Crossing Pictures of 'Determination' in Wittgenstein's Remarks on Rule-Following

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Abstract
In PI 189 Wittgenstein's interlocutor asks, "But are the steps then *not* determined by the algebraic formula?". Wittgenstein responds, "The question contains a mistake". What is the mistake contained in the interlocutor's question? Wittgenstein's elaboration is neither explicit nor its intended upshot transparent. In this paper, I offer a reading on which the interlocutor's question arises from illicitly crossing different pictures of 'determination'. I begin by working through Wittgenstein's machine analogy in PI 193, which illustrates picture-crossing in our ways of talking about a machine. Using the lessons from this analogy, I show how the interlocutor's "mistake" can be diagnosed in similar terms: their confusion about the power of a rule to determine its applications rests on mistakenly crossing a behavioral and a mathematical sense of 'determine' – thereby concocting a mystifying picture of rule-following.

> When you get the picture of "being determined" out of your mind, then you get rid of the puzzle. – But still one can say the algebraic expression determines his actions – and perfectly correctly. But now you have got rid of the cramp.[1]

1. Introduction

In PI 188, Wittgenstein addresses a certain conception, adopted by an imagined interlocutor, of one's meaning something or other by a simple arithmetical rule such as '+2'.

> Here I'd like to say first of all: your idea was that this *meaning the order* had in its own way already taken all those steps: that in meaning it, your mind, as it were, flew ahead and took all the steps before you physically arrived at this or that one.[2]

This conception leads the interlocutor to describe the process of teaching and learning such a rule in a rather surprising way.

> So you were inclined to use such expressions as "The steps are *really* already taken, even before I take them in writing or in speech or in thought". And it seemed as if they were in some *unique* way

[1] Wittgenstein (1984: 24).
[2] Wittgenstein (2009: §188).

> predetermined, anticipated – in the way that only meaning something could anticipate reality.[3]

The interlocutor's expression is an instance of what Wittgenstein soon after calls being "seduced into using a super-expression. (It might be called a philosophical superlative.)"[4] Part of the "super-ness" of this expression comes through in the emphasis "*really*", i.e., it is not enough for the interlocutor to merely say that "The steps are already taken", but that instead "The steps are *really* already taken". This emphasis signifies that there is something powerful and mysterious happening when one gives the order to add 2 ("as if [the steps] were in some *unique* way predetermined"), by contrast with the mundane fact that someone or other has (of course, many people have) already counted through this series (though not *all* of it!) before the student goes on to do so.[5] Instead, it is assumed that *the teacher's mind* has taken *all* of these steps in advance once they understand '+2' and mean something specific by it in giving the order.

It is not difficult to see why such an idea would perplex the interlocutor, since if the mind *really* does take all these steps in advance, then our presumably finite minds are capable of containing infinite quantities and performing an infinite number of steps "in a flash", or as the interlocutor puts it, "It is as if we could grasp the whole use of the word [or rule] at a stroke".[6] The exotic nature of such an idea should give us pause. Where does this idea and its attending expressions come from? Why is the interlocutor tempted to use such expressions in describing something as mundane as teaching and learning the simple rule '+2'? According to Wittgenstein,

> [T]his mode of expression suggests itself to us [e.g., "It is as if we could grasp the whole use of the word at a stroke" or "The steps are *really* already taken"]. As a result of the crossing of different pictures.[7]

These expressions are thus not to be taken with much authority, but rather seen as the product of "crossing different pictures". The interlocutor has been, "*seduced* into using a super-expression".[8]

However, it might seem (to the interlocutor or anyone sympathetic with them) that Wittgenstein is here *denying* the very possibility of meaning something by a rule or of a rule's determining its applications.[9] This is why the interlocutor asks

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[3] Ibid.
[4] Wittgenstein (2009: §192).
[5] C.f., Wittgenstein (1983: §22).
[6] Wittgenstein (2009: §191).
[7] Ibid.
[8] Wittgenstein (2009: §192, my emphasis).
[9] Compare Kripke (or, rather, "Kripkenstein"): "There can be no such thing as meaning anything by any word. Each new application we make is a leap in the dark; any present intention could be interpreted so as to accord with anything we may choose to do. So there can be no accord, nor conflict. This is what Wittgenstein said in §201" (Kripke (1982: 55)). See also Fogelin (1987) and Wright (1980). The general philosophical problem that arises here (with varying degrees of connection to Wittgenstein's own remarks on it) has been a matter of continuing discussion. See especially Miller & Wright (2006) and Kusch (2006) for more recent discussions of the Kripkensteinian paradox.

him, after Wittgenstein begins to note the "expressions" that the interlocutor is (merely) "inclined to use": "But are the steps then *not* determined by the algebraic formula?".[10] "The formula *must* determine these steps!", thinks the interlocutor, since to say otherwise would imply that one could legitimately count in whatever way one likes when given the rule, or that there is nothing whatsoever to explain why folks go on as they do when they are given the order to follow it. Wittgenstein responds, however, that, "The [interlocutor's] question contains a mistake".[11]

What is the mistake "contained in the question"? Wittgenstein's intentions in PI 189 are not exactly transparent.[12] Rather than saying explicitly what is the "mistake" in the question, he instead transitions into a discussion of how one might use the expression "The steps are determined by the formula . . .". On the one hand, Wittgenstein suggests, it can be used to describe how people react or respond to an arithmetical formula such as $y = x^2$. On the other hand, it can be used as a way of categorizing different kinds of formulae, e.g., to differentiate formulae that 'determine' a value of y given x (e.g., $y = x^2$) from those that do not (e.g., $y \neq x^2$). But how exactly are these uses of the expression meant to illustrate the "mistake" contained in the interlocutor's question – especially if the interlocutor had an entirely different use of 'determine' in mind? (If that were so, Wittgenstein's elaboration would appear to be a complete non-sequitur.) Wittgenstein apparently left this matter as an exercise for the reader (c.f., "I should not like my writing to spare other people the trouble of thinking"[13]).

In the most careful reading of PI 189 currently available, Juliet Floyd[14] has proposed the following. The interlocutor presumes that they understand what they mean by 'determine' when they ask, "But are the steps then *not* determined by the algebraic formula?". In fact, there is no antecedently fixed notion of 'determination' (or of 'necessity' as it is used to describe the truths of logic or mathematics). When Wittgenstein lists some possible uses of 'determine', he is placing a burden on the interlocutor to explain, more precisely, what they have in mind. At the moment, however, this is not at all clear. Until the interlocutor can tell us more precisely what it means for a rule to 'determine' its applications, Wittgenstein cannot say one way or another whether this is something he would deny. This is disruptive of a tempting (and not uncommon) philosophical trajectory that assumes there is a unified notion of 'determination' or 'necessity' in mathematics that is amenable to philosophical theorizing. The "mistake" is thus, on Floyd's reading, that the interlocutor presumes they mean something specific by "The steps are determined by the formula …" when in fact they do not – which disrupts the tendency of philosophers to theorize mathematical or logical 'necessity' or 'determination' as if they had something clear in mind at the outset. They can select amongst one of the uses Wittgenstein has offered, or (if none of those will suffice) they should tell us more clearly what they have in mind.

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[10] Wittgenstein (2009: §189).
[11] Ibid.
[12] As noted by Floyd (1991: 151).
[13] Wittgenstein (2009: 4).
[14] Floyd (1991: 155-157).

This reading saves Wittgenstein's elaboration in PI 189 from being a non-sequitur, though it does make his case against the interlocutor rather indecisive. Such indecision does not, on the face of it, seem to match the tone we find in the surrounding remarks (or perhaps even of the accusation that the interlocutor's question contains a "mistake"). Given that in PI 188, Wittgenstein is criticizing the interlocutor's "idea" that, "meaning the order has in its own way already taken all those steps", it seems most natural to see Wittgenstein as committing his interlocutor to something *specific enough* to be labelled in the following ways: a "result of crossing pictures",[15] encouraging a "super-expression",[16] or a "philosophical superlative",[17] a "misunderstanding of the use of a word"[18] which inclines them to take it as signifying "an odd kind of process",[19] and, last but certainly not least, a "false interpretation" put on the talk of "civilized people" from which the interlocutor draws "the oddest conclusions".[20] Thus there is *some* idea here (or a thought, or a picture) that the interlocutor has in mind, however vague it may be; an idea that inspires them to seek out a deeper explanation of the allegedly mysterious phenomenon of following a rule.[21] Wittgenstein seeks to remove, undermine, or dissolve this idea by diagnosing it as a product of confusion, i.e., a result of "crossing different pictures".

This is not to suggest against Floyd's reading that the "idea" the interlocutor has in mind is clear enough, by Wittgenstein's lights, to justify initiating a systematic theory of mathematical 'determination' or 'necessity'. Floyd is completely right that Wittgenstein would deny this and it is entirely plausible that he would criticize a philosopher on such grounds. For instance, to go beyond Floyd's suggestion a bit, it is very natural in this context to invoke the Wittgensteinian staple that 'determine' (among most other concepts in ordinary language) is a "family-resemblance concept", not a rigidly unified category that adheres to strict rules and boundaries.[22] If that were so, then one is better off describing examples of 'determination' and noting likenesses (as well as differences) between them ("All *explanation* must disappear, and description alone must take its place"[23]; "Philosophy must not interfere in any way with the actual use of language, so it can in the end only describe it"[24]).[25] I won't deny, then, that the considerations Floyd raises are the *sorts*

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[15] Wittgenstein (2009: §191).
[16] Wittgenstein (2009: §192).
[17] Ibid.
[18] Wittgenstein (2009: §196).
[19] Ibid.
[20] Wittgenstein (2009: §194).
[21] Compare McDowell (1998: 223): "What this suggests is something we might anyway have expected: that Wittgenstein's target is not the very idea that a present state of understanding embodies commitments with respect to the future, but rather a certain seductive misconception of that idea."
[22] Wittgenstein (2009: §§66-7). As Floyd (2021: 51) puts it, ""Family resemblance" characterizes the generality of certain concepts. A single property, a fixed-for-all-cases criterion, an explicit set of grammatical rules – these are not required. A concept may hold together – like a family – with a variegated, evolving series of properties."
[23] Wittgenstein (2009: §109).
[24] Wittgenstein (2009: §124).
[25] Pears (1988: 218-19) likewise notes the fundamental importance of description for Wittgenstein's later conception of philosophy.

of criticisms Wittgenstein would have issued against philosophers of mind or mathematics, and I myself will appeal to some of these considerations in developing the reading that follows.

However, I will offer a reading of PI 189 on which the intended critique goes somewhat deeper than this. (So, without assuming Floyd would agree to the details I offer, it can be seen as a crucial supplement to her reading.) Although the interlocutor's idea is vague and unsystematic, it is surprising and exotic enough to invoke philosophical perplexity of the very sort that Wittgenstein would like to diagnose and remove ("The philosopher treats a question; like an illness"[26]). It is the sort of "picture" that holds one "captive" while they are doing philosophy.[27] To reveal the confusions inherent in such an idea is thus apt to diminish the interlocutor's motivation to theorize 'meaning' something or other by a rule – it removes the sort of wonder that Aristotle famously regarded as being the root of philosophy.[28]

The rest of the paper will proceed as follows. In Section 2, I will take a close look at what I call "the machine analogy" in PI 193, as this is meant to shed light on the interlocutor's earlier confusion about 'determination' in PI 189. In Section 3, I will use the machine analogy to explain how the interlocutor's "mistake" in PI 189 is a mistaken conception of 'determination' that results from crossing different pictures.[29]

2. The Machine Analogy in PI 193

Something that might seem to upset my reading of PI 189 as an instance of "crossing pictures" is that Wittgenstein does not mention this in the passage itself. As we noted earlier, he simply lists some uses of the expression "The steps are determined by the formula . . ." and then pithily mentions that, "it is not clear offhand what we are to make of the question, 'Is $y = x^2$ a formula which determines y for a given x?'". This seems to support Floyd's reading that Wittgenstein's only intention here is to show that the interlocutor does not have anything clear in mind, thus placing a burden on them to explain more precisely what they mean by 'determine' in this context.

An indication that there is something more to Wittgenstein's attribution of a "mistake" to the interlocutor comes from the fact that Wittgenstein's discussion of the word 'determine' does not end in PI 189, but is taken up explicitly in PI 193. In this later passage, Wittgenstein discusses, "A machine as a symbol of its mode of operation" and compares it with what we might call an "actual" (say, physical) machine. PI 193 is nestled into Wittgenstein's remarks on how a rule can

[26] Wittgenstein (2009: §255).
[27] Wittgenstein (2009: §115).
[28] Aristotle (2016: 982b).
[29] The importance of "crossing pictures" for the diagnosis of the interlocutor's mistake is noted briefly in Baker & Hacker's (2009: 103-110) exegesis. The reading offered in this paper can be seen as an elaboration and explanation (with some minor departures from Baker & Hacker) of how such a diagnosis is meant to work in these texts. See Bold (2022: 1-18, 67-71) for a detailed reading of "crossing pictures" in Wittgenstein's later therapeutic conception of philosophy.

'determine' its applications. It is thus highly unlikely that Wittgenstein presented his machine example for its own sake – the example is presumably an analogy that is meant to shed light on the interlocutor's perplexity about rule-following. The intended connection between PI 189 and PI 193 is also clearly indicated by the fact that immediately following Wittgenstein's discussion of the machine, his interlocutor responds with the following.

> "But I don't mean that what I do now (in grasping the whole use of a word) determines the future use *causally* and as a matter of experience, but that, in a *strange* way, the use itself is in some sense present."[30]

Further, given that PI 193 explores the idea that, "if we know the machine, everything else – that is the movements it will make – seem already completely determined", and that, "The machine seems already to contain its own mode of operation", Wittgenstein presumably intends us to compare this example with the interlocutor's earlier idea that, "The steps [following from the rule '+2'] are *really* already taken", as if they were, "in some *unique* way predetermined".[31] For these reasons, I will refer to the example in PI 193 as "the machine analogy", i.e., an analogy that is meant to shed light on the "mistake" contained in the interlocutor's question of PI 189. In this section I will focus on the machine analogy. In the next section I will show how this indeed sheds light on the interlocutor's earlier mistake alluded to in PI 189.

First, let's get clear on what the example in the machine analogy is supposed to be an example of. Wittgenstein opens the passage as follows.

> A machine as a symbol of its mode of operation. The machine, I might say for a start, seems already to contain its own mode of operation. What does it mean? – If we know the machine, everything else – that is the movements it will make – seem to be already completely determined.[32]

Wittgenstein has us consider "a machine as a symbol of its mode of operation", or, put another way, a machine as a symbol of the way it operates. What does Wittgenstein mean by this? Wittgenstein elaborates somewhat in the next paragraph.

> We use a machine, or a picture of a machine, as a symbol of its mode of operation. For instance, we give someone such a picture and assume that he will derive the successive movements of the parts from it. (Just as we can give someone a number by telling him that it is the twenty-fifth in the series 1, 4, 9, 16, . . .).[33]

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[30] Wittgenstein (2009: §195).
[31] Wittgenstein (2009: §188).
[32] Wittgenstein (2009: §193).
[33] Ibid.

So, the kind of picture Wittgenstein has in mind (at least in one instance) is something like a machine blueprint or diagram. This is the sort of picture that a teacher could give an engineering student on a test and ask them to explain its successive movements from this initial state given basic kinematic principles. It is, as Wittgenstein notes, akin to giving, "someone a number by telling him that it is the twenty-fifth in the series 1, 4, 9, 16 …", thereby likening their use of the "machine as symbol" to a simple arithmetical exercise. (One should immediately compare this detail with PI 189).

But Wittgenstein says that this could be either "a picture of a machine" or "a machine" (e.g., one made of steel and not mere scribbles on paper), the intention being that this would have to be a machine *taken in a certain way*, i.e., "as a symbol of its mode of operation". For instance, the same teacher could give their engineering student the following exercise: Open the machine on your desk, examine its parts without turning its crank, and explain what the successive movements of its parts will be given the structure of its initial state – you can assume for the exercise that the parts in the machine are perfectly rigid. As the machine is used in this exercise, this is likewise an instance of "a machine as its mode of operation", i.e., a machine taken as a symbol of how it will operate, where this can be inferred (as it were *a priori*) from what we might call its initial state: "a machine, or a picture of it, is the first in a series of pictures which we have learnt to derive from this one".[34]

"A machine as a symbol of its mode of operation" is contrasted in this section with what Wittgenstein refers to as "an actual machine": "[I]t may now look as if the way it moves must be contained in the machine *qua* symbol still more determinately than in the actual machine".[35] In the machine *qua* symbol, we were able to derive its successive movements simply by looking at the structure of its initial state (the sort of thing that can be represented in a blueprint or diagram). The "actual machine", however, is subject to, "the possibility of [its parts] bending, breaking off, melting, and so on".[36] In the case of the actual machine, "we do not in general forget the possibility of a distortion of the parts and so on", i.e., "when this is a matter of predicting the actual behavior of a machine".[37]

For instance, the engineering student might have a blueprint from which (as we saw earlier) they can derive successive movements according to basic kinematics. Their teacher might then provide a physical machine and show them that its parts correspond with the blueprint, asking them whether *this* machine will have the same successive movements as in our earlier derivations. The student hastily answers, "Yes!", and they fail the test. Why? Because they didn't check to see what the parts are made of. As it turns out, one of the gears is made out of soft clay and thus when the crank is turned, that gear is crushed and the entire machine collapses. Given the mismatch between the successive movements of the blueprint and the physical machine, the teacher explains, "We assumed in our earlier derivations that the parts of the machine were perfectly rigid – the parts in *this machine* are not." In

[34] Ibid.
[35] Ibid.
[36] Ibid.
[37] Ibid.

Wittgenstein's less familiar way of putting things (though making essentially the same point), the student has failed to distinguish the machine *qua* symbol of its mode of operation from the physical machine, which can bend, break, melt, and so on.

With a clearer sense of this distinction to hand, it is easier to see the intended upshot of the passage. The idea that the machine, "seems already to contain its own mode of operation", soon leads to confusion.

> We talk as if these parts could only move in this way, as if they could not do anything else. Is this how it is? Do we forget the possibility of their bending, breaking off, melting, and so on?[38]

In other words, we have the beginnings of a paradox on our hands: the movements seem to be completely determined (given its initial state), and yet, at the same time, we know that the machine can bend or break. So, the machine is completely determined and yet it is also *not* completely determined! But this rests on crossing two importantly different uses of "the machine" and thus different senses in which its successive movements might be 'determined'. Indeed, there is a use of "the machine" where, "in many cases, we don't think of [the machine bending or breaking] at all".[39] This is "the machine as its mode of operation". It is the use of "the machine" in this sense (*qua* symbol) that encourages a seemingly odd expression.

> "The machine seems already to contain its own mode of operation" means: we are inclined to compare the future movements of the machine in their definiteness to objects which have been lying in a drawer and which we now take out.[40]

However, Wittgenstein chimes in to say that this odd way of putting things is not encouraged when we consider "the machine" in the sense relevant to "predicting the actual behavior of a machine".[41] The idea that, "the future movements of the machine are akin to objects which have been lying in a drawer and which we now take out", might nonetheless be encouraged when we consider the machine as a symbol of its mode of operation.

> We *do* talk like that, however, when we are wondering at the way we can use a machine as a symbol of some way of moving – since [the actual, physical machine], can, after all, also move quite differently.
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> We might also say that a machine, or a picture of it, is the first of a series of pictures which we have learnt to derive from this one.[42]

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[38] Ibid.
[39] Ibid.
[40] Ibid.
[41] Ibid.
[42] Ibid.

So, the machine *qua* symbol encourages a picture on which its future movements are like objects lying in a drawer waiting to be pulled out. But Wittgenstein insists that it is not at all fitting to consider a physical machine in these terms, "since it can, after all, also move quite differently". To say that its movements were already sitting in a drawer, waiting to be pulled out, would be to ignore this possibility.

This clarification, on its own, however, does not resolve the confusion at hand, since it only leads us to wonder at how the machine *qua* symbol can involve a yet more powerful kind of determination than we might find in the actual physical machine.

> But when we reflect that the [actual, physical] machine could have moved differently, it may now look as if the way it moves must be contained in the machine *qua* symbol still more determinately than in the actual machine. As if it were not enough for the movements in question to be empirically predetermined, but they had to be really – in a mysterious sense – already *present*.[43]

Thus, the distinction between 'determination' as we find it in the machine *qua* symbol by contrast with the mere empirical predetermination of the movements in the actual, physical machine leads us to wonder at a kind of 'super-determination' involved in the machine as symbol. How can the machine *qua* symbol determine its future movements in this way – as if its movements were contained in it at the outset – as if its movements were like objects sitting in a drawer waiting to be taken out? Wittgenstein's sober response brings us back down to earth.

> And it is quite true: the movement of the machine *qua* symbol is predetermined in a different way from how the movement of any given actual machine is.[44]

There is nothing exotic in this difference, since it is easily seen in the contrasting uses of "the machine *qua* symbol" and "the physical machine". With respect to the machine *qua* symbol, the student can crank out its successive movements with the assumption of perfect rigidity, and thus can completely disregard how the make-up of its parts might lead to complications. Their results are derived from basic kinematics and inferred in a way akin to mathematical rules (e.g., calculating the successive steps in the pattern 1, 4, 9, 16, and so on). As Wittgenstein puts it in the RFM,

> "If the parts were perfectly rigid this is how they would move", is that [an empirical] hypothesis? It seems not. For when we say: "Kinematics describes the movements of the mechanism on the assumption that its parts are perfectly rigid", on the one hand we are admitting that this assumption never squares with reality, and on the other hand it is not supposed to be in any way doubtful that

[43] Ibid.
[44] Ibid.

> completely rigid parts would move in this way. But whence this certainty? The question here is not really one of certainty *but of something stipulated by us*.[45]

By contrast, the physical machine is understood to depend crucially on its physical make-up and other (physical) conditions surrounding its operation. The future movements of an actual, physical machine are thus not stipulated or a matter of convention. The distinction between the machine *qua* symbol and the actual, physical machine nonetheless allows for useful comparisons between them, not to mention predictions made on the basis of the machine *qua* symbol albeit with caution and perhaps some relevant qualifiers about its probability.

PI 193 thus illustrates the resolution of at least three important confusions, each of which involves "crossing different pictures" of "the machine" as well as different senses of 'determine' associated with them.

First, there is a relevant distinction between the machine *qua* symbol and an actual physical machine. The successive movements of the former are determined as a matter of mathematical convention or stipulation,[46] whereas the successive movements of the latter depend on complicated physical conditions which allow for the possibility of bending, breaking, and so on. This resolves completely the apparent paradox of how the machine's movements can *only* move this or that way (i.e., in the sense of the machine *qua* symbol) while also being subject to bending or breaking (i.e., in the sense of an actual, physical machine).

Second, given that an actual, physical machine is subject to the possibility of bending or breaking, it is misleading at best to consider the successive movements of the actual machine as akin to objects, "lying in a drawer waiting to be taken out". Physical conditions are complicated and various things might happen – they are not merely "contained" in the initial state. To think otherwise would be to impose a natural picture of the machine *qua* symbol onto the actual, physical machine, which does not simply "contain" its future movements.

Thirdly, and lastly, any resulting perplexity about the mysterious way in which a machine *qua* symbol determines its movements can be resolved by noting the obvious differences in use between 'determine' as it pertains to "the machine *qua* symbol" and "the actual machine". The former is 'determined' as a matter of mathematical convention or stipulation, the latter is 'determined' as a matter of complicated physical conditions. Ignoring this distinction might incline one to (mistakenly) think that the machine *qua* symbol is a machine made from "material harder and more rigid than any other"[47] that obeys quasi-physical principles – principles somehow akin to those of the physical machine, but lying in some outer and more perfect realm, involving perfectly rigid machines with their successive movements timelessly awaiting discovery (Plato's heaven for machines and engineering, as it were). Crossing the disparate uses and pictures of "the machine" thus risks seducing one into super-expressions or philosophical superlatives. Teasing

[45] Wittgenstein (1983: I, §120).
[46] Ibid.
[47] Wittgenstein (1983: I, §119).

these pictures apart allows us to see this as misbegotten, thereby undermining their seductive quality.

3. Crossing Pictures of 'Determination' in PI 189: The Interlocutor's "Mistake"

Let's return now to PI 189. Wittgenstein's machine analogy – which illustrates confusions that might arise from crossing different senses of 'determine' in our talk about "a machine" – is, on my reading, intended to shed light on the "mistake" contained in the interlocutor's earlier question, "But are the steps then *not* determined by the algebraic formula?". As we saw earlier, the interlocutor's idea that "…the steps are really already taken" is diagnosed by Wittgenstein as a super-expression that results from crossing different pictures. PI 189, however, only provides a list of different uses of 'determine' as it might apply to an algebraic formula, thus initially making it unclear what the mistake is supposed to be and how this might involve "the crossing of different pictures". But with the help of the machine analogy, it is not difficult to see how this might go.

Generally put, the two major uses of 'determine' as it applies to an algebraic formula are being crossed, thus resulting in a misbegotten picture of a formula's determining its successive steps. Just as there is a distinction between two uses of 'determine' in the machine analogy, there is also such a distinction relative to an "algebraic formula": what we might call a "behavioral" and a "mathematical" use of 'determine'.[48] The former "behavioral" sense pertains to the behavior of some people, i.e., how they in fact respond to a specified formula, say, uttered by a teacher or written with chalk. The latter "mathematical" sense pertains to basic conventions regarding the classification of different formulae, i.e., those that determine a value of y given x by contrast with those that do not. The "behavioral" sense is akin to our talk about the "behavior" of an actual physical machine, whereas the "mathematical" sense is akin to our talk about the machine *qua* symbol of its mode of operation. Following the lessons from the machine analogy, the interlocutor's confusions arise from crossing the behavioral and mathematical senses of 'determine', just as confusion might arise from crossing the machine as a mode of its operation with the actual, physical machine. In this section I will explain how this is so.[49]

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[48] Compare Baker & Hacker (2009: 103)'s exegesis according to which Wittgenstein was concerned with the crossing of "empirical" and "grammatical" senses of 'determine': "An example of such 'crossing of pictures' was in effect given in §189, where the interlocutor's misconception of 'determining the steps to be taken' derived from crossing the empirical sense of 'determines' with the grammatical sense."

[49] A discussion of how these considerations might bear on "Kripkenstein's" famous skeptical argument would require a separate article – and other articles still for the many other questions and problems that have been raised in the wake of Kripke's massively influential book (see Miller & Wright (2002) for a useful survey). But one immediate connection to Kripke's presentation of the so-called "skeptical paradox" relates to the following crucial premise of that argument: "An answer to the sceptic must satisfy two conditions. First, it must give an account of what fact it is (about my mental state) that constitutes my meaning plus, not quus. But further, there is a condition that any

After mentioning the interlocutor's question and claiming that it contains a mistake, Wittgenstein proceeds to examine some uses of the expression "The steps are determined by the formula".

> We use the expression "The steps are determined by the formula . . .". *How* is the expression used? – We may perhaps mention that people are brought by their education (training) so to use the formula $y = x^2$, that they all work out the same number for y when they substitute the same number for x. Or we may say: "These people are so trained that they all take the same step at the same point when they receive the order '+3'." We might express this by saying "For these people the order '+3' completely determines every step from one number to the next". (By contrast with other people who do not know what they are to do on receiving this order, or who react to it with perfect certainty, but each one in a different way.)[50]

As Wittgenstein puts it in the LFM, the general sense of 'determine' in this paragraph (even with its slight variations) is a matter of the "description of the behavior of people"[51] and serves as an answer to the question, "Do most people act in the same way in this connection?".[52] It is not an issue for this sense of 'determine' that people can and do sometimes count differently, skip a step, make what they would call a "miscalculation", and so on. These are exceptions that, as we might say, prove the rule – as shown in the various behaviors people engage in to "correct" such deviations from the general practice. It is also possible in this sense of 'determine' that, despite the fact that this is how all or most people respond to '+3', someone *might* still misinterpret the rule in a variety of ways that would be corrected,

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putative candidate for such a fact must satisfy. It must, in some sense, show how I am justified in giving the answer '125' to '68 + 57'. The 'directions' mentioned in the previous paragraph, that *determine* [my emphasis] what I should do in each instance, must somehow be 'contained' in any candidate for the fact as to what I meant" (Kripke (1982: 11)). So, "Kripkenstein" requires a "fact" that *both* determines "what one means" and why one is "justified" in going on as they do. For Wittgenstein, "what one means" is found in the general use of an expression, i.e., the behavior of some people (including how they "check" or "correct" each other's responses, defer to one another in cases of uncertainty, etc.). Matters of "justification" or "giving reasons" for one's answer to a mathematical problem are part of the language-game of mathematics. Hence, assuming the reading offered in this paper is correct, Wittgenstein would have likely accused Kripkenstein of crossing pictures of 'determination': i.e., a behavioral ("what one means") with a mathematical ("justification") sense of 'determine'. Crossing these senses of 'determine' is what allows the "paradox" to get off the ground. The reason is that whatever fact one cites to explain their behavior does not seem adequate as a mathematical justification ('68 + 57 = 125' is not true *because* I was trained to say so (c.f., Wittgenstein (2009: §241)); and whatever fact one cites to mathematically justify the answer given does not seem adequate for determining – with mathematical necessity, as it were – their behavior (since it is logically possible that they might see or hear the mathematical 'directions' Kripke refers to above, but nonetheless go on differently). Undermining this crucial premise in the skeptical argument (viz., "An answer to the sceptic must satisfy two conditions", etc.) would thus dissolve the famous Kripkensteinian paradox, at least as it is formulated by Kripke in the quote above.

[50] Wittgenstein (2009: §189).

[51] Wittgenstein (1976: 29).

[52] Wittgenstein (1976: 28).

e.g., by an ordinary school teacher. This use of 'determine' is thus akin to the "actual, physical machine" discussed in PI 193, which might bend or break despite the fact that, generally speaking, it won't do so when it is properly set up. Human beings in their use of "the formula" might also "bend or break" when given an algebraic formula and told to calculate a series of steps from it. This generally doesn't happen, i.e., people generally do not disagree in their algebraic calculations; if they do, they re-calculate until they get the same result (or simply assume that one or the other made an error). This is a simple empirical matter of how people contingently behave, i.e., how they respond to a formula provided in writing or speech. We might just as easily call these matters of "anthropological fact".[53]

Wittgenstein then proceeds to describe a different family of uses of the expression, "The steps are determined by the formula . . .".

> On the other hand, we may contrast different kinds of formula, and the different kinds of use (different kinds of training) appropriate to them. Then we *call* formulae of a particular kind (with the appropriate method of use) "formulae which determine a number *y* for a given value of *x*", and formulae of another kind, ones which "do not determine the number *y* for a given value of *x*." ($y = x^2$ would be of the first kind, $y \neq x^2$ of the second.)[54]

These uses of 'determine' pertain to the formulae themselves and are a matter of mathematical stipulation or convention.[55] They have nothing to do, *per se*, with the behavior of people as in the sense of 'determine' discussed in the previous paragraph. To the extent that they are relevant to the behavior of people, this is a matter of which kinds of training are "*appropriate*", i.e., consistent with pre-established mathematical conventions. In this "mathematical" sense of determine, different formulae are categorized either as determining a value of *y* given *x* or as not. More directly, as Wittgenstein puts it in the LFM, this is a matter of "a description of the formula" and serves as an answer to the question, "Is it a formula of this kind or that?".[56] These categorizations, by contrast with a description of the behavior of people, are not ones that, as it were, allow for "bending or breaking". $y = x^2$ does not determine *y* given a value for *x* on Tuesday, but not on Wednesday.

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[53] As Goldfarb (1985: 486) highlights, the alleged inadequacy of such empirical details is a crucial element of the interlocutor's perplexity: "We give a rule, some examples of its application, and perhaps some further explanations. Yet, for all that, a person 'could' go on in different ways and take himself to be going on the same. This seems to indicate that what we give is insufficient to tell, or to justify, how to go on; and we demand something more. The demand is not for that which in fact succeeds in showing a person, in particular circumstances, how to go on. It is rather for that which picks out the correct continuation in some unconditioned way, by giving that in which the same really consists." See also Maddy (2014: 72).

[54] Ibid.

[55] Wittgenstein (1976: 29) notes a similarity between these uses of 'determine' and a related distinction that might apply to different senses in which 'pointing' can determine how one goes (i.e., how people happen to behave vs. a convention of distinguishing different kinds of pointing).

[56] Wittgenstein (1976: 29).

It does not determine *y* given *x* for Paul, but not for Suzy. (If it does, then we might say that they were actually talking about different formulae.[57])

The sense of 'determine' as it applies to the categorization of mathematical formulae is thus akin to "the machine as a symbol of its mode of operation". Just as the steps of the machine qua symbol can be derived without attention to the steps taken by an actual, physical machine, the steps of the formula and its classification as either determining a value for *y* or not can be derived without attention to how Paul or Suzy happens to apply or categorize it (i.e., unless we are taking them as perfectly demonstrating the convention). These are matters of "mathematics", and, as Wittgenstein often puts it in other writings, they are "non-temporal" and thus not subject to change.[58] By contrast, descriptions about the behavior of people are "temporal", regarding how people behave at some time or other, and are subject to the possibility of variation, e.g., with the obvious possibility of miscalculation and even the possibility of alternative mathematical practices in other cultures.

So far, then, we have a distinction between at least two different uses of 'determine': one use that pertains to the behavior of people, and another use that pertains to the classification of different formulae and which takes place in the context of doing or talking about mathematics. Now we have to consider how crossing these different uses might yield the interlocutor's confusion.

Recall that in PI 188, the interlocutor's idea was that,

> this *meaning the order* had in its own way already taken all those steps: that in meaning it, your mind, as it were, flew ahead and took all the steps before you physically arrived at this or that one.[59]

This idea inclined them to, "use such expressions as 'The steps are *really* already taken, even before I take them in writing or in speech or in thought'".[60] According to this idea, it seemed that the steps, "were in some *unique* way pre-determined, anticipated – in the way that only meaning something could anticipate reality".[61] Thus the interlocutor has a suspicious picture of 'determination': one on which a formula determining its steps seems to require that the steps are *really* already taken by one's mind when they give the order to add 2.

The interlocutor's picture is completely analogous to the super-expression about the machine according to which, "the machine seems already to contain its own mode of operation", which means, "we are inclined to compare the future movements of the machine in their definiteness to objects which have been lying in a drawer and which we now take out".[62] This confusion about the machine resulted from crossing pictures of the "machine qua symbol" and the "actual, physical machine". The interlocutor's confusion at PI 188 is likewise a result of crossing different pictures, this time regarding "the formula" and the different senses in which it can 'determine its steps' – as articulated in PI 189. There is the behavioral

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[57] C.f., Wittgenstein (2009: §190).
[58] See especially Wittgenstein (1983: I, §23, §27, §§101-103; VI, §2).
[59] Wittgenstein (2009: §188).
[60] Ibid.
[61] Ibid.
[62] Wittgenstein (2009: §193).

sense according to which the teacher writes the formula on the chalkboard, and thereby 'determines' the steps taken in that those steps are then written uniformly by his students in response. Alternatively, if (some or all of) the students struggle to give the desired response, the teacher might write down the series for them on the chalkboard, erase it, and then request that they do the same from memory.[63] The picture of "the steps *really* already being taken" is inappropriate in this context, since it is clear that human beings are imperfect and the situation could go a variety of different ways (just as "the movements being contained in the machine" was inappropriate when talking about the actual, physical machine, which can bend or break). To think that the possibility of non-conforming behavior is incompatible with "the formula's determining its steps" would be to cross the behavioral sense of 'determine' with the non-temporal and mathematical sense of 'determine', which does not admit of exceptions or variations. This is a matter of mathematical stipulation and takes place in the language-games of doing and discussing mathematics – it does not have anything to do *per se* with the sense in which the formula might determine a student's behavior as a result of their training in a classroom, or the teacher's behavior as a result of their past training. We can imagine the interlocutor insisting, "'+2' determines its steps without any possibility of variation. So it must be true that the steps are *really* already taken in the teacher's mind!". The response from Wittgenstein would be that this picture might be encouraged by the mathematical sense of determine, but it is not at all appropriate in the behavioral sense.

Alas, this might leave us with a residual worry about the mathematical sense of 'determine'. As we saw in the machine analogy, even when one grants that an actual, physical machine might bend and break, and thus that the picture of its steps being contained in advance is inappropriate, one might then be mystified by the sense in which a machine as symbol can determine its steps in advance, "it may now look as if the way it moves must be contained in the machine *qua* symbol still more determinately than in the actual machine".[64] An analogous confusion might undoubtedly result for the interlocutor regarding the 'mathematical' sense of determine, which seems to contain its steps "still more determinately" than in the teacher's and the student's responses to the formula, say, as it is written on the blackboard. This confusion pertains not to a perplexity about how human beings respond to formulae in the ways that we do, but how the *must* of logic or mathematics is so much as possible.[65] Wittgenstein's answer here is that this is a matter of convention or stipulation – thus there is no special mystery regarding the distinctive way in which the steps might be mathematically pre-determined.

However, if one projects the behavioral sense of 'determine' onto the mathematical, then it can appear as if what the teacher does on the chalkboard is somehow performed timelessly in mathematical reality. Whereas the teacher can

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[63] C.f., Wittgenstein (1983: I, §22).

[64] Wittgenstein (2009: §193).

[65] This topic is explored in detail by Maddy (2014). Pears (2006: 65) puts what I'm calling the residual worry in the following way: "[T]he application of any general word might well have been different from what it now is, and it is often easy to imagine circumstances in which it really would have been different. […] But logic [and mathematics] seems to be made of harder stuff."

write out the series 2, 4, 6, 8, 10, and so on with chalk and subsequently erase it, the series "written out", as it were, in abstract mathematical reality persists timelessly and cannot be erased. (And now we really are talking about Plato's heaven in the traditional sense). This is, again, a crossing of different pictures of 'determine'. Such a crossing of the behavioral into the mathematical, resulting in an ethereal picture of mathematical reality, is noted explicitly in the RFM. Wittgenstein first describes pictures on which logical and mathematical applications are somehow "already completed" before any one of us performs them.

> In his fundamental law Russell seems to be saying of a proposition: "It already follows—all I still have to do is, to infer it". Thus Frege somewhere says that the straight line which connects any two points is really already there before we draw it; and it is the same when we say that the transitions, say in the series +2, have really already been made before we make them orally or in writing—as it were tracing them.[66]

Wittgenstein's response is that someone seduced by these expressions (such as Russell or Frege perhaps) is illicitly projecting a picture of behavior onto logic and mathematics.

> One might reply to someone who said this: Here you are using a picture. One *can determine* [in the behavioral sense] the transitions which someone is to make in a series, by doing them for him first. E.g. by writing down in another notation the series which he is to write, so that all that remains for him to do is to translate it; or by actually writing it down very faint, and he has to trace it. In the first case we can also say that we don't write down *the* series that he has to write, and so that we do not ourselves make the transitions of that series; but in the second case we shall certainly say that the series which he is to write *is already there* [my emphasis]. We should also say this if we *dictate* what he has to write down, although then we are producing a series of sounds and he a series of written signs. It is at any rate a sure way of *determining* [albeit in the behavioral sense] the transitions that someone has to make, if we in some sense make them first.[67]

Thus when one projects the behavioral sense of 'determine' onto the mathematical sense, this suggests a picture of the mathematical realm where there is a series somehow akin to that written on the chalkboard by the teacher, but different in that the objects of the series are timeless, eternal, unchanging, and awaiting discovery by mathematicians in some special realm.

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[66] Wittgenstein (1983: I, §21).
[67] Wittgenstein (1983: I, §22).

> Here what is before our minds in a vague way is that this reality is something very abstract, very general and very rigid. Logic is a kind of ultra-physics, the description of the 'logical structure' of the world, which we perceive through a kind of ultra-experience (with the understanding e.g.).[68]

A seductive picture, indeed, as the history of the philosophy of mathematics has shown. Wittgenstein's method aims to dismantle it by revealing one of its sources, namely, an illicit crossing of two disparate uses of 'determine'.[69]

The interlocutor tellingly infers from the machine analogy in PI 193 that Wittgenstein is claiming that the 'determination' involved in one's understanding of the formula is merely "causal" or "a matter of experience" (akin to the 'determination' involved in the actual, physical machine).

> "But I don't mean that what I do now (in grasping the whole use of a word) determines the future use *causally* and as a matter of experience, but that, in a *strange* way, the use itself is in some sense present."[70]

The interlocutor insists to the contrary that on *his* idea of determination, the use is determined "in a strange way", such that it makes sense to say "the steps are already taken". Wittgenstein's response makes clear, yet again, that the interlocutor's idea relies on an illicit crossing of pictures.

> —But of course it is, 'in *some* sense'! Really, the only thing wrong with what you say is the expression "in an odd way". The rest is right; and the sentence seems odd only when one imagines it to belong to a different language-game from the one in which we actually use it. (Someone once told me that as a child he had been amazed that a tailor could 'sew a dress' – he thought this meant that a dress was produced by sewing alone, by sewing one thread onto another.)[71]

Just as the child alluded to has concocted a mystifying picture of 'sewing a dress' by confusing two different senses of this expression (i.e., the sense in which it can be done from scratch and the sense in which it is done "by sewing one thread onto another"), likewise the interlocutor has concocted on "odd" picture of 'determination' by crossing disparate uses of the expression. Wittgenstein has already provided at least two different senses in which the formula can indeed determine its successive steps: a behavioral sense and a mathematical sense. Neither of these is odd. The behavioral sense of 'determination' is illustrated quite easily via

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[68] Wittgenstein (1983: I, §8).
[69] See Bold (2022) for various treatments of this and related pictures of mathematical reality in Wittgenstein's later philosophy.
[70] Wittgenstein (2009: §195). C.f., Wittgenstein (1970: §296).
[71] Ibid.

our earlier descriptions of an ordinary classroom setting. The mathematical sense of 'determination' takes place within the language-game of doing or discussing mathematics, a fairly banal distinction between different types of algebraic formulae. The interlocutor apparently takes issue with having his idea of 'determination' lumped into the behavioral sense (as this makes it out to be merely "causal" or "a matter of experience"). Wittgenstein's diagnosis is that the interlocutor is imagining the expression "the formula determines its steps" to take place in a language-game different from any of those in which it is actually used, namely, a language-game in which a formula 'determines' its steps both in the behavioral and mathematical senses. This is at best puzzling and at worst paradoxical: behavior takes place in time, and yet mathematics is timeless, implying a "strange" sense of 'determination' on which my grasping the use *in time* involves the *timeless* performance of all the steps in an infinite series.[72] This leads to a major conclusion about the interlocutor in PI 196.

> In misunderstanding the use of the word, one takes it to signify an odd *process*. (As one thinks of time as a strange medium, of the mind as an odd kind of being.)[73]

Thus we have a fairly decisive diagnosis of the interlocutor's misbegotten idea. Their idea – on which the steps are *really* already taken in one's understanding of the rule and thus 'determined' in a strange sense – is the result of crossing different uses of 'determine'. Crossing these uses leads to a fantastic picture of an individual's mental life, one which is counteracted via an ordinary description of people's behavior when they specify and respond to a mathematical formula. It also leads to a fantastic picture of the mathematical realm, in which something like our "counting" takes place perfectly, timelessly, completely, and so on – a picture that is counteracted by the reminder that, within mathematics, the series following from '+2' is a matter of stipulation or convention. The wonder that might have inspired the interlocutor to further theorizing or speculation is thereby diminished.

When Wittgenstein begins to diagnose the interlocutor's idea in PI 188 by noting the expressions they are merely inclined to use, the interlocutor takes this to mean that Wittgenstein is denying that a formula can determine its steps – hence his question that opens PI 189. The mistake in the interlocutor's question is a mistaken conception of 'determination', one which leads to perplexity and paradox. The source of this mistaken idea is, according to Wittgenstein, an illicit crossing of different uses of the word 'determine', which are laid out in PI 189. The crossing of these uses leads to their super-expressions about following a simple mathematical rule. Without this mistaken conception of 'determine', the interlocutor's question dissolves – along with the downstream puzzles and paradoxes their idea might have

[72] Wittgenstein (2009: §138).
[73] Wittgenstein (2009: §196).

encouraged ("Problems are solved (difficulties eliminated), not a single problem"[74]).[75]

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