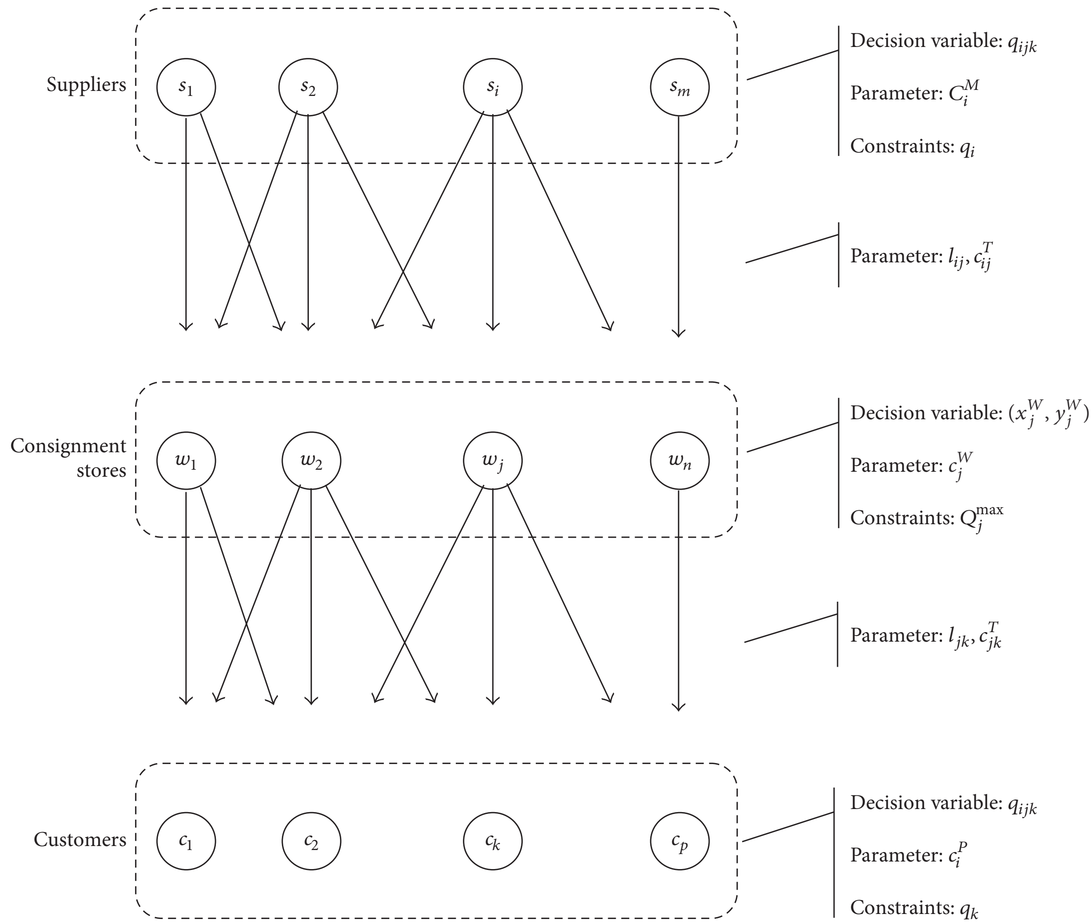

Figure 1: Model framework of consignment-store-based supply chain.

transportation route between the jth consignment store and kth customer.

The values of the products supplied, stored, and bought are limited by the following three constraints.

Constraint 1. Each time interval, no more than the capacity of the consignment store may be transported from suppliers to consignment stores (see the following equation):

$$\sum_{i=1}^{m}\sum_{k=1}^{p} q_{ijk} \leq Q_j^{\max} \quad j \in (1, 2, \ldots, n), \tag{5}$$

where $Q_j^{\max}$ is the capacity of the jth consignment store.

Constraint 2. All products manufactured by suppliers should be transported to consignment stores each time interval (see the following equation):

$$\sum_{j=1}^{n}\sum_{k=1}^{p} q_{ijk} = q_i \quad i \in (1, 2, \ldots, m), \tag{6}$$

where q_i is the total amount of product produced by the ith supplier.

Constraint 3. Each time interval, amount of purchased products must reach the total demand of customers (see the following equation):

$$\sum_{i=1}^{m}\sum_{j=1}^{n} q_{ijk} = q_k \quad k \in (1, 2, \ldots, p), \tag{7}$$

where q_k is the total required amount of the kth customer in the planning period.

The decision variables can only assume nonnegative values, so we associate sign restrictions with the abovementioned decision variables (see the following equation):

$$q_{ijk}, x_j^W, y_j^W \geq 0. \tag{8}$$

4. Black Hole Algorithm

Black holes are places in the outer space where the gravitation force is so high that no particles even light can get out. Black holes are born when stars die. The environment of black holes can be analyzed, but the black holes are invisible. The Schwarzschild radius is the radius of the event horizon. If the distance between a particle (star, proton, electron, photon, etc.) is much higher than the Schwarzschild radius, then the particle can move in any direction. If this distance is larger

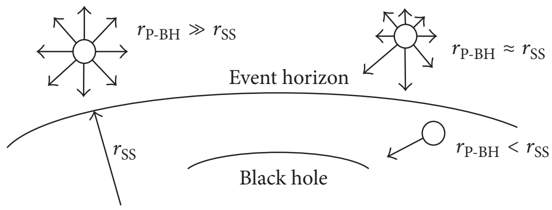

Figure 2: Behavior of particles depending on the distance from black hole.

than the Schwarzschild radius but this difference is not too much, the space-time is deformed, and more particles are moving towards the center of the black hole than in other directions. If a particle reaches the Schwarzschild radius, then it can move only towards the center of the black hole (Figure 2). The black hole optimization is based on this phenomenon of black holes.

The first phase of the black hole optimization is the so called big-bang, when a new generation of stars is generated in the search space. Each star represents one solution of the optimization problem. The coordinates of the ith star in the n-dimensional search space represent the decision variables of the n-dimensional optimization problem.

$$\overrightarrow{x}_j^{s_i} = \left(x_1^{S_i}, x_2^{S_i}, \ldots, x_n^{S_i}\right). \tag{9}$$

The second phase of the algorithm is the evaluation of the stars. The stars are evaluated with the value of the objective functions (gravity force represented by the star).

$$f^{S_i} = f^{S_i}\left(x_1^{S_i}, x_2^{S_i}, \ldots, x_n^{S_i}\right). \tag{10}$$

The third phase is to choose one or more black holes. Black holes are the stars with the highest gravity force.

$$f^{\text{BH}} = \max_i \left(f^{S_i}\right). \tag{11}$$

The fourth phase of the algorithm is to move the stars towards the black holes in the search space. There are different operators to calculate the new locations of the stars. The basic operator uses only the gravity force of the black holes and the gravity force of stars is not taken into consideration. There are two main types of operators: the first type calculates the new position of the stars depending on the gravity forces among stars and black holes, and the second type does not take into account the gravity forces (see the following equation):

$$x_j^{S_i}(t + \Delta t) = x_j^{S_i}(t) + \text{Rnd} \cdot \left(x_j^{\text{BH}}(t) - x_j^{S_i}(t)\right). \tag{12}$$

The movement of stars towards the black hole changes the decision variables of the solution represented by the moving star so that the decision variables will move to the decision variables of the best solution represented by the black hole (Figure 3).

Stars reaching the event horizon will be absorbed and a new star is generated in the search space. The radius of the event horizon (the Schwarzschild radius) is calculated as follows:

$$R^{\text{EH}} = \frac{f^{\text{BH}}}{\sum_{i=1}^{n} f^{S_i}}, \tag{13}$$

where R^{EH} is the radius of event horizon, f^{BH} is the gravity force of the black hole, and f^{S_i} is the gravity force of the ith star.

The fifth phase is the evaluation of stars. Stars with the best gravity force will be the new black holes, and the old black holes become stars. This role of this fifth phase is the same as the role of the mutation operator of genetic algorithms: to avoid the local optimum. Termination criteria of the algorithm can be the number of iteration steps, computational time, or the measure of convergence.

If the location of the optimum is inside the event horizon, it is impossible to find it, because all stars inside the event horizon are absorbed. Stephen Hawking published a theoretical argument for the existence of blackbody radiation [41]. Virtual particle-antiparticle pairs, like photons or neutrinos, are being created near the event horizon of the black hole. These particle-antiparticle pairs annihilate each other or one of them falls into the black hole and the other one escapes as Hawking radiation due to quantum effects. The black hole loses a part of its energy and its mass. This Hawking radiation is called black hole evaporation. It is possible to apply this black hole evaporation to search inside the event horizon [42]. This application means that a little change occurs in the location of the black hole, so that a small part of the old event horizon is available for the stars.

$$x_j^{\text{BH}}(t + \Delta t) = x_j^{\text{BH}}(t) + \varepsilon, \quad |\varepsilon| \ll R^{\text{EH}}. \tag{14}$$

Another way to open the event horizon of black holes for the stars to search for the best solution is to decrease the measure of event horizon. The pseudocode shows the developed approach (Pseudocode 1).

The described Pseudocode 1 makes it possible to replicate the implementation. It is also possible to have more than one black hole; in this case, the movement of stars towards the black holes is similar to the gravity search algorithm [43].

5. Sensitivity Analysis

Within the frame of this chapter, the sensitivity analysis of the black hole algorithm is described. The following 10 different benchmark functions were used to evaluate the above-described black hole algorithm:

(i) The n-dimensional nonconvex Shifted sphere function is evaluated on the hypercube $x_i \in [-100, 100]$. It has a global minimum at $f_1(s_1, \ldots, s_i, \ldots, s_d) = b$.

(ii) The n-dimensional shifted Schwefel function is evaluated on the hypercube $x_i \in [-100, 100]$. It has a global minimum at $f_2(s_1, \ldots, s_i, \ldots, s_d) = b$.

(iii) The n-dimensional shifted elliptic function is evaluated on the hypercube $x_i \in [-100, 100]$. It has a global minimum at $f_3(s_1, \ldots, s_i, \ldots, s_d) = b$.

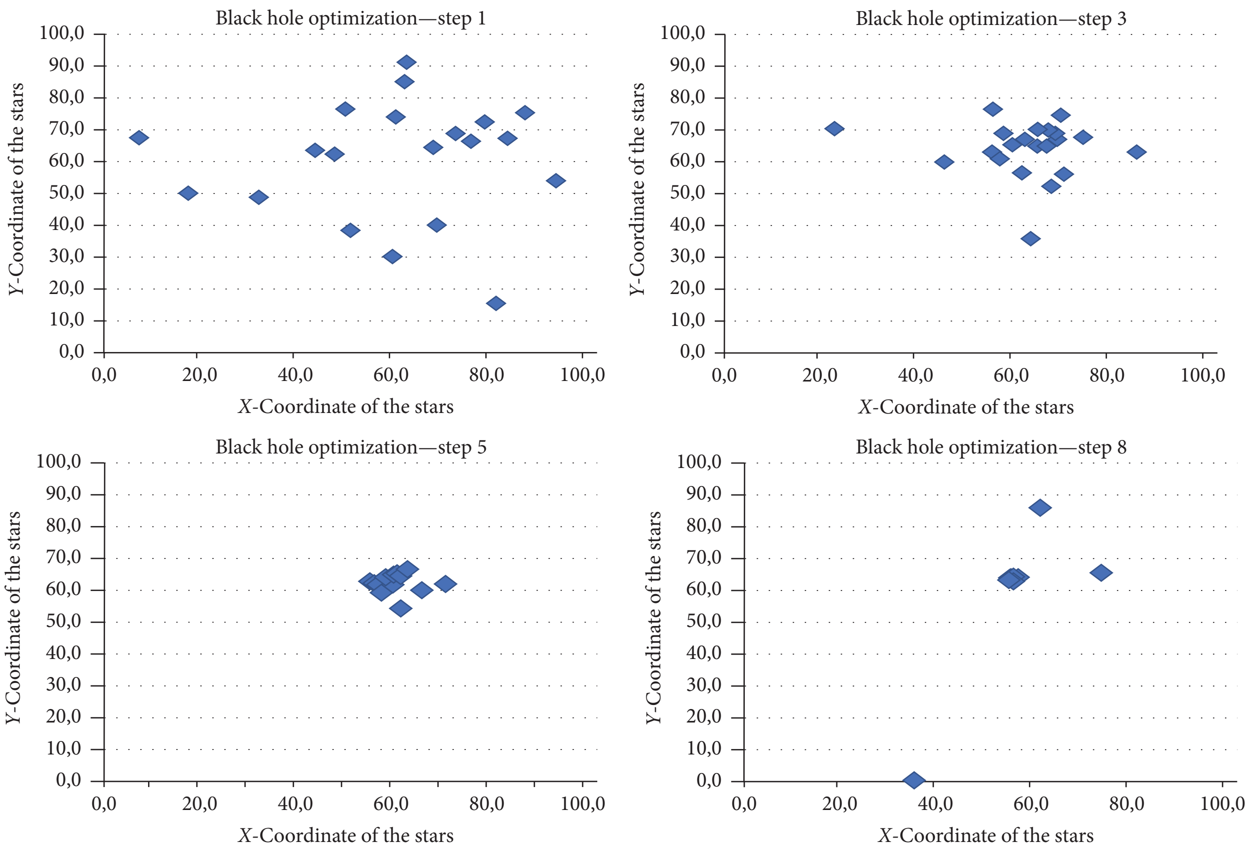

Figure 3: *Moving of stars in BHO* (step 1: initialization of stars; step 3: stars are moving in the direction of the black hole; step 5: stars are nearing the event horizon; step 8: some stars cross the Schwarzschild radius, they are absorbed, and new stars are generated in the search space).

Input: number of stars, objective function, constraints, sign restrictions, termination criteria
Output: optimal solution
// Initialization
(1) generate feasible solutions randomly in the *n*-dimensional search space (9)
// Pre-evaluation
(2) for each stars, evaluate the objective function ((2)–(4), (10))
// Loop until the termination criteria satisfy
While (termination criteria satisfy) **do**
// Selection of the black hole
(3) select the best star that has the best value to become a black hole (11)
// Hawking radiation
(4) change the position of the black hole (14)
// movement of stars towards the black hole
(5) move the stars towards the black holes (12) while constraints ((5)–(8)) are taken into consideration
// Check the position of stars
(6) if star is inside the Schwarzschild radius
absorb the star and generate a new one in the search space (13)
end if
// Evaluation
(7) for each stars, evaluate the objective function ((2)–(4), (10))
End of while

Pseudocode 1: Pseudocode of BHA.

Table 2: Error values of BHO in the case of 10 benchmark functions after 100 iteration steps.

Evaluation function	Standard BHO	BHO with moving black hole location	BHO with decreasing event horizon	Complex BHO
Shifted sphere function $f_1(x_1,\ldots,x_i,\ldots,x_d)=\sum_{i=1}^{d}(x_i-s_i)^2+b$	1,02$E-7$	1,02$E-7$	9,55$E-8$	4,76$E-8$
Shifted Schwefel function $f_2(x_1,\ldots,x_i,\ldots,x_d)=\sum_{i=1}^{d}\left(\sum_{j=1}^{i}(x_i-s_i)\right)^2+b$	8,82$E-6$	8,34$E-6$	7,21$E-6$	4,02$E-6$
Shifted elliptic function $f_3(x_1,\ldots,x_i,\ldots,x_d)=\sum_{i=1}^{d}(10^6)^{(i-1)/(d-1)}(x_i-s_i)^2+b$	4,32$E-7$	4,12$E-7$	2,62$E-7$	1,92$E-7$
Styblinski-Tang function $f_4(x_1,\ldots,x_i,\ldots,x_d)=\frac{1}{2}\sum_{i=1}^{d}(x_i^4-16x_i^2+5x_i)$	2,91$E-6$	1,38$E-6$	6,12$E-7$	4,65$E-7$
Rosenbrock function $f_5(x_1,\ldots,x_i,\ldots,x_d)=\sum_{i=1}^{d-1}\left[100(x_{i+1}-x_i^2)^2+(x_i-1)^2\right]$	5,50$E-6$	5,44$E-6$	2,92$E-6$	2,02$E-6$
Rastrigin function $f_6(x_1,\ldots,x_i,\ldots,x_d)=10+\sum_{i=1}^{d}\left[x_i^2-10\cos 2\pi x_i\right]$	2,45$E-6$	2,32$E-6$	1,07$E-6$	9,69$E-7$
Ackley function $f_7(x,y)=-20\cdot e^{-0.2\sqrt{0.5(x^2+y^2)}}-e^{(0.5(\cos 2\pi x)+\cos 2\pi y)}+e+20$	1,14$E-7$	1,01$E-7$	9,63$E-8$	7,17$E-8$
Beale function $f_8(x,y)=(1.5-x+xy)^2+(2.25-x+xy^2)^2+(2.625-x+xy^3)^2$	2,29$E-7$	2,26$E-7$	1,91$E-7$	1,21$E-7$
Booth function $f_9(x,y)=(x+2y-7)^2+(2x+y-5)^2$	5,31$E-6$	2,11$E-6$	5,51$E-7$	3,26$E-7$
Goldstein-Price function $f_{10}(x,y)=[1+(x+y+1)^2(19-14x+3x^2-14y+6xy+3y^2)][30+(2x-3y)^2(18-32x+12x^2+48y-36xy+27y^2)]$	3,34$E-8$	8,22$E-9$	6,31$E-9$	3,12$E-9$

(iv) The n-dimensional Styblinski-Tang function is evaluated on the hypercube $x_i \in [-5,5]$. It has a global minimum at $f_4(-2.903534,\ldots,-2.903534) = -39.16599$.

(v) The n-dimensional Rosenbrock function is evaluated on the hypercube $x_i \in [-\infty,\infty]$. It has a global minimum at $f_5(1,\ldots,1,\ldots,1) = 0$.

(vi) The n-dimensional Rastrigin function is evaluated on the hypercube $x_i \in [-5.12, 5.12]$. It has a global minimum at $f_6(0,\ldots,0,\ldots,0) = 0$.

(vii) The 2-dimensional Ackley function is evaluated on the $x_i \in [-5,5]$ square. It has a global minimum at $f_7(0,0) = 0$.

(viii) The 2-dimensional Beale function is evaluated on the $x_i \in [-4.5, 4.5]$ square. It has a global minimum at $f_8(3, 0.5) = 0$.

(ix) The 2-dimensional Booth function is evaluated on the $x_i \in [-10,10]$ square. It has a global minimum at $f_9(1,3) = 0$.

(x) The 2-dimensional Goldstein-Price function is evaluated on the $x_i \in [-2,2]$ square. It has a global minimum at $f_{10}(0,-1) = 3$.

The aim of this evaluation is to analyze the effect of permanently decreased Schwarzschild radius and the changes occurred in the location of black holes.

As Table 2 demonstrates, the permanently decreased Schwarzschild radius and the moving black hole effect decreased the error value after 100 iteration steps. The problem size was fixed as a 10-dimensional problem in the case of f_1 to f_6. The efficiency of the algorithm in the case of different problem sizes (different search space dimensions) is demonstrated in the next chapter. The average error value was reduced by 36% with a deviation of 19% (Table 3).

In order to demonstrate how black hole implementation performs when problem size increases, we tested the algorithm both with test functions and with the consignment-store-based supply chain problem.

As Table 4 shows, the increased size of the problem led to the increase of the required iteration steps to reach the predefined error value that is based on the performance of

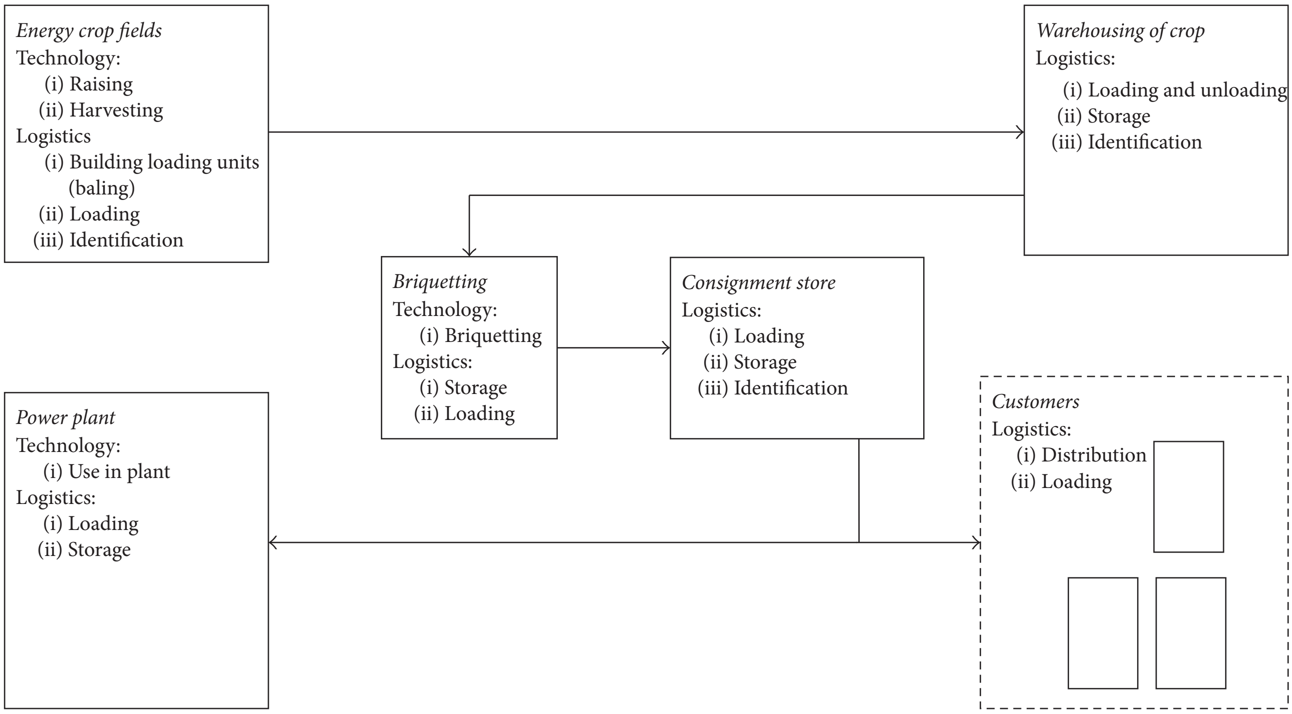

Figure 4: Complex power plant supply chain.

Table 3: Error value decline using moving black holes and permanently decreasing Schwarzschild radius.

Evaluation function	Error value decline
Shifted sphere function	47%
Shifted Schwefel function	46%
Shifted elliptic function	44%
Styblinski-Tang function	16%
Rosenbrock function	37%
Rastrigin function	40%
Ackley function	63%
Beale function	53%
Booth function	6%
Goldstein-Price function	9%

the algorithm in the case of a 10-dimensional search space (Table 2). The highest iteration step was required in the case of the consignment-store-based supply chain problem because of the specific constraints and sign restrictions.

6. Numerical Analysis of Consignment-Store-Based Supply Chain

Within the frame of this chapter, a case study will be analyzed. The aim of this chapter is to analyze the design of an energy crop supply chain of biomass-fired power plants, especially from the point of view of integrated facility location. The model shown in Figure 4 includes the whole energy crop supply chain from harvesting crop in the crop fields through briquetting plants to the distribution for power plants and customers through consignment stores. The following parameters are taken into consideration: the total amount of harvested energy crop, required briquette amount by each power plant and the cluster of customers, transportation distances, specific transportation costs, type of transportation devices (average truck capacity), and location of crop fields, power plants, and customers. The objective function is a cost function based on the transportation processes from energy crop fields to briquetting plants and from briquetting plants to power plants and communal customers. The decision variables are the following: (a) how many energy crops from crop field through consignment stores to customers and power plants should be transported; (b) location of each consignment store.

Figure 5 demonstrates the results of the optimization of the above-mentioned complex supply chain. The datasets represent simple specification of the system so that the results of the optimization can be checked.

In the first case, customers and power plants need the same quantity of briquette; therefore the central position of both consignment stores is correct. The first power plant and the first and second customer's clusters are assigned to the first consignment store, while the second power plant and the third and the fourth customer's clusters are assigned to the second consignment store.

In the second case, the third customer's cluster has a greater demand of briquette. In this case, the second consignment store's location is not centralized in order to minimize the materials handling costs of the whole supply chain.

Table 4: Number of required iteration steps to reach the predefined error value (PEV).

Evaluation function	Problem size		
	$d = 5$	$d = 10$	$d = 25$
Shifted sphere function $f_1(x_1,\ldots,x_i,\ldots,x_d) = \sum_{i=1}^{d}(x_i - s_i)^2 + b$ PEV = 4,76E − 8	94	100	122
Shifted Schwefel function $f_2(x_1,\ldots,x_i,\ldots,x_d) = \sum_{i=1}^{d}(\sum_{j=1}^{i}(x_i - s_i))^2 + b$ PEV = 4,02E − 6	92	100	134
Shifted elliptic function $f_3(x_1,\ldots,x_i,\ldots,x_d) = \sum_{i=1}^{d}(10^6)^{(i-1)/(d-1)}(x_i - s_i)^2 + b$ PEV = 1,92E − 7	81	100	156
Styblinski-Tang function $f_4(x_1,\ldots,x_i,\ldots,x_d) = \frac{1}{2}\sum_{i=1}^{d}(x_i^4 - 16x_i^2 + 5x_i)$ PEV = 4,65E − 7	85	100	162
Rosenbrock function $f_5(x_1,\ldots,x_i,\ldots,x_d) = \sum_{i=1}^{d-1}[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$ PEV = 2,02E − 6	93	100	133
Rastrigin function $f_6(x_1,\ldots,x_i,\ldots,x_d) = 10 + \sum_{i=1}^{d}[x_i^2 - 10\cos 2\pi x_i]$ PEV = 9,69E − 7	84	100	144
Consignment-store-based supply chain problem PEV = 5,63E − 7	89	100	168

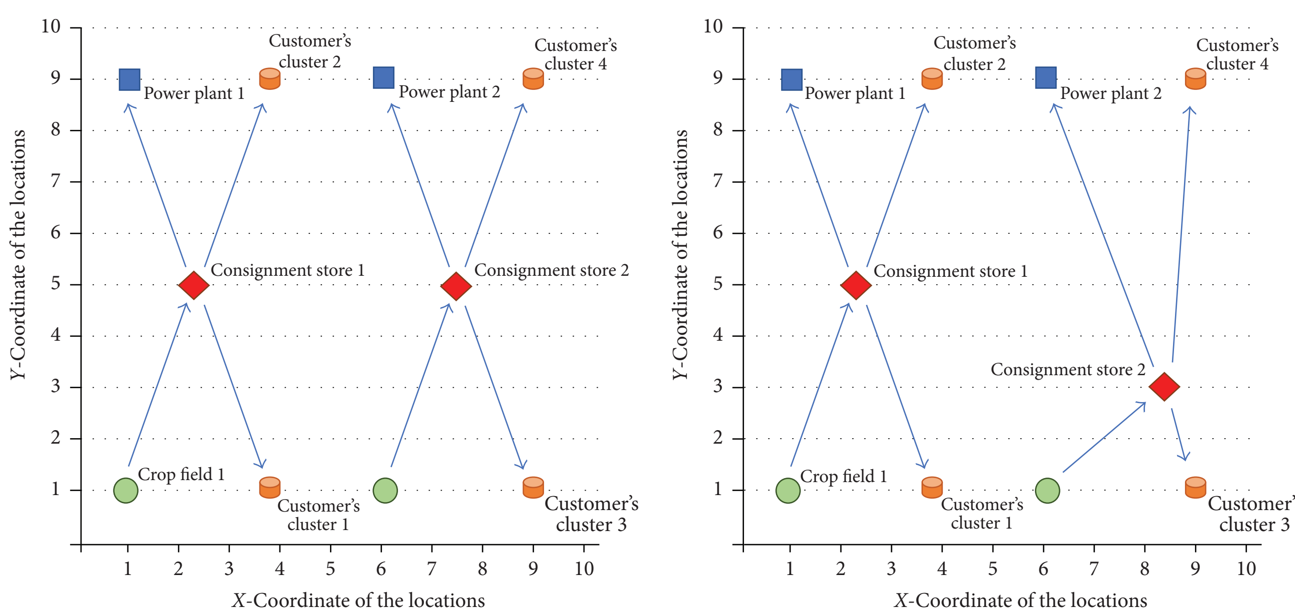

Figure 5: Results of the facility location and assignment problem of energy crop supply chain.

The optimization of this complex supply chain problem can lead to the decrease of different costs, like transportation costs, warehousing costs, and materials handling costs (packaging, loading, unloading, and building of loading units).

7. Conclusions and Further Research Directions

This study developed a methodological approach for design of consignment-store-based supply chains. In this paper, firstly, we reviewed and systematically categorized the recent works presented for consignment-store-based supply chain optimization. Then, motivated by the gaps in the literature, a model for companies performing their purchasing through consignment stores is developed. Two models were proposed: the framework model shows the levels of supply chain, while the second model as a case study focuses on power plant supply. The integrated model included facility location and assignment problems, which were solved with black hole optimization algorithm. The sensitivity analysis showed the efficiency of two advanced BHO operators and a numerical example shows the efficiency of the algorithm.

The scientific contributions of this paper are the following: integrated model for consignment-store-based supply chain, black-hole-optimization-based heuristic algorithm with enhanced convergence through integration of phenomena of real black holes, like dynamic black hole location, and decreased event horizon. The results can be generalized, because the model can be applied for in-plant supply, especially in the case of milk-run-based just-in-sequence supply. The described methods make it possible to support managerial decisions; the operation strategy of the supply chain and the consignment contract can be influenced by the results of the above-described contribution.

However, there are also directions for further research. First, although the transportation routes as distances among the locations are considered in this paper, the capacities of vehicles are not taken into consideration. In further studies, the model can be extended to a more complex model including capacities of vehicles and store capacities of locations. Second, this study only considered the black hole optimization method as possible solution algorithm for the described NP-hard problem. In reality, other heuristic methods can be also suitable for the solution of the problem.

Third, the convergence of the described algorithm can be improved using other operators and the behavior of BHO to other optimization approaches can be tested. However, there is a great body of research dealing with testing of performance of different metaheuristic optimization methods, especially from the point of view of "novel" algorithm, but these tests are sometimes inconsistent. This inconsistency can be caused by the optimization behavior. For example, the comparison of black hole algorithm and particle swarm optimization showed that the performance of BHO is poorer than the performance of PSO [44], while the test in another source showed that the performance of BHO is better than the performance of genetic algorithm or PSO [45]. This should be also considered in the future research.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Grant Agreement no. 691942. This research was partially carried out within the framework of the Center of Excellence of Mechatronics and Logistics at the University of Miskolc.

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