# INTENSITY OF COMPETITION AND THE CHOICE BETWEEN PRODUCT AND PROCESS INNOVATION * 

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#### Abstract

Two questions are examined within a model of vertical differentiation. The first is whether cost-reducing innovations are more likely to be observed in regimes of more intense or less intense competition. Following Delbono and Denicolo (1990) and Bester and Petrakis (1993) we compare two identical industries that differ only in the regime of competition: Bertrand versus Cournot. Since Cournot competition leads to lower output and higher prices, it can be thought of as a regime of less intense competition. We find that the increase in profits associated with any given cost reduction is higher in the case of Cournot competition than in the case of Bertrand competition. Thus there are cost-reducing innovations that would be pursued under Cournot competition but not under Bertrand competition.

The second question, which so far has not been analyzed in the literature, is what factors might be important in a firm's decision whether to invest in product innovation (improvement in the quality of its product) or process innovation (cost reduction). We show that the regime of competition might be one such factor. For the high quality firm our result is that if there is a difference between the choice made by a Bertrand competitor and the choice made by a Cournot competitor, then the former will opt for product innovation, while the latter will prefer process innovation. For the low-quality firm, on the other hand, the result is reversed: whenever there is a difference, the Bertrand competitor will favor process innovation, while the Cournot competitor will favor product innovation.


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## 1. Introduction

There is a vast literature on the economic aspects of innovation. A wide spectrum of issues has been analyzed, from the timing of innovative ventures, to expenditure patterns in R\&D races, to spillover effects and their impact (for an excellent survey of the latter see De Bondt, 1995). The issue we address in this paper is the relationship between intensity of competition and the profitability of innovative activity. A traditional line of reasoning, associated with Schumpeter (1943), is that market concentration is a stimulus to innovation. An early challenge to this view came from Arrow (1962), who sought to establish the reverse proposition that more competitive environments would give a greater incentive to innovate. Arrow considered the case of a firm undertaking a costreducing investment that cannot be imitated by competitors. He compared a monopoly with a perfectly competitive industry, under the same demand and cost conditions, and showed that the gain from a cost-reducing innovation is higher for a firm in the latter than for the monopolist. A more interesting comparison would be between two oligopolistic industries. It is not clear, however, how "intensity of competition" can be measured in such a setting. Delbono and Denicolo (1990) and Bester and Petrakis (1993) suggested comparing two industries (with the same number of firms and the same linear demand and cost functions) under different regimes of competition: Cournot (where firms' decision variables are output levels) and Bertrand (where firms' decision variables are prices). Since Cournot competition normally leads to lower output and higher prices than Bertrand competition, one can think of the former as a situation where competition is less intense. Delbono and Denicolo (1990) showed that, under the assumption of a homogeneous product, the incentive to introduce a cost-reducing innovation is greater for a Bertrand competitor than for a Cournot competitor: an "Arrow-like" result. Bester and Petrakis (1993), on the other hand, considered the case of differentiated products and
obtained a mixed result: if the degree of differentiation is "large", the incentive to introduce a cost-reducing innovation is higher for the Cournot competitor, while if the degree of differentiation is "small", then the incentive is higher for a Bertrand competitor.

Bester and Petrakis's model is one of horizontal differentiation (when prices are equal both products enjoy positive demand). In the first part of the paper we re-examine the issue within a model of vertical differentiation (if prices are equal, only one product -the higher quality one - enjoys positive demand) and show that the increase in profits associated with any given cost reduction is higher in the case of Cournot competition than in the case of Bertrand competition, and this is true no matter how small the degree of differentiation (thus even if the products are virtually homogeneous). It follows that there are cost-reducing innovations that would be pursued under Cournot competition but not under Bertrand competition (a "Schumpeter-like" result).

In the second part of the paper we address a related issue, which - somewhat surprisingly - has received very little attention in the literature. It is customary to distinguish between two types of innovation: product and process innovation. The former consists in the creation of new goods and services, while the latter leads to a reduction in the cost of producing existing products. The literature has dealt primarily with overall innovative activity (that is, the sum of product and process innovation) or one specific type of innovative activity (either process or product innovation). ${ }^{1}$ There have been no attempts to explain what factors might be important in a firm's decision whether to direct $R \& D$ expenditure towards product innovation or towards process innovation ${ }^{2}$. In this

[^1]paper we take a first step in the direction of filling this gap, by providing an explanation based on the type of competitive regime in which the firms find themselves (Cournot vs. Bertrand). We shall think of product innovation as an improvement in the quality of a firm's product (e.g. the introduction of a faster computer chip). Process innovation will be interpreted as a reduction in the firm's costs. We show that, if the choice is between a given cost reduction or a given quality improvement and the innovator is the high quality firm, one of three things can happen: (1) both the Cournot competitor and the Bertrand competitor choose the cost reduction, or (2) both choose the quality improvement, or (3) they make different choices, in which case the Cournot competitor chooses the cost reduction, while the Bertrand competitor chooses the quality improvement. That is, if Bertrand competition and Cournot competition lead to different choices, then the Bertrand competitor will favor product innovation, while the Cournot competitor will opt for process innovation. On the other hand, if the innovator is the low quality firm, then the opposite is true: whenever the two regimes of competition yield different choices, the Bertrand competitor will choose process innovation, while the Cournot competitor will choose product innovation.

The paper is organized as follows. Section 2 develops the model, Section 3 deals with cost-reducing innovations, while Section 4 is concerned with the choice between process and product innovation. Section 5 contains some final remarks and a conclusion.

The proofs of all the results are omitted and can be obtained from the authors.

## 2. A model of vertical differentiation

We use a model of vertical differentiation introduced by Mussa and Rosen (1978). There are N consumers with the same income, denoted by E , but different values of the
taste parameter $\theta$. Each consumer buys at most one unit. If a consumer does not buy the product, her utility is equal to her income E . If a consumer with parameter $\theta$ buys one unit of a good of quality $k$, at price $p$, her utility is equal to $E-p+\theta k$. The parameter $\theta$ is uniformly distributed in the interval $(0,1]$. It follows that, for every $\mathrm{x} \in(0,1]$, the number of consumers with parameter $\theta$ less than or equal to x is xN . We consider the case where there are two firms. Firm $H$ sells a product of quality $\mathrm{k}_{\mathrm{H}}$ while firm L sells a product of quality $\mathrm{k}_{\mathrm{L}}$, with $\mathrm{k}_{\mathrm{H}}>\mathrm{k}_{\mathrm{L}}>0$ (thus ' H ' stands for 'high quality' and ' L ' for 'low quality'). Let $p_{i}$ be the price charged by firm $\mathrm{i}(\mathrm{i}=\mathrm{H}, \mathrm{L})$. The demand functions are obtained as follows. Let $\theta_{0}$ be the value of $\theta$ for which the corresponding consumer is indifferent between consuming nothing and consuming the low-quality product. Then $\theta_{0}$ is the solution to the equation

$$
\mathrm{E}=\mathrm{E}-\mathrm{p}_{\mathrm{L}}+\theta \mathrm{k}_{\mathrm{L}} .
$$

Thus $\theta_{0}=\frac{p_{L}}{k_{L}}$. Let $\theta_{1}$ be the value of $\theta$ for which the corresponding consumer is indifferent between buying the low-quality product and the high-quality one. Then $\theta_{1}$ is the solution to the equation

$$
\mathrm{E}-\mathrm{p}_{\mathrm{L}}+\theta \mathrm{k}_{\mathrm{L}}=\mathrm{E}-\mathrm{p}_{\mathrm{H}}+\theta \mathrm{k}_{\mathrm{H}} .
$$

Thus $\theta_{1}=\frac{p_{H}-p_{L}}{k_{H}-k_{L}}$. Hence the (direct) demand functions are given by

$$
\begin{aligned}
& D_{H}\left(p_{H}, p_{L}\right)=\left(1-\theta_{1}\right) N=\left(1-\frac{p_{H}-p_{L}}{k_{H}-k_{L}}\right) N \\
& D_{L}\left(p_{H}, p_{L}\right)=\left(\theta_{1}-\theta_{0}\right) N=\left(\frac{p_{H}-p_{L}}{k_{H}-k_{L}}-\frac{p_{L}}{k_{L}}\right) N
\end{aligned}
$$

Like Bester and Petrakis (1993) and Rosenkrantz (1995) we assume that the two firms operate under constant returns to scale. Thus firm $i(i=H, L)$ has a cost function of the
form $C_{i}\left(q_{i}\right)=c_{i} q_{i}$ with $c_{i}>0$. We also assume that higher quality is associated with higher costs: $\mathrm{c}_{\mathrm{H}}>\mathrm{c}_{\mathrm{L}}$. Finally, we assume that $\mathrm{c}_{\mathrm{H}}$ and $\mathrm{c}_{\mathrm{L}}$ are such that both demands are positive when the two products are sold at unit cost (i.e. when $p_{H}=c_{H}$ and $p_{L}=c_{L}$ ). ${ }^{3}$ It is easy to see that this is the case if and only if the following two conditions are satisfied ${ }^{4}$

$$
\begin{equation*}
\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}>\mathrm{c}_{\mathrm{H}}-\mathrm{c}_{\mathrm{L}} \tag{1a}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}>\mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}} \tag{1b}
\end{equation*}
$$

The inverse demand functions are given by (where $q_{H}$ denotes the output of firm $H$ and $q_{L}$ the output of firm L)

$$
\begin{aligned}
& \mathrm{f}_{\mathrm{H}}\left(\mathrm{q}_{\mathrm{H}}, \mathrm{q}_{\mathrm{L}}\right)=\frac{\mathrm{Nk}_{\mathrm{H}}-\mathrm{k}_{\mathrm{H}} \mathrm{q}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}} \mathrm{q}_{\mathrm{L}}}{\mathrm{~N}} \\
& \mathrm{f}_{\mathrm{L}}\left(\mathrm{q}_{\mathrm{H}}, \mathrm{q}_{\mathrm{L}}\right)=\frac{\mathrm{k}_{\mathrm{L}}\left(\mathrm{~N}-\mathrm{q}_{\mathrm{H}}-\mathrm{q}_{\mathrm{L}}\right)}{\mathrm{N}}
\end{aligned}
$$

We consider two cases: the Bertrand case (decision variables are prices) and the Cournot case (decision variables are output levels). We shall use superscript ' $B$ ' for the Bertrand case and superscript ' $C$ ' for the Cournot case.

In the Bertrand case the profit functions are given by

[^2]\[

$$
\begin{align*}
& \Pi_{\mathrm{H}}^{\mathrm{B}}\left(\mathrm{p}_{\mathrm{H}}, \mathrm{p}_{\mathrm{L}}\right)=\mathrm{N}\left(\mathrm{p}_{\mathrm{H}}-\mathrm{c}_{\mathrm{H}}\right)\left(1-\frac{\mathrm{p}_{\mathrm{H}}-\mathrm{p}_{\mathrm{L}}}{\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}}\right) \\
& \Pi_{\mathrm{L}}^{\mathrm{B}}\left(\mathrm{p}_{\mathrm{H}}, \mathrm{p}_{\mathrm{L}}\right)=\mathrm{N}\left(\mathrm{p}_{\mathrm{L}}-\mathrm{c}_{\mathrm{L}}\right)\left(\frac{\mathrm{p}_{\mathrm{H}}-\mathrm{p}_{\mathrm{L}}}{\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}}-\frac{\mathrm{p}_{\mathrm{L}}}{\mathrm{k}_{\mathrm{L}}}\right) \tag{2}
\end{align*}
$$
\]

Prices and output levels at the Bertrand-Nash equilibrium are given by

$$
\begin{array}{r}
\mathrm{p}_{\mathrm{H}}^{\mathrm{B}}=\frac{\mathrm{k}_{\mathrm{H}}\left(2 \mathrm{k}_{\mathrm{H}}-2 \mathrm{k}_{\mathrm{L}}+2 \mathrm{c}_{\mathrm{H}}+\mathrm{c}_{\mathrm{L}}\right)}{4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}} \\
\mathrm{p}_{\mathrm{L}}^{\mathrm{B}}=\frac{\left(\mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}-\mathrm{k}_{\mathrm{L}}^{2}+\mathrm{c}_{\mathrm{H}} k_{\mathrm{L}}+2 \mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}\right)}{4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}} \\
\mathrm{q}_{\mathrm{H}}^{\mathrm{B}}=\mathrm{N} \frac{\left(2 \mathrm{k}_{\mathrm{H}}^{2}-2 \mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}-2 \mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{H}}+\mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}+\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}\right)}{\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)}  \tag{3}\\
\mathrm{q}_{\mathrm{L}}^{\mathrm{B}}=\mathrm{N} \frac{\mathrm{k}_{\mathrm{H}}\left(\mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}-\mathrm{k}_{\mathrm{L}}^{2}+\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}-2 \mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}+\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{L}}\right)}{\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right) \mathrm{k}_{\mathrm{L}}}
\end{array}
$$

giving the following expressions for the equilibrium profits of firms H and L :

$$
\begin{align*}
& \pi_{\mathrm{H}}^{\mathrm{B}}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}\right)=\mathrm{N} \frac{\left[2 \mathrm{k}_{\mathrm{H}}^{2}-2 \mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}-2 \mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{H}}+\mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}+\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}\right]^{2}}{\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{2}\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)}=\frac{\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}}{\mathrm{~N}}\left(\mathrm{q}_{\mathrm{H}}^{\mathrm{B}}\right)^{2} \\
& \pi_{\mathrm{L}}^{\mathrm{B}}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}\right)=\mathrm{Nk}_{\mathrm{H}} \frac{\left[\mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}-\mathrm{k}_{\mathrm{L}}^{2}-2 \mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}+\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{L}}+\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}\right]^{2}}{\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{2}\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right) \mathrm{k}_{\mathrm{L}}}=\frac{\mathrm{k}_{\mathrm{L}}\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)}{\mathrm{k}_{\mathrm{H}} \mathrm{~N}}\left(\mathrm{q}_{\mathrm{L}}^{\mathrm{B}}\right)^{2} \tag{4}
\end{align*}
$$

We now move to the Cournot case, where the profit functions are given by

$$
\begin{aligned}
& \Pi_{\mathrm{H}}^{\mathrm{C}}\left(\mathrm{q}_{\mathrm{H}}, \mathrm{q}_{\mathrm{L}}\right)=\mathrm{q}_{\mathrm{H}}\left(\frac{\mathrm{Nk} \mathrm{H}_{\mathrm{H}}-\mathrm{q}_{\mathrm{H}} \mathrm{k}_{\mathrm{H}}-\mathrm{q}_{\mathrm{L}} \mathrm{k}_{\mathrm{L}}}{\mathrm{~N}}-\mathrm{c}_{\mathrm{H}}\right) \\
& \Pi_{\mathrm{L}}^{\mathrm{C}}\left(\mathrm{q}_{\mathrm{H}}, \mathrm{q}_{\mathrm{L}}\right)=\mathrm{q}_{\mathrm{L}}\left(\frac{\mathrm{k}_{\mathrm{L}}\left(\mathrm{~N}-\mathrm{q}_{\mathrm{H}}-\mathrm{q}_{\mathrm{L}}\right)}{\mathrm{N}}-\mathrm{c}_{\mathrm{L}}\right)
\end{aligned}
$$

Prices and output levels at the Cournot-Nash equilibrium are given by

$$
\begin{array}{ll}
\mathrm{p}_{\mathrm{H}}^{\mathrm{c}}=\frac{2 \mathrm{k}_{\mathrm{H}}^{2}-\mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}+2 \mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{H}}+\mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}-\mathrm{c}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}}{4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}} & \mathrm{q}_{\mathrm{H}}^{\mathrm{c}}=\mathrm{N} \frac{2 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}-2 \mathrm{c}_{\mathrm{H}}+\mathrm{c}_{\mathrm{L}}}{4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}} \\
\mathrm{p}_{\mathrm{L}}^{\mathrm{c}}=\frac{\mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}+2 \mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}+\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{L}}}{4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}} & \mathrm{q}_{\mathrm{L}}^{\mathrm{C}}=\mathrm{N} \frac{\mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}+\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}-2 \mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}}{\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right) \mathrm{k}_{\mathrm{L}}} \tag{5}
\end{array}
$$

yielding the following expressions for the equilibrium profits of firms H and L :

$$
\begin{align*}
& \pi_{\mathrm{H}}^{\mathrm{C}}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}\right)=\mathrm{N} \frac{\mathrm{k}_{\mathrm{H}}\left(2 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}-2 \mathrm{c}_{\mathrm{H}}+\mathrm{c}_{\mathrm{L}}\right)^{2}}{\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{2}}=\frac{\mathrm{k}_{\mathrm{H}}}{\mathrm{~N}}\left(\mathrm{q}_{\mathrm{H}}^{\mathrm{C}}\right)^{2} \\
& \pi_{\mathrm{L}}^{\mathrm{C}}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}\right)=\mathrm{N} \frac{\left(\mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}-2 \mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}+\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}\right)^{2}}{\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{2} \mathrm{k}_{\mathrm{L}}}=\frac{\mathrm{k}_{\mathrm{L}}}{\mathrm{~N}}\left(\mathrm{q}_{\mathrm{L}}^{\mathrm{C}}\right)^{2} \tag{6}
\end{align*}
$$

REMARK 1. The following facts can be checked easily. If the parameter restrictions (1a) and (1b) are satisfied, then, for each firm $\mathrm{i}(\mathrm{i}=\mathrm{H}, \mathrm{L})$, Cournot output is smaller than Bertrand output $\left(q_{i}^{C}<q_{i}^{B}\right)$, Cournot price is higher than Bertrand price $\left(p_{i}^{C}>p_{i}^{B}\right)$, and Cournot profit is higher than Bertrand profit $\left(\pi_{i}^{C}>\pi_{i}^{B}\right)$. Furthermore, all these quantities are positive and equilibrium prices are greater than unit cost $\left(\mathrm{p}_{\mathrm{i}}^{\mathrm{C}}>\mathrm{p}_{\mathrm{i}}^{\mathrm{B}}>\mathrm{c}_{\mathrm{i}}\right)$.

## 3. Intensity of competition and the profitability of cost-reducing innovations

In this section we compare the incentives for a given cost reduction between a Bertrand competitor and a Cournot competitor and show that the latter is larger. Let $\Delta_{\mathrm{H}}>$ 0 be a non-drastic cost reduction for firm H and $\Delta_{\mathrm{L}}>0$ a non-drastic cost reduction for firm L, where "non-drastic" means that after the cost reduction the innovator cannot drive the other firm out of the market by charging a price close to unit cost. That is, we assume
that $\Delta_{\mathrm{H}}$ and $\Delta_{\mathrm{L}}$ are sufficiently small for inequalities corresponding to (1a) and (1b) to be satisfied:

$$
\begin{align*}
& \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}>\left(\mathrm{c}_{\mathrm{H}}-\Delta_{\mathrm{H}}\right)-\mathrm{c}_{\mathrm{L}}  \tag{7a}\\
& \mathrm{k}_{\mathrm{L}}\left(\mathrm{c}_{\mathrm{H}}-\Delta_{\mathrm{H}}\right)>\mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}  \tag{7b}\\
& \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}>\mathrm{c}_{\mathrm{H}}-\left(\mathrm{c}_{\mathrm{L}}-\Delta_{\mathrm{L}}\right)  \tag{7c}\\
& \mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}>\mathrm{k}_{\mathrm{H}}\left(\mathrm{c}_{\mathrm{L}}-\Delta_{\mathrm{L}}\right) \tag{7d}
\end{align*}
$$

For each firm $\mathrm{i}(\mathrm{i}=\mathrm{H}, \mathrm{L})$, let $\Delta \pi_{\mathrm{i}}^{\mathrm{C}}$ be the increase in profits expected from the given cost reduction in the case of Cournot competition and $\Delta \pi_{\mathrm{i}}^{\mathrm{B}}$ the increase in profits expected from the given cost reduction in the case of Bertrand competition:

$$
\begin{gather*}
\Delta \pi_{\mathrm{H}}^{\mathrm{C}}=\pi_{\mathrm{H}}^{\mathrm{C}}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}-\Delta_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}\right)-\pi_{\mathrm{H}}^{\mathrm{C}}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}\right)  \tag{8a}\\
\Delta \pi_{\mathrm{H}}^{\mathrm{B}}=\pi_{\mathrm{H}}^{\mathrm{B}}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}-\Delta_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}\right)-\pi_{\mathrm{H}}^{\mathrm{B}}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}\right)  \tag{8b}\\
\Delta \pi_{\mathrm{L}}^{\mathrm{C}}=\pi_{\mathrm{L}}^{\mathrm{C}}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}-\Delta_{\mathrm{L}}\right)-\pi_{\mathrm{L}}^{\mathrm{C}}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}\right)  \tag{8c}\\
\Delta \pi_{\mathrm{L}}^{\mathrm{B}}=\pi_{\mathrm{L}}^{\mathrm{B}}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}-\Delta_{\mathrm{L}}\right)-\pi_{\mathrm{L}}^{\mathrm{B}}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}\right) \tag{8d}
\end{gather*}
$$

where $\pi_{\mathrm{H}}^{\mathrm{C}}$ and $\pi_{\mathrm{L}}^{\mathrm{C}}$ are given by (6) and $\pi_{\mathrm{H}}^{\mathrm{B}}$ and $\pi_{\mathrm{L}}^{\mathrm{B}}$ are given by (4).
The following remark confirms Bester and Petrakis's result (1993, p. 525, Proposition 1) that the marginal return on investment in a cost reduction is increasing.

REMARK 2. $\Delta \pi_{\mathrm{H}}^{\mathrm{C}}$ and $\Delta \pi_{\mathrm{H}}^{\mathrm{B}}$ are decreasing in $\mathrm{c}_{\mathrm{H}}$ and $\Delta \pi_{\mathrm{L}}^{\mathrm{C}}$ and $\Delta \pi_{\mathrm{L}}^{\mathrm{B}}$ are decreasing in $\mathrm{c}_{\mathrm{L}}$.

Proposition 1 below gives a "Schumpeter-like" result: less intense competition is associated with a greater propensity to introduce cost-reducing innovations. Define a cost-
reducing investment opportunity for firm $\mathrm{i}(\mathrm{i}=\mathrm{H}, \mathrm{L})$ as a pair $\left(\Delta_{\mathrm{i}}, \alpha\right)$ where $\alpha$ is the cost of implementing the innovation and $\Delta_{\mathrm{i}}$ is the reduction in unit cost expected from the innovation. It is clear that firm i will carry out the investment if and only if the expected increase in profits is greater than the implementation cost, that is, if and only if $\Delta \pi_{\mathrm{i}}>\alpha$.

PROPOSITION 1. For each $\mathrm{i}(\mathrm{i}=\mathrm{H}, \mathrm{L})$ there are cost-reducing investment opportunities that are carried out by firm if it operates in a regime of Cournot competition but not if it operates in a regime of Bertrand competition. On the other hand, every cost-reducing investment carried out under Bertrand competition is also carried out under Cournot competition.

Proposition 1 follows directly from the following fact: for all $\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}, \Delta_{\mathrm{H}}$ and $\Delta_{\mathrm{L}}$ that satisfy (7), and for every $\mathrm{i}=\mathrm{H}, \mathrm{L}, \Delta \pi_{\mathrm{i}}^{\mathrm{C}}>\Delta \pi_{\mathrm{i}}^{\mathrm{B}}$. The intuition behind Proposition 1 is as follows. A cost reduction by firm i has a direct (positive) effect on the profits of firm i as well as a strategic or indirect effect through the change it induces in the choice variable of the competitor. In a Bertrand regime the strategic effect is negative: the competitor will respond to a reduction in $\mathrm{c}_{\mathrm{i}}$ by reducing its own price, thereby increasing the intensity of competition ${ }^{5}$. In a Cournot regime on the other hand, a cost
${ }^{5}$ The strategic effect is given by $\frac{\partial \Pi_{i}^{B}}{\partial p_{j}} \frac{\partial p_{j}^{B}}{\partial c_{i}}$ where $\mathrm{i} \neq \mathrm{j}, \Pi_{i}^{B}$ is given by (2) and $p_{i}^{B}$ is given by (3). It is straightforward to verify that $\frac{\partial \Pi_{i}^{B}}{\partial p_{j}}>0$ and $\frac{\partial p_{j}^{B}}{\partial c_{i}}>0$ so that $\frac{\partial \Pi_{i}^{B}}{\partial p_{j}} \frac{\partial p_{j}^{B}}{\partial c_{i}}>0$. In the terminology of Bulow et al (1985), in the Bertrand case (with linear demand) prices are strategic complements: a reduction in $c_{i}$ leads to a reduction in $p_{i}$ which in turn leads to a reduction in $p_{j}$, that is, an "aggressive" response by the competitor.
reduction has positive strategic effects, that is, it leads to a softening of competition ${ }^{6}$.
Note that Proposition 1 holds no matter how small the degree of product differentiation, that is, no matter how close $\mathrm{k}_{\mathrm{L}}$ is to $\mathrm{k}_{\mathrm{H}}$. Thus in a model of vertical differentiation the mixed result obtained by Bester and Petrakis (1993) does not hold.

## 4. On the choice between process and product innovation

We now turn to the choice between process and product innovation. Assume that one of the two firms, say firm H, has invested in R\&D (e.g. it has hired a team of engineers) and the corresponding cost is sunk. Suppose that the firm has two options:
(1) it can instruct its researchers to pursue product innovation, expected to lead to an increase in the quality of the firm's product from $\hat{\mathrm{k}}_{\mathrm{H}}$ to $\hat{\mathrm{k}}_{\mathrm{H}}+\Delta \mathrm{k}$ (with $\Delta \mathrm{k}>0$ ); or
(2) it can instruct them to pursue process innovation, expected to lead to a reduction in the firm's unit cost from $\hat{\mathrm{c}}_{\mathrm{H}}$ to $\hat{\mathrm{c}}_{\mathrm{H}}-\Delta \mathrm{c}$ (with $0<\Delta \mathrm{c} \leq \hat{\mathrm{c}}_{\mathrm{H}}$ ).

Assume that there are no other costs involved in the implementation of the innovation. The choice facing the firm is illustrated in Figure 1. For example, the firm's product could be a computer chip with quality represented by the operating speed (measured in MHz ) and the choice could be between increasing the speed from 166 MHz to 200 MHz or reducing the unit cost of its present product (the 166 MHz chip) from $\$ 800$ to $\$ 720$. Define a product/process investment opportunity as a triple $(\Delta \mathrm{c}, \Delta \mathrm{k}, \alpha)$ where $\alpha$ is the

[^3]cost of implementing the innovation (e.g. the cost of hiring a team of researchers), which is the same for both types of innovation, $\Delta \mathrm{c}$ is the expected reduction in unit cost if process innovation is pursued (e.g. if the researchers are instructed to seek a cheaper production process for the existing product) and $\Delta \mathrm{k}$ is the expected quality increase if product innovation is pursued (e.g. if the researchers are instructed to improve the quality of the product). A process/product investment opportunity $(\Delta \mathrm{c}, \Delta \mathrm{k}, \alpha)$ is profitable if the expected increase in profits from at least one of the two types of innovation (cost reduction or quality improvement) is greater than $\alpha$, the (common) cost of implementing the innovation. We shall first consider the case where the innovator is the high quality firm. The following proposition states that a Bertrand competitor is more prone to choose product innovation, while a Cournot competitor is more prone to choose process innovation.

PROPOSITION 2. The following is true for the high-quality firm. Given a profitable product/process investment opportunity $(\Delta \mathrm{c}, \Delta \mathrm{k}, \alpha)$, either both the Bertrand and the Cournot competitor choose the same type of innovation or, if they make different choices then the Bertrand competitor chooses product innovation, while the Cournot competitor chooses process innovation.

As illustrated in Figure 1, Proposition 2 follows from the following fact which applies to the high-quality firm. Fix arbitrary $\overline{\mathrm{k}}_{\mathrm{H}}, \overline{\mathrm{k}}_{\mathrm{L}}, \overline{\mathrm{c}}_{\mathrm{H}}$ and $\overline{\mathrm{c}}_{\mathrm{L}}$ that satisfy restrictions (1a) and (1b); then in the ( $\mathrm{k}_{\mathrm{H}}, \mathrm{c}_{\mathrm{H}}$ )-plane both the Bertrand isoprofit curve [obtained from (4)] and the Cournot isoprofit curve [obtained from (6)] that go through the point ( $\overline{\mathrm{k}}_{\mathrm{H}}$, $\overline{\mathrm{c}}_{\mathrm{H}}$ ) are increasing; furthermore, the Bertrand isoprofit curve is steeper (at that point) than the Cournot isoprofit curve. It follows that the two isoprofit curves cannot cross
more than once. Figure 1 shows the three possible cases ${ }^{7}$. Case 1 (Figure 1a): both the Bertrand competitor and the Cournot competitor choose product innovation. Case 2 (Figure 1b): both the Bertrand competitor and the Cournot competitor choose process innovation. Case 3 (Figure 1c): the Bertrand competitor and the Cournot competitor make different choices. In this case the Bertrand competitor opts for product innovation, while the Cournot competitor chooses process innovation.


Figure 1a

[^4]

Figure 1b


Figure 1c

The proof of Proposition 2 involves a number of rather complex algebraic manipulations which are hard to interpret. To obtain some intuition as to why a Bertrand competitor has a propensity to favor product over process innovation, recall that in a Bertrand regime a cost reduction has a negative strategic effect, in that it leads to an intensification of competition (see Section 2), with the consequence that at the equilibrium following process innovation both firms charge lower prices than at the pre-innovation equilibrium. Product innovation, on the other hand, will always lead to an increase in the price of firm H (the innovator), even though the equilibrium price of firm $L$ (the competitor) may increase or decrease, as shown in Figure 2. Of course, this intuitive explanation is only partially correct for three reasons: (1) as shown in Figure 1b, even a Bertrand competitor will choose process innovation over product innovation if the former "dominates" the latter (thus one can only speak of a tendency of Bertrand competitors to favor product innovation), (2) the analogous intuition for the Cournot competitor cannot be established, since both product and process innovation have a positive strategic effect, as shown in Figure 3 and (3) it is easier to understand a comparison between regimes of competition holding the type of innovation fixed (as we did in Section 2) than a comparison of different types of innovation holding the regime of competition fixed (as we are doing here), because there is no obvious way of making a change in quality (e.g. an increase of 34 MHz ) directly comparable with a cost reduction (e.g. \$80).


Figure 2
The effect of product innovation in the Bertrand case when the innovator is firm H
output of firm L


Figure 3a
The effect of process innovation in the Cournot case when the innovator is firm H
output of firm L


Figure 3b
The effect of product innovation in the Cournot case when the innovator is firm H

We now turn to the case where the innovator is the low quality firm.

PROPOSITION 3. The following is true for the low-quality firm. Given a profitable product/process investment opportunity $(\Delta \mathrm{c}, \Delta \mathrm{k}, \alpha)$, either both the Bertrand and the Cournot competitor choose the same type of innovation or, if they make different choices then the Bertrand competitor chooses process innovation, while the Cournot competitor chooses product innovation.

Proposition 3 follows from the following fact which applies to the low-quality firm. Fix arbitrary $\overline{\mathrm{k}}_{\mathrm{H}}, \quad \overline{\mathrm{k}}_{\mathrm{L}}, \overline{\mathrm{c}}_{\mathrm{H}}$ and $\overline{\mathrm{c}}_{\mathrm{L}}$ that satisfy restrictions (1a) and (1b). Then in the $\left(k_{L}, c_{L}\right)$-plane the Cournot isoprofit curve [obtained from (6)] that goes through the point $\left(\overline{\mathrm{k}}_{\mathrm{L}}, \overline{\mathrm{c}}_{\mathrm{L}}\right)$ is increasing and steeper (at that point) than the Bertrand isoprofit curve [obtained from (4)] that goes through the same point. Note that, while the Cournot isoprofit curve is always increasing, the Bertrand isoprofit curve might not be (it will be increasing if the degree of differentiation is not too small). Indeed, it has been shown in the literature (Gabszewicz and Thisse, 1979, 1980; Shaked and Sutton, 1982 ) that when there is Bertrand competition a low-quality firm might refrain from increasing the quality of its product even if it could do so at zero cost. This will happen when the degree of differentiation is very small. On the other hand, when competition is Cournot style, the low-quality firm does have an incentive to increase the quality of its product (Bonanno, 1986). The comparison between process and product innovation is therefore interesting mainly in the case where the low quality firm would profit from a costless quality improvement (that is, when the Bertrand iso-profit curve is increasing). In this case we have a reversal of the result of Proposition 2: when the innovator is the low-quality firm and the Bertrand competitor makes a different choice from the Cournot competitor, then
the latter will opt for product innovation, while the former will choose process innovation.

As for the case of Proposition 2, the proof of Proposition 3 involves a number of complex algebraic manipulations which are hard to interpret. Some intuition for the result can be obtained by examining the strategic effects. Consider, for example, the case of Bertrand competition. Process innovation by the low-quality firm has negative strategic effects, since it induces the innovator to reduce its price (firm L's reaction curve shifts down) and the competitor (firm H) will respond by also lowering its price. Product innovation by firm L , on the other hand, would potentially have positive strategic effects, since it shifts the innovator's reaction curve up. However, unlike the case of Proposition 2 - where a quality improvement by the high quality firm increased the degree of differentiation - here a quality improvement by firm L reduces the degree of differentiation and induces an aggressive response by the competitor: the reaction curve of firm H shifts to the left. To put it differently, a cost reduction for firm L has only an indirect effect on firm H's profits, through a reduction in the price of the innovator. A quality improvement by firm L , on the other hand, has a direct effect on the competitor's profits (it reduces firm H's revenue) and therefore induces a more aggressive response by firm H .

## 5. Conclusion

Within a model of vertical differentiation (due to Mussa and Rosen, 1978) we examined two issues. The first, which has received considerable attention in the literature, is whether more intense competition is associated with a stronger or weaker incentive to introduce a cost-reducing innovation. Following Delbono and Denicolo (1990) and Bester
and Petrakis (1993) we compared two identical industries (same demand and cost functions, same number of firms) that differed only in the regime of competition: Bertrand style versus Cournot style. Since Cournot competition leads to lower output and higher prices than Bertrand competition, it can be thought of as a regime of less intense competition. Our finding was that the incentive to introduce a cost-reducing innovation is stronger for a Cournot competitor.

We then turned to an issue that so far has received little attention in the literature, namely what factors might be important in a firm's decision whether to invest in product innovation (improvement in the quality of its product) or process innovation (cost reduction). We found that the regime of competition might be one such factor. For the high quality firm our result is that if there is a difference between the choice made by a Bertrand competitor and the choice made by a Cournot competitor, then the former will opt for product innovation, while the latter will prefer process innovation. For the lowquality firm, on the other hand, the result is reversed: whenever there is a difference, the Bertrand competitor will favor process innovation, while the Cournot competitor will favor product innovation.

As far as we know, the only other paper in the literature that deals with the choice between process and product innovation is Rosenkrantz (1995). She considers a model of horizontal differentiation, similar to the model used by Bester and Petrakis (1993). A two-stage Cournot duopoly model is considered where in stage 1 the firms simultaneously choose their unit cost $c_{i}$ and their product characteristic $d_{i}$ (the choice of $c_{i}$ is called process innovation and the choice of $d_{i}$ is called product innovation); in the second stage the firms choose outputs. Note, therefore, the following substantial differences: (1) for us product innovation means an improvement in the quality of the product (ours is a model of vertical differentiation), while for Rosenkrantz product innovation means a change in the horizontal characteristic of the product; (2) while we
compare the investment choice of one firm in different regimes of competition (Bertrand versus Cournot), Rosenkrantz analyzes the simultaneous choices of both firms within the same regime of competition (Cournot); (3) while we assume that the firm is faced with the choice between product and process innovation, Rosenkrantz allows each firm to mix both types of innovation and is interested in studying how the optimal mix varies with the parameters of the model (in particular the consumers' reservation price).

A natural question to ask is: how robust are these results? The answer to this question is two-fold. First of all, one cannot hope to obtain any results whatsoever in a very general model where properties of demand and costs are specified only qualitatively. The reason is that one needs to compare equilibria and in order to do so one needs to be able to compute them. Indeed the model used in this paper is as "general" as the models used in the literature on this topic (e.g. Delbono and Denicolo, 1990, Bester and Petrakis, 1993 and Rosenkrantz, 1995). The type of issues considered can only be analyzed in models that have a lot of structure and the richer the structure the less general the model. Secondly, although the model is rather specific, the results can be understood (fully, as in the case of Proposition 1 or only partially, as in the case of Propositions 2 and 3) in terms of qualitative properties, such as the strategic effects of different types of innovation.

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## Proofs of results in Bonanno-Haworth "Intensity of competition and the choice between product and process innovation" (for Referees' use: not meant for publication)

Proof of Proposition 1. We need to show that $\Delta \pi_{\mathrm{H}}^{\mathrm{C}}-\Delta \pi_{\mathrm{H}}^{\mathrm{B}}>0$ (where $\Delta \pi_{\mathrm{H}}^{\mathrm{C}}$ and $\Delta \pi_{\mathrm{H}}^{\mathrm{B}}$ are given by (8a) and (8b), respectively) and $\Delta \pi_{\mathrm{L}}^{\mathrm{C}}-\Delta \pi_{\mathrm{L}}^{\mathrm{B}}>0$ (where $\Delta \pi_{\mathrm{L}}^{\mathrm{C}}$ and $\Delta \pi_{\mathrm{L}}^{\mathrm{B}}$ are given by ( 8 c ) and ( 8 d ), respectively). Straightforward manipulations lead to:

$$
\begin{array}{r}
\Delta \pi_{\mathrm{H}}^{\mathrm{C}}-\Delta \pi_{\mathrm{H}}^{\mathrm{B}}=-\frac{N k_{L} \Delta_{H}}{\left(4 k_{H}-k_{L}\right)^{2}\left(k_{H}-k_{L}\right)}\left[\left(k_{H} c_{L}-k_{L} c_{H}\right)+\left(k_{H} c_{L}-k_{L} c_{H}+k_{L} \Delta_{H}\right)\right] \\
\Delta \pi_{\mathrm{L}}^{\mathrm{C}}-\Delta \pi_{\mathrm{L}}^{\mathrm{B}}=\frac{\mathrm{Nk}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}} \Delta_{\mathrm{L}}}{\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{2}\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)}\left[2\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)-2\left(\mathrm{c}_{\mathrm{H}}-\mathrm{c}_{\mathrm{L}}\right)-\Delta_{\mathrm{L}}\right] \tag{A1b}
\end{array}
$$

The fractions in both expressions are positive ( $\operatorname{since} \mathrm{k}_{\mathrm{H}}>\mathrm{k}_{\mathrm{L}}>0$ ). The term in square brackets in (A1a) is negative, since its first term is negative by (1b) and its second term is negative by (7b). The term in square brackets in (A1b) is positive since it is greater than $2\left[\left(k_{H}-k_{L}\right)-\left(c_{H}-c_{L}+\Delta_{L}\right)\right]$, which is positive by (7c).

Proof of Proposition 2. Fix arbitrary $\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}$ subject to the restrictions (1). Using the implicit function theorem the slope of the Bertrand isoprofit curve at this point is given by:

$$
\begin{equation*}
-\frac{\frac{\partial \pi_{\mathrm{H}}^{\mathrm{B}}}{\partial \mathrm{k}_{\mathrm{H}}}}{\frac{\partial \pi_{\mathrm{H}}^{\mathrm{B}}}{\partial \mathrm{c}_{\mathrm{H}}}} \tag{A2}
\end{equation*}
$$

where $\pi_{\mathrm{H}}^{\mathrm{B}}$ is given by (4). Similarly, the slope of the Cournot isoprofit curve is equal to (where $\pi_{\mathrm{H}}^{\mathrm{C}}$ is given by (6))

$$
\begin{equation*}
-\frac{\frac{\partial \pi_{\mathrm{H}}^{\mathrm{C}}}{\partial \mathrm{k}_{\mathrm{H}}}}{\frac{\partial \pi_{\mathrm{H}}^{\mathrm{C}}}{\partial \mathrm{c}_{\mathrm{H}}}} \tag{A3}
\end{equation*}
$$

We want to show that (A2) and (A3) are positive and that (A2) is greater than (A3). From (4) we get that

$$
\begin{equation*}
\frac{\partial \pi_{\mathrm{H}}^{\mathrm{B}}}{\partial \mathrm{c}_{\mathrm{H}}}=-\Phi \frac{2 \mathrm{~N}\left(2 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)}{\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{2}\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)} \tag{A4}
\end{equation*}
$$

where

$$
\begin{equation*}
\Phi=2 \mathrm{k}_{\mathrm{H}}^{2}-2 \mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{H}}-2 \mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}+\mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}+\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}} \tag{A5}
\end{equation*}
$$

Since the fraction in (A4) is positive (because $\mathrm{k}_{\mathrm{H}}>\mathrm{k}_{\mathrm{L}}$ ), we have that (A4) is negative if and only if (A5) is positive. By inspection, (A5) is indeed positive. In fact, since, by ( 1 b ), $\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}>\mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}$, (A5) is greater than $2 \mathrm{k}_{\mathrm{H}}\left[\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)-\left(\mathrm{c}_{\mathrm{H}}-\mathrm{c}_{\mathrm{L}}\right)\right]$ which is positive by (1a). Thus in order to prove that (A2) is positive (that is, that the Bertrand isoprofit curve of firm H is increasing) it only remains to prove that $\frac{\partial \pi_{H}^{B}}{\partial \mathrm{k}_{\mathrm{H}}}>0$. Now, from (4) we get

$$
\begin{equation*}
\frac{\partial \pi_{\mathrm{H}}^{\mathrm{B}}}{\partial \mathrm{k}_{\mathrm{H}}}=\frac{\mathrm{N} \Phi}{\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{3}\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{2}} \Psi \tag{A6}
\end{equation*}
$$

where $\Phi>0$ is given by (A5) and

$$
\begin{equation*}
\Psi=\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)\left(8 \mathrm{k}_{\mathrm{H}}^{2}-6 \mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}+4 \mathrm{k}_{\mathrm{L}}^{2}\right)+\Theta \tag{A7}
\end{equation*}
$$

with

$$
\begin{equation*}
\Theta=8 \mathrm{k}_{\mathrm{H}}^{2} \mathrm{c}_{\mathrm{H}}+2 \mathrm{k}_{\mathrm{L}}^{2} \mathrm{c}_{\mathrm{L}}-4 \mathrm{k}_{\mathrm{H}}^{2} \mathrm{c}_{\mathrm{L}}-\mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{L}}+5 \mathrm{k}_{\mathrm{L}}^{2} \mathrm{c}_{\mathrm{H}}-10 \mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}} \tag{A8}
\end{equation*}
$$

Since $k_{H}>k_{L}$, the fraction in (A6) and the first term in (A7) are positive. Thus (A6) is positive if $\Theta$ is positive. Note that $\frac{\partial \Theta}{\partial c_{H}}=5\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{2}+3 \mathrm{k}_{\mathrm{H}}^{2}>0$. Hence $\Theta$ is increasing in $\mathrm{c}_{\mathrm{H}}$ and to show that $\Theta>0$ it is enough to show that $\Theta$ is positive at the minimum value of $c_{H}$, which, by (1b), is equal to $\frac{k_{H} c_{L}}{k_{L}}$. Now substituting $\frac{k_{H} c_{L}}{k_{L}}$ for $c_{H}$ in (A8) and simplifying we obtain

$$
\frac{2\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right) \mathrm{c}_{\mathrm{L}}\left(4 \mathrm{k}_{\mathrm{H}}^{2}-3 \mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}-\mathrm{k}_{\mathrm{L}}^{2}\right)}{\mathrm{k}_{\mathrm{L}}}>0
$$

Thus we have proved that the Bertrand isoprofit curve of firm H is increasing.
Next we prove that (A3) is positive. From (6) we get

$$
\begin{equation*}
\frac{\partial \pi_{\mathrm{H}}^{\mathrm{C}}}{\partial \mathrm{c}_{\mathrm{H}}}=-\frac{4 \mathrm{Nk}_{\mathrm{H}}\left(2 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}-2 \mathrm{c}_{\mathrm{H}}+\mathrm{c}_{\mathrm{L}}\right)}{\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{2}} \tag{A9}
\end{equation*}
$$

which is negative since the expression in brackets in the numerator is equal to the sum of $\left(\mathrm{k}_{\mathrm{H}}-\mathrm{c}_{\mathrm{H}}\right)$, which is positive (cf. footnote 3 ), and $\left[\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)-\left(\mathrm{c}_{\mathrm{H}}-\mathrm{c}_{\mathrm{L}}\right)\right]$, which is positive by (1a).
Thus to prove that (A3) is positive it only remains to prove that $\frac{\partial \pi_{H}^{C}}{\partial \mathrm{k}_{\mathrm{H}}}>0$. From (6) we obtain

$$
\begin{equation*}
\frac{\partial \pi_{\mathrm{H}}^{\mathrm{C}}}{\partial \mathrm{k}_{\mathrm{H}}}=\frac{\mathrm{N}\left(2 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}-2 \mathrm{c}_{\mathrm{H}}+\mathrm{c}_{\mathrm{L}}\right)}{\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{3}} \Gamma \tag{A10}
\end{equation*}
$$

where

$$
\begin{equation*}
\Gamma=\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{2}+\left(\mathrm{c}_{\mathrm{H}}-\mathrm{c}_{\mathrm{L}}\right)\left(4 \mathrm{k}_{\mathrm{H}}+\mathrm{k}_{\mathrm{L}}\right)+7 \mathrm{k}_{\mathrm{H}}^{2}+4 \mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{H}}+\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}} \tag{A11}
\end{equation*}
$$

Since the numerator of (A10) is positive (cf. the remark after (A9)) and $\Gamma>0$ (since $\mathrm{k}_{\mathrm{H}}>\mathrm{k}_{\mathrm{L}}$ and $c_{H}>c_{L}$ ) it follows that (A3) is positive, that is, the Cournot isoprofit curve of firm H is increasing.

To complete the proof of Proposition 2 we need to show that (A2) is greater than (A3). Simple manipulations lead to the following expression for (A2) minus (A3):

$$
\begin{equation*}
\frac{\mathrm{k}_{\mathrm{L}}}{4 \mathrm{k}_{\mathrm{H}}\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)\left(2 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)} \mathrm{F}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}\right) \tag{A12}
\end{equation*}
$$

where
$\mathrm{F}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}\right)=5 \mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{L}}-12 \mathrm{k}_{\mathrm{H}}^{2} \mathrm{c}_{\mathrm{L}}+4 \mathrm{k}_{\mathrm{H}}^{2} \mathrm{k}_{\mathrm{L}}+8 \mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}-3 \mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}^{2}-2 \mathrm{k}_{\mathrm{L}}^{2} \mathrm{c}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}^{3}+\mathrm{k}_{\mathrm{L}}^{2} \mathrm{c}_{\mathrm{L}}$
Our objective is to show that (A12) is positive. Since the fraction in (A12) is positive (because $\mathrm{k}_{\mathrm{H}}>\mathrm{k}_{\mathrm{L}}>0$ ), this is equivalent to (A13) being positive. Note that F is increasing in $\mathrm{c}_{\mathrm{H}}$, since $\frac{\partial \mathrm{F}}{\partial \mathrm{c}_{\mathrm{H}}}=8 \mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}-2 \mathrm{k}_{\mathrm{L}}^{2}>0$. Thus it is enough to show that F is positive at the minimum value of $\mathrm{c}_{\mathrm{H}}$, which, by (1b), is given by $\frac{k_{H} c_{L}}{k_{L}}$. Substituting $\frac{k_{H} c_{L}}{k_{L}}$ for $c_{H}$ in (A13) and simplifying we obtain

$$
\mathrm{F}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \frac{\mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}}{\mathrm{k}_{\mathrm{L}}}, \mathrm{c}_{\mathrm{L}}\right)=\left(\mathrm{k}_{\mathrm{L}}-\mathrm{c}_{\mathrm{L}}\right)\left(4 \mathrm{k}_{\mathrm{H}}^{2}-3 \mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}-\mathrm{k}_{\mathrm{L}}^{2}\right)
$$

which is positive since $\mathrm{k}_{\mathrm{H}}>\mathrm{k}_{\mathrm{L}}>0$ and (cf. footnote 3 ) $\mathrm{k}_{\mathrm{L}}>\mathrm{c}_{\mathrm{L}}$.

Proof of Proposition 3. Fix arbitrary $\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}$ subject to the restrictions (1). Using the implicit function theorem the slope of the Bertrand isoprofit curve at this point is given by:

$$
\begin{equation*}
-\frac{\frac{\partial \pi_{\mathrm{L}}^{\mathrm{B}}}{\partial \mathrm{k}_{\mathrm{L}}}}{\frac{\partial \pi_{\mathrm{L}}^{\mathrm{B}}}{\partial \mathrm{c}_{\mathrm{L}}}} \tag{A14}
\end{equation*}
$$

where $\pi_{\mathrm{L}}^{\mathrm{B}}$ is given by (4). Similarly, the slope of the Cournot isoprofit curve is equal to (where $\pi_{\mathrm{L}}^{\mathrm{C}}$ is given by (6))

$$
\begin{equation*}
-\frac{\frac{\partial \pi_{\mathrm{L}}^{\mathrm{C}}}{\partial \mathrm{k}_{\mathrm{L}}}}{\frac{\partial \pi_{\mathrm{L}}^{\mathrm{C}}}{\partial \mathrm{c}_{\mathrm{L}}}} \tag{A15}
\end{equation*}
$$

First we show that (A15) is positive. From (6) we get

$$
\begin{equation*}
\frac{\partial \pi_{\mathrm{L}}^{\mathrm{C}}}{\partial \mathrm{c}_{\mathrm{L}}}=-\frac{4 \mathrm{Nk}_{\mathrm{H}}\left(\mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}+\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}-2 \mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}\right)}{\mathrm{k}_{\mathrm{L}}\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{2}} \tag{A16}
\end{equation*}
$$

which is negative, since the expression in brackets in the numerator is positive. In fact, since, by (1b), $\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}>\mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}$, that expression is greater than $\mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}-\mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}=\mathrm{k}_{\mathrm{H}}\left(\mathrm{k}_{\mathrm{L}}-\mathrm{c}_{\mathrm{L}}\right)>0$ (since $\mathrm{k}_{\mathrm{L}}>\mathrm{c}_{\mathrm{L}}:$ cf. footnote 3 ).

Again from (6) we get

$$
\frac{\partial \pi_{\mathrm{L}}^{\mathrm{C}}}{\partial \mathrm{k}_{\mathrm{L}}}=\frac{\mathrm{N}\left(\mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}+\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}-2 \mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}\right)}{\mathrm{k}_{\mathrm{L}}^{2}\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{3}}\left[4 \mathrm{k}_{\mathrm{H}}^{2} \mathrm{k}_{\mathrm{L}}+\mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}^{2}+4 \mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}+\mathrm{k}_{\mathrm{L}}^{2} \mathrm{c}_{\mathrm{H}}+2 \mathrm{k}_{\mathrm{H}}^{2} \mathrm{c}_{\mathrm{L}}+6 \mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)\right]
$$

which is positive since $\mathrm{k}_{\mathrm{H}}>\mathrm{k}_{\mathrm{L}}$ (for positivity of the numerator see the remark after (A16)).
Thus we have proved that (A15) is positive, that is, the Cournot isoprofit curve of firm L is increasing.

Now we turn to (A14). From (4) we get

$$
\begin{equation*}
\frac{\partial \pi_{\mathrm{L}}^{\mathrm{B}}}{\partial \mathrm{c}_{\mathrm{L}}}=-\frac{2 \mathrm{Nk}_{\mathrm{H}}\left(2 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)}{\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{2}\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right) \mathrm{k}_{\mathrm{L}}} \Xi \tag{A17}
\end{equation*}
$$

where

$$
\begin{equation*}
\Xi=\mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}-\mathrm{k}_{\mathrm{L}}^{2}-2 \mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}+\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}+\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{L}} \tag{A18}
\end{equation*}
$$

The fraction in (A17) is positive, since $\mathrm{k}_{\mathrm{H}}>\mathrm{k}_{\mathrm{L}}$. Thus (A17) is negative since (A18) is positive. In fact, (A18) is (since, by (1b), $\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}>\mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}$ ) greater than $\mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}-\mathrm{k}_{\mathrm{L}}^{2}-\mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}+\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{L}}=\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)\left(\mathrm{k}_{\mathrm{L}}-\mathrm{c}_{\mathrm{L}}\right)>0$, since $\mathrm{k}_{\mathrm{L}}>\mathrm{c}_{\mathrm{L}}$ (see footnote 3).

It follows that the sign of (A14) is equal to the sign of $\frac{\partial \pi_{\mathrm{L}}^{\mathrm{B}}}{\partial \mathrm{k}_{\mathrm{L}}}$. From (4) we get

$$
\begin{equation*}
\frac{\partial \pi_{\mathrm{L}}^{\mathrm{B}}}{\partial \mathrm{k}_{\mathrm{L}}}=\frac{\mathrm{N} \mathrm{k}_{\mathrm{H}} \Xi}{\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{3}\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)^{2} \mathrm{k}_{\mathrm{L}}^{2}}\left(\Omega_{1}+\Omega_{2}\right) \tag{A19}
\end{equation*}
$$

where $\Xi>0$ is given by (A18) and

$$
\begin{gather*}
\Omega_{1}=k_{H} k_{L}\left(k_{H}-k_{L}\right)\left(4 k_{H}-7 k_{L}\right)  \tag{A20}\\
\Omega_{2}=k_{H} c_{L}\left(k_{H}-k_{L}\right)\left(8 k_{H}-9 k_{L}\right)-k_{H}^{2} k_{L} c_{L}+k_{H} k_{L}^{2} c_{H}-2 k_{L}^{3} c_{H}-2 k_{L}^{3} c_{L}+4 k_{H}^{2} k_{L} c_{H} \tag{A21}
\end{gather*}
$$

Since, by (1b), $k_{L} c_{H}>k_{H} c_{L}$, (A21) is greater than (A22), which is obtained by replacing $4 \mathrm{k}_{\mathrm{H}}^{2} \mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}$ in (A21) with $2 \mathrm{k}_{\mathrm{H}}^{2} \mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}+2 \mathrm{k}_{\mathrm{H}}^{3} \mathrm{c}_{\mathrm{L}}$ :

$$
\begin{equation*}
\mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)\left(8 \mathrm{k}_{\mathrm{H}}-9 \mathrm{k}_{\mathrm{L}}\right)+2 \mathrm{c}_{\mathrm{L}}\left(\mathrm{k}_{\mathrm{H}}^{3}-\mathrm{k}_{\mathrm{L}}^{3}\right)+2 \mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}\left(\mathrm{k}_{\mathrm{H}}^{2}-\mathrm{k}_{\mathrm{L}}^{2}\right)+\mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}\left(\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}-\mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}\right) \tag{A22}
\end{equation*}
$$

Since $k_{H}>k_{L}$ and, by (1b), $\mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}>\mathrm{k}_{\mathrm{H}} \mathrm{c}_{\mathrm{L}}$, a sufficient condition for (A22), and therefore (A21), to be positive is $8 \mathrm{k}_{\mathrm{H}}-9 \mathrm{k}_{\mathrm{L}}>0$. On the other hand, a sufficient condition for $\Omega_{1}$ to be positive is $4 \mathrm{k}_{\mathrm{H}}-7 \mathrm{k}_{\mathrm{L}}>0$, which implies the previous one. Hence we can conclude that a sufficient (although not necessary) condition for (A14) to be positive, that is, for the Bertrand isoprofit curve of firm L to be increasing is:

$$
\mathrm{k}_{\mathrm{H}}>\frac{7}{4} \mathrm{k}_{\mathrm{L}}
$$

That is, the degree of differentiation should be not too small.

In order to complete the proof of Proposition 3 we only need to show that (A14) is less than (A15). Simple manipulations lead to the following expression for (A14) minus (A15):

$$
\begin{equation*}
\frac{\mathrm{k}_{\mathrm{L}}}{4 \mathrm{k}_{\mathrm{H}}\left(4 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)\left(2 \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)\left(\mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}\right)} \mathrm{G}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}\right) \tag{A23}
\end{equation*}
$$

where
$\mathrm{G}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{c}_{\mathrm{H}}, \mathrm{c}_{\mathrm{L}}\right)=-5 \mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{H}}-12 \mathrm{k}_{\mathrm{H}}^{3}+13 \mathrm{k}_{\mathrm{H}}^{2} \mathrm{k}_{\mathrm{L}}+2 \mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}} \mathrm{c}_{\mathrm{L}}+12 \mathrm{k}_{\mathrm{H}}^{2} \mathrm{c}_{\mathrm{H}}-8 \mathrm{k}_{\mathrm{H}}^{2} \mathrm{c}_{\mathrm{L}}-\mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}^{2}-\mathrm{k}_{\mathrm{L}}^{2} \mathrm{c}_{\mathrm{H}}$
Our objective is to show that (A23) is negative. Since the fraction in (A23) is positive (since $k_{H}>$ $\mathrm{k}_{\mathrm{L}}>0$ ), this is equivalent to (A24) being negative. Note that $G$ is increasing in $\mathrm{c}_{\mathrm{H}}$, since $\frac{\partial \mathrm{G}}{\partial \mathrm{c}_{\mathrm{H}}}=12 \mathrm{k}_{\mathrm{H}}^{2}-5 \mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}-\mathrm{k}_{\mathrm{L}}^{2}>0\left(\right.$ since $\mathrm{k}_{\mathrm{H}}>\mathrm{k}_{\mathrm{L}}$ ). Thus it is enough to show that G is negative at the maximum value of $c_{H}$, which by (1a), is given by $k_{H}-k_{L}+c_{L}$. Substituting $k_{H}-k_{L}+c_{L}$ for $c_{H}$ in (A24) and simplifying we obtain

$$
\mathrm{G}\left(\mathrm{k}_{\mathrm{H}}, \mathrm{k}_{\mathrm{L}}, \mathrm{k}_{\mathrm{H}}-\mathrm{k}_{\mathrm{L}}+\mathrm{c}_{\mathrm{L}}, \mathrm{c}_{\mathrm{L}}\right)=-\left(\mathrm{k}_{\mathrm{L}}-\mathrm{c}_{\mathrm{L}}\right)\left(4 \mathrm{k}_{\mathrm{H}}^{2}-3 \mathrm{k}_{\mathrm{H}} \mathrm{k}_{\mathrm{L}}-\mathrm{k}_{\mathrm{L}}^{2}\right)
$$

which is negative since $k_{H}>k_{L}>0$ and (cf. footnote 4) $k_{L}>c_{L}$.


[^0]:    * We are grateful to two anonymous referees, Raymond De Bondt, Louis Makowski and Klaus Nehring for helpful comments and suggestions.

[^1]:    ${ }^{1}$ See surveys by Kamien and Schwartz (1975), Baldwin and Scott (1987), Cohen and Levin (1989), Scherer and Ross (1990) and Tirole (1988).
    ${ }^{2}$ An exception is Rosenkrantz (1995) which is discussed in Section 5. We are grateful to Raymond De Bondt for bringing this paper to our attention.

[^2]:    ${ }^{3}$ This assumption guarantees that at all the equilibria we consider, prices and output levels are positive: cf. Remark 1 below.
    ${ }^{4}$ Note that (1a) and (1b) imply that $\mathrm{c}_{\mathrm{H}}<\mathrm{k}_{\mathrm{H}}$ and $\quad \mathrm{c}_{\mathrm{L}}<\mathrm{k}_{\mathrm{L}}$.

[^3]:    ${ }^{6}$ The strategic effect is given by $\frac{\partial \Pi_{i}^{c}}{\partial q_{j}} \frac{\partial q_{j}^{c}}{\partial c_{i}}$ which is negative, as one can easily verify (cf., in particular, (5)). In the terminology of Bulow et al (1985), in the Cournot case (with linear demand) output levels are strategic substitutes: a reduction in $c_{i}$ leads to an increase in $q_{i}$ which in turn leads to a reduction in $q_{j}$, that is, a "submissive" response by the competitor.

[^4]:    ${ }^{7}$ It is useful to consider not the isoprofit curve that goes through the status quo (or pre-innovation) point, but rather the isoprofit curve that goes through the point that represents process innovation.

