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PERIPATETIC HYPOTHETICAL SYLLOGISTIC IN GALEN – PROPOSITIONAL LOGIC OFF THE RAILS?

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Galen's *Introduction to Logic* (*Institutio Logica*, *IL*) is the only introduction to logic in Greek that has survived from antiquity. In it we find – among other things – a theory that bears some resemblance to propositional logic. The theory is commonly understood as being essentially Stoic.[1] However, this understanding of the text leaves us with a large number of seeming inconsistencies and oddities. What I offer in this paper is an alternative comprehensive interpretation of the theory. I suggest that it is Peripatetic at base, and has drawn on Stoic elements, but adapted them – partly deliberately, partly unintentionally – to an overall decidedly non-Stoic conception of logic and language, a conception that is indebted to Aristotelian logic in many ways. This interpretation makes it possible to cut down dramatically on the seeming inconsistencies, a fact that may be counted as evidence for its historical accuracy. The Peripatetic theory on which Galen draws was possibly developed in the first century BC. One important aspect in which it differs from Stoic logic is that it shuns the latter's syntactic approach, and considers certain linguistic

[1] Some scholars consider passages in Galen on hypothetical syllogistic as Stoic, although they are not attributed to the Stoics (e.g. Łukasiewicz (1935), Kneale and Kneale (1962), 160, 162, Mates (1953), 53, 55–7; Dillon (1993), 83). Other scholars are more careful and allow for the possibility that (where neither a Stoic nor the 'ancients', i.e. Theophrastus and Eudemus, are mentioned) what we have is Stoic theory in Peripatetic garb. Still, they often interpret the hypothetical syllogistic as if it functions just like they think the Stoic logic of assertibles (ἀξιώματα) did. (Cf. e.g. the commentaries on *IL* chapters 3–6 and 14–15 by Mau (1960) and Kieffer (1964). Note also that both von Arnim and Hülser include the passages in their collections of Stoic fragments and testimonies.) The purpose of this article is to show that the Peripatetics did not simply use different terms and made some cosmetic changes, all the while, in essence, preserving Stoic theory; but rather that what Galen has preserved for us is a radically different way of understanding compound propositions and the basic arguments that can be formed with them.

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assumptions and language conventions as part of the logical theory itself. The reconstruction of the theory in Galen offered in this paper results in a logic of propositions which differs wildly both from Stoic logic, and from the 'classical' propositional logic of the 20th century. Modern logicians may shudder at the sight of it; or, perhaps, they may delight in its quirkiness. In any event, the theory in Galen shows that the ancients grappled with a number of problems that are still a matter of debate among contemporary logicians.

1. Historical Ancestors

The historical impacts on the propositional logic in Galen's *Institutio Logica* are manifold, and interconnected. The main influences are three: the early Peripatetic theory of syllogisms from a hypothesis; Stoic propositional logic (both Chrysippean and post-Chrysippean); and Aristotelian logic. Some of these may have found their way to Galen via contemporary Peripatetic or Platonist logic.

The most relevant part of Stoic logic in this context is the theory of the five types of indemonstrable syllogisms (ἀναπόδεικτοι συλλογισμοί):[2]

(S1) If p, q
Now p
Hence q

(S2) If p, q
Now not q
Hence not p

(S3) Not (p and q)
Now p
Hence not q

(S4) p or q
Now p
Hence not q

(S5) p or q
Now not p
Hence q

The early Peripatetics distinguished four types of syllogisms from a hypothesis (συλλογισμοὶ ἐξ ὑποθέσεως), the hypothetical premisses of the

[2] For details of Stoic syllogistic see Bobzien (1996), Bobzien (1999), 127–151, Frede (1974) 124–201.

first two based on a relation of connection (συνέχεια), those of the last two based on a relation of division (διαίρεσις) of things. I believe their forms were similar to the following:[3]

(EP1)	If something is F, it is G	or	If *a* is F, it is G
	Now *a* is F		Now *a* is F
	Hence *a* is G		Hence *a* is G
(EP2)	If something is G, it is F	or	If *a* is G, it is F
	Now *a* is not F		Now *a* is not F
	Hence *a* is not G		Hence *a* is not G
(EP3)	Everything is either F or G	or	*a* is either F or G
	Now *a* is F		Now *a* is F
	Hence *a* is not G		Hence *a* is not G
(EP4)	Everything is either F or G	or	*a* is either F or G
	Now *a* is not F		Now *a* is not F
	Hence *a* is G		Hence *a* is G

Further relevant elements from Stoic and early Peripatetic logic, as well as from Aristotelian logic, will be mentioned as I go along.

In the *Institutio Logica*, Galen draws on different sources, and he does not always work them into one coherent theory. Sometimes he simply reports a certain philosopher's or school's view; thus he presents part of Chrysippus' view at *IL* 6.6, and he presents some of the views of the ancients (οἱ παλαιοί), which I take to be early Peripatetics such as Theophrastus and Eudemus, at *IL* 3.1–3, 14.2 and 14.10.[4] In most of the passages on hypothetical syllogistic, however, he presents the theory as his own view, or in any case as a theory he approves of.[5] This is – I argue – mainly a post-Chrysippean Peripatetic theory, which may have originated with 'middle' Peripatetics such as Aristo and Boethus (1st BC), and into which Galen incorporates a number of post-Chrysippean Stoic elements. Galen repeatedly treats the differences between these two ancestor theories as if they were merely terminological, and thereby glosses over important philosophical discrepancies. As a result, his theory is not homogeneous, and sometimes borders on the incoherent. However, I

3 Cf. Bobzien (2002a).
4 Cf. Bobzien (2002b).
5 This theory dominates *IL* chapters 4, 5, 14 and 15; and parts of chapter 3; and parts of sections 6.1, 6.2, 6.4, 6.7, 7.1, 7.4, 8.2.

believe that the theories he drew from had a high degree of coherence. In the following I set out, on the one hand, the post-Chrysippean Peripatetic theory, which for brevity I shall refer to as Peripatetic; and on the other, the overall theory Galen presents, which I shall refer to as Galen's view. Naturally, those two theories overlap considerably; and it is not always clear what Galen takes over from other theories, and what he adds himself.

2. Basic Semantics

This part of the Peripatetic (and Galen's) theory is obscure and rather unsatisfactory. The basic parameters of the semantic theory are speaker-listeners, meaningful sound (λέξις) and things (πράγματα). *Meaningful sound* includes (i) words (singular terms, such as 'Dion', and general terms, such as 'animal'), (ii) simple sentences (with singular terms in subject position, such as 'Dion is a human being', and quantified, such as 'Every human being is an animal', etc.), and (iii) complex sentences (such as 'if Dion is a human being, then Dion is an animal'). Propositions (προτάσεις) are linguistic items. They are sentences (λόγοι) insofar as they are significant. Truth and falsehood are properties of propositions, hence of linguistic items. The *things* include (i) particular things, such as Dion, and generic things, such as 'animal', and (ii) simple states of affairs, such as *that Dion is an animal* (or Dion's being an animal).[6] Speakers make assertions *with* linguistic items *about* things and relations between things. By contrast, the Stoics have as basic parameters speaker-listeners, utterance (λόγος (or signifier (σημαῖνον)), what is meant (λεκτόν) (or what is signified (σημαινόμενον)), and things (τυγχάνοντα). A speaker, in uttering a declarative sentence asserts an assertible (ἀξίωμα), which is either true or false. Assertibles are not linguistic items. Yet, Galen – wrongly – treats Peripatetic propositions and Stoic assertibles as if they were the same sort of thing.

We can distinguish three levels of relations between speaker-listeners, meaningful sound and things in the Peripatetic (and Galen's) theory:

(1) The word 'Dion' indicates the thing Dion, and the word 'animal' indicates the thing animal .[7]

(2) At the level of simple sentences, the relation is more complex: A simple sentence or proposition does not directly indicate a thing of the state of affairs kind, but indicates either its obtaining (holding, subsisting) or its not

[6] The Greek typically has accusative with infinitive constructions.

[7] Whether there are things like goat-stags, and whether the word 'goat-stag' would indicate the thing goat-stag is unclear.

obtaining. Thus the affirmative proposition 'Dion is an animal' indicates the obtaining of the state of affairs *that Dion is an animal*, whereas the negative proposition 'Dion is not an animal' indicates the not-obtaining of the same state of affairs. Simple sentences are also called categorical (κατηγορικός, *IL* 2.1–2). The speaker makes an assertion *with* a simple proposition *about* the obtaining of the things (cf. *IL* 3.1). Thus, both with 'Dion is an animal' and with 'Dion is not an animal', I make an assertion about the obtaining of the state of affairs *that Dion is an animal*, namely in the first case that it obtains, and in the second that it does not obtain. Hence at this level, the things indubitably include both states of affairs that obtain and states of affairs that do not obtain. The things are thus *possibilia* rather than *actualia*.[8] As regards truth-values, a simple proposition is true, if what is asserted with it to obtain obtains, and if what is asserted with it not to obtain, does not obtain. It is false, if what is asserted with it to obtain, does not obtain, and if what is asserted with it not to obtain obtains (cf. *IL* 17.6). Affirmative and negative simple sentences are treated as logically on a par. There is nothing resembling a negation operator; nor is there a rule of double negation. Instead – in Aristotelian manner – a relation of contradictoriness is defined between pairs of simple affirmative and negative sentences (*IL* 6.2).

(3) At the level of complex sentences, the relation becomes more complex still: A complex sentence or proposition does not indicate the obtaining or not-obtaining of a state of affairs. Rather, at least in the case of hypothetical propositions, it indicates a relation between states of affairs. For instance, the complex proposition 'if Dion is a human being, Dion is an animal' indicates a relation of consequence between the state of affairs *that Dion is a human being* and the state of affairs *that Dion is an animal.* Again, the speaker is taken to make an assertion *with* the complex proposition *about* a relation of consequence between the states of affairs *that Dion is a human being* and *that Dion is an animal* – presumably the assertion that such a relation of consequence holds.[9]

3. The five basic relations between things

Galen's classification of non-simple propositions is largely based on a distinction between five kinds of relations that may hold between states of affairs.

[8] This is in the Aristotelian spirit.

[9] Cf. Alcinous *Didasc.* 158.16–17, Ammonius *Int*. 3.11–14 and 3.32–4.3, see also Bobzien (2002c).

Starting from the three relations of (i) consequence (ἀκολουθία), (ii) conflict (μάχη) and (iii) neither consequence nor conflict, and sub-distinguishing two sub-types of consequence and conflict,[10] Galen obtains the following relations:

- complete conflict (τελεία μάχη)
- incomplete conflict (ἐλλιπὴς μάχη)
- complete consequence (τελεία ἀκολουθία)
- incomplete consequence (ἐλλιπὴς ἀκολουθία)
- absence of both consequence and conflict.

The theory is based on the pair of notions of consequence and conflict. The expressions ἀκολουθία and μάχη, used as a pair for logical relations, seem to be of Stoic provenance. They were, however, appropriated by Peripatetic and Platonist logicians (besides Galen, e.g. Alexander *An.Pr.* 11.19–20 and Alcinous *Didasc.* 158.16–17) and were put by them to a different use in line with their general conception of logic.[11] Consequence and conflict seem to have taken the place connection (συνέχεια) and division (διαίρεσις) had with the early Peripatetics.[12] Like the latter, they are considered to be two kinds of relations between things;[13] these things are not bearers of truth-values. Consequence and conflict are fundamental 'ontic' relations and neither can be reduced to the other. I suspect, this view of consequence and conflict evolved in analogy to Aristotle's *Metaphysics Theta* (Θ.10 1051a34-b17, cf. E.4), where Aristotle considers the relations between being and not-being, false and true, being combined and being separated.

Galen explains the five ontic relations in two ways. In chapter 4 he uses accounts based on modal notions; in chapter 14 he presents accounts based on temporal notions. The modal explanations are as follows:

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[10] Whether the sub-distinctions are Galen's doing, or, at least in part, taken from other sources is debatable. The only other source I know of that explicitly makes a sub-distinction of complete and incomplete consequence is Al-Farabi, *Paraphr. Cat.* 56–9. (In Boethius, *Hyp.Syll.* 2.2.4–5, 2.3.6, 2.4.1–3 and 3.10.3–11.7 we find discussed something similar to complete consequence.)

[11] In Stoic logic, 'to follow' (ἀκολουθεῖν) and 'to conflict' (μάχεσθαι) are relations between assertibles, and thus between the Stoic (non-linguistic) truth-value bearers. Moreover, at least in Chrysippus' view, the notion of consequence is reducible to that of conflict. Neither point holds for the Peripatetic concepts.

[12] Cf. e.g. the un-Stoic formulations κατὰ τὴν μάχην and κατὰ τὴν ἀκολουθίαν (*IL* 14.7) with the early Peripatetic κατὰ συνέχειαν and κατὰ διαίρεσιν (*IL* 14.2, 3.5, 4.1).

[13] Cf. *IL* 14.7. I take πραγμάτων τε καὶ λόγων in *IL* 14.5 as juxtaposing the Peripatetic and the Stoic understanding of μάχεσθαι.

(i) two or more things stand in a relation of *conflict* if and only if they cannot obtain together (συνυπάρχειν, *IL* 4.2)

- the conflict is *complete*, if it is also impossible that the things should together not obtain (*IL* 4.2), i.e. if it is also impossible that neither obtains.
- it is *incomplete*, if it is possible that the things should together not obtain (*IL* 4.2), i.e. if it is possible that neither obtains.

(ii) two things stand in a relation of *consequence* if and only if they necessarily obtain together (inferred from *IL* 14.7 and Al-Farabi *Paraphr. Cat*. 56)[14]-

- the consequence is *complete*, if when one thing obtains, necessarily so does the other, and when the other obtains, necessarily so does the one (Al-Farabi *Paraphr. Cat.* 56–7)
- it is *incomplete*, if when one thing obtains, necessarily so does the other, but if the other obtains, it is not necessary that the one does (Al-Farabi *Paraphr. Cat.* 56–7)

(iii) two or more things stand in *neither conflict nor consequence*, if and only if they all can together obtain and they all can together not obtain (inferred from *IL* 14.7).

All five relations thus contain a modal element. Galen cashes out the modal element temporally at *IL* 14.7.[15]

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[14] Al-Farabi could have taken these distinctions from Galen's lost work *On Demonstration*. Cf. Zimmermann, (1981), lxxxi-lxxxiii.

[15] The temporal interpretation is thus:

(i) two or more things stand in a relation of *conflict* if and only if they never obtain together (*IL* 14.7).

- the conflict is *complete*, if the things also never not obtain together (inferred from *IL* 14.7 and 4.2)
- the conflict is *incomplete*, if the things sometimes not obtain together (ditto)

(ii) two things stand in a relation of *consequence* if and only if they always obtain together (*IL* 14.7, text corrupt)

- the consequence is *complete*, if whenever one thing obtains, so does the other, and whenever the other obtains, so does the one (inferred from IL 14.7 and Al-Farabi)
- the consequence is incomplete, if whenever one thing obtains, so does the other, but if the other obtains, the one does not always obtain (ditto)

(iii) two or more things stand in neither conflict nor consequence, if and only if they sometimes obtain together and sometimes do not obtain together (*IL* 4.4 and 14.7, lacuna, text corrupt).

4. Complex Propositions

In Stoic logic, non-simple assertibles are constructed from simple ones in accordance with certain rules, similar to modern logic: let p, q, r ... be simple assertibles. Then, if φ is an assertible, so is 'not φ'; if φ, ψ are assertibles (possibly the same!), so are 'both φ and ψ'; 'either φ or ψ'; 'if φ then ψ'. The type of assertible to which a non-simple assertible belongs is determined by the connective with the largest scope. In syllogisms, i.e. formally valid arguments, uniform substitution of simple or non-simple assertibles for simple assertibles will always be validity preserving.[16] Two assertibles φ, ψ can be combined into more than one true non-simple assertibles. For example, it could be that both 'if Dion is walking, Theon is talking' is true and 'not: both Dion is walking and not: Theon is talking' is true. All these points also hold in standard modern propositional logic.

The Peripatetic idea of complex propositions is grounded on the five basic relations between things laid out above, and consequently, the Peripatetic (and Galen's) understanding of complex propositions is strikingly different from the Stoic. Complex propositions are not considered as composites of simple ones. Rather, each type of complex proposition indicates a specific kind of relation between things . The resulting theory of complex propositions appears to have the following features:

(i) Any two (or more) things can stand only in one of the five basic relations.

(ii) No thing stands in any of these relations to itself (cf. e.g. *IL* 3.1)

(iii) There is exactly one type of proposition for each type of relation.[17]

(iv) A complex proposition is true if and only if the relation it indicates holds. Thus, we can understand why we obtain no explicit truth-conditions for complex propositions in the *Institutio Logica*.[18] The reason is that given that any complex proposition is true precisely when the relation it indicates holds, the truth-conditions can be 'read off' immediately from

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16 By contrast, the Peripatetics (and Galen) seem to have had substitutivity only in the very restricted sense that if one substitutes a complex proposition for a simple one in a hypothetical proposition, the result is again a hypothetical proposition (cf. e.g. *IL* 15.7–8, 10; Galen, *On Semen* II 1.69).

17 The case is not entirely clear for the conditionals, see below.

18 Nor do we get explicit truth-conditions in any other passages on Peripatetic hypothetical syllogistic. 'The truth and falsehood of hypothetical propositions depends on consequence and conflict' is all we get (Alex. *An.Pr.* 11.19–20).

the specification of the relation which is given for each type of proposition; they hence need not be spelled out separately.

(v) It follows from (i) – (iv) that any two (or more) simple propositions can form only one complex proposition that is true, namely the one that indicates the relation in which the two (or more) things of the simple propositions stand .[19]

(vi) The relations that are of special logical interest are those which enable us to construct valid inferences from them. Propositions which indicate such relations are called hypothetical. Hypothetical syllogistic is concerned with setting out those syllogisms that contain hypothetical propositions (in logically relevant positions).

Galen introduces the following types of complex propositions: disjunctions (*διεζευγμένα*), which indicate complete conflict; quasi-disjunctions (*ὅμοια διεζευγμένοις*), which indicate incomplete conflict; conditionals (*συνημμένα*) which indicate complete or incomplete consequence; affirmative and negative conjunctions (*συμπεπλεγμένα*) which each indicate neither consequence nor conflict; and, adopted from Stoic logic, para-disjunctions (*παραδιεζευγμένα*).

The Stoics introduced one canonical formulation for each type of complex assertible. Peripatetic propositional logic permits both that the same kind of complex proposition should be formulated in different ways (i.e. with different sentence connectors) and that different kinds of complex propositions should be formulated in the same way (i.e. with the same sentence connectors). This fact forces the Peripatetics to distinguish between the *logical* category to which a complex proposition belongs (which is the important one, and which corresponds to one of the basic relations between things), and the *grammatical* or *linguistic* category to which a complex proposition belongs (which is determined by the connectors used, and is arbitrary up to a point). In the following, I use 'disjunction', 'disjunctive proposition', 'conditional', 'conditional proposition', etc., to refer to the logical categories, and 'disjunctive sentence', 'conditional sentence', etc. to refer to the grammatical categories. This does not reflect Galen's usage,[20] but is convenient.

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[19] For instance, when a number of things stand in the relation of complete conflict with each other, they do not stand in the relation of incomplete conflict, and vice versa. Thus the proposition 'Two is either odd or even', which is considered to be a true proposition of the kind that indicates complete conflict, is not a true proposition of the kind that indicates incomplete conflict.

[20] Galen uses tags such as *τῇ λέξει* and *ἐν σχήματι λέξεως* in order to mark the difference.

Conjunctions, disjunctions, quasi-disjunctions and para-disjunctions can all be composed of two *or more* component propositions. This may reflect the Peripatetic assumption that the underlying ontic relations can hold between two or more things. In modern truth-functional logic, conjunctions and disjunctions with multiple conjuncts or disjuncts are considered as being constructed with two-place connectors by way of bracketing, for instances:

‘p v q v r’ is short (or sloppy) either for ‘[p v q] v r’ or for ‘p v [q v r]’

The corresponding ancient connectors on the other hand seem all to have been regarded as genuine n-place connectors, with variable $n \geq 2$. This fact could be expressed with the n-place connector placed before the set of (what is indicated by the) conjuncts, disjuncts, etc.:

AffConj $\{p_1, p_2, \ldots p_n\}$	for	p_1 and p_2 and ... and p_n
NegConj $\{p_1, p_2, \ldots p_n\}$	for	not (p_1 and p_2 and ... and p_n)
Disj $\{p_1, p_2, \ldots p_n\}$	for	p_1 or p_2 or ... or p_n
Q-Disj $\{p_1, p_2, \ldots p_n\}$	for	not (p_1 and p_2 and ... and p_n)
P-Disj $\{p_1, p_2, \ldots p_n\}$	for	p_1 or p_2 or ... or p_n
ConjNeg $\{p_1, p_2, \ldots p_n\}$	for	neither p_1 nor p_2 nor ... nor p_n

Alternatively, the Peripatetics (and Galen) may have considered the connectors as some kind of quantifying expressions that quantify over things, i.e. states of affairs:

All of $\{s_1, s_2, \ldots s_n\}$	hold	for	s_1 and s_2 and ... and s_n
Not all of $\{s_1, s_2, \ldots s_n\}$	hold	for	not (s_1 and s_2 and ... and s_n)
Precisely one of $\{s_1, s_2, \ldots s_n\}$	holds	for	s_1 or s_2 or ... or s_n
At most one of $\{s_1, s_2, \ldots s_n\}$	holds	for	not (s_1 and s_2 and ... and s_n)
At least one of $\{s_1, s_2, \ldots s_n\}$	holds	for	s_1 or s_2 or ... or s_n
None of $\{s_1, s_2, \ldots s_n\}$	holds	for	neither s_1 nor s_2 nor ... nor s_n

Complex propositions with two conjuncts, disjuncts, etc. are then the simplest case of such propositions, and not the elements from which multiple conjunct, etc., complex propositions are constructed. I do not suggest that the Peripatetics (or Galen) had any systematic theory of this kind; rather that this may have been their general way of thinking of such propositions, which they may have arrived at by taking the quantificational approach of Aristotle’s categorical syllogistic as template . In any case, the above assignment of quantificational expressions to types of complex propositions provides no more than a rough guide. In order to capture the full notion of these types of propositions, several modal elements need to be added to some of them.

4.1 Conjunctions

I begin with conjunctions, since they best illustrate the difference between the Peripatetic and Galen's view on the one hand, and the Stoic and similar views on the other. For the Stoics, a conjunctive assertible is one formed with the conjunctive particle 'both ... and ... and ... ...' (καὶ ... καὶ ... καὶ ... ...). It is true when and only when all component assertibles are true. Stoic conjunctive assertibles are thus truth-functional. Galen expressly disagrees with the Stoic view of conjunction (*IL* 4.6). He holds that not everything that is a conjunctive sentence is a conjunctive proposition;[21] the complex proposition is a conjunction only when the things whose obtaining or not obtaining is indicated in the component propositions do not stand in consequence with each other. In other words, a conjunction indicates that a number of things do or do not obtain at the same time *without* standing in a relation of consequence (*IL* 4.4, 14.7). Conjunctions are hence not truth-functional. They can also not be used as leading premisses in hypothetical syllogisms and thus are not hypothetical propositions (*IL* 14.4–8). There are affirmative and negative conjunctions.

Affirmative and negative conjunctions: An affirmative conjunction is a complex proposition which indicates of each of two or more things either that it obtains (i.e. when the conjunct is affirmative) or that it does not obtain (i.e. when the conjunct is negative), while these things do not stand to each other in a relation of consequence. It is true, when what it indicates is the case (i.e. when all its conjuncts are true *and* the things do not stand in a relation of consequence).[22] Galen's example is 'Dion is walking and Theon is talking' (*IL* 4.4, 14.7). A negative conjunction is a complex proposition which indicates that it is not the case that, at the same time, of each of two or more things it holds that it obtains (i.e. when the conjunct is affirmative) or that it does not obtain (i.e. when the conjunct is negative), and in which these things do not stand in any relation of conflict. It is true, when what it indicates is the case (i.e. when not all its conjuncts are true *and* the things stand not in a relation of conflict).[23] Galen's example is 'not: Dion is walking and Theon is talking' (*IL* 14.7, text corrupt). It follows that, although it is impossible that both an affirmative conjunction and its negative counterpart are true at the same time, it is possible that both are false. They are contraries, not contradictories. This

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21 Cf. my terminological distinctions in the previous section.

22 Or, if one conjunct is affirmative, the other negative, not in a relation of conflict.

23 Or, if one conjunct is affirmative, the other negative, not in a relation of consequence.

may seem surprising. However, from an Aristotelian point of view, it would hardly be: an affirmative universal 'All A are B' and a negative one 'No A are B' are contraries, not contradictories, so why should not negative and affirmative conjunctions, too?

Conjunction of negations versus negative conjunction: The Peripatetics call the proposition 'not: Dion is walking and Theon is talking' a 'negative conjunction'. The Stoics would call it a 'negation of conjunction'. This is telling. For the Stoics, the connector with the largest scope (here 'not') determines the *type* of complex assertible expressed with a sentence. The Peripatetics (and Galen) seem to take the distinction between affirmative and negative propositions as basic, and apply it to conjunctions. On the other hand, *they* use for conjunctions that are composed entirely of negative propositions (e.g. 'not p and not q')[24] phrases such as 'negative (proposition) concerning two'.[25] This suggests that such complex propositions were regarded as negations of sorts, whereas for the Stoics they would be conjunctions. Perhaps the Peripatetics thought that the connection in a conjunction exists only on the linguistic level; 'not p and not q and not r' is then rather like 'not p, not q, not r', and in this sense a negating of everything.[26] (To avoid confusion, I shall however refer to such propositions as 'conjunction of negations'.)

Propositions in the form of conjunctive sentences which are not conjunctions: A sentence of conjunctive form that indicates a conflict or consequence is not a conjunction (*IL* 4.6, 14.4, 14.7). For instance, a negated conjunctive sentence of the form 'not: p and q and r and ...' that indicates incomplete conflict is a quasi-disjunction (*IL* 14. 6, see below). Similarly, the end of *IL* 4.6 suggests that sentences of the form 'both p and q', if they indicate a consequence, are not affirmative conjunctions, but some other kind of complex proposition. An example would be 'Dion is walking and Dion is moving.' Thus for Galen 'Dion is walking and Theon is talking' is a conjunction, whereas 'Dion is walking and Dion is moving' is not, but is taken to indicate some kind of consequence.

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[24] Or 'b is not F and (b is) not G'.

[25] *IL* 15.3; or 'we negate all' (*IL* 5.4), 'none obtains' (*IL* 6.7, 15.9).

[26] Such propositions may have been conceived of as having an operator 'none of' in front: 'none of: p, q, r', as opposed to 'all of: p, q, r'. Then there would be, as it were, two kinds of negations for conjunctions ('none of' (ConjNeg) and 'not all of' (NegConj)), as there are two kinds of negations of statements involving two terms A, B, namely 'No A are B' and 'Some A are not B'. Again, Aristotle's logic would have provided the model.

For in this case whenever the first thing obtains, so does the second. Before declaring Galen a nut-case, we should reflect on how odd it is to say something like 'Dion is walking and Dion is moving' (assuming that the moving of Dion at issue is exactly that involved in Dion's walking). A conditional sentence, such as 'If Dion is walking, Dion is moving' seems also not right, though, to express that two things hold, if we presuppose that the connection between them is one of consequence. Perhaps this is where the Peripatetic complex propositions formulated with 'since' have their place (cf. Theophrastus frg. 112c Fortenbaugh); in this case, if Dion is walking and (thus) moving, the right way to express this was thought to be 'since Dion is walking, Dion is moving'.

Summary: The case of conjunction thus illustrates the following differences to the Stoic and the standard contemporary understanding of propositional logic: (i) what sort of complex proposition a sentence is depends in part on the content of the proposition; more precisely, on the relation between the things indicated by the component propositions; (ii) two simple propositions that would make a true conjunction cannot also make a true quasi-disjunction, and vice versa; (iii) the Peripatetics (and Galen) did not define complex propositions recursively; (iv) the connectors are not truth-functional.

4.2 Disjunctions

Galen provides neither definitions of the disjunction, quasi-disjunction and para-disjunction, nor an explicit account of what they indicate, or of their truth-conditions; we have to reconstruct these from the scattered information in the *Institutio Logica*. We obtain the following relevant information about disjunctions: they can be compounded of two or more simple propositions (*IL* 4.1, 5.1–2, 5.3–4). 'Either it is day or it is night' (*IL* 3.4), 'If it is not day, it is night' (*IL* 3.5), and 'Dion either walks or sits or lies or runs or stands' (*IL* 5.2) are disjunctions. A disjunction indicates complete conflict.[27] If there are more than two disjuncts, then any one stands with any other in incomplete conflict, and each stands in complete conflict to all others taken together (*IL* 5.2). It is necessary that one of the things obtains, and that all the others do not obtain (*IL* 5.2). Thus precisely one of the disjuncts in a disjunction is true (*IL* 4.1)[28] and disjunctions are exclusive and exhaustive. Moreover, it seems that no complex proposition

[27] In fact, Galen says that the nature of the things (indicated by a disjunctive proposition) indicates complete conflict (*IL* 3.5–4.1).

[28] This could mean either that a disjunction indicates this, or that it is a necessary condition for its truth.

can indicate both complete and incomplete conflict, and accordingly that no combination of propositions can be both a true disjunction and a true quasi-disjunction (implied by *IL* 5.1, see section 4.3). Two common interpretations of the truth-conditions for disjunctions can then be ruled out:

(a) a simple truth-functional interpretation:

p or q = $[p \vee q] \,\&\, \neg[p \,\&\, q]$
p or q or r = $[p \vee [q \vee r]] \,\&\, \neg[p \,\&\, q] \,\&\, \neg[q \,\&\, r] \,\&\, \neg[p \,\&\, r]$
p or q or r or s = etc.

This interpretation cannot be right, first because the text says that it is necessary, and not just the case, that (exactly) one disjunct is true; second, because this interpretation implies that e.g. 'p or q' and 'p or q or r' could be true at the same time (namely when r is false), hence could both indicate complete conflict, and this also cannot be.

(b) a modal extension on the truth-functional interpretation:

p or q = $\Box\, [[p \vee q] \,\&\, \neg[p \,\&\, q]]$
p or q or r = $\Box\, [[p \vee [q \vee r]] \,\&\, \neg[p \,\&\, q] \,\&\, \neg[q \,\&\, r] \,\&\, \neg[p \,\&\, r]]$
p or q or r or s = $\Box$ [etc.

This cannot be right either, since again both 'p or q' and 'p or q or r' could be true at the same time, namely e.g. when p is true at all possible worlds and q and r are at none; or when in some worlds we have $[p \,\&\, \neg q]] \,\&\, \neg r]$, and in others $[[q \,\&\, \neg p] \,\&\, \neg r]]$. (Also, propositions like 'either 2 is even, or 2 is odd, or triangles are round' would satisfy these truth-conditions, but would most certainly not have been accepted by Galen or any Peripatetic logicians as a true disjunction.)

In consideration of the fact that the preceding suggestions do not work, I submit that the whole traditional approach of interpreting Galen's hypothetical syllogistic as a kind of propositional logic in the manner of Stoic or modern propositional logic is mistaken. Instead, I suggest, all ('middle' Peripatetic and most Galenic) hypothetical propositions indicate (i) a relation of classes[29] and (ii) the belonging of an individual to certain classes; and that all disjunction-type propositions are based on the Platonic-Aristotelian notion of division or *diairesis* (διαίρεσις). In order for there to be a *diairesis*, there has to be a class of things which with regard to some feature N is completely dividable into *non-empty* subclasses, and where the division is based on non-contingent properties

[29] Or predicates or properties – I ignore the differences here.

of the things. Often plausible candidates for N would be sortals. In any event, N must be more generic than F, G, H, etc. In the case of the disjunction, the *diairesis* based interpretation comes out as something like:

> '2 is even or 2 is odd' is true if and only if necessarily every number is precisely either odd or even, there are (can be)[30] both odd and even numbers, and 2 is a number.

Or, more formally:

$$\text{Fa or Ga} = \forall x[Nx \rightarrow \Box[[Fx \vee Gx] \,\&\, \neg\, [Fx \,\&\, Gx]]] \,\&\, \exists x\, [Nx \,\&\, \Diamond Fx] \,\&\, \exists x\, [Nx \,\&\, \Diamond Gx] \,\&\, Na$$

(With 'Nx' for ' x is a number'; 'Fx' for 'x is even', 'Gx' for 'x is odd' and 'a' for '2')

This interpretation has none of the shortcomings of the propositional interpretations. First, it is precluded that both (i) 'Fa or Ga' and (ii) 'Fa or Ga or Ha' are true at the same time. For (i) implies $\neg\exists x\, [Nx \,\&\, \Diamond[\neg Fx \,\&\, \neg Gx]]$, whereas (ii) implies $\exists x\, [Nx \,\&\, \Diamond[Hx \,\&\, \neg Fx \,\&\, \neg Gx]]$,[31] and thus also $\exists x\, [Nx \,\&\, \Diamond[\neg Fx \,\&\, \neg Gx]]$. On the assumption that the same N is chosen for (i) and (ii), (i) and (ii) are hence incompatible. Second, disjunctions such as 'Either 2 is even or 2 is odd' would come out as true (see example above).

4.3 Quasi-Disjunctions

Galen provides the following information: quasi-disjunctions can be compounded of two or more simple propositions (*IL* 4.1, 5.1, 5.3–4). 'If Dion is in Athens, Dion is not on the Isthmus'[32] and 'It is not the case that Dion is in Athens and on the Isthmus' are quasi-disjunctions. A quasi-disjunction has or indicates incomplete conflict (*IL* 4.4, 5.1, cf. 14.11). It is customarily expressed in the form of a negative conjunctive sentence (*IL* 4.4). It is impossible that any of the conflicting things obtain together, but it is possible that they do not

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[30] Our sources suggest that in the case of mathematical and similar propositions that are not time-dependent, the requirement is one of existence ('there are'), whereas in case of time-dependent propositions the requirement is of the possibility of existence ('there can be').

[31] I have derived this formula from the account of

$$\text{Fa or Ga or Ha} = \underline{\forall x\, [Nx \rightarrow \Box}[[Fx \vee Gx \vee Hx] \,\&\, \neg[Fx \,\&\, Gx] \,\&\, \underline{\neg[Fx \,\&\, Hx] \,\&\, \neg[Gx \,\&\, Hx]]]} \,\&\, \exists x\, [Nx \,\&\, \Diamond Fx] \,\&\, \exists x\, [Nx \,\&\, \Diamond Gx] \,\&\, \underline{\exists x\, [Nx \,\&\, \Diamond Hx]} \,\&\, Na$$

by making use of the parts that are underlined.

[32] For this formulation in a conditional sentence see below, section 5.4.

obtain together (*IL* 4.2). At most one of the quasi-disjuncts is true (cf. *IL* 5.4).[33] Quasi-disjunctions are thus exclusive but not exhaustive.

As (i) disjunctions indicate complete conflict and quasi-disjunctions incomplete conflict, and (ii) complete and incomplete conflict are mutually exclusive, and (iii) a proposition is true if what it indicates obtains, it follows that what is a true disjunction cannot be a true quasi-disjunction, and vice versa. This is confirmed by Galen's saying that in the case of the disjunction it is impossible that the things do together not obtain, whereas in the case of the quasi-disjunction it is possible that they do together not obtain (*IL* 4.2). Thus no compound of propositions can be both a true disjunction and a true quasi-disjunction. Note again how this goes against the grain of modern propositional logic.

What are the truth-conditions of quasi-disjunctions? Again, we can rule out several possibilities. (I use 'not ... and ...' in order to distinguish quasi-disjunctions from negated conjunctions.)

(a) a simple truth-functional interpretation:

not (p and q) = ¬[p & q]
not (p and q and r) = ¬[p & q] & ¬[q & r] & ¬[p & r] etc.

This interpretation cannot be correct, since Galen emphatically distinguishes the quasi-disjunction from the negated conjunction, and moreover explains the quasi-disjunctions in modal terms.

(b) a modal extension of the truth-functional interpretation:

not (p and q) = □¬[p & q]
not (p and q and r) = □¬[p & q] & □¬[q & r] & □¬[p & r] etc.

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[33] There is a problem with the quasi-disjunctions which becomes apparent only in cases with more than two quasi-disjuncts. The truth-functional analogue of the two-disjunct quasi-disjunction allows, in principle, for two expansions into three-or-more-disjunct quasi-disjunctions: 'at most one of {p, q, r ...}' and 'not all of {p, q, r ...}'. The list of possible inferences from quasi-disjunctive leading premisses at *IL* 5.4 leaves no doubt that 'at most one' is intended by Galen (see below, section 5.4). Incomplete conflict between more than two things implies then that any two things stand in incomplete conflict with each other, but none of them stands in complete conflict with the rest. However, the 'at most' reading sits uncomfortably with Galen's claim that quasi-disjunctions are customarily expressed in negated conjunctive sentences. For although this is convincing for the case of two-disjunct quasi-disjunctions, it is not for three-or-more disjunct ones. A sentence 'not: p and q and r' would normally be understood as 'not all of p, q, r (hold)', not as 'at most one of p, q, r (holds)'.

This cannot be right either, for the following reasons: First, according to this interpretation 'at most one of {2 is odd, 2 is even}' would be a true quasi-disjunction. But it is not, since it is a (true) disjunction. Second, according to this interpretation, e.g. 'at most one of {Dion is talking, circles are square, humans have no soul}' would be true, which is most unlikely to have been the Peripatetic view. For a conflict, whether complete or incomplete, some kind of connection between the disjuncts seems to have been assumed.

Thus, again, I suggest that a traditional propositional logical approach is mistaken, and that quasi-disjunctions, too, have an Aristotelian flair in that they indicate (i) a relation of classes, and (ii) the belonging of an individual to certain classes:

> 'Not: Dion is in Athens and on the Isthmus' is true if and only if it holds for all humans that they cannot be both in Athens and on the Isthmus, and it is possible that they are neither in Athens nor on the Isthmus, and it is possible that someone is in Athens, and it is possible that someone is on the Isthmus, and Dion is a human being.

Or, more formally:[34]

$$\text{not (Fa and Ga)} = \forall x\,[Nx \rightarrow \Box\neg[Fx \,\&\, Gx]] \,\&\, \exists x\,[Nx \,\&\, \Diamond[\neg Fx \,\&\, \neg Gx]] \,\&\, \exists x\,[Nx \,\&\, \Diamond Fx] \,\&\, \exists x\,[Nx \,\&\, \Diamond Gx] \,\&\, Na$$

$$\text{not (Fa and Ga and Ha)} = \forall x\,[Nx \rightarrow [\Box\neg[Fx \,\&\, Gx] \,\&\, \Box\neg[Fx \,\&\, Hx] \,\&\, \Box\neg[Gx \,\&\, Hx]]] \,\&\, \exists x\,[Nx \,\&\, \Diamond[\neg Fx \,\&\, \neg Gx \,\&\, \neg Hx]] \,\&\, \exists x\,[Nx \,\&\, \Diamond Fx] \,\&\, \exists x\,[Nx \,\&\, \Diamond Gx] \,\&\, \exists x\,[Nx \,\&\, \Diamond Hx] \,\&\, Na$$

$$\text{not (Fa and Ga and Ha and Ja)} = \text{etc.}$$

This avoids all problems from propositional interpretations, and also squares with Galen's examples, as they all have shared subject terms.

4.4 Para-disjunctions

Galen seems to have taken the para-disjunctions in the *Institutio Logica* from a Stoic (or Stoicizing) context. The term 'para-disjunction' seems to be of

[34] The complexity of these and other formalizations I offer provides no argument against my suggested interpretations. The formal language I make use of has been developed for a different approach to dealing with complex sentences, and the reason for my using it is an attempt to make reasonably precise and comprehensible my suggestion for those who are versed in modern predicate logic; nothing more.

Stoic origin and when they first occur, at *IL* 5.1, Galen uses the Stoic expression 'assertible' (ἀξίωμα) instead of the Peripatetic 'proposition' (πρότασις); their second occurrence, at *IL* 6.7, is just after the Stoic schemata of the five indemonstrables have been introduced; and where they occur in the context of hypothetical syllogistic (*IL* 15.1–6), Galen reverts to the use of 'assertible' (*IL* 15. 3, 4 and 5). Galen adapts the Stoic para-disjunctions to his Peripatetic-style theory. For the Stoics, there are two types of para-disjunctive assertibles: 'at-most-para-disjunctions', in which the para-disjuncts cannot be true together, and 'at-least-para-disjunctions', in which the contradictories of the para-disjuncts cannot be true together (Gellius, *NA* 16.8.14). As the Peripatetic quasi-disjunctions already fill the role of the Stoic 'at-most-para-disjunctions', Galen restricts the term 'para-disjunction' to what would have been the Stoic 'at-least-para-disjunctions'. But even then, the para-disjunctions do not really fit the Peripatetic framework, since they involve neither conflict nor consequence in the Peripatetic sense. Peripatetic conflict is at the level of things, not truth-bearers, and in the case of para-disjunctions there are no things that are in conflict with each other.[35]

Galen provides the following relevant information for the truth-conditions of the para-disjunctions: Para-disjunctions can be compounded of two or more simple propositions (implied *IL* 5.1 and 15.1–6). Examples are 'The distribution of nourishment from the belly to the whole body occurs either by the food being carried along of its own motion, or by being digested by the stomach, or by being attracted by the parts of the body, or by being conducted by the veins' (*IL* 15.1) and 'Alcibiades knows justice either by having learnt it or by having discovered it himself' (*IL* 15.11). The use of medical and Platonic examples confirms that here we have Galen's own theory. Twice we get some sort of account: at *IL* 5.1 'it is possible that some or all <simple assertibles obtain>, and it is necessary that one obtains[36]'. At *IL* 15.2 'it is possible that all obtain together; it is necessary that one is, and it is possible that one of the others or all the remaining ones obtain together'.[37] These accounts are ambiguous.

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[35] Except if one assumed they were taken to indicate a conflict between the 'negative things' not-F and not-G, say; however, there is no evidence for this. On the other hand, for the Stoics, there seems to be no difference in logical status between their versions of at-least-disjunctions and at-most-disjunctions (both called 'para-disjunctions'). Either involves a conflict in the Stoic sense: at-most-para-disjuncts cannot be true together, and in the case of at-least-para-disjuncts their contradictories cannot be true together; cf. Gellius, *NA* 16.8.14.

[36] ὑπάρχειν – Galen mixes terminologies.

[37] I assume something like καταλελειμμένα instead of the κατειλημμένα from the MS.

(a) a truth-functional interpretation would interpret them as: 'it is a necessary condition for the truth of a para-disjunction that one para-disjunct obtains and it is not a necessary condition that not more than one obtain'. The truth-conditions would then, in short, be 'at least one of the para-disjuncts is true'. (I use '$\underline{\text{or}}$' in order to distinguish para-disjunctions from disjunctions.)

$$p\ \underline{\text{or}}\ q \quad = \quad p \vee q$$
$$p\ \underline{\text{or}}\ q\ \underline{\text{or}}\ r \quad = \quad [p \vee q] \vee r \text{ etc.}$$

(b) a modal extension of the truth-functional interpretation would have to read the text differently: 'a para-disjunction is true if and only if it is necessary that one para-disjunct obtains and possible that more than one obtains'[38]

$$p\ \underline{\text{or}}\ q \quad = \quad \Box[p \vee q]\ \&\ \Diamond[p\ \&\ q]$$
$$p\ \underline{\text{or}}\ q\ \underline{\text{or}}\ r \quad = \quad \Box[[p \vee q] \vee r]\ \&\ \Diamond[[p\ \&\ q] \vee [q\ \&\ r] \vee [p\ \&\ r]] \text{ etc.}$$

(c) a predicate logical interpretation in line with those given for disjunction and quasi-disjunction would read the text in the same way as (b), but interpret it differently:

'Alcibiades knows justice either by having learnt it or by having discovered it himself' is true if and only if necessarily every person knows justice either by learning or by discovering and it is possible that they know justice by both; and there are people who can know it by learning and people who can know it by discovering, and Alcibiades is a person.

or more formally:

$$Fa\ \underline{\text{or}}\ Ga \quad = \quad \forall x\ [Nx \rightarrow \Box[Fx \vee Gx]]\ \&\ \exists x\ [Nx\ \&\ \Diamond[Fx\ \&\ Gx]]\ \&\ Na$$

$$Fa\ \underline{\text{or}}\ Ga\ \underline{\text{or}}\ Ha = \quad \forall x\ [Nx \rightarrow \Box[[Fx \vee Gx] \vee Hx]]\ \&\ \exists x[Nx\ \&\ \Diamond[[Fx\ \&\ Gx] \vee [Gx\ \&\ Hx] \vee [Fx\ \&\ Hx]]]\ \&\ Na$$

$$Fa\ \underline{\text{or}}\ Ga\ \underline{\text{or}}\ Ha\ \underline{\text{or}}\ Ia = \text{etc.}$$

From both interpretation (b) and (c) it follows that the same component propositions can never make both a true disjunction and a true para-disjunction. For in a true disjunction it is impossible that more than one disjunct is true (*IL* 15.2), whereas in a true para-disjunction it is possible that more than one disjunct is true. Since the para-disjunction seems to be Galen's addition to the set of Peripatetic complex propositions, (a), (b) and (c) are all

[38] $\Box$[p v q] alone seems not an option, since at *IL* 5.1 the two modal expressions 'it is possible' and 'it is necessary' are co-ordinated in the account by *μέν* and *δέ*.

possible candidates for their truth-conditions. We will see later, that Galen almost certainly worked with (a).

4.5 Conditionals

Galen provides no truth-conditions for the conditional (*συνημμένα*) either. Again, I assume that a 'Middle' Peripatetic conditional is true if and only if the relation it indicates holds. But here we encounter an obvious problem: why does Galen only ever talk about *the* conditional, without distinguishing two types of conditionals,[39] one that indicates complete consequence and one that indicates incomplete consequence? The easy answer would be that Galen mixed Peripatetic and Stoic elements, and never quite thought through the implications. Although there may be some truth in this, it can be shown that Galen is at least coherent in his 'mixing'. For clarity, I shall call a conditional that indicates complete consequence a 'two-way conditional', and one that indicates incomplete consequence a 'one-way conditional'. I assume that the formulation with 'if ... then ...' was the same for two-way and one-way conditionals; and that a conditional is one that indicates complete consequence only if the two predicates are such that whenever one predicate holds of something, the other holds of the same thing, and vice versa; and that a conditional is one that indicates incomplete consequence only if the predicates are such that whenever one predicate holds of something, the other holds of the same thing, but not vice versa. I shall express the two-way conditional by 'if ... then ...' and the one-way conditional by '<u>if</u> ... <u>then</u> ...'. Thus 'if it is day, then the sun is above the earth'[40] (*IL* 3.4) is a two-way conditional , and '<u>if</u> it is day, <u>then</u> it is light' (*IL* 6.4) is a one-way conditional. The truth-conditions I suggest are in line with those proposed for the disjunction and its lesser brethren, although I am less certain here whether Galen assumed predicate logical truth-conditions. For the one-way conditional we would have:

> '<u>If</u> Dion is walking, he is moving' is true if and only if necessarily every person who is walking is moving, and not necessarily every person who is moving is walking,[41] and there are people who can walk and people who are able to not move, and Dion is a person.

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[39] Or a conditional and a quasi-conditional.

[40] In this case and similar ones that seem to be borrowed from the Stoics, we can imagine implicit quantification over times.

[41] More precisely: '... if and only if, if someone is a person, then necessarily if they are walking, they are moving, and not necessarily, if they are moving, they are walking, ...'.

Or more formally:

If Fa, then Ga = $\forall x\, [Nx \rightarrow [\Box[Fx \rightarrow Gx]\ \&\ \neg\Box[Gx \rightarrow Fx]]]\ \&$
$\exists x\, [Nx\ \&\ \Diamond Fx]\ \&\ \exists x\, [Nx\ \&\ \Diamond\neg Gx]\ \&\ Na$

And for the two-way conditional:

'If this is a human being, he is a rational mortal being' is true if and only if necessarily every living being who is a human being is rational and mortal, and vice versa, and there are living beings who can be human beings, and living beings who are able not to be human beings, and this is a living being.

Or more formally

If Fa, then Ga = $\forall x\, [Nx \rightarrow \Box[Fx \rightarrow Gx]\ \&\ \Box[Gx \rightarrow Fx]]\ \&\ \exists x\, [Nx\ \&\ \Diamond Fx]\ \&$
$\exists x\, [Nx\ \&\ \Diamond\neg Fx]\ \&\ Na.$

It follows from these accounts that the two-way conditional 'If Fa, then Ga' cannot be defined in terms of the one-way conditional as 'If Fa, then Ga and if Ga, then Fa', since the former contains the formula $\neg\Box[Gx \rightarrow Fx]$ whereas the latter contains the formula $\Box[Gx \rightarrow Fx]$ in the scope of the universal quantifier. This result should not surprise, if we assume that it was a basic tenet of the Peripatetics that no two objects can stand in more than one of the five basic relations (above, section 3). In section 5.7 it should become clearer why for the Peripatetics there was no need for distinguishing *two* types of conditionals in the context of hypothetical syllogistic.

In Galen's treatment of the conditional the incompatibility of Stoic and Peripatetic tradition becomes particularly apparent. The Stoics work only with a concept of asymmetrical consequence, expressed by the conditional, presumably since this is sufficient for their syllogistic. In their theory it is not precluded that if a conditional <p, q> is true, the conditional <q, p> is also true. The Peripatetic heritage provides the distinction between complete and incomplete connection or consequence (ἀκολούθησις), and the assumption that if a true conditional <p, q> indicates incomplete consequence, then there cannot be a true conditional <q, p>. (The Peripatetics would have to classify some of the Stoic examples of conditionals as indicating complete consequence, others as indicating incomplete consequence, and still others as not being conditionals at all.)

4.6 The rejection of formalism

We can now tackle the seeming problems that the 'Middle' Peripatetic (and Galen's) propositional logic allows us both (i) to formulate the same kind of complex propositions in different ways (i.e. with a different sentence connector), and (ii) to formulate different kinds of complex propositions in the same way (i.e. with the same sentence connector).

In case (i), there are two possibilities. We may have (a) one quasi-disjunction {p, q} which has the grammatical form of a negative conjunctive sentence 'not both p and q', and another quasi-disjunction {p, q} which has the grammatical form of a conditional sentence 'if p, not q' (cf. *IL* 4.1). Or, we may have (b) one disjunction {p, q} which has the grammatical form of a disjunctive sentence 'either p or q', and another which has the grammatical form of a conditional sentence 'if p, not q' (*IL* 3.4–5). [42] Case (i) is not really problematic, as long as the accepted grammatical forms for a type of complex proposition are all known. In any event, it seems that the conditional formulations in (a) and (b) were not regarded as canonical, but used in order to signal the deductive power of those types of propositions; ('if p, not q' signals that if you have 'either p or q' and 'p', then you can deduce 'not q', etc.).[43]

In case (ii), there are the following possibilites: (a) A conditional sentence could indicate either a consequence or a conflict. (b) If a conditional indicates a consequence, this could be either complete or incomplete. (c) If a conditional indicates a conflict, this could be either complete or incomplete. (d) A disjunctive sentence could be either a disjunction or a para-disjunction. (e) A negative conjunctive sentence 'not both p and q' could be either a negative conjunction or a quasi-disjunction. Case (ii) raises two problems: First, if one is confronted with an isolated sentence of a complex grammatical form, how is one to know which type of complex propositions is (or is intended to be) manifested in it? Since the form of the sentence seems not to help here, one would have to look to its content. But in that case, we get the derivative problem, how does the identity criterion for a certain type of complex proposition differ from its truth criterion? For example, it seems that in order to ascertain that a particular sentence of the linguistic form 'not both p and q' is a quasi-disjunction (as opposed to a negated conjunction), we may need to determine that the things indicated by p and q stand in the relation of incomplete conflict. But this seems to be exactly what we need to do in order to find out whether the sentence is a *true* quasi-disjunction.

However, both problems are apparent only. To explain why, I restate some of the specific features of Peripatetic logic (1.–3.), and add two common sense assumptions (4. and 5.) for which there is some support in the ancient texts:

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[42] 'if p, not q' and 'either p or q' (both taken to indicate complete conflict) count not as alternative expressions of the same disjunctive proposition, but as two different, but equivalent, disjunctive propositions (see e.g. *IL* 3.4–5).

[43] Cf. Bobzien (2002a).

1. The relations of consequence and conflict are defined as holding between things (or 'states of affairs'), not between affirmations and negations.
2. The complex propositions are defined in such a way that any combination of two or more simple propositions can at most produce a true complex proposition of one kind.
3. The truth-conditions of the complex propositions are not truth-functional.
4. In the pairings (ii) (a)-(c) above, each time one kind of complex proposition was regarded as 'stronger' than the other. These were the two-way conditional, the disjunction, and the quasi-disjunction (cf. their accounts above).[44]
5. A version of the Principle of Charity was followed in deciding what type of complex proposition is manifested in a complex sentence in which, in principle, two different types could be manifested: that is, (i) if the complex proposition can be interpreted in such a way that it is true, this interpretation is taken as intended by the speaker (writer, text, etc.), and (ii), the test procedure starts always with checking the stronger of two possible interpretations.

The presumed problems then disappear as follows:

- Point 1. makes it possible to determine unambiguously whether a complex sentence of if...then – form indicates a conflict or consequence. When both component propositions are affirmative we have a case of a consequence being indicated; and so we do when both are negative, because of the rule of contraposition, which the Peripatetics accepted.[45] When, on the other hand, one component proposition is affirmative and the other negative, the proposition indicates a conflict (cf. Galen *IL* 3.1, etc.). This solves case (a).
- Cases (b)-(e) are decided by using the Principle of Charity in tandem with the distinction of (relatively) weaker and stronger complex propositions. First, it is assumed – as some sort of implicature – that a speaker intends to make the strongest possible assertion; e.g. in the case of a sentence of the linguistic form 'either p or q', a disjunction rather than a para-disjunction. Then the Principle of Charity is applied, in the sense that if the sentence is

44 I rely here on an intuitive sense of 'stronger'.

45 See also below, note 60.

recognized to be a false disjunction, it is assumed to be the weaker type of statement, in this case a para-disjunction, instead. It can then either be a true para-disjunction, or a false one.[46]

We are left with a residual problem: Does it not follow that all 'stronger' complex propositions, i.e. disjunctions, quasi-disjunctions, and two-way conditionals turn out to be true? And hence that (at least for these kinds of propositions), after all, identity criteria and truth criteria coincide? It does not. It is possible for there to be false disjunctions, etc. Thus, (i) the mere fact that someone identifies a complex proposition as a disjunction because they believe it to be a true disjunction, does not entail that it is a true disjunction. But if it happens to be false, this does not entail that it is not a disjunction. Moreover, the context may imply that it is a disjunction. (ii) If someone *intends* to utter or mention, for instance, a false disjunction, this can still be done in various ways, by discharging the 'implicature' somehow, e.g. by way of calling it a 'disjunction', or by embedding it into an unambiguous linguistic or non-linguistic context.[47]

5. Hypothetical syllogisms

The purpose of Galen's discussion of complex propositions is as a preliminary to hypothetical syllogistic. Of the five basic types of complex propositions four are called hypothetical propositions (conditional, disjunction, quasi-disjunction and – at least at one point – para-disjunction). Hypothetical propositions are those that can function as hypothetical or leading premisses in hypothetical syllogisms. Hypothetical syllogisms are those syllogisms which come to be from hypothetical premisses. The hypothetical syllogisms discussed in the *Institutio Logica* all consist of one hypothetical proposition as leading assumption (ἡγεμονικὸν λῆμμα, *IL* 7.2), one co-assumption (*πρόσληψις*, *IL* 4.3), and a conclusion (*συμπέρασμα*, *IL* 1.4). Hypothetical syllogisms are completed by transition from one thing to another (*IL* 14.10, viz. by adding co-assumption and

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46 The Principle of Charity has more plausibility for the Peripatetic theory than for modern propositional logic: if the sentence 'either p or q' expresses a true disjunction (*διεζευγμένον*), then it cannot express a true para-disjunction, so there is no need for further investigation. In modern logic, when someone says: 'either two is even, or two is odd', they could be expressing either a true exclusive disjunction, or a true inclusive disjunction, and hence the Principle of Charity on its own does not help to decide which one is intended.

47 e.g. 'let "either Dion is walking or Dion is sitting" be a disjunction'; ' "either p or q" is the first premiss in the following fourth hypothetical syllogism'.

conclusion to the hypothetical premiss). It is a special feature of hypothetical – as opposed to categorical – syllogisms, that their leading premisses fully determine what further premisses would permit drawing a conclusion (*IL* 7.1, 7.4). Galen discusses five types of hypothetical syllogisms which resemble Chrysippus' indemonstrables (in *IL* 14.1–11) and two types based on para-disjunctions (*IL* 15.1–6, 9, 11). Here is first of all a list of all the types of syllogisms:

(G1) The 'first hypothetical syllogism', in which a relation of consequence[48] between p and q allows the transition from p to q.

If p, q	conditional (one-way or two-way)
now p	co-assumption
therefore q	conclusion

(G2) The 'second hypothetical syllogism', in which a relation of consequence between p and q allows the transition from not q to not p.

If p, q	conditional (one-way or two-way)
now not q	
therefore not p	

(G3) The 'third hypothetical syllogism', in which a relation of incomplete conflict between two or more things allows the transition from one of the conflicting things to the negation of the other, or (if there are more than two quasi-disjuncts in the leading premiss) to the conjunction of negations of the rest. The linguistic form of the leading premiss is that of a negative conjunctive sentence, but its logical form is that of a quasi-disjunction. For example (with $p_i \neq p_j \neq p_k$ for G3–G7):[49]

Not: p_1 and p_2 and p_3	quasi-disjunction	At most one holds of $\{p_1, p_2, p_3\}$
now p_i		p_i
therefore neither p_j nor p_k	conj. of negations	None holds of $\{p_j, p_k\}$

(G4) The 'fourth hypothetical syllogism', in which a relation of complete conflict between the two or more things allows the transition from one of the conflicting things to the negation of the other, or (if there are more than two disjuncts in the leading premiss) to the conjunction of the negations of the rest. For example:

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48 For the question of whether this consequence is complete or incomplete or either, see section 5.7 below.

49 For reasons of brevity and simplicity I restrict myself to examples of three-component complex propositions. The quantifier versions of the arguments in the third column ('At most one …', etc.) are added solely to provide the reader with some aide-mémoire for the different types of syllogism G4–G7. They don't take any of the modal elements into account.

Either p_1 or p_2 or p_3	disjunction	Precisely one holds of $\{p_1, p_2, p_3\}$
now p_i		p_i
therefore neither p_j nor p_k	conj. of negations	None holds of $\{p_j, p_k\}$

(G5) The 'fifth hypothetical syllogism', in which a relation of complete conflict between the two or more things allows the transition to one of the conflicting things from the negation of the other, or (where there are more than two disjuncts in the leading premiss) from the conjunction of the negations of the rest. For example:

Either p_1 or p_2 or p_3	disjunction	Precisely one of $\{p_1, p_2, p_3\}$
now neither p_j nor p_k	conj. of negations	None of $\{p_j, p_k\}$
therefore p_i		p_i

(G6) The first para-disjunctive syllogism,[50] in which the transition is to one of the conflicting things from the negation of the other, or (where there are more than two para-disjuncts in the leading premiss) from the conjunction of the negations of the rest. For example:

Either p_1 or p_2 or p_3	para-disjunction	At least one of $\{p_1, p_2, p_3\}$
now neither p_j nor p_k	conj. of negations	None of $\{p_j, p_k\}$
therefore p_i		p_i

(G7) The second para-disjunctive syllogism, in which the transition is from the negation of one of the conflicting things to the other, or (where there are more than two para-disjuncts in the leading premiss) to the inclusive disjunction of the rest. For example:

Either p_1 or p_2 or p_3	para-disjunction	At least one of $\{p_1, p_2, p_3\}$
now not p_i	Not p_i	
therefore either pj or pk	para-disjunction	At least one of {pj, pk}

It can be easily shown, that (G1)–(G7) are valid with the interpretations of the hypothetical propositions given in sections 4.2–4.5 above.[51]

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[50] This is my, not Galen's, way of referring to (G6) and (G7).

[51] The reader may prefer to start with hypothetical propositions with two disjuncts, quasi-disjuncts, etc. Thus for G4 we have:

Ga or Fa = $\forall x[Nx \rightarrow \Box[[Fx \vee Gx] \& \neg[Fx \& Gx]]] \& \exists x [Nx \& \Diamond Fx] \& \exists x [Nx \& \Diamond Gx] \& Na$
Now Ga
Therefore not Fa

We can disregard the existential conjuncts in the hypothetical premiss for obvious reasons. The conjunct 'Na' allows as to get to ' $\Box[[Fa \vee Ga] \& \neg[Fa \& Ga]]$'. If to this we add 'Ga' as a second premiss, we can infer 'not Fa'. The validity of the other cases, with two or more components, can be shown in the same way.

At *IL* 14.2–3 and 10–11, Galen conveys that there are precisely five types of hypothetical syllogisms that are useful for proof (ἀπόδειξις), viz. those whose leading premiss indicates either a conflict or a consequence (i.e. G1–5). Galen refers to these five *types* as 'first, second, etc. hypothetical syllogism'.[52] Their linguistic form is close to that of the Stoic types of indemonstrables (see above section 1), and they come in the same order. They are modelled on the indemonstrables, as Galen freely admits (e.g. *IL* 14.11). There are, however, plenty of differences. Most of these can be explained as being the result of modifications of the Stoic system from a Peripatetic perspective. (This is a major difference from early Peripatetic 'propositional logic'.) First, there are a number of non-Stoic, mainly Peripatetic, elements which are germane to the general understanding of hypothetical syllogistic:

- Galen's name for these syllogisms with complex first premiss is 'hypothetical syllogism', and not 'indemonstrables'.
- They are based on relations between states of affairs or things (πράγματα), not between truth-bearers.
- The set of the five types (G1–G5) is subdivided into two groups according to whether the underlying relation is one of consequence (G1 and G2) or one of conflict (G3–G5)), as is clear from Galen's insistence that the latter three all indicate a conflict of sorts (*IL* 14.11). This division corresponds to the early Peripatetic twofold division between connection and division.[53]
- The examples do not show the Stoic rigour of formulation, and often do not follow Stoic conventions of formulation. Negations are not always at the beginning of the sentence (*IL* 15.3). Most examples have the same subject term in all their component sentences. Often, there are not even independent component sentences (e.g. *IL* 15.7), or in any case the subject term is not repeated in the second and next following component sentences. As a result, those examples are open to predicate logical interpretations.

[52] These are the standard names in later Peripatetic and Platonist passages; in Stoic texts the corresponding *arguments* are called 'first, second, etc. indemonstrables'. For details see Bobzien (1996), section 1. At times Galen seems to use the two types of names interchangeably (*IL* 15.8–9).

[53] See Bobzien (2002a). It becomes standard later. The Stoics would divide their indemonstrables into three groups, depending on whether the leading premiss is a conditional, a negation of a conjunction, or a disjunction.

In the Peripatetic derived syllogisms G1–5, all leading premisses are hypothetical propositions, and as such contain elements of necessity. The co-assumptions are either simple affirmatives (G1, G3, G4) or simple negatives (G2), or conjunctions of negations (G5), and the conclusions fall in the same categories (simple affirmatives: G1, G5; simple negatives: G2; conjunctions of negations: (G3, G4)). Thus, the hypothetical syllogisms G1-G5 have no hypothetical propositions in either co-assumptions or conclusions. Hence, the hypothetical syllogisms do not allow repeated use of a syllogistic form in the same proof – except in those proofs in which a component sentence is a hypothetical proposition that remains unanalyzed in the first application of one of the types of hypothetical syllogisms. For instance, we can use the conclusion of an argument of type G1, with the hypothetical premiss 'if p, then either q or r', as the two-disjunct hypothetical premiss of an argument of type G4 or G5 (see next section).

In accordance with their Stoic origin, Galen's para-disjunctive syllogisms (G6 and G7) seem to have functioned differently. Their first premisses do not indicate a conflict or consequence in the Peripatetic sense. As a consequence, it seems that we could, for instance, use the conclusion of a three-para-disjunct argument of type G7 as the premiss of a further para-disjunctive syllogism.

5.1 Reduction of hypothetical syllogisms:

Stoic syllogistic was a system of five types of axiomatic syllogisms, and four inference rules which made it possible to reduce all other syllogisms to axiomatic ones.[54] In this respect it resembles modern argumental deductive systems. No mention is made of such a systematic reducibility of all complex syllogisms in Galen (or in any Peripatetic or Platonist sources). I assume that generally for the proof of a proposition it was considered possible to use several syllogisms in a row (the conclusion of one serving as one of the premisses of the next);[55] moreover, that certain inference rules were known and accepted. Cut rules, i.e. inference rules such as

$$\frac{A, B \vdash C \qquad C, D \vdash E}{A, B, D \vdash E}$$

would be used as a means of 'abbreviating' chains of arguments, and were known to the Peripatetics, and used by Galen.[56] Another kind of inference rule

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[54] See Bobzien (1996), or Bobzien (1999), 127–151.

[55] Cf. e.g. Galen, *On Semen* II 1.68–9.

[56] Cf. e.g. Galen, *On Semen* II 1.69 and Alex. *An.Pr.* 283–4.

was based on 'contraposition of syllogisms'. At *IL* 6.5 we learn that two two-premiss syllogisms stand in contraposition with each other if they share one premiss, and the second premiss of each is the contradictory of the conclusion of the other syllogism. And in *IL* 6.6 Galen says that any argument that stands in contraposition to a sound argument is itself sound, and any argument form that stands in contraposition to a syllogistic argument form is itself syllogistic:[57]

$$\frac{A, B \vdash C}{A, \text{ctrd. } C \vdash \text{ctrd. } B}$$

Such rules are sometimes called 'Antilogism'.

In the Stoic system, multiple disjunct fourth indemonstrables and multiple conjunct third indemonstrables can be reduced to ordinary Stoic indemonstrables. For Peripatetic propositional logic there is no evidence that any syllogisms with three-or-more disjuncts (quasi-disjuncts, para-disjuncts) in their leading premiss could be reduced to syllogisms with two disjuncts (quasi-disjuncts, para-disjuncts) in their leading premisses. If the logicians considered the underlying ontic relations of conflict and consequence as basic and unanalysable, this may explain why the third, fourth, and fifth hypothetical syllogisms (and perhaps the para-disjunctive ones) were *defined* with reference to multiple disjuncts.

A further difference from Stoic syllogistic is that while the Stoics consider any first or second indemonstrables as 'indemonstrable' (i.e. not in need of proof), and as on all fours with each other,[58] for the Peripatetics, the first hypothetical syllogism is prior to the second. Galen remarks that the second hypothetical syllogism is not indemonstrable, but requires proof (*IL* 8.2). He does not present the proof, but we can guess from *IL* 6.5 (see above) that it made use of the antilogism rule. Thus, on the assumption that 'If p, q; now p; therefore q' is a syllogism, so is 'If p, q; now not q; therefore not p', and I believe that this was the proof of the second hypothetical syllogism.[59] But equally, of course, if the latter is a syllogism, so is the former. So why does the second

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57 Cf. also Alex. *An.Pr.* 29.7–13.

58 Any second indemonstrable can in principle be reduced to a first, and any first to a second by application of the first *thema*. But the rule seems to have been to stop reduction as soon as an indemonstrable has been reached.

59 This is also suggested by the later description or name of the second hypothetical syllogism as 'conversion with contraposition' (σὺν ἀντιθήσει ἀντιστροφή), see e.g. [Ammonius] *An.Pr.* 68.

hypothetical syllogism stand in need proof, but not the first? The answer, I suspect, can be found once more by resorting to the basic relations between things. The relation of consequence requires that whenever the first thing obtains, necessarily the second obtains as well. Thus, when one has established that such a consequence holds, one can infer *directly* that if the first thing obtains, so does the second. One cannot infer directly that if the contradictory of the second obtains, so does the contradictory of the first.[60]

5.2 What is a syllogism?

Galen's account of the term 'syllogism' at *IL* 1.4 has almost entirely fallen victim to mould in our single surviving manuscript. However, we can recapture Galen's conception of syllogismhood up to a point by drawing on the *Institutio Logica* as a whole, on other of his writings, and on the general views of syllogisms of his time.

- A syllogism is an *argument* (λόγος), consisting of premisses and conclusion.
- It is a *valid* argument in the sense that the conclusion follows from the premisses.
- It satisfies the standard condition for validity that no argument that is a syllogism has/can have a false conclusion when its premisses are all true.
- It has a specifiable syllogistic form.

Thus it becomes important to know what qualifies as a syllogistic form. I suggest that for the 'Middle' Peripatetics Galen is drawing from, a syllogistic argument form is a (standardly) valid argument form which is useful for proof, i.e. which can be used in order to prove a thesis or problem.[61] The following

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60 On the other hand, I assume that a syllogism of the form 'if not q, not p; now p; therefore q' was not proved by Antilogism, since such a proof would fail to lead to a syllogism with a conditional that indicates a consequence. Rather it would have been proved by contraposition of the first premiss, which *would* have that effect. Such use of contraposition to prove syllogisms was common among the Peripatetics in the case of the wholly hypothetical syllogisms, see Bobzien (2000a), sections 1 and 6.

61 *IL* 14.3 (charitable reading), 14.10. In any case, for Alexander, only arguments that can be used for proof are syllogisms, cf. e.g. Alex. *Top.* 9.25–31, *An.Pr*. 18.12–21, and this may suggest a Peripatetic tradition for this view. Galen is perhaps not entirely clear on the point whether being useful for proof is a necessary condition for syllogismhood. If it is not, then at least being useful for proof was a necessary condition for an argument form's being included in an introduction to syllogistic and proof such as the *Institutio Logica*, and this would suffice for my purposes.

list of criteria for an argument's having a syllogistic form is partly taken from Galen, partly constructed on the basis that it is compatible with Galen's text, finds evidence in related texts, and makes it possible to explain a considerable number of oddities surrounding Galen's Peripatetic-influenced 'propositional logic'.

1. Arguments of syllogistic form need to have at least two premisses.[62]

2. The form must be such that there is at least one argument of that form in which the premisses and conclusion are all true together (or, in other words, the form must be such that there is at least one sound argument of that form). [63]

3. The form must be such that it is not possible that in an argument of this form (and because of this form) coming to know the truth of any one premiss (or proper subset of premisses, including the null set) is sufficient for coming to know the truth of the conclusion. (This requirement precludes redundant premisses, including the case that one premiss entails another.) [64]

4. Moreover – and this is a combination of the previous two requirements – the form must be such that it does not follow from it that when one has come to know on its own the truth of a premiss (or proper subset of premisses), then *either* one has come to know the truth of the conclusion, *or* it has become impossible to get a sound argument.

Thus the following argument forms, all valid in classical propositional calculus, would fail syllogismhood for the 'Middle' Peripatetics in Galen's *Institutio Logica*: 'p and q. Therefore p' (because of 1 & 3); 'p. Therefore p' (ditto); 'p. not p. Therefore q' (because of 2); 'p. q. Therefore r or not r' (because of 3); 'p. q. Therefore p' (ditto); 'Not both p and q. p. Therefore not q' (because of 4 – I explain this case below). In these requirements for what counts as a syllogistic form, epistemic elements seem to encroach on logical ones. This

62 Cf. Aristotle's definition of the syllogism (*An.Pr*. 24b18–20), but this holds also for the Stoics (with the exception of Antipater).

63 Cf. Alexander, *An.Pr*. 20.10ff, where he claims that a conditional premiss that does not permit taking a second (true) premiss is not suitable for a syllogism. Remember also that for the Peripatetics, logic is a tool, and is thus destined for use. Cf. also Galen, *De plac. Hipp. et Plat*. II 3.18 (CMG V 4.1.2 p.114.1–2) and Galen, *IL* 19.6 together with Barnes (1993), 38–41.

64 Cf. again Aristotle's definition of the syllogism: ἕτερον τι τῶν κειμένων. Cf. also Alex. *Top.* 9–11, *An.Pr*. 18–20, 164, Amm. *An.Pr*. 27–8.

is not unusual in ancient logic. At the same time, the requirements keep out many of the seemingly paradoxical theorems which relevance logicians have aimed at avoiding.[65] For Galen's 'Middle' Peripatetics, the key to validity or syllogismhood appears to be to capture knowledge introduction or knowledge extension rather than truth-preservation.[66]

5.3 The application of hypothetical syllogistic

The Peripatetic way of using hypothetical syllogistic is derivative of Aristotle's *Topics* – which is where the early Peripatetic theory of 'propositional logic' had its origins.[67] A discussant or investigator selects a thesis they wish to establish. Hypothetical syllogistic provides a logical system which can be used for this goal, especially (but not solely) in cases in which categorical syllogistic is unhelpful.[68]

The *logical system* consists of

- the types of hypothetical propositions and their 'adequacy conditions' (by which I mean the accounts of what they indicate)[69] or their truth-conditions; and
- a list of types of (basic) hypothetical syllogisms, or of the corresponding syllogistic forms; this list is classified into groups according to (i) their hypothetical premisses; and (ii) the possible kinds of co-assumptions.
- rules such as Cut and Antilogism (see section 5.1).

The *application* of the system is then as follows:

- you select a *thesis* which you want to establish. (In the basic case this will be a simple proposition, e.g. 'Dion is alive', and in any case a proposition that remains unanalyzed in the course of the argument.)

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65 E.g. the so-called paradoxes of material and strict implication.

66 This is not so alien to some 20th century logicians: Cf. von Wright (1957), 181: '*p* entails *q*, if and only if, by means of logic, it is possible to come to know the truth of $p \supset q$ without coming to know the falsehood of *p* or the truth of *q*,' (note especially the similarity to my point 3. in the main text); Geach (1958), 164: 'I maintain that *p* entails *q* if and only if there is an a priori way of getting to know that *Cpq* which is not a way of getting to know whether *p* or whether *q*.'. See also Smiley (1959). All three quoted in Anderson and Belnap (1975), 152–3.

67 Cf. Bobzien (2002a).

68 Galen *IL* 14.1 suggests that these are in particular questions of the existence of things, e.g. whether providence or the gods exist. These are a kind of statement not catered for by categorical syllogistic. (Cf. also *IL* 2.1) But this does not exhaust the realm of application, and Galen himself uses examples of quite different kinds (e.g. *IL* 16.11).

69 See above, section 4.

- you enquire whether there holds any relation of conflict or consequence between the thing your thesis is about and *other* things. (You may look for relations such as that breathing has being alive as a consequence, or that being odd is in conflict with being even.)
- if there is such a relation, you express it in the adequate hypothetical proposition. (For instance, 'If Dion is breathing, Dion is alive', 'Two is either odd or even'.) For this you need to know the 'adequacy conditions' or the truth-conditions of the hypothetical propositions.
- you check whether the truth of any simple proposition indicating the obtaining or not obtaining of the *other* thing(s) in the hypothetical proposition(s) has been established. (For instance, the truth of 'Dion is breathing'.)
- you check the list of 'hypothetical syllogisms' for possible inferences with your hypothetical proposition(s) and those additional (true) simple propositions, and if you find a type of syllogism in which any of the other simple propositions which have been established can be used (singly or together) as co-assumption, you do so. (For instance, the first hypothetical syllogism.)
- you are then in a position to establish (or to refute) your thesis by making the inferential transition from your co-assumption ('Dion is breathing') to the conclusion ('Dion is alive').

Peripatetic (and Galen's) hypothetical syllogistic would most probably have been used without anyone systematically going through all the above steps one by one. My purpose of setting them out here is rather to give a general idea of the function Peripatetic hypothetical syllogistic, as described by Galen, seems to have had.[70]

This interpretation of the Peripatetic-influenced hypothetical syllogistic in Galen and of its application enables us to explain most of the oddities and apparent incoherence in the relevant sections of the *Institutio Logica*. I

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70 Sometimes, in particular when the hypothetical proposition contains more than two simple propositions, two successive applications of hypothetical syllogisms may be required. For instance, you want to prove q, and realize that a conflict between the things *that q* and *that r* would be the consequence of p, and accordingly formulate the hypothetical proposition 'If p then either q or r'. If p is established, you can then in a first step infer 'either q or r'. And if 'not r' is established, you can then in a second step infer q. Or you can do it all in one step, using a Cut rule, as Galen seems to have done in *On Semen* II 1.69.

address a number of these in the following remarks about the various types of syllogisms.

5.4 The third hypothetical syllogism:

Galen pursues two aims with his discussion of the third hypothetical syllogisms (G3 above, section 5): (i) to show what sort of syllogism they are (*IL* 4.3, 5.3–4, 7.1, 14.11). (ii) to show that a negated conjunction as leading premiss does not provide a third hypothetical syllogism (*IL* 4.4, 14.3, 7–8). I take these points in turn.

(i) Third hypothetical syllogisms are those with a quasi-disjunction with two or more quasi-disjuncts as leading premiss (*IL* 5.3–4, 14.11). There is only one kind of co-assumption (*IL* 4.3, 5.3–4, 7.1, 14.11). This is affirmative (*IL* 14.11); it is the assertion of one quasi-disjunct (implied at *IL* 4.3), or of the obtaining of one of the conflicting things (*IL* 5.3). There is also only one kind of conclusion, which is negative (implied at *IL* 4.3). It negates the holding of the remaining thing or things (*IL* 5.3–4).

The properties ascribed to the third hypothetical syllogisms leave little doubt that Galen and his Peripatetic source understood quasi-disjunctions in the sense of 'at most one (out of two or more)' rather than 'not all (of two or more)': The conclusion is described as 'we negate the remaining number' (*IL* 5.4). This can only mean that we negate each of the remaining 'conjuncts' of the quasi-disjunction[71] – exactly what the 'at most one' reading requires. By contrast, 'not all' would require that we obtain as conclusion a negated conjunction, formed of the remaining quasi-disjuncts.

The purpose of the alternative formulation of the quasi-disjunctions as conditional sentences of the form 'If p, not q' (*IL* 4.1) now also becomes clear. Quasi-disjunctions are those which say that 'if this is, that is not' (*IL* 3.1), and hence allow a transition (*IL* 14.10) from the obtaining of this to the not obtaining of that. Thus third hypothetical syllogisms with two quasi-disjuncts can be seen to have the underlying structure of *modus ponendo ponens* with a negative consequent: 'If p, not q; now p; therefore not q'. Third hypothetical syllogisms with more than two quasi-disjuncts can also be paraphrased in a way that makes an underlying structure of *modus ponendo ponens* apparent:

if p_i, then neither p_j nor p_k nor ... nor p_n
now p_i
therefore neither p_j nor p_k nor ... nor p_n.

[71] Cf. similar formulations for disjunctions, e.g. at *IL* 5.4.

The formulation as a negative conjunctive sentence directly expresses the relation of incomplete conflict between things, whereas the formulation as a conditional sentence brings out the inferential force of the hypothetical premiss in a third hypothetical syllogism.[72]

(ii) Galen's second aim is to explain why, unlike a quasi-disjunction, a negative conjunction does not provide a syllogism (or is not useful for proof). Galen's reasoning is roughly this: a negative conjunction indicates neither a conflict nor a consequence between things (*IL* 4.4). Only complex propositions that indicate a consequence or a conflict are suitable for proof, since only they can warrant a transition from co-assumption to conclusion (*IL* 14.10). Hence a negative conjunction cannot function as the leading premiss in a syllogism.

Recalling the above interpretation of hypothetical syllogisms and their application, we can see why Galen thinks this: a true negative conjunction does not indicate a relation of conflict. Suppose that we have an argument

not (p and q)
p
———————
not q

with such a negative conjunction as first premises. It then follows from the argument form that when one has come to know on its own the truth of one premiss, then *either* one has come to know the truth of the conclusion, *or* it has become impossible to get a sound argument. For, if the first premiss is true, then either (i) p is false or (ii) q is false.

(i) If p is false, no sound argument can be had, since this fact rules out that the second premiss, p, can be true.

(ii) If q is false, then we know already that the conclusion is true before we draw the inference, or, at least, we can get to it without taking the second premiss into account.

Take Galen's example 'not: Dion is walking and Theon is talking; Dion is walking; hence Theon is not talking'. In order to know the truth of the leading

[72] This explains why we find no evidence in antiquity that the relation of consequence was ever expressed as a disjunction with one affirmative and one negative disjunct. For in the cases of the first and second hypothetical syllogisms the two functions of (i) directly expressing the relation of consequence and (ii) bringing out the inferential force, fall together in the formulation as a conditional; cf. on this point Bobzien (2002a).

premiss we need to know either (i) that Dion is not walking or (ii) that Theon is not talking.

(i) If we know that Dion is not walking (and he hence is not walking), no sound argument can be had, since the second premiss cannot be true.

(ii) If we know that Theon is not talking, we already know the (truth of) the conclusion, or in any case could draw it without taking the second premiss into account.

Such reasoning works *mutatis mutandis* for negated conjunctions with more than two conjuncts (in fact, regardless of whether an 'at most' or a 'not all' interpretation is chosen). From the point of view of standard contemporary logic, this result (or perhaps rather the above requirement 4 from section 5.2 on which it is based) is somewhat perplexing.[73] However, it is very close to the von Wright-Geach-Smiley approach to entailment[74] and, more importantly, it fits Galen's and the Peripatetic way of thinking about syllogisms excellently.

5.5 The fourth and fifth hypothetical syllogisms:

Galen allows for only two types of syllogisms from disjunctions with multiple disjuncts: either by affirming that any one disjunct holds we will negate the rest (above G4); or by negating this rest (above G5), we will affirm that the one holds (*IL* 5.4). Why does he not mention the following pair of argument forms?

(*G8)	Either p_1 or p_2 or p_3		disjunction
	now p_j or p_k		disjunction
	therefore not p_i	(with $p_i \neq p_k \neq p_j$)	
(*G9)	Either p_1 or p_2 or p_3		disjunction
	now not p_i		
	therefore p_j or p_k	(with $p_i \neq p_k \neq p_j$)	disjunction

These kinds of arguments come out as standardly valid (i.e. can't/don't ever have a false conclusion when the premises are true) with a truth-functional interpretation, with a modal propositional interpretation, and also with the above predicate-logical interpretation of the truth-conditions of the disjunction

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73 In fact, the same kind of argumentation works for any kind of propositional argument composed only of truth-functional propositions, with a complex proposition as leading premiss – whether the connectors are '$\vee$', '&', '$\rightarrow$' or '$\leftrightarrow$'. Thus, no argument form valid in non-modal classical propositional logic would be a valid syllogistic form in the above-given sense.

74 See above, note 66.

(section 4.2). Did Galen forget to mention them? This is rather unlikely, since in the case of the para-disjunctions he lists <u>all</u> the different possibilities (*IL* 15.3–6, see next section). A more satisfying answer can be given if one (i) chooses the predicate-logical interpretation suggested in section 4.2 and (ii) assumes that Galen had a conception of syllogismhood like the one presented above (section 5.2), which goes beyond standard validity in that only those argument forms qualify as syllogistic that can in principle be used for proving something. The requirement for syllogismhood relevant here is requirement 2 from section 5.2, that a syllogistic form must be such that it is possible for logical reasons[75] that all the premisses and the conclusion are true together. From assumptions (i) and (ii) taken together it then follows that (8) and (9) are not syllogistic: For in (8) whenever its first premiss is true, its second premiss is false, and vice versa; and in (9) whenever its first premiss is true, its conclusion is false, and vice versa, and this is so for logical reasons, i.e. owing to the truth-conditions for disjunctions.

5.6 The para-disjunctive syllogisms:

Para-disjunctive syllogisms have as co-assumptions either a negative simple proposition or a conjunction of negative simple propositions; as conclusion either an affirmative simple proposition or a para-disjunction. Disregarding permutations of the simple propositions, a para-disjunctive syllogism with n para-disjuncts in its leading premiss will have n–1 possible co-assumptions: not p; not p and not q; not p and not q and not r; etc.[76] These

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[75] i.e. for reasons of the definitions of the logical constants and logical terms of the theory at issue.

[76] At *IL* 7.1 Galen says that para-disjunctive syllogisms can only have two kinds of co-assumptions; at *IL* 6.7 they are specified as those with a simple negative as co-assumption and a disjunction (of sorts) as conclusion, and those with a conjunction of negatives as co-assumption and a simple affirmative as conclusion. This distinction covers neither para-disjunctive syllogisms with a two-para-disjunct leading premiss nor, in cases with more than three para-disjuncts in the leading premiss, those with a conjunction of simple negatives as co-assumption and a disjunction (of sorts) as conclusion. At *IL* 15.3–6 Galen appears to classify the latter with those that have a simple negative as co-assumption. I consider it likely that, in Peripatetic manner, one classification criterion was based on the types of co-assumptions (thus *IL* 15.3–6 comes out right), and that at *IL* 6.7 Galen added the types of conclusions without having thought beyond cases with three para-disjuncts in the leading premises. One may also think that one type of co-assumptions (a simple negative) would suffice, since one could simply use G7 repeatedly in cases with more than two para-disjuncts. No such repeated use is possible in the cases of G1–G5.

kinds of arguments presumably originated with post-Chrysippean Stoics. Galen's use of the term 'assertible' rather than 'proposition' in his discussion of para-disjunctive syllogisms suggests that he adopted these arguments from Stoic philosophy and tried to integrate them into the Peripatetic system of hypothetical syllogisms.

Galen's treatment of the para-disjunctive syllogisms makes it possible for us to decide between the three possible interpretations (a), (b) and (c) of the para-disjunctive propositions that I introduced above in section 4.4. At *IL* 15.4 and 15.6 Galen says explicitly that the conclusion of arguments of type G7 is a para-disjunction.[77] Thus we have to choose interpretation (a), i.e. the interpretation of the para-disjunction as a truth-functional inclusive disjunction. For with non-truth-functional interpretations like (b) and (c) the argument would come out as invalid. Moreover, we should expect the conclusion to be a truth-functional inclusive disjunction, because by contraposition of syllogisms of type G6 we get

<u>Either</u> p_1 <u>or</u> p_2 <u>or</u> p_3
Now not p_i
Therefore not (not p_j and not p_k)

and here the conclusion seems to be equivalent to a truth-functional inclusive 'or'.

Are the para-disjunctive syllogisms hypothetical syllogisms? There is no clear-cut answer to this question. At *IL* 14.3 Galen claims that there are only five types of hypothetical syllogisms, or arguments with hypothetical premisses suitable for proof, namely G1 to G5 from section 5 above. The – Peripatetic – criterion for syllogismhood given at *IL* 14.10 is that the leading premisses of hypothetical syllogisms must indicate conflict or consequence. The para-disjunctive premisses do not indicate either.[78] On the other hand, Galen calls the para-disjunctions 'hypothetical propositions' (at *IL* 6.7), and in chapter 15 calls para-disjunctive arguments syllogisms. Thus it is at least possible that he considered them to be hypothetical syllogisms. *IL* 15.11, read in conjunction with *IL* 15.10, could, but need not, be read as implying that Galen regarded them as useful for proof.[79] In any event, if the truth-conditions for para-disjunctions are truth-functional (as suggested in interpretation (a) in section 4.4), the para-disjunctive syllogisms do not satisfy the strict Peripatetic criteria

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77 And at *IL* 6.7, in the last line, this would be a natural emendation of the text.
78 See above, section 4.4.
79 In some of his other writings Galen himself appears to use para-disjunctive hypothetical syllogisms in order to prove things.

for proof which Alexander of Aphrodisias reports, and which Galen seems to endorse in chapter 14 of the *Institutio Logica*.

Note however that Galen, qua medical researcher, has good reasons for adding the para-disjunctive arguments to his arsenal of formally valid arguments. The *Peripatetic* program may have been to collect and systematize non-accidental relations between things (or at least those of scientific interest). Eternal classification schemes that permit us to put *things* in specific *classes* are looked for, the purpose being, broadly speaking, ontological. Galen, qua doctor, is looking for premisses useful for diagnostic, for which it can almost never be ruled out that there is, as it were, over-determination or co-causation of symptoms. One typical case of diagnostic is that in which a symptom can be caused by a variety of diseases. (In such cases, over-determination of the symptom is usually possible, even if actually rare.) By taking into account additional pieces of information, the number of diseases a specific patient may be suffering from can be reduced. The para-disjunctive syllogism is ideally suited for such cases.[80]

5.7 The first and second hypothetical syllogisms

Institutio Logica 14.10 suggests that Galen thought that there are hypothetical syllogisms that come to be from complete consequence and hypothetical syllogisms that come to be from incomplete consequence. At *IL* 14.11 we learn that there are two types of syllogisms from complete consequence; that Galen wants to call them first and second hypothetical syllogisms, in line with Chrysippus' numbering; and that there are two types of co-assumptions in syllogisms from complete consequence: affirmative ones and negative ones. This raises three questions: (1) what about hypothetical syllogisms that come to be from incomplete consequence? (2) why have the hypothetical syllogisms from complete consequence only two types of co-assumptions? (3) why does Galen never mention a distinction of two types of conditionals (a two-way and a one-way one, in my terminology of section 4.5 above), parallel with the two types of propositions that indicate complete and incomplete conflict?

(1) How this question should be answered is difficult to say because the text is corrupt at the crucial place. I believe that Galen, right after saying that syllogisms from complete consequence have two types of co-assumptions, added something about those from incomplete consequence. The manuscript

[80] For Galen's use of logic in medical science generally see Barnes (1988) and Barnes (1993).

has ‘and like complete consequence there is <also> incomplete consequence’.[81] This could indicate that there is no noteworthy difference for syllogistic between the two; alternatively, there could be a lacuna in the text after this clause, in which Galen mentioned how many and what co-assumptions there would be in the case of syllogisms from incomplete consequence. If either of these suggestions is correct, it is likely that Galen thought that there were also two types of syllogisms from incomplete consequence: those with an affirmative co-assumption, and those with a negative one. We can then also assume that the two types of syllogisms from incomplete consequence had the linguistic forms of G1 and G2 (section 5 above), and that their hypothetical premisses indicated incomplete consequence.

(2) Why have the hypothetical syllogisms from complete consequence only two types of co-assumptions? One might have expected Galen to introduce four, not two, types of syllogisms from complete consequence, corresponding roughly to:

(i)	(ii)	(iii)	(iv)
If p, q and if q, p	If q, p and if p, q	If not p, not q and if not q, not p	If not q, not p and if not p, not q
p	q	Not p	Not q
q	p	Not q	Not p

It is worth looking at Galen’s wording: he says that the syllogisms ‘from complete consequence and from complete conflict’ each have two co-assumptions, whereas that from incomplete consequence has only one, which is affirmative (*IL* 14.11, cf. *IL* 5.3, 5.5, 7.1). Thus Galen appears to assume that there can be *at most* two types of co-assumptions per hypothetical leading premiss,[82] viz. affirmative and negative ones. And that, derivative from this classification, there are at most two types of hypothetical syllogisms for any type of leading premiss. This leaves one wonder why he chose to classify types of hypothetical syllogisms *in this way*. Here again we fare best when we assume Galen took a ‘Peripatetic perspective’.

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[81] According to Kalbfleisch, the Greek MS has οὔσης δὲ καὶ ἀκολουθίας ὡς τελείας ἐλλιποῦς. I would suggest to add καὶ after τελείας, or to transpose the earlier καὶ to that place. In any case it would be very odd if Galen had said nothing about syllogisms from incomplete consequence after he had just announced that they exist at *IL* 14.10.

[82] In arguments with two simple (or unanalyzed) propositions in the leading premiss.

To see this, the comparison between complete conflict between two things and complete consequence is instructive. For, in the case of complete conflict, one might expect four types of syllogisms too, which could be expressed roughly thus:

(v)	(vi)	(vii)	(viii)
If p, not q	If q, not p	If not p, q	If not q, p
and if not p, q	and if not q, p	and if p, not q	and if q, not p
p	q	Not p	Not q
not q	not p	q	p

Of these Galen would take (v) and (vi) on the one hand, and (vii) and (viii) on the other, as belonging to the same type of syllogism, the first two having an affirmative, the second two a negative co-assumption. Similarly, I assume, in the case of complete consequence, Galen would take the above listed (i) and (ii) on the one hand, and (iii) and (iv) on the other as belonging to the same type of syllogism. But why?

First, each inference of types (i) to (viii) utilises only 'one half' of the complete consequence or conflict indicated by the first premisses. Second, if one regards only the half used, the inferences of the pairs (i) and (ii), etc., come out as structurally the same: (with the 'active' half in bold, I give as examples the pairs (i), (ii) and (vii), (viii))

(i)	(ii)	(vii)	(viii)
If p, q and if q, p	**If q, p** and if p, q	**If not p, q**	**If not q, p**
		and if p, not q	and if q, not p
p	q	Not p	Not q
q	p	q	p

We can understand why such structural identity provides sufficient reason for Galen for not distinguishing between the members of the pairs (i) and (ii), etc., if we recall the function of Peripatetic syllogistic. You have a thesis which you intend to establish (or refute), and some additional information in the form of propositions. Combining these, you look for a type of hypothetical premiss you can use for making the transition from your additional information (in the co-assumption) to your thesis (expressed in the conclusion). That is, you look what 'ontic' relation – if any – that would allow such a transition holds between the things, and then select a hypothetical premiss that indicates that relation. From this perspective of application, you have only two options in the cases at issue. Your thesis (say p) stands in complete consequence with q. You can then either establish p, by using the pattern shared by (i), (ii); or refute p, by using the pattern exemplified in (iii), (iv); there will be no difference between

(i) and (ii), or between (iii) and (iv), since, given that you start with your thesis p, it is already determined which 'half' of the complete consequence you can use, if any. And depending on whether your additional information provides you with q or with not q, you will be able either to establish or to refute your thesis. Thus we can explain why Galen says there are 'two co-assumptions', and hence two types of syllogisms, in the case of *complete* consequence, and *mutatis mutandis* in the case of complete conflict.

(3) It is now easy to answer the third question, why Galen never mentions a distinction of two types of conditionals corresponding to the distinction of the two types of hypothetical propositions that indicate complete and incomplete conflict. In the case of complete and incomplete conflict, there was a difference in the number of types of co-assumptions: those with a quasi-disjunction as leading premiss can only have affirmative co-assumptions. By contrast, in the case of leading premisses indicating complete or incomplete consequence, there is no difference in their syllogistic function. For the 'syllogistically active' parts are the same in both cases. What differs are the 'adequacy conditions' and the truth-conditions of the hypothetical premisses. In the case of complete consequence, to make the validity of the argument transparent discussants must ensure that they take the 'active half' as model for the formulation of the leading premiss, i.e.

If it is day, the sun is above the earth.
It is day.
Therefore the sun is above the earth.

rather than

If the sun is above the earth, it is day.
It is day.
Therefore the sun is above the earth.

Of course, it may also be that Galen never quite worked out the relation between Stoic conditionals, Peripatetic conditionals, and complete and incomplete consequence. In that case, the remarks in this section show that he could have done so in a coherent way.

6. Conclusion

Is the Peripatetic hypothetical syllogistic in Galen's *Institutio Logica* a propositional logic off the rails? The answer should have become evident: despite superficial similarities to systems of propositional logic, the Peripatetic theory is not a propositional logic at all. It lacks all the defining elements of a propositional logic. Not only are its hypothetical propositions not truth-functional,[83] and does it seem that *none* of the logical truths of classical propositional calculus hold in the Peripatetic system. More importantly, Galen's Peripatetics seem to have had no conception of propositional connectives as logical operators and the idea that uniform propositional substitution preserves validity appears to have been alien to them. However, we should not conclude that the Peripatetic hypothetical syllogistic in Galen is 'off the rails'. It is a very modest, but well-thought out theory of deduction, based on the idea that the key to formal validity of arguments is to capture knowledge introduction or knowledge extension rather than truth-preservation; as such it avoids the so-called paradoxes of implication and similar counterintuitive logical truths of the classical propositional calculus. And we should not forget that several eminent logicians of the 20th century have devised theories along the same general lines and on the same basic assumptions.

You may of course still regard as unusual the interpretation offered in this paper. Yet, all the points in which my interpretation of the Peripatetic hypothetical propositions and syllogisms differ from Stoic non-simple assertibles and Stoic syllogisms have been justified by resort to Aristotelian and early Peripatetic logic. These include, among others, the use of affirmations and negations as on a par and the absence of a negation operator (section 2); the understanding of conflict and consequence as fundamental 'ontic' relations, neither of which can be reduced to the other (section 3); the use of the quantificational approach of Aristotle's categorical syllogistic as a template for the interpretation of complex propositions (section 4); the understanding of affirmative and negative conjunctions as contraries rather than contradictories (section 4.1); the Platonist-Aristotelian concept of *diairesis* as underlying the understanding of disjunction and quasi-disjunction (sections 4.2 and 4.3); and the application of hypothetical syllogistic as based on tenets from Aristotle's *Topics* (section 5.3).

Moreover, the resulting overall interpretation of Peripatetic and Galenic hypothetical syllogistic made it possible to offer solutions to almost all of the many puzzles raised by the passages on complex propositions and hypothetical syllogisms in the *Institutio Logica*, a point which should speak in its favour. In particular, the following questions have obtained answers:

- Why the terminology in the passages on hypothetical propositions and syllogisms is not Stoic in most places where neither Chrysippus nor the Stoics in general are mentioned (*passim*).

[83] I do not consider truth-functionality as a necessary requirement for a propositional logic.

- Why, in those passages, Galen talks about hypothetical propositions and syllogisms rather than about non-simple assertibles, first, second, etc., indemonstrables, and just syllogisms, as the Stoics did (e.g. *IL* 3.1, 6.4, 7.2, 7.4, 14.2).
- Why 'when it is not day, it is night' counts as a disjunction (*IL* 3.5).
- Why a sentence that is a true disjunction cannot be a true quasi-disjunction (*IL* 4.2–3, 5.1).
- Why Galen need not be accused of confounding accounts of hypothetical propositions with their truth-conditions (*passim*).
- Why two constitutive propositions which stand in conflict or consequence to each other cannot form a conjunction (*IL* 4.4).
- Why none of the hypothetical propositions based on conflict or consequence are truth-functional.
- Why conjunctions are not hypothetical propositions (*IL* 14.3–9).
- Why there are no syllogisms with affirmative or negative conjunctions as hypothetical premisses.
- Why Galen did not include any of the following as hypothetical syllogisms (with $p_i \neq p_k \neq p_j$):

(*G8)	Either p_1 or p_2 or p_3 now p_j or p_k therefore not p_i	(*G9)	Either p_1 or p_2 or p_3 now not p_i therefore p_j or p_k

- Why there was a distinction between complete and incomplete consequence and between hypothetical syllogisms with complete and incomplete consequence (*IL* 14.10).
- Why, nonetheless, Galen did not treat separately hypothetical syllogisms with complete and hypothetical syllogisms with incomplete consequence as hypothetical premiss (*IL* 14.11). [84]

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