Reflective inquiry and "The Fate of Reason"

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What particular privilege has this little Agitation of the Brain which we call Thought, that we must make it the Model of the whole Universe? (Hume, *Dialogues Concerning Natural Religion*, 1976, p. 168)

...at once it struck me, what quality went to form a Man (sic) of Achievement especially in Literature and which Shakespeare possessed so enormously—I mean *Negative Capability*, that is when someone is capable of being in uncertainties, Mysteries, doubts without any irritable reaching after fact and reason. (Keats 1959, p. 261)

Die menschliche Vernunft hat das besondere Schicksal in ihrer Gattung ihrer Erkenntnisse: dass sie durch Fragen belästigt wird, die sie nicht abweisen kann; denn sie sind ihr durch die Natur der Vernunft selbst aufgegeben, die sie aber auch nicht beantworten kann; denn sie übersteigen alles Vermögen der menschlichen Vernunft. (Opening lines of the *Kritik der reinen Vernunft* (1956), A VII; the italics in the text are mine)

Human reason has the particular *fate*¹ in one branch of its investigations: that it is harassed by questions which it cannot dismiss out of hand, for they are posed to it by the nature of reason itself; but which it also cannot answer, for they exceed all capacity of human reason.

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¹ It may be metaphysically relevant in this well-known context that "fatum"—before the Romans hijacked the expression for the 'decrees' of their 'gods'—was simply "what was spoken", and in that sense "deemed", or "judged" (from "fari", to speak, akin to phemi and phasko in Greek).

Der Verstand macht für die Vernunft ebenso einen Gegenstand aus, als die Sinnlichkeit für den Verstand. Die Einheit aller möglichen empirischen Verstandeshandlungen systematisch zu machen, ist ein Geschäft der Vernunft, sowie der Verstand das Mannigfaltige der Erscheinungen durch Begriffe verknüpft und unter empirische Gesetze bringt. Die Verstandeshandlungen aber, ohne Schemata der Sinnlichkeit, sind unbestimmt; ebenso ist die Vernunfteinheit auch in Ansehung der Bedingungen, unter denen, und des Grades, wie weit, der Verstand seine Begriff systematisch verbinden soll, an sich selbst unbestimmt. Allein, obgleich für die durchgängige systematische Einheit aller Verstandesbegriffe kein Schema in der Anschauung ausfindig gemacht werden kann, so kann und muß doch ein Analogon eines solchen Schema gegeben werden, welches die Idee des Maximum der Abteilung und der Vereinigung der Verstandeserkenntnis in einem Prinzip ist. (Kritik der reinen Vernunft, B692; the italics are again mine)

Understanding constitutes an object for reason just as the sensory [does] for understanding. To render the unity of all possible operations of the understanding systematic is an affair of reason, [just] as understanding structures and brings under empirical laws the manifold of appearances. The operations of the understanding, however, without schemata of what is sensory, are undetermined; just as the unity of reason in itself is undetermined with respect to the conditions under which and degree to which understanding is supposed to structure and interrelate its concepts. But though no comprehensive schema of systematic unity of all concepts of the understanding can be found in intuition, an analogon of such a schema, namely the idea of the maximum of (sub)division and unification of conceptual knowledge in a principle, can and must be found

Abstract

In this essay, I

- 1 conjecture that Galileo's "book of philosophy" (not "nature") may not be written in Galileo's mathematics, but in Hilbert's and Gödel's metamathematics;
- 2 outline a common metalogical analysis of four 'cogito'-like arguments: Descartes' original, Berkeley's (1979) 'master-argument', and Kant's 'metaphysical' and 'transcendental' 'deductions';
- 3 advocate a 'skeptical' form of 'transcendental idealism', which 'locally' distinguishes 'experiential' assertions from metatheoretic 'preconditions' for their 'intended' interpretation;
- 4 and argue
 - (i) that "experience" is a rational but inherently 'problematic' regulative ideal (cf. KdrV, 1956, B100);
 - (ii) that the *integrity of 'essentially incomplete' but 'intelligible' inquiry* is the only 'unity' such "experience" could have; and

² The "skeptics" were "reflective" ("skeptikoi") "seekers" ("zetetikoi"), who "sought" to "see (more)" ("skeptesthai"). What they "sought" was not 'the' "truth"—ironically, derived in English from a Germanic word for "trust" or "belief"—but 'locally' adequate "vantage-points" ("skopoi", "skopia") of "insight" ("skemma") and conceptual 'tranquility' ("ataraxia").



(iii) that 'almost all' 'maximal limits' to which such 'experiential' inquiry might converge would be intrinsically 'unintelligible'. ("Es ist das Mystische...")

1 Introduction

'Neo-Kantians' such as Hermann Cohen and Ernst Cassirer 'relativised' Kant's attempts to 'ground' "Erfahrung" (experience) in an "Architektonik" derived from Newtonian spacetime, in an effort to refine his 'critical' presuppositions, and accommodate more recent mathematical and scientific inquiries into the 'hermeneutics' of differential geometry and mathematical physics undertaken by Gauss, Lobachevsky, Poincaré, Lorentz, Hilbert, Einstein and others.

In this essay and a pair of sequels I 'relativise' Kant's attempts to explore ascending levels of 'reflective inquiry' and 'regulative principles' in his three Critiques, in an effort to refine his "dialectical", "practical" and "teleological" extensions of the first's 'theoretical' "Architektonik" of (Newtonian scientific) 'experience', and accommodate more recent metalogical and metamathematical inquiries into the 'hermeneutics' of concept-formation undertaken by Hilbert, Skolem, Gödel, Tarski, Keisler, Scott, Solovay and others.

In potted-historical terms, a preliminary rationale for such a 'relativisation', and for the 'skeptical' as well as 'locally transcendental' idealism' I will derive from it, might be sketched as follows.

There is a sense—to which people like Cassirer were particularly attentive—in which Kant 'merely' emulated Leibniz' and Leonhard Euler's impassioned attempts to respond to the *metaphysical* resonance of Galileo's (often-misquoted) suggestion in the *Assayer* (*Opere* 6:23:2): that the "vast book" of "philosophy" would be "written in the language of mathematics".

More than a century later, working scientists and historians of science among Cassirer's predecessors and contemporaries such as Hertz, Helmholtz, Duhem and others also attempted to respond to Galileo's dictum in new ways, and Peano, Frege and Russell more or less simultaneously formalised the 'neo-Leibnizian' recursive framework of a quantificational logic, which gave rise rather quickly to intriguing "semantic-paradoxical" refinements of Kant's "mathematische und dynamische Antinomien".

During the twentieth century, 'logicists' and 'logical positivists'—most of them convinced that such paradoxes would be swept aside by a new 'empiricist' synthesis—drew on work of Duhem and others to propose 'reductive' translations of Galileo's "vast book" into rudimentary forms of first-order logic, Russellian type-theory and Zermelian set theory, and sidestep skeptical implications of these antinomies implicit in the work of Skolem and Gödel.

Against the grain of this (admittedly partial as well as summary) historical sketch, I wish to suggest that

1 efforts on the part of the logical positivists' more nuanced 'analytical' successors to titrate conventional empiricism with a bit of 'logic' have not yielded a stable response to these and other antinomies (and never will),

and propose a conjectural 'metalogische Wende', or (in more pretentious emulation of Kant's preface to the first Critique (cf. B XVI ff.)), "Revolution": that



2 future 'books' which record inquiries into boundaries of (inquiries into (boundaries of (.... 'philosophy'))) will be 'written in the languages of metamathematics';

and that

3 the 'texts' of such 'books' will be 'intelligible' (recursively axiomatisable, essentially incomplete) theories, representable as 'closed' subsets of a 'universal' topological and measure-space adumbrated in the 'dyadic' writings of Leibniz (1978), and reintroduced in nineteenth- and twentieth-century guises by Cantor (1933) and Marshall Stone.

In opposition to Kant and his empiricist successors' efforts to 'define' or 'demarcate' a 'canonical' or 'surveyable' "Bereich der Erfahrung" ("range of experience")—via 'critical methods', 'transcendental' analyses or more alembicated ad hoc methodological 'titrations' of 'empiricism' of the sort just mentioned—I will also

- 4 outline '*locally transcendental*' rationales for the epistemic and metaphysical adequacy of *essentially incomplete* (first-order) *theories*;
- 5 interpret Kant's "kritische Methode" and 'constitutive'/'regulative' distinction as partial anticipations of metalogicians' 'theory'/'metatheory' distinctions;
- 6 invoke well-studied arguments to argue that such 'local' distinctions and methodological principles generate 'heuristic' *iterative hierarchies of concept-formation*; and
- 7 characterise (but not 'define') "experience" as a 'heuristic', distributive, 'problematic' (cf. KdrV, B100) and 'merely regulative' temporal process of ramified 'ascents' within such hierarchies.

A few preliminary remarks may anticipate immediate and obvious objections to these proposals, and my eclectic and 'impressionist' use in them of metalogical terms-of-art such as 'object-' and 'metatheory'. Philosophers often find such 'technical' terms-of-art reductive; and logicians, for their part, find philosophical uses of them jejune. Perhaps eclectic uses of them fall into the sort of conceptual intermundium C. P. Snow wrote about many years ago.

Be that as it may, both the views just cited seem to me prejudices, as the proposals in 2 and 3 make clear. If these proposals have heuristic value, the methodological partitions and preconceptions just cited will erode as needs to

- 8 provide coherent accounts of 'artificial' as well as 'natural' sentience, and
- 9 develop natural-scientific analyses of notions such as 'existence', 'complexity', 'randomness' and 'experimental 'isolation'.

become more apparent.

Kant, in any case, was no more acquainted with 'first-order metatheoretical hierarchies' than he was with 'time-orientable locally Lorentzian manifolds'. He *did*, however, make a number of suggestive attempts to distinguish "*regulative*" from "*constitutive*" applications ("*Gebräuche*") of broadly problematic 'higher-order' 'principles' and metaphysical assertions (cf. the passage *KdrV*, B692, quoted at length above).

Such distinctions—or superpositions of distinctions—first appeared in B536–B611 of the *Transcendental Dialectic*. Kant then took them more or less for granted thereafter, and applied them *passim* to more abstruse and underdetermined ranges of *prac*-



tical (ethical) as well as teleological assertions and principles in *Dialectic* as well as the second and third *Critiques*.

At a kind of asymptotic limit of such 'principles', he even appealed at various points in all three *Critiques* and their ancillary texts to the notion of a *regulative Ideal*—an interpretative *Grenzidee* which seems to me as 'reflexive', suggestive and conceptually underdetermined as Aristotle's *theoria*, and shadowy "aggregates" ("Vielheiten") of the sort Georg Cantor aptly called "inkonsistent" ('the' paradoxical "aggregate" of 'all' consistent recursively axiomatisable first-order theories, for example; cf. Cantor 1933, pp. 443–444, reprinted in Van Heijenoort 1967, p. 114).

In Sect. 2 below, I will consider certain attempts to coopt and normalise ancient skeptical notions of 'ta phainomena' in ways which might

- 10 ensure that a fixed 'maximal' realm or "aggregate" of such phainomena 'must' exist, and
- 11 furnish 'design'-arguments for the 'existence' and 'unicity' of 'experience' (rather than 'god').

In Sections 3 and 4, I will

- 12 examine 'pre-theoretical' or 'pre-metalogical' characterisations of "'cogito'-like" arguments;
- 13 argue that such arguments can be construed as informal prototypes of "fixed-point arguments"; and
- 14 'test' these anachronistic characterisations against four historical 'examples':
 - (i) Descartes' "cogito"-argument, and its stoic and Augustinian antecedents;
 - (ii) *Berkeley*'s more original but strangely ill-regarded "*master-argument*"; and in somewhat greater detail in the next section,
 - (iii) Kant's "transcendental" and "metaphysical deductions" of the "necessity of the possibility" of "synthetic a priori³ judgment(s)", and
 - (iv) 'the' allegedly unique structures of "experience" which such judgments "determine" when they are "constitutive", and 'underdetermine' when they are "('merely') (regulative)".

In Sections 5–8, I will

15 focus on *metalogically syntactical* and *semantic* as well as informal interpretations for the '*unicity*' of '*rule*'-bound theoretical '*experience*' Kant hoped to derive from his "*transzendentale Deduktion*".

I do this in part because

⁽iv) 'adverbial' phrases which qualify 'processive' verbs and adjectives ("judge" (or "juger") "a priori"; "decree ex cathedra"; "estimate off the cuff"; "prima facie plausible").



³ I will attach little or no epistemic or metaphysical significance in the sequel to 'adverbial' and 'adjectival' readings of the ablative phrase "a priori".

⁽i) there is little or no syntactical distinction between such usages in German; and in part because

⁽ii) there is little or no *semantic* distinction, at least in Latin, English, French or German, between

⁽iii) 'adjectival' phrases which qualify 'processive' nouns ("judgment") (or "judgement") a priori; "ex cathedra decree"; "off the cuff estimate"; "prima facie plausibility"); and

- 16 argue for *metamathematical* interpretations of such "*rules*", and offer 'skeptically transcendental' rationales for the recurrent and interpretive uses of *first-order theories* in the metatheoretical hierarchies mentioned in 6 and 7 above, and
- 17 adduce a modest result of metalogical 'folklore': that no 'faithful' syntactical interpretation of an 'experiential' 'object-theory' can be defined in any metatheory of "concepts of the understanding" ("Verstandesbegriffe") which proves that such an object-theory is consistent.

On the one hand, the metamathematical observation in 15 suggests that

- 18 no metalogically tenable 'transcendental deduction' of the 'existence' of a 'faith-ful' interpretation of a provably consistent 'experiential' object-theory can be formulated.
- 19 On the other, it also suggests (to me at least) that
 - (i) indeterminate as well as underdetermined recourses to 'higher'-order metatheories, which semantically interpret ('lower' levels or stages of) 'experience' introduce ever-higher-order mathematical as well as metamathematical analogues of
 - (ii) Kant's 'merely regulative' "intelligible Ursachen" ('intelligible causes') and "Vernunftbegriffe" ('concepts of reason') into the object-theoretical processes they interpret, and that
 - (iii) the resulting 'eternal return'—or more precisely, 'eternal recurrence' of alternating 'object'- and 'metatheoretic' ('epistemic' and 'metaphysical') 'boundary-conditions'—is indeed the "fate of reason". But may also be—in several senses—its "grace"....
- 2 [Reflective inquiry into 'ultimate' limits of [Reflective inquiry into 'ultimate' limits of [Reflective inquiry into 'ultimate' limits of [....]]]]

Pyrrhonist skeptics often claimed to follow 'undogmatically' what they called *ta phainomena*—a Greek neuter plural participle which is usually 'objectified' in (oddly quasi-Kantian) English as 'the appearances'. (It might be more accurately rendered as 'whatever things "seem (to be)"'.)

Whatever the translation, *relational* and *intentional* implications of such usages—'seem' or 'appear', under *what circumstances*, and to *what*?—have always fostered 'idealist' interpretations of 'ta phainomena' as ('mere') 'intentional objects'.

These interpretations, in turn, crossed metaphysical boundaries in interesting ways: 'dogmatic' proto-'*cogito*'-arguments might be found in them, as well as rationales for (otherwise rather puzzling) eighteenth-century claims that George Berkeley was a 'skeptic'.

This ambiguity itself has also been historically *generative*, in ways genuine 'suspenders-of-judgment' and 'skeptics' ('observers', 'searchers') might applaud. Much of the history of early modern philosophy as well as the science which crystalised out of it might be read as a series of brilliantly informative attempts to *coopt*



skeptics' 'phenomenalism' for 'dogmatic' purposes, with the aid of arguments which 'normalised' or 'uniformised' their 'phainomena'.

Galileo, for example, sought such uniformities in the syntax and semantics of *mathematics*, in the passage from the *Assayer* cited earlier. And Descartes, Leibniz, Newton and Berkeley sought them in the hermeneutic (and mathematical) capacities of that great 'analyst', 'god'.

Hume—even less plausibly, in my view—sought them in the persistence of human "Custom or Habit". Kant, finally, sought to 'deduce' them, in a great secular Newtonian-scientific synthesis whose 'transcendental' adaequatio would be 'complete' enough to determine (or dispense with) its limiting res (or 'Dinge an sich').

Intricately interrelated refinements in the languages of Galileo's mathematical 'book', in short, have provided, and will continue to provide, heuristic ways to explore endless ramifications of 'mere' phenomena.

But these 'ramifications' also bring into focus two deeper (and closely interrelated) skeptical reservations, which philosophical efforts to coopt skeptical phainomena have left unanswered (and, in my opinion at least, almost untouched).

The first is

1 that postulation of 'unique' metaphysical *limits* of these practices (whatever they may be called) amount to secular—mathematical and 'physical'—refinements of 'arguments from design'—not for the 'existence' of 'god', but for the existence of 'the' phainomena (a slightly ironic echo, perhaps, of "deus sive natura").

(Notice once again in this context the extent to which

2 the 'structural' and 'constitutive' uniformity and intertranslatability *Kant* attributed to his 'phenomena' ('*Erscheinungen*')

may be assimilated to

3 the 'structural' and 'constitutive' uniformity and intertranslatability Berkeley attributed to his all-seeing 'God' or "*The Author of Nature*".)

A second and (again in my opinion) deeper reservation derives from one of the ancient skeptics' more acute insights and anti-stoic 'tropoi' ('turns'):

- 4 that the very notion of 'completeness' may therefore be conceptually 'liminal' and 'theory-relative', and in that sense incomplete; and in particular,
- 5 that conceptual as well as phenomenal 'relationality' and 'intentionality' may be *iterable* 'eis apeiron'—'into (what is) infinite', and 'into (what is) untried, or unexperienced' (both readings are etymologically defensible).

There are essentially two ways to respond to this extended skeptical 'regress' (or 'ascent'), and block '*iteration*', 'ramification' and 'indefinition' of the hierarchies to which it gives rise:

- 6 to 'bound' such iterations in a unique 'limit'; and
- 7 to 'diagonalise' over them, in allegedly 'canonical' ways which appeal to informal counterparts of metalogical 'fixed-point'-arguments.

I will devote the most of the rest of this section to the first, and the next to the second. 'Semantic monist' commitments of any sort to the effect that



8 there 'must' 'exist' one and exactly one 'universal' interpretation of a sufficiently 'comprehensive' (as well as consistent) theory T of 'experiential' phenomena (which must, then, be the 'intended' interpretation of T, if there is one)

have traditionally had (at least) three venerable but fallacious arguments or argumentforms at their disposal, in my view:

- 9 'existence-of-maximality'-arguments, often analysable as instances of objecttheoretic quantifier-error;
- 10 'existence-of-determination'-arguments, which I will interpret as subtler and more plausible variants of metatheoretic quantifier-error; and finally
- 11 'uniqueness-of-maximality-and-determination'-arguments, which may be analysed as misreadings of informal patterns of metatheoretic Stone-'duality.'

In 'existence of maximality'-arguments, for example, one 'derives' the 'existence' of a unique 'maximal' bearer of a property ('maximal' in the sense of some 'practical' or 'metaphysical' ordering) from assumptions (often begged) about the structure of an ordering or hierarchy.

One might, for example, 'infer'

- 12 '(necessary) existence' of a 'maximal' relational entity ('cause'; 'explanation'; 'perfection';), from
- 13 corresponding 'medial' premises (every 'effect' has a 'cause'; every 'phenomenon' has an 'explanation'; every 'quality' 'has' less 'perfection' than some other; ...).

I characterised certain instances of 'existence of maximality'-arguments in 9 and 10 above as 'quantifier-errors'. Kant had no symbolic treatments of such formal 'error' at his disposal, in part because precise formal studies of indefinite quantifier-alternation came very late to 'syllogistic' logic.

But his analyses of 'transcendent' "Regressus der Bedingungen" in the Antinomies of the first Critique made it clear that he saw through several 'classical' 'mathematical' and 'dynamical' 'existence-of-maximality'-arguments (at least in 'constitutive' contexts: cf., e.g., KdrV, B692).

Subtler and more 'metatheoretic' 'existence-of-determination'-arguments have been adduced since antiquity to justify, in some elusively absolute theory-marginal sense, the 'existence' of a definitive 'criterion of truth', ultimate 'intended interpretation' or other extratheoretic form of semantic closure.

Roughly speaking, such 'existence-of-determination'-arguments derive protometatheoretic claims about 'final causality' (for example) from tacit or explicit counterparts of the quantifier-errors 'existence-of-maximality'-arguments applied to their 'formal' or 'efficient' counterparts.

I will call this argument-type in both cases (for lack of any other name) the *metathe-oretic EA-argument*, from a standard logical usage for *sentences with existential-universal* ('exists-for all') *quantifier-structure*: sentences, in other words, of the form 'there exists an x such that for all y (s(x,y)). (*AE-* or *universal-existential* sentences, by contrast, have the form 'for all x there exists a y such that (s(x,y))).

Here, for example, are two thumbnail templates of such arguments, formulated in terms of "reasons" rather than "causes" ("aitia" meant both in Aristotle's Greek):



- 14 if for everything a *contextual* reason may be found, a *noncontextual* reason 'must' (allegedly) be found for Everything;
- 15 if for everything a 'better' reason may be found, a 'best' reason 'must' (allegedly) be found for Everything.

More often than not, such *EA*-arguments were (and still are) formulated in '*elenctic*', *contrapositive* forms. Such an argument played a prominent role, for example, in one of Socrates' 'refutations' of the 'vitiated' semantic relativity Plato (1969) attributed to Protagoras in the *Theaetetus*, where Socrates fallaciously claimed (in effect) that anyone who *rejects* the *EA*-claim,

- 16 that there 'must' exist a single 'best' objective or intersubjective standard for all human judgments,
 - 'must' also reject a cognate, but quantifier-reversed and weaker, AE-claim,
- 17 that for **a**ll human judgments, 'better' contextual standards **e**xist for evaluation of those judgments' truth.

More subtly fallacious counterparts of such *EA-arguments*—to the effect that "not (there exists an x such that for all y s(x,y))" would 'imply' "not (for all x there exists a y such that s(x,y))"—seem to me to enable 'cogito-like'-arguments which are also implicit in Kant's 'transcendental deduction', a view I will elaborate in the next section.

In a wide range of classical and contemporary metaphysical contexts (justifications of mathematical 'realism', for example), the specious attractiveness of *semantic EAarguments* is roughly equivalent to the specious attractiveness of '*semantic monist*' views they support.

When such 'existence'-assertions are formulated in formally or informally 'stronger' theoretical contexts U, however, (as they are in the next section's 'cogito'-like arguments), parity of reasoning raises analogous 'criterial' questions about the semantic credentials of U. And iterations of such questions and responses generate 'natural' informal counterparts of the introduction's 'metatheoretic ascents'.

Such 'regressive' hierarchies have typically been blocked by (fallacious as well as semantically equivocal) '*modal*' EA-assertions that

18 there 'must' 'exist' a 'unique' last metatheory **U** for **T** which uniquely interprets [itself], and therefore 'proves' or 'grounds' [its own existence]

which 'modally' beg the semantic-monist desiderata in question (with or without the Gödelian 'coding brackets' "[...]", which 'reduce' semantic 'use' to syntactical 'mention').

As the parenthetical remarks in 16–18 suggest, their existence-claims have also been informally begged on the basis of the very 'comprehensiveness' of the theories in question (an attractive but thoroughly dubious enthymeme, as Cantor saw when he called class-theoretic counterparts of such 'comprehension' "inkonsistent").

It *is correct*, for example, that a theory *in a given fixed language* has *fewer* interpretations the more '*complete*' it is (the more assertions it *decides*). This is the fundamental underlying *datum* of a metalogical pattern called '*Stone duality*' (cf., e.g., Bell and Machover 1977, pp. 141–149).

19 But the same 'dual' analysis also establishes that



- (i) theories in *fixed* languages have *more* interpretations the fewer assertions they '*decide*' (the more '*abstract*' they are); and as a corollary of Craig's Theorem (cf., e.g., Chang and Keisler 1973/1972, pp. 84–87), that
- (ii) theories in 'augmented' languages have more interpretations when the terms in which their additional axioms are formulated are 'new'.

Sometimes, however, cognate 'monist' arguments have been begged on the basis of a subtler, tacitly *meta*theoretic argument which anticipated 'cogito'-like arguments (including Kant's 'transzendentale Deduktionen') considered in greater detail in the next section:

- 20 that the position I called *semantic monism* above '*must*' be accepted; for if it is *not*, then
- 21 *incoherence* would be the consequence, and we would not even be able then to *query the claim*.
 - (Cf. the miniature reconstruction of the *Theaetetus*-argument in 16 and 17 above, and related claims that semantic-hierarchical relativism is 'self-refuting', and therefore incoherent).
- 22 Dialectically attractive though such arguments may seem, they were really little more than 'transcendental' second-order *petitiones* and quasi-metatheoretic reassertions of the *supposition* (closely related to 18 above):
 - (i) that any 'sufficiently' 'deep' and 'comprehensive' as well as consistent theory must have a unique interpretation; or its contraposition,
 - (ii) that any 'sufficiently' 'deep' and 'comprehensive' theory without a unique interpretation would be inconsistent.

For not only is the 'inference' just sketched *refutable*, for a wide range of philosophically relevant theories **T** (cf. 28–31 below), in any theoretical venue in which it can be *expressed*. It is essentially an amorphously '*modal*' type-raised *counterpart* of what is being '*deduced*'.

Subtler but analogously dubious 'existence of determination'-arguments also appeared at several points in Kant's 'speculative', 'practical' and 'teleological' 'Deduktionen', usually in contrapositive forms.

Compare, for example,

- 23 'Experience' 'must' have a unique 'transcendental' structure. For many instances of it would otherwise be 'merely empirical'.
- 24 'Practical' judgment 'must' have a unique 'transcendental' structure (at least 'regulatively'). For otherwise, 'our' actions would be governed by 'mere interest'. Notice that
- 25 the first is a 'critical' paraphrase of the simple but dubious argument that 'experience' 'must' be unique, and therefore 'determinate' (for otherwise particular instances of it would be 'mere chance');
- 26 the second is the 'merely' regulative (but comparably anacoluthic) argument that 'the' moral order' 'must' be unique, and therefore 'determinate' (for otherwise particular instances of it would be 'mere casuistry').



(In each of the formulations of 23–26, it may also be worth pointing out that "bestimmen" in ordinary German means "define" as well as "determine").

Notice also that these arguments have (or at least strongly suggest) a tacit quantifier-structure—in the cases of 23 and 25, for example, the 'inference' is that

- 27 if every *particular* experience has a metatheoretic 'determination', a master-metatheoretic 'determination' of 'all' 'experience' must 'exist'; and in 24, that
- 28 if every *particular* action has a metatheoretically defined moral 'valuation', a master- metatheoretic moral 'valuation' of 'all' actions must 'exist'.

(The **bold face** usages in 27 and 28 and the sequel are intended to reflect '**hypostatic**' shifts in each case—from particular object-theoretic '*experiences*', for example, to an 'ultimate' self-referential *Grenzidee* called '**experience**'.)

Observe, finally, that the *quantifier-structures* I have imputed to these 'transcendental' arguments closely resemble that of traditional theological 'arguments from design' (or more accurately, **design**):

29 if every *particular* phenomenon has a metatheoretically defined '*design*', a (*numinous*) master-metatheoretic '**design**' of '*all*' phenomena must 'exist'.

In none of these *limit-arguments* (or '**limit**'-arguments) could one seriously argue that the inference in question is (or 'must' be) metalogically valid.

Whether any of them is a tenable 'deduction' in some broader forensic sense of the word is open to question. But any hypostatic 'limit' of such senses would be deeply 'problematic', in Kant's usage.

Consider, for example, the *analogies in* 23–28. If *they* are (*'forensically'*) '*deducible*' in some 'limiting' hypostatic sense, why not their '*physico-theological' counterparts* in 29 as well?

Finally, one might perhaps hope to recover the validity of such 'deductions' with the aid of 'uniqueness-of-maximality-and-determination'-arguments (cf. 11 above).

If, for example, semantic interpretations of theoretical designs (of whatever sort) formed (what is called) a 'direct system', mathematical arguments would indeed yield a unique 'direct limit' of such a system.

A good historical case can be made that Platonic, Stoic, Neoplatonic and other 'dogmatic' metaphysicians believed that theoretical 'cognition' 'must' be directed with respect to mutual consistency.

But if such 'cognition' is 'intelligible', in a sense outlined below, it is not. To see this, call a 'philosophically relevant' (first-order) theory T 'intelligible' if and only if it is

- 30 'parsable' (its language is countable, and the metatheoretically defined set of Gödel-codes of its axioms is recursive; roughly speaking, it has an arithmetically decidable 'axiomatisation'); and
- 31 'autological': (it syntactically interprets a theory of finite sets or theory of arithmetic 'strong' enough to include a mathematical-induction scheme which permits 'encoding' of its proofs; roughly speaking, it has an arithmetically enumerable 'proof-structure').

The 'philosophical relevance' of



- 32 'parsability' is that no inscrutable metatheoretic 'oracle' (a word actually used in recursion theory) would be needed to recognise the theory's premises; and of
- 33 'autologicality' that the theory can trace out, step by step, its consequence-relations, and pose (but not, Gödel discovered, answer) 'internally' coded counterparts of semantic questions about 'itself' ("Am 'I' 'consistent"'? "Do 'I' 'exist"'?) (cf., once again the first paragraph of the KdrV, cited above).

Together, these two conditions—which are most readily and neutrally formulated in first-order *set*- or *class-metatheories* which permit semantic interpretations of **T**—imply that Gödel's *incompleteness-theorems apply to* **T**.

For from Gödel's insights, it follows then that if one

34 partially orders a 'system' S_T of (metatheoretically) consistent 'intelligible' extensions of a given 'intelligible' theory T by setting $T' \prec T''$ if and only if T' is a subtheory of T'',

Then

- 35 'inverse' (but not 'direct)' limits or maximal 'threads' of the system S_T exist (they are the 'models' or semantic interpretations of T) but
- 36 *such 'limits'*—or '*worlds*' in which *T 'holds'*—*are never unique*. For *any* such *T*, in fact, there will be *as many* such *semantic interpretations* of *T* as there are *real numbers in the* (tacit metatheory's) *set-theoretic continuum*.

Admittedly, the conditions in 32 and 33 are

37 intrinsically metatheoretic with respect to T.

But they also

38 apply equally well to the *set*- or *class*-metatheories just mentioned, as well as a wide variety of other theories which are not first-order.

Efforts to make sense of first- and non-first-order theories **T** would give rise quite naturally, therefore, to

39 'interleaved' or 'interpolated' first-order semantic hierarchies, in which every consistent theory—first-order or not—would 'eventually' find an interpretation.

Such hierarchies offer a first rationale for enlargement of Galileo's 'book' to the metatheoretic *compendia of interleaved commentaries* proposed above, in which *paths* of inquiry would *not* be *directed*, but *ramify ad indefinitum*.

Would inquiry in such hierarchies be dialectically clever, but 'practically', 'speculatively' and 'teleologically' jejune, as Socrates suggested Protagoras' sophistry would have to be?

In the following sections, I will continue to argue that it would not.

3 Reflective inquiry and 'transcendent' "[self]-validation"

As I mentioned in Sect. 2, one way to respond to dispel skeptical hierarchies is to 'bound' them in (allegedly) unique and determinate 'limits', and another to 'diagonalise' over them in (allegedly) stable and determinate 'fixed-points'.

To clarify the latter, I will begin with a sense in which Kant (I believe) proposed



- 1 a 'cogito'-like fixed-point argument (or 'necessity-of-the possibility of'-argument) to 'refute' skepticism about the 'necessity' of a 'uniquely' 'presupposed' structure of a generic 'self,' whose 'intentions' he identified with 'Erkenntnis überhaupt' (the 'metaphysical deduction', usually associated with Kant's claims for the 'completeness' of his 'logical' categories); and
- 2 a second ('cogito'-like) fixed-point argument (or 'necessity-of-the possibility of'-argument) to 'prove' that a uniquely 'schematised' cognitive structure 'must' 'determine' isomorphic patterns in the 'appearances' it 'must' 'experience' (cf. the 'transcendental deduction', usually associated with wider templates of 'intuition' and 'the understanding' which also 'determine' 'the' structures of space and time).

Put somewhat differently, Kant

- 3 'deduced' from the first fixed-point argument above the 'necessity of the possibility' of a uniform structure for 'the' mind's (and 'the minds'') 'cognition' (the 'existence' of which Berkeley had attributed to 'God', and the agnostic Hume later acknowledged in the appendix to the *Treatise* he had essentially begged); and
- 4 'deduced' from the second fixed-point argument the 'necessity of the possibility' of Hume's petitio, as a 'unique' 'precondition' or 'presupposition' for the 'apodeictic' 'necessity' of four-dimensional Newtonian spacetime (which Kant—I will later argue—tacitly acknowledged in the third Critique he had essentially begged).

In effect, I am arguing that Kant believed he had devised more innovative 'dialectical' Augustinian/Cartesian fixed-point-arguments, which would modulate

- 5 assertions a 'Newton of the Moral Sciences' had made about highly normalised forms of "Custom or Habit" into
- 6 more rigorous assertions an 'architect' of a 'Copernican/Newtonian Turn' hoped to make about an even more highly normalised "Architektonik" 'der' Erfahrung.

After I develop this interpretation (which can be considered and accepted or rejected in informal, non-'metalogical' terms), I will

- 7 consider straightforward but non-Kantian *metamathematical* glosses of "*deduction*" proposed in 1, and
- 8 prove a straighforward metalogical proposition which bears on the tenability of metalogical analogues of the 'results' Kant thought he had achieved, as well as the 'transcendental methods' he devised to 'deduce' them.

I will begin with an attempt to explain what I meant above by a "cogito'-like" argument", and outline the four historical examples mentioned in the essay's introduction.

I will call an (formal or informal) argument "'cogito'-like" if it 'satisfies' the following conditions (cf. 10 through 15 below).

The argument employs

- 9 notions (tacitly or explicitly) posited in a 'theoretical' framework **T** and 'wider' 'theoretical' framework **U**; and
- 10 assertions about 'existence' and 'interpretation' of T and notions in T, which may be formulated in T as well as the 'wider' framework U.



Moreover, the argument's

- 11 'derivations' and 'demonstrations' of assertions in **T** (from 'obvious' or 'universally granted' 'first principles' of **T**) also hold in **U**; and its
- 12 'epistemic' and 'ontological' assertions about what is 'conceivable' in **T** or 'exists' in **T** (e.g.) can be formulated in **T** as well as **U**, and sometimes 'proved' or 'refuted' (from equally 'obvious' 'first principles') in **U**; Finally, the argument (ostensibly)
- 13 'derives' the 'epistemic' and/or 'ontological' assertion Φ about T from 'first principles' of U (or 'confutes' the 'skeptical' negation Σ of Φ in U);
- 14 'concludes' from this that the 'epistemic' and/or 'ontological' assertion Φ is also 'derivable' from 'first principles' of T (so Σ 'must' also be 'refutable' in T).

Admittedly, this 'definition' is long and convoluted, but so, I would argue, were its historical prototypes. Part of my larger argument in the sequel will be that conceptual (or 'critical') 'theory/metatheory'-distinctions offer the only tenable remedies for such convolutions and ambiguities.

The best informal rationale or plausibility-argument for the characterisation may be the reconstructions it provides for study of the three or four well-known 'arguments' from the history of early modern philosophy, mentioned earlier: Descartes' (1973-1978) "cogito"-argument, Berkeley's 'master argument' and the two best-known Kantian 'transcendental arguments' (or 'deductions') in the Analytic of the first Critique.

Before offering these reconstructions, I would like to apologise in advance for my particularly extravagant use in them of single ('scare')-quotation-marks.

They are not, I believe, an affectation. For my underlying point in 15 through 26 below will be that such 'scare-quoted' nouns, pronouns, adjectives and adverbs may be subjected to conceptually relevant forms of metalogical relativisation and metatheoretic ascent.

In the case of *Descartes*' (not entirely) original "cogito",

- 15 **T** is 'my' theory of 'myself';
- 16 U is 'god''s counterpart of 'this' theory; and
- 17 Φ is an assertion of the 'ontological' 'existence' and unique interpretability of 'my' theory of 'myself' (which 'I' had epistemically 'doubted').

In the case of **Berkeley**'s (more original) 'master argument',

- 18 **T** is 'the' (theoretical) '*idea*' of a given 'external object';
- 19 **U** is 'the' theoretical framework in which '*god*' or another 'spirit' confers 'existence' on the 'object' of **T** by 'perceiving' (or 'conceiving', or 'interpreting') it; and
- 20 Φ asserts the 'existence' or 'interpretability' of the 'object' of this well-defined 'idea' T (the complex 'object' determined by a "mite"'s idea of its "foot", for example, in one of his more whimsical examples).
 - In the case of *Kant*'s (implicitly 'Cartesian') '*metaphysical deduction*',



- 21 **T** is 'the' theoretical framework of 'forms of intuition' and 'categories of the understanding' which 'constitute' 'Erkenntnis' ("knowledge");
- 22 U is 'the' ('bloß regulative') "systematische Einheit aller möglichen empirischen Verstandeshandlungen" (('merely' regulative) "systematic unity of all possible actions of the understanding") which Kant characterised as an "affair of reason" "Geschäft der Vernunft" at KdrV, B692, cited above); and
- 23 Φ asserts 'the' 'unity', 'completeness' and consequent 'existence' of 21's 'forms of intuition' and 'categories of the understanding' **T** which 'determine' or 'constitute' corresponding 'interpretations' of 'experience'.
 - In the case of *Kant*'s (implicitly 'Newtonian') 'transcendental deduction',
- 24 **T** is 'the' theoretical framework or conceptual structure of 'the' 'necessary' *Newtonian-scientific 'preconditions*' which '*constitute*' '*Erkenntnis*';
- 25 U (once again) is 'the' ('bloß regulative') "systematische Einheit aller möglichen empirischen Verstandeshandlungen"; and
- 26 Φ asserts 'the' 'existence', 'completeness' and 'transzendentale Einheit' of 24's Newtonian-scientific 'preconditions' for the framework T, which 'determine' or 'constitute' unique 'interpretations' of this framework in 'experience'.

In each of the cases just canvassed, I will argue, the assertion Φ 's *initial plausibility* derives from a *dual premise*:

- 27 that such an implicitly 'metatheoretic' U 'exists'; and
- 28 that U's 'attributes' ensure that all the relevant assertions of 'existence', 'unicity' and 'interpretability' of T encoded in Φ are 'provable' or 'derivable' or 'demonstrable' in U:

In *Descartes*' case, these premises followed (via the 'diagonal' *pons asinorum* of the 'circle') from the *veracity* and '*perfection*' of (Augustine's, Anselm's and Aquinas') 'god' (1986; 1978).

In *Berkeley*'s, their counterparts followed from the kindness and hermeneutic agility of a somewhat more 'personal' Anglican 'god', whose 'existence' he inferred from a 'design'-argument.

In Kant's case,

- 29 the ('merely' regulative) 'existence' of the theoretical framework **U** (a secular *Vernunftidee*) is effectively secured by a 'critical' 'design'-argument, and
- 30 the 'existence', ('transcendental') 'interpretability' and ('constitutive') 'unity' of 'experience' Φ attributes to T is secured in U by the 'unicity', 'completeness' and 'universality' Kant hoped he had 'deduced' from
- 31 the 'unicity', 'completeness' and 'universality' of **T**'s 'forms' and 'categories' (in the 'metaphysical' deduction), and
- 32 the 'unicity', 'completeness' and 'universality' of the theoretical framework **T** of Newtonian science (in the 'transcendental' deduction).

Whatever the merits of such "cogito'-like arguments" assertions that the 'existence'-and 'unicity'-claims Φ are 'demonstrable' in U, I will also argue that

33 the 'probative' force and philosophical purchase of these arguments derive from tacit 'metatheoretic' assumptions in each case that $[\Phi']$'s demonstrability in U]



implies [its demonstrability in T] (for metalogical counterparts of these assertions, cf. 4.6 and 4.32 below).

Shamans and high priests have told us since time immemorial, after all (but without philosophical 'proof'), that 'gods' and other higher 'unities' U assure us 'we' exist, and that our 'experiential' theories T 'make sense' in such U.

The 'philosophical' (as opposed to confessional) 'purchase' of 'cogito'-like arguments, by contrast, is to formulate subtler and more 'secular' 'experiential' conceptual apparatus T which 'provably' ensure us that 'we' exist, and that T 'makes sense' in T.

The underlying project of 'cogito'-like arguments, in other words, is to broaden conceptual and sensory 'experience' so that it 'validates' and semantically 'interprets' 'itself'.

Be that as it may, the relevance of the tacit premise 33 in the case of *Descartes*' "*cogito*" seems to me supported, or at least illustrated, by

- 34 the centrality of Descartes' otherwise abstruse (and classically quite 'dogmatic') attempts to secure a '*criterion of truth*', where 'truth' is identified *de facto* with 'demonstrability in U');
- 35 the tenacity of his efforts to 'demonstrate' that the epistemically and ontologically more 'perfect' 'god' (or more 'divine' 'theory' U) does not 'conceal' such 'truths' from its less comprehensive 'image and likeness' T; and
- 36 the swift emergence and enduring prominence of debates about the rhetorically begged 'Cartesian circle', construed in the present context as a kind of 'meta-verification' (whose formal counterpart would have to be formulated in a metatheory for \mathbf{U}) that [demonstrability of $\mathbf{\Phi}$ in \mathbf{U}] implies [demonstrability of $\mathbf{\Phi}$ in \mathbf{T}].

(In the case of 35, for example, the 'Cartesian circle' may be paraphrased as a 'fixed-point'-assertion that "'god' (or U) does not deceive us that ['god' (or U) does not deceive us]").

In *Berkeley*'s "*master argument*", the "tacit premise" sketched in 18–20 above is a tacit conflation of

- 37 what 'you' or 'I' or **T** (may hypothetically be posited or 'thought' to) "perceive" or "conceive" in **T** ("[that] a tree ['exists'] with 'nobody by to perceive it"); and
- 38 what 'god' (or **U**) (presumably) "perceives" or "conceives" about this thought-experimental framework **T** (that "you yourself [did] perceive it all the while").

To see that this 'tacit premise' is *problematic* in Berkeley's system, recall that a finite "*spirit*" can only have a "*notion*" of (but cannot "perceive" or "conceive") another "spirit" s "perception" or "conception".

The 'master argument's triumphantly adduced auxiliary assumption, therefore—that "you yourself did perceive it all the while"—may or may not be 'demonstrable' in **U**. But is not even 'expressible', much less 'demonstrable' in **T**, on Berkeley's own principles. (In quasi-Berkeleyan terms, **T** might at most have a semantically underdetermined 'notional' 'sign' for this 'second-order' self-referential assertion.)

The relevant Φ in the 'master argument'—which seems to have little intrinsically to do with "*trees*"—might also be paraphrased as an assertion that "I" (or "you" or 'god') "perceive" or "conceive (something)" (and thereby confer 'existence' on it).



If so, then ('idealist') assertions that such 'intentional' acts confer 'existence' might be interpreted as natural generalisations of ('rationalist') 'cogito-arguments'.

Berkeley, finally—who acutely critiqued the contradictions implicit in semiformal attempts to 'define' Leibniz' 'infinitesimals' and Newton's 'fluxions'—also

- 39 offered interesting ('sign'-theoretic) 'coherentist' reasons to accept the framework of Newtonian science, in the form of
- 40 a *grand scheme of intersubjective interpretations* between 'all' 'our' (otherwise disparate) 'signs', "perceptions" and "conceptions" coordinated by his kindly Anglican god (the '*design*'-argument mentioned earlier).

To the extent *Kant* acknowledged that (what he called) "*Erkenntnis*"—which is not quite the same as 'knowledge' ("*Wissen*") in German—might '*presuppose*' secular forms of *intersubjective interpretation* between cognitive agents' '*Begriffe*' (and did not simply *beg* their conceptual or intersubjective '*Einheit*'),

- 41 there may therefore have been something 'Berkeleyan' about the 'design'-argument I attributed to Kant in 29 and 30 above; and
- 42 Berkeley might offer another conceptual bridge from Descartes' and Leibniz' 'rationalisme' to Kant's "Kritizismus", despite the latter's 'refutation' of his "schwärmender Idealismus".

In the next section, I will consider in some detail the senses in which Kant's "deductions" seem to me to fit 'cogito'-like templates of the sort sketched above.

Here I will close with a suggestion that his 'transzendentale Methode' in general (and the 'deductions' in particular) fit another, complementary 'cogito'-like template, or prescription: that of an 'Archimedean' - or 'fixed-point' confutation of 'regressive' skeptical 'doubt'.

More precisely, I will attempt to interpret this 'Methode' as a 'dialectical'

- 43 'proof' (in some sense) that (something called) 'synthetic a priori cognition' is a 'proof' (or 'sufficient reason', or 'transcendental' 'Archimedean point') for [itself];
 - or, in more or less equivalent 'adverbial' terms, as a collection of roughly cognate 'proofs' that
- 44 cognitive agents synthetically a priori 'prove' or 'determine' that [cognitive agents synthetically a priori 'prove' or 'determine'] (cf. the gloss of the 'Cartesian circle' following 36 above).

For if not.

45 [cognitive agents fail synthetically a priori to 'prove' or 'determine' that

[cognitive agents fail synthetically a priori to 'prove' or 'determine' that

[cognitive agents fail synthetically a priori to 'prove' or 'determine' that

[cognitive agents fail synthetically a priori to 'prove' or 'determine' that [....]]]].



But such a "*Regressus*" (a 'descending' variant of ancient academic skeptics' 'ascending' 'problem of the criterion') would (Kant believed)

46 render 'synthetic a priori' 'proof' or 'determination' 'impossible' (i.e. counterfactual, Kant's standard 'critical' gloss of 'impossibility' in such contexts; cf., e.g., B 101, 137 and 628).

But

47 this—Kant argued—would be 'absurd'. For 'synthetic priori' 'determination' is not only regulative (and in this sense 'possible'). It is also "konstitutiv" (and in this sense 'necessary'). For its 'constitutivity' is instantiated by the ['synthetic a priori' capacity (Vermögen) to formulate this argument]'

In effect, I believe, such a 'fixed-point'-argument may be construed as an attempt reconfigure 45's non-well-founded 'descent' into a kind of 'transcendental circle', in which the T of 33 above 'validates' T's 'experience' in T. (More of this later.)

In metalogical opposition to such metaphysical 'circles' (attempts to 'collapse' 'regresses' such as the one which would otherwise emerge in 36), I will argue in the next section that

48 notions of 'doubt', 'certainty', 'perception', 'conception', knowledge', 'syntheticity', 'apriority', 'cognition' and 'determination', for example, cited earlier—are 'liminal', in the sense that they straddle the 'limines' or 'thresholds' of whatever they 'intend' or 'qualify'.

In quasi-Kantian terms, one might call them 'locally' or ('situationally') "transzendent". Alternatively, in language Georg Cantor introduced in a letter to David Hilbert more than a century later, one might call them 'locally' or 'situationally' "inkonsistent".

To the extent one can make *metalogical* counterparts of such notions precise (more possible than many readers might think), [self]referential attributions of such properties 'generate' 'semantic paradoxes' and corresponding patterns of iterated metatheoretic 'ascent' and (what might be called) intentional 'descent'.

In Kant's case—and in particular in the allusion to "[this very argument]" just above—such notions will only be 'counterfactual' (or non-'experiential'). They will be ('locally') transzendente Vernunftideen, or perhaps more accurately 'Vernunftprädikate' (predicates of 'pure reason').

In plain German: "'die' **Rahmenbedingungen** der Kantischen Argumente über 'die' Bedingungen 'der' 'Möglichkeit' 'der' Erfahrung sind lauter **Vernunftideen**..."

(In even plainer English: "'the' boundary-conditions of Kantian arguments about 'the' conditions of 'the' 'possibility' of experience are a bunch of ideas-of-reason...")

'Worse' (or in my view, 'better'): metatheoretically defined counterparts of such 'frame'- or 'boundary-conditions' will always be 'locally transzendent', in relative terms, with respect to any given 'level' of what they (e.g.) 'doubt', 'conceive', 'cognise' or 'determine' to 'be the case'.

In particular, if one tried to 'halt', with Kant and his neo-Kantian successors, the 'object-theoretic descents' in 44 and 45—or the 'metatheoretic ascents' they mirrored—in an effort to 'define' 'experience',

49 the terms in which one claimed to do so would become (*locally*) '*transcendent*' with respect to *any* fixed 'neo-Kantian' object-theoretic notion of '*experience*' one



might propose, in the sense that

50 the terms in which one proposed to 'define' a given 'critically' demarcated notion of 'experience' would not only fail to be 'experiential'; they would be 'experientially' inexpressible, and 'paradoxically' as well as 'conceptually' underdetermined by what they defined.

4 Reflective inquiry and "das Schicksal der Vernunft"

In this section, I will assimilate the 'locality' and essential incompleteness of 'critical' inquiry to Kant's 'fate of reason', poignantly characterised above in the opening lines of the Kritik der reinen Vernunft.

I will attempt in this section to

- 1 interpret more carefully the 'begged' implication in section 3 for the "'cogito'-like" reconstructions of *Kant's 'transcendental' method* and 'deductions';
- 2 characterise the views and analyses attributed above to Descartes, Berkeley and Kant as '*idealisms*' with successively broader and more '*deductive ranges*' of what is '*conceived*' in 'experiential' **T** and 'higher' **U**;
- 3 offer 'skeptical' as well as 'transcendental' arguments for assimilation of such analyses' 'assertions' and 'demonstrations' in T and U to counterparts of them in first-order metatheoretical hierarchies;
- 4 review well-known proofs that metalogical counterparts of the 'dialectical' inferences and implications in 3.14 and 3.33 are either 'locally' refutable or 'locally' inexpressible in such hierarchies' 'theoretical frameworks' T; and
- 5 adduce (and sketch the proof of) a simple but relevant bit of *metalogical 'folklore*': that *no 'faithful' interpretation of a 'weaker' theory* **T** *can be found in a 'higher' theory* **U** *which is 'strong' enough to prove the consistency of* **T**.

The breadth of the "Rechtmäßigkeit" (cf. B116) Kant associated with "Deduktion" is well-known, as is the notion's association with 'mediated' forms of reasoning (cf. B761), and the role of "transzendentaler Deduktion" in the (curiously 'subsumptive' or 'reflective') "Erklärung der Art, wie sich Begriffe a priori auf Gegenstände beziehen können" (B117).

In what follows, I will draw in fact on the latter quotation to argue

6 that (ostensibly) more latitudinarian notions of Kantian 'deduction' do, after all, tend to close down on straightforward first-order consequence-relations in metatheoretical hierarchies which interpret such 'predication' and 'subsumption';

and that a good way to sustain Kantian 'critical' insights (between the 'T's and 'U's of Sect. 3, for example) might be to

- 7 'localise' and iterate them, in the form of 'locally critical' theorylmetatheory distinctions;
- 8 exploit them as generators of 'heuristic' notions of object- and concept-formation in the metatheoretic hierarchies they generate; and



9 interpret their unending extensions and ramifications in such hierarchies as 'regulative' traces and counterparts of Kant's "fate of reason" ("Schicksal der Vernunft").

I suggested earlier that Kantian or quasi-Kantian reconstructions of the generic 'cogito'-like assertions Φ in Sect. 3 might be

- 10 ('metaphysically' 'deductive') claims that 'the' 'completeness' and 'unity' of 'forms of intuition' and 'categories of the understanding' 'determine' (or 'constitute') T and its 'unique' isomorphic counterparts in 'experience'; and
- 11 roughly cognate 'transcendentally' 'deductive' claims that 'the' 'completeness' and 'transzendentale Einheit' of 'intuition' and 'understanding' 'determine' (or 'constitute') T and its 'unique' isomorphic counterparts in 'experience'.

I also suggested that *all* of the *four* 'cogito-like' arguments outlined in Sect. 3 decompose into

- 12 (relatively uninteresting) 'demonstrations' that interpretations of 'thought', 'perception' and 'conception' in T (may) 'exist' in 'higher-order' frameworks U; and
- 13 (interesting but begged) 'demonstrations' that corresponding interpretations of 'thought', 'perception' and 'conception' in T ('must') 'exist' in the 'experiential' framework T.

In his 'metaphysical' and 'transcendental' variants of these arguments in the Analytik, Kant sought to work in a resolutely secular U, and dispense with conceptual 'mathematical' and 'dynamical' 'physicotheological' arguments in his 'constitutive' rationales for the "transcendental unity" of "Erkenntnis" to "Erfahrung".

Yet he insisted over and over again in the *Dialektik* of the first *Critique* that the "systematic unity" of 'Reason' at KdrV, B692—the theoretical framework I have called U—'must' remain a 'merely regulative' but conceptually significant "Aufgabe", an *Ideal* which zieht uns hinan (so to speak) at the marginal horizons of such (scientific) 'experience'.

Logicians familiar with the *semantic purchase* of (relatively) 'higher-order metatheories' such as **U** know that part of this 'purchase' is their 'ability' to 'define' 'internally canonical' interpretations of 'weaker' subtheories such as **T**, and it is in that sense that such **U** may be 'locally' 'strong' enough to account for 'scientific' 'boundary-conditions' of 'experience' and 'experiment(ation)'.

But they *also* know, on the evidence of Gödel's results, if nothing else (cf. the proof of 38 below) that

14 the 'strength' of such theories is 'theory-relative' (or at least theory-mediated')—
if one stipulates that U is '(intrinsically) stronger' than T iff U proves the consistency of T, the (nonlinear) ordering of such 'strength' is neither reflexive nor symmetric.

In particular, the [self]-sufficiency (and implicit [self]-validation) of Laplace's famous remark (quoted third-hand in Victor Hugo's autobiography) that "[s]ire, je



n'avait pas besoin de cette hypothèse(-*la*)" might therefore have to be more carefully calibrated than Laplace knew.

If all Laplace meant was that he had no 'need' of (secular *or* numinous) versions of (what Kant called) "*the unconditioned*" ("*das Unbedingte*"), he was quite *right*. But if he (or Kant) meant

15 that his own or anyone else's intelligible formalisations of mathematics or mathematical physics could uniquely 'determine' their own (mathematical or conceptual) boundary-conditions, much less their own 'intended' semantics,

he was subtly but deeply *wrong* (and Descartes, Berkeley and Kant with him). In the sequel, therefore, I will also continue to interpret

- 16 Kant's 'transcendental deductions' as failed attempts to find such an '*Archimedean point*' of [self]-validation on the rasor's edge between Newtonian science and its elusive 'hypotheses' (which Newton had tried to disavow); and
- 17 the *second* and *third Critique*'s qualifications and relativisations of such claims as tacit admissions that his efforts to locate this 'transcendental' limit had turned out to be more '*problematic*' (a Kantian term of art) than he had hoped.

Whatever the merits of Sect. 3's "cogito'-like arguments", therefore, the tacit petitio of their claims that

18 [demonstrability of semantic assertions **Φ about T** in **U**] 'followed' from [demonstrability of such semantic assertions **Φ about T** in **T**]

remained present in Kant's Architektonik.

In the remarks following 13 in this section, I appealed to analogies with *metalogical hierarchies* which have run in the background throughout the essay's first two sections, and it is past time to offer the rationale for these analogies.

It would follow from the heuristic principles I have sketched that *no one*—on pain of circularity or semantic paradox—could claim to offer a 'hypothesis-free' 'deduction' that first-order 'deductions' (much less metatheoretical hierarchies which employ it) are 'canonical'.

One can, however, offer 'skeptical' variants of Kant's 'critical' or 'transcendental analysis' which suggest that 'eventual' recourse to first-order theories in 'ramified' metatheoretical hierarchies may be ineluctable.

The 'analysis' I have in mind is

- 19 'skeptical' in that it 'suspends judgment' between (alternative interpretations of) 'abstract logics' and their 'consequence relations';
- 20 'critical' in that it postulates (on pain of semantic paradox) theory/metatheory-distinctions to 'make (syntactical) sense' of such interpretations;
- 21 'transcendental' in that recurrent theory/metatheory-distinctions of this sort 'regulate' (but do not 'determine') (semantic) efforts to assert such alternative interpretations' 'existence';
- 22 'eventual' in that such interpretive predications of semantic 'existence' may 'eventually' and 'economically' be adjudicated in first-order metatheoretic set- or class-theories;



- 23 'ramified' in that 'partial' interpretations of any 'intelligible' theories which 'encode' their own syntax and 'consequence-relations' in 'inductive' ways branch without limit; and finally
- 24 '*ineluctable*' in that '*all*' *sufficiently* '*finitary*' theories introduced to examine and interpret *other* theories in such hierarchies are semantically *indistinguishable from first- order counterparts*, by a remarkable theorem of Per Lindström. In somewhat plainer English:
- 25 the simplest and most 'natural' way to *understand* abstruse theoretical frameworks in metatheoretical hierarchies is to *interpolate* set- or class-theories which *interpret* them:
- 26 'intelligible' set- or class-theories which satisfy 'finitarity'-conditions are semantically indiscernible (via Lindström's theorem) from their first-order counterparts;
- 27 hierarchies of recursively axiomatisable first-order languages offer flexible, communicable and ontologically neutral 'common languages' and frameworks of interpretation for 'rational' discourse; and finally,
- 28 such languages' 'ontological neutrality'(cf. 27) and 'skeptical' 'suspension of judgment' (cf. 19) offer 'natural' conceptual interpretations of Kantian "Reinheit" and "apriority", as well as physicists' (equally elusive) ideals of 'controlled experimentation' and 'experimental isolation'.

There are many reasons, of course to dismiss such analogical conjectures and 'skeptical transcendental arguments' out of hand.

The first is that 'ordinary language' is 'obviously' not first-order, so claims that 'it' is' are absurdly and obviously reductive.

Of course 'ordinary language' is not first-order, at any given level of (potential) syntactic and semantic interpretation.

My *conjectures* in 19–24 above—which I clearly cannot '*prove*' ('Chinese rooms' are lurking about here somewhere)—are

- 29 that the limitless syntactical and semantic complexities of 'ordinary linguistic' usages may be *interpretable* in comparably limitless syntactical and semantic complexities of *first-order hierarchies*; ('hand over hand', so to speak); and
- 30 that 'sufficiently' skilled speakers of 'ordinary languages' often *improvise 'locally'* adequate *finitary substructures of such hierarchies in 'real time*' (otherwise—as non-native speakers well know—their interlocutors will 'give up' on them).

A second objection is that one has seen unconvincing versions of such views before—in the writings of logical positivists, for example, or in 'Quinean' assertions that 'everything' 'can be expressed' in 'set theory'.

On the account I wish to defend, 'experience'—including linguistic experience—is certainly much broader than anything I am aware of in the writings of logical positivists. It is also broader than "set theory", or at least any particular set theory.

For 'set theories' (in my view at least) are just ontologically neutral way-stations for attempts to talk about 'predication'—what Kant would have called "Subsumtion", or "das Besondere als enthalten unter dem Allgemeine zu denken" in B XXVI and B XXXII of the third Critique.



And 'critical' theory/metatheory distinctions, I would argue, are inevitable concomitants of attempts to *iterate* such '*predication*', and heuristic metatheories can be *interpolated* in such iterations almost at will.

Such iterations occur 'naturally' in hierarchies of 'stronger' (possibly non-first-order) metatheories, with the proviso that assertions about such metatheories' consistency are 'liminal' and remain open to further ('skeptical') inquiry.

The most 'natural' way to clarify such consistency-questions, in turn, is to seek semantic interpretations for them in yet-'stronger' theories of 'predication'. And the simplest and most 'natural' venues for such searches are 'stronger' as well as linguistically augmented first-order set-theories.

Russell's early *type-hierarchies* were '*linear*' and begged questions about '*the*' limits of their '*types*' in ways which were anticipated by his own *paradox* and Cantor's observations about '*Inkonsistenz*', and were clarified later by metalogical observations of Skolem, Gödel and Henkin.

Wondrously subtle refinements of such hierarchies have been and will continue to be devised. But the more *open* and *flexible* they were, the more 'natural' their interpretations in the aforementioned linguistically enhanced set-theories became.

Such remarks are obviously 'heuristic' and subject to correction and revision. But they may make a provisional case for a conjecture outlined earlier: that 'everything' we talk about and endeavor to interpret 'is' (or 'may as well be') 'eventually' first-order. In defense of this heuristic conjecture, finally, I would

- 31 appeal again to the *openness*, *generality and intensional interpolability* of metatheoretic hierarchies, given that *extensional* canvasses of potentially infinite evidence are beyond finitary "*seekers*" ken; and
- 32 appeal finally to their *pragmatic utility*, in 'ordinary' as well as metaphysical'discourse', as naturally '*interleaved*' venues for *critique* and *interpretation* of everyday 'intentionality'.

(My favorite thumbnail-critique of this sort is Wittgenstein's (1972) uncharacteristically disarming remark in Über Gewißheit (ÜG, 12), that "[m]an vergißt eben immer den Ausdruck 'Ich glaubte, ich wüßte es'" ("one always forgets the expression 'I thought I knew"'…)

The 'subreptively' self-referential problem Kant sought to solve may be characterised as follows.

A genuinely 'a priori' 'Deduktion' would have to be

- 33 *a conceptually idealised or 'isolated'* (*thought*)-'*experiment*', independent of 'all' 'experience' and 'all' 'external' ('merely empirical') verification;
- 34 *a 'deductive' act* (in accordance with some sort of generalised 'consequence'-relation) *invariant* under 'all' changes of metatheoretic boundary-conditions; and most '*problematically*' (once again)
- 35 a 'fixed point', 'Archimedean point' or 'self-validating' 'demonstration' (in the sense once again of some sort of generalised 'consequence' relation), with respect to 'all' attempts to 'refute' it or 'call it into question'.

(It might also be assumed to 'prove' its own ['provability']—a curiously unproblematic assumption in metalogical contexts, as Löb observed (cf. Smorynski 1977,



p. 845), but one which also turns out to have little 'transcendental' force or proof-theoretic purchase).

What it *could not 'prove*', or even 'express', remained, as before, its own 'existence', 'interpretability' or 'consistency', much less the 'necessity' of the 'possibility' of its own 'existence', 'interpretability' or 'consistency'.... (Kant's cogent observations about the *ontological ambiguity of 'existence*' might be more relevant to 'transcendental' analyses than he realised).

To the extent such 'proofs' were truly 'isolated', for example, 'we' could not even adduce stable thought-experimental evidence to 'prove' that they would not also be nonstandard (in quasi-Kantian jargon, that non-well-founded "intensive Gröβen"—metalogical counterparts of Hume's 'problem of induction'—might occur in their formulation).

I will conclude this section with a brief semiformal sketch of the 'folklore'-result promised above in 1.17 and 4.5, which may clarify the nature of the barriers devisers of philosophical 'cogito'-like arguments have faced. I've elided inductive definitions and blurred notational distinctions to simplify the exposition, but any competent logician can bring them into sharper focus.

The definitions first. A 'syntactical interpretation' E of a (not necessarily first-order) theory T in another theory U is a systematic translation of formulae ϕ of T into counterparts ϕ^E in U such that

36 [**T** proves φ] (metatheoretically) *implies that* [**U** proves $\varphi^{\mathbf{E}}$] in $\mathbf{T}^{\mathbf{E}}$, where $\mathbf{T}^{\mathbf{E}}$ is the collection of such translates $\varphi^{\mathbf{E}}$ for φ in **T**.

Such a syntactical interpretation E is faithful if and only if

37 [T proves ϕ] if and only if [U proves ϕ^E] for all ϕ in T.

Since the 'begged' metatheoretical assumptions of Sect. 3's 'cogito-like' arguments asserted that the identity translation faithfully interprets **T** in **U**, the following may be relevant.

38 **Proposition**.

Suppose **T** is a consistent first-order theory which *syntactically interprets Peano's arithmetic*, and **U** is a (metatheoretically) '*stronger*' consistent theory which *proves* that [**T** is consistent]. *Then no syntactical interpretation* **E** *of* **T** *in* **U** *faithfully interprets the assertion that* [**T** is consistent] *in* **U**.

Proof (Sketch)

Suppose (for contradiction) that a given syntactical interpretation \mathbf{E} did *faithfully* interpret the assertion that [\mathbf{T} is consistent] in \mathbf{U} . Then [\mathbf{T} is consistent] would be provable in \mathbf{U} , by hypothesis, and the biconditionals [[\mathbf{T} is consistent] *iff* [$\mathbf{T}^{\mathbf{E}}$ is consistent] *iff* [\mathbf{T} is consistent] *iff* [\mathbf{T} is consistent].

So

39 $[T \text{ is consistent}]^E$ would be provable in U.

But it is a consequence of Gödel's original incompleteness results (cf. Bell and Machover 1977, or Smorynski, pp. 821–865), that [T is consistent] is **not** provable in the theory T.

The assumption that ${\bf E}$ is *faithful* and the remarks in the first paragraph would therefore yield that



40 [T is consistent]^E is **not** provable in U,

so we have derived a (metatheoretic) *contradiction* from the assumption that such a faithful interpretation **E** of **T** into **U** exists.

The *informal* originals of the assertions Φ in the proposition made *informal semantic claims*—in 'ordinary languages' (whatever they are)—about "existence", "interpretation" and "conceivability" of 'experiential' frameworks, and I have assimilated such frameworks to *formal theories* \mathbf{T} .

'Analytic realists' who dismiss such assimilations would presumably argue that such assimilations are inapt ('category-mistakes', so to speak)—that "analogy" and "assimilation", for example, are not "identity".

(The ancient skeptics' response to this particular argument was that 'we' might not be able to discern the difference—arguably an anticipation of 'equivalence-class'-arguments employed in the proof of Leon Henkin's completeness-theorem (Chang and Keisler 1973/1972, pp. 61–67)).

I would respond that the intense *eristic attention* devoted to Descartes' 'cogito', Berkeley's 'master argument' and Kant's 'deduction(s)' tacitly suggests that they were

- 41 the most *vulnerable* as well as '*ambitious*' thought-experiments in their authors' work,
 - to which I have added a sustained argument that
- 42 the premises 'begged' in their 'proofs' and 'deductions' become refutable in reasonable metalogical counterparts.
 - In the case at hand, moreover:
- 43 the notions of *syntactical interpretation* I have introduced are rather broad (they apply to theories which are *not* necessarily first-order, for example);
- 44 there are 'good (metalogical) reasons' to assimilate the 'semantic' claims of the informal assertions Φ to consistency-assertions about first-order counterparts of such T; and
- 45 'skeptically transcendental' arguments for the recurrence of first-order interpretation in 'ramified' metatheoretical hierarchies offer correlative 'good reasons' to consider the methodological relevance of such constructions.

I will finish in the same spirit with a final 'heuristic' argument (which may be all an honest skeptic should try to offer).

As I've suggested earlier, two interrelated hypotheses have animated this essay: that

- 46 'precise' 'transcendental' assertions of [self]-validation are "transcendental illusions";
 - and that
- 47 *semantic paradoxes* and *metalogical analyses* devised to accommodate them have *hermeneutic value* as thought-experimental *refinements* of their metaphysical and epistemological ancestors.

Such 'hypotheses'—along with their companion-of-the-route, the "metalogical turn" postulated above in 1.1–1.3—cannot be 'proved'. Measured against [their own] criteria, they may be partially and provisionally sustained, but they will never be 'conclusively' (much less 'ultimately') 'secured'.

But these reflections suggest two more *hypotheses*. The *third* is that



48 *critical awareness itself* (and in particular *critical [self]-awareness*) may be a (*'merely regulative'*) *value*, and Keats' *"negative capability*" (Keats, p. 277) one of its *marks*.

Such an 'awareness', for example, might (partially) reconcile

- 49 the apparent *incompleteness* of Kant's '*practical reason*' with the *underdetermination* of its '*speculative*' counterpart;
- 50 the 'regulative Ideen' of Kant's 'moral law within us' with our 'desires' to 'understand' the 'starry heaven above us'; and
- 51 the Würde and Schicksal of Kant's "reasonable beings" ("vernünftige Wesen"), with the 'dignity' and 'fate' of Pascal's roseaux pensants ("thinking reeds") in a "univers" which "n'en sait rien".

The last hypothesis is that

- 52 ideals of 'heuristic' inquiry and 'intelligible' verification and concept-formation may be preconditions for 'freedom', 'autonomy' and mutual 'respect'; and therefore that
- 53 recourses to such 'merely' regulative ideals may be 'reasonable beings' cradle gifts as well as their 'fate(s)'.

5 Reflective inquiry and 'Das' Reich 'der' Zwecke

Consider the following (fairly representative) passage, taken from a section in the second *Critique* entitled:

Wie eine Erweiterung der reinen Vernunft in praktischer Absicht, ohne damit ihre Erkenntnis als spekulativ zugleich zu erweitern, zu denken möglich sei....

Hier werden sie [praktische Ideen, wie Freiheit, Unsterblichkeit, Gott und das höchste Gut] immanent und konstitutiv, indem sie Gründe der Möglichkeit sind, das notwendige Objekt der reinen Praktischen Vernunft (das höchste Gut) wirklich zu machen, da sie ohne dies transzendent und bloß regulative Prinzipien der spekulativen Vernunft sind, die ihr nicht ein neues Objekt über die Erfahrung hinaus anzunehmen, sondern nur ihren Gebrauch in der Erfahrung der Vollständigkeit zu nähern auferlegen.

KdpV, B 241, 244 (last emphasis mine)

How an Extension of Pure Reason in Practical Intention is Possible to Be Thought, without at the Same Time extending its Cognition as Speculative....

Here they [practical ideas, such as freedom, immortality, god and the highest good] become immanent and constitutive, in that they are grounds of the possibility for making the necessary object of pure practical reason (the highest good) *real*, since without this they are *transcendent* and merely *regulative* principles of



speculative reason, which impose on it not the obligation to accept a new object beyond experience, but only to bring its use in experience *closer to completeness*.

Such appeals to "transcendent and merely regulative principles" provided later commentators and metaphysicians with a textual basis for assorted 'two-truths'-interpretations of Kantian metaphysics, and the late-nineteenth century Kant-scholar Hans Vaihinger, in particular, with one of the prototypes for his "Philosophie des Als Ob" (the most striking basis for such readings may perhaps be found in KdU, 1974 §76, B 339–344).

In the first *Critique*, Kant employed such "merely *regulative* principles" as *ladders* to '*higher*' forms of 'truth' (cf., e.g., B 83–87, 185, 269 and 670). But he never explicitly acknowledged—except for the *Dialectic*-passages devoted to '*intelligible causes*' (cf. *KdrV*, B366 ff.) ('critical' counterparts, so to speak, of epicurean 'swerves')—that

- 1 'practical' recourse to regulative/constitutive distinctions might shift the ground of 'speculative' (scientific) 'experience', or (in the jargon of the last section) that
- 2 'scientific' experience might not be 'faithfully' interpreted in wider 'regulative' frames for its 'conscious' 'practical' extensions ("...without at the same time extending its cognition as speculative....").

In the opening section's 'potted history', I also

- 3 canvassed straightforward implications of the neo-Kantian observation that many "new objects" (and "new relations") have become part of 'experience' in mathematics and mathematical physics, and
- 4 suggested that metalogical margins of 'experience' and its thought-experimental counterparts might someday—'should' someday, if our descendants have the wisdom for it—encompass patterns of recurrently metatheoretical 'object'- and concept-formation undreamt of in Kant's philosophy (or mine).

Such arguments and the semantic *pluralism* which underlie them suggest alternative readings for certain remarks in the passage quoted above, and skeptical interpretations, in particular, for 'practical' aspects of Kant's *monist* assumptions about 'immanent'/'transcendent'- and 'constitutive'/'regulative'-dichotomy.

Suppose one accepted, for example that 'local' claims of 'speculative', 'practical' or 'teleological' '*completeness*' (Kant's "*Vollständigkeit*") are only '*locally*' expressible, in essentially *incomplete* and therefore *plurally interpretable* metatheories.

Then

- 5 the 'ladders' mentioned earlier—an ancient skeptical image which appeared in the pyrrhonist writings of Sextus Empiricus (cf. 1976, M VIII, p. 481), and reappeared in Wittgenstein's 'dogmatic' Tractatus—might branch and diverge 'forever' in plurally interpretable metatheoretic patterns; and
- 6 the 'practical' remarks in *KdpV*, B 241 and 244 and 'teleological' counterparts of them in *KdU*, B 301, 342 and 344 ff.) might begin to resemble ancient *skeptical* claims to engage in 'dialectical' (and plurally interpretable) speculation "adoxastōs"—"undogmatically", "undoctrinally", and "not in the manner of the dogmatists" (cf., e. g., Sextus Empiricus, 1976, PH I, 24/16). In what follows, I will argue



- 7 that the principal *bar* to such 'skeptical' readings of Kant's non-skeptical texts is his assumption that the 'hierarchies' of his 'constitutive'/regulative' and 'immanent'/'transcendent' distinctions *have exactly two levels*;
- 8 that Kant worked with 'speculative', 'practical' and 'teleological' counterparts of this linear two-stage ascent as a ground-bass in an extraordinary variety of assertions about 'die' Grenzbestimmung zwischen Immanenz und Transzendenz (cf., e.g., Prolegomena, 1976, pp. 350–362 and KdrV, B 786); and
- 9 that this 'capped', two-stage linearity of Kant's had an obvious historical precedent: the 'cave parable', the most influential account of 'practical' and 'epistemic' perspective-shift ever written, in which Plato simply assumed that emergence from 'the' cave would take place only once, in one 'direction', and would never be iterated.

In Kant's case, as well as Plato's, why?

Why did Plato and Kant, both of whom seemed to countenance other sorts of 'infinitum' and 'indefinitum' readily enough, reject stages of semantic (re)interpretation and hermeneutic reflection out of hand?

Why did both assume there was only one 'line', or 'ladder', or 'form of the good', or 'realm of ends', when partial realisations of noetic enlightenment would have accorded equally well with the 'conversational' and self-'critical' aspects of Socratic dialektike and Kant's 'Dialektik'?

Why, above all, did both construe epistemic, metaphysical and practical 'truth' and 'goodness' as second-order templates, rather than hierarchies of relational interpretations elucidated in 'heuristic' metatheories (or 'caves' illuminated by 'heuristic' 'suns')?

In the case of Kant's 'critical' work in general, and his '*practical*' philosophy in particular, the answer, I believe, had little to do with his alleged temperamental preoccupations with '*duty*' and dour fascination with unitary '*law*'. He was a kindly teacher and brilliant conversationalist, who enjoyed musical entertainment and held down a place at his local *Stammtisch* (when he could afford it).

If Kant felt a deeper sense of philosophical 'duty' or 'commitment', it was to 'hieratic' philosophical traditions which enjoin the sorts of unicity and canonicity of interpretation I have assimilated to nominally secular 'design-arguments'.

I have already argued that certain scientific ('speculative') counterparts of such arguments are metalogically untenable efforts to finesse the 'fate' of 'ontological' and epistemological 'reason'.

Here I would suggest that such arguments are indeed *more* untenable—

10 *not* because ethical and 'practical' 'experience(s)' and (thought-)experiments are *less* complex than their scientific counterparts, but because they are *vastly more so*.

(Consider, for example, that

- 11 ethicists are expected to acknowledge the potential "relevance" of illimitable complex "initial" and "boundary" conditions to moral "experience", whereas
- 12 *scientists* are expected to *require* that experiments ("*expériences*" in French) be as "*simple*" (or at least "*controlled*", "*isolated*" and "*replicable*") as possible.)



In both cases, therefore (but a fortiori in the 'realm' of 'the practical'),

13 "intelligibility" and heuristic [self]-reference (however incomplete) are 'regulative' 'preconditions' of forms of communication and [self]-understanding worthy of the names.

The 'regulative' force of such ideals of processive inquiry—without an awareness that such inquiry must be ramified if it is to be 'intelligible'—clearly animated Descartes', Berkeley's and Kant's hopes that 'design'-based 'uniqueness-of-maximality-and-determination'-arguments might be 'provable';

But it could also provide a framework—mindful of the greater 'depth' evoked in 10 above—for

14 acceptance of 'localised' forms of Kant's 'Primat des Praktischen', but rejection of 'the' inexpressible 'unicity' of his 'Reich der Zwecke' ('realm of ends').

The rest of the section will be given over to an attempt to outline such a 'regulative' and 'experiential' interpretation of Kant's Primat, in which significant notions of 'Zweck(e)' and 'Autonomie' will never 'converge' to or eventuate in a unique moral or teleological 'Reich'.

On my account, the 'primacy of the practical' reflects

- 15 a 'natural' *desire* to interpret one's experience in something '*deeper*'; more '*conscious*'; more '*contemplative*' (cf. 'Aristotle's *theoretikon*' (1975/1977; 1975)); and more '*comprehensive*' (and *compréhensive*); and
- 16 an 'autonomously' self-imposed obligation to 'respect' that desire in other "reasonable" (or at least "sentient") beings which 'experience' it, in the sense that they can propose and test counterfactual' 'purposes' ("Zwecke") they hope might 'realise' (or 'fulfill') it.
- 17 An "end" or "purpose", in this 'practically' shifted context, is simply a (syntactically or semantically) interpretable object-theoretic assertion (the German 'Sachverhalt').
- 18 A "maxim" ("Maxime") is simply an object-theoretic assertion that certain object-theoretic 'means' or actions imply or 'bring about' a particular end.
- 19 A (moral) "law" ("Gesetz"), finally, is a modal as well as an (intelligible) metatheoretic assertion that certain 'means' or actions 'should'imply (or 'morally entail') a particular object-theoretic 'end'.

In "Virtual Modality" (2003b), I defined an adequate 'virtual' semantics for alethic modal assertions in a given theory T, in which 'worlds' W_T are boolean-valued extensions of an 'initial' placeholder-'world' 'generated' by a 'virtual' structure which interprets T.

For what it is worth, one could define deontological refinements of such alethic semantics which could be fitted with *deontological refinements* which would single out

20 'practical' subclasses W_T of such 'worlds' or 'realm of ends' W_T,

characterised by "reasonable" constraints on the 'permissibility' or 'proportionality' of means and 'legitimacy' recognised in them, and in which 'reality' 'would' be as it 'ought' to be (by the lights of W_T).



The *ranges* of each such class W_T of counterfactual 'deontological' interpretations would be

- 21 metatheoretic with respect to the first-order set-theory in which the 'virtual' modal T mentioned above is interpreted; and
- 22 *metameta*theoretic with respect to the original theory **T**, in which quasi-Rawlsian '*ends*' or '*purposes*' might '*initially*' be identified and debated,

but the semantic intricacies of such a deontological 'stratification' might reflect the *illimitable complexities* associated with the "*primacy of the practical*" above.

But is conspicuously, notoriously (and perhaps inevitably) *absent* from any 'deontological' refinements of modal sematics (the one sketched earlier or any other) which are *guidelines* for 'choices' of the hypothetical classes W_T .

I will devote the rest of the section therefore to an impredicative effort to sketch a few *minimal* and *incomplete* criteria which 'ought' to be met by such 'realms', and argue on good skeptical grounds (and close alignment in this case with Kant) that more 'complete' decision-procedures would be 'inexpressible' in the theoretical contexts to which they applied.

Consider, for example, the following (rough) 'marks' of 'awareness', in ascending order of metatheoretic complexity.

- 23 It is a *mark* (indeed a characterisation) *of a well-defined* (idealised) *Turing machine* that it can *implement* object-theoretically feasible and well-demarcated '*induction-schemes*' of inferences, and '*know*', in this sense, '*how to go on*' with them.
- 24 It is a *mark of epistemic 'intelligence*' to '*understand*' metatheoretically (and perhaps even '*know*') that one does *not* '*know how to go on*' with respect to indefinitely extendible ranges of such (more and more complexly defined metatheoretic) schemes.
- 25 It is a 'deeper' and more 'zetetic' ('searching') mark of metatheoretic 'insight' to search (perhaps in vain) for 'reasons why' we do not 'know how to go on'; make attempts to extend or modify such schemes (or 'theories'); and propose new 'characterisations' of them in other (more complexly schematic) 'metatheories'.
- 26 It is a still deeper and more 'zetetic' mark of sentience, ethical insight and quasi-Kantian 'reasonable faith' to seek to understand why and when 'we' 'should' and 'should not' 'be expected to' 'know how to go on' with our attempts to query and refine current schemes; and formulate (in full awareness of our limited moral and conceptual capacities) quasi-Kantian practical 'Gebote' ('commandments' or moral 'injunctions').

The principle such "Gebot" might be called Pascal's imperative: to

- 27 value 'sentience' and its preconditions, and 'respect' 'cognitive agents' enmeshed in its dilemmas;
 and
- 28 attribute to such sentient agents 'rights' and 'responsibilities'—to the best of 'our' abilities and 'theirs' (cf. 24 above)—as (potentially) 'reasonable beings' in need of mutual aid.



(We often seem to forget the obvious: not only do 'we' not 'know how to go on'—'we' do not go on.)

If certain actions clearly seem to be *incompatible* with the '*respect*' and its '*attributions*' just invoked, we have ('*autonomous*') *duties* to abjure them, if nothing within 'our' physical or conceptual horizon constrains us.

And *if*—by parity of reasoning, so to speak—such forms of 'respect' and their 'attributions' clearly seem to *entail* certain actions or resorts to particular means, we have equally '*autonomous*' *duties* to carry them out.

Exactly because these deliberations *are* autonomous, however—'legislated' by an elusive and underdetermined 'self'—they *do not have sharp metatheoretic boundaries,* and their conceptual horizons elude our view (cf. Leibniz' hypothesis that "[c]haque âme connaît l' infini, connaît tout, mais confusément", in the "Principles of Nature and of Grace", GVI, 604).

Metatheoretically nuanced efforts to *interpret* aspects of 'experience' may *extend* them, and even "bring ...[them] *closer to completeness*" in this sense. But if they are intelligible—and in that sense communicable—they will not be able to '*determine*' 'ultimate' boundary-conditions of practical experience.

What endeavors to 'prescribe', in short, cannot itself be 'ultimately' 'prescribed'. What 'it' can do (and 'should' do), is (try to) enjoin itself to (try to) be more comprehensive (and compréhensif).

One could, of course, attempt dogmatically to assert the 'existence' of an—essentially indefinable—*EA* notion of ('human') *consciousness*—a sort of intentional 'last metatheory', of the sort mentioned earlier.

But the arguments offered in prior sections suggest that such a 'consciousness' could not *express*, much less '*know*' what [it itself] *is*, much less 'know' with sovereign certainty that the thing behind the curtain is not [it].

(For a first indication that something like this might be inevitable, consider the 'impredicative' nature of attempts to sketch criteria which 'ought' to be met by 'criteria'—such as the ' W_T ' sketched earlier—of what 'ought' to be countenanced.)

There are, in short, meta-deontological counterparts of 'autological' dilemmas, and there is no reason to believe that they can be finessed or ('ethically') suppressed.

Kant, for example—to his credit—essentially acknowledged, in the *Grundlegung der Metaphysik der Sitten*, that a "purely good will" is indefinable and indiscernible in human experience (cf., e.g., *Gr*, 1965, pp. 459–463).

This seems to me methodologically right, on the analogies I have sketched. But the arguments of prior sections suggest (at least to me) that no metatheoretically stable *Analogon* of a 'transcendental deduction' which would ensure a will's '*existence*'—even 'regulatively'—could be formulated.

They also suggest another, somewhat more oblique analogy, between traditional *theodicies* and Kant's so-called *ascription-problem*—the ambiguities that arise when one tries to choose among various initial and boundary-conditions for particular applications of 'the' categorical imperative.

For Kant felt compelled to *acknowledge* that the activities and 'motives' of the good will, and its operative "*Prinzip*", the categorical imperative, may be unsurveyable and (internally) indefinable, so that he could postulate for it the utter detachment from merely empirical and hypothetical ('internal') ectypes that Leibniz *sought* in his 'god'.



This may sound less surprising, if one reformulates this putative 'analogy' as follows.

Aristotle, Anselm, Aquinas, Leibniz and others wished to 'ground' (or end) 'merely' relational regresses—of origination, design and conception, for example—in the (allegedly) reflexive [self]-origination, [self]-design and [self]-conception of 'god', along the lines sketched in prior sections.

Kant, by contrast, wished to employ 'practical' [self]-referential or *fixed-point arguments* to 'ground' (end) certain other 'merely' relational regresses—of actuation, purpose-seeking and other-directed *Heteronomie*—in the (alleged) [self]-actuation, [Selbst]zweck and [Auto]nomie of the noumenal Wille, and its equally noumenal Prinzip.

He sought, in other words, to provide in his ethics (among many other things) a 'first cause'-argument for a "religion of humanity" (or 'ultimate' "purpose than which no more ultimate purpose can be conceived").

This analogy may help explain why most of us find something numinous but thoroughly indeterminate in the resonant language of Kant's supposedly so affectless ethics.

It might also help explain why Wöllner, Woltersdorff and the other opportunist and fundamentalist *Zensoren Seiner Durchlaucht Friedrich Wilhelms II* went after him with such ferocity in the 1790s, after he published *Religion innerhalb der Grenzen der bloßen Vernunft*.

Along with many others, I find the ideal of a Kantian *Reich der Zwecke*—and its more conceptually accessible Rawlsian counterpart—moving and 'liminally' persuasive.

But the culture-bound rigidity and parochiality of the notorious four 'applications' Kant offered in the *Grundlegung* for the first two versions of the categorical imperative—not to mention the even more notorious strains in his late essay "Über ein vermeintliches Recht, aus Menschenliebe zu lügen"—seem to require suspension of judgment, and a measure of (what might be called) noetic restraint.

For the very *reflexivity* of Kant's 'Selbstzweck(e)', 'Selbstgesetzgebung' and *reine* Selbsttätigkeit (pure self-activity or -actuation, the quality Aristotle attributed to *theo-ria*; cf. *Gr*, p. 452) poses a (literal) *dilemma*: 'practical' *Grenzideen*, on the analyses of the foregoing sections, are either

- (i) *indiscernible* to themselves (and therefore of little 'practical' use); or
- (ii) plurally interpretable in ethically defensible 'realms' of branching 'Zweck-mäßigkeiten ohne ('ultimate') Zweck'.

Such dilemmas would not be surprising if one construed Kant's 'deduction' of practical Autonomie (self-lawgiving; cf., e.g., Gr, pp. 453–454) as an 'individualisation' of traditional 'first'- and 'self-cause'-characterisations of 'god', or at least

(iii) "[etwas] zu aller Reihen der Bedingungen notwendig ... Unbedingtes, mithin auch eine sich gänzlich von selbst bestimmte Kausalität...." (KdpV, B 83–84).

As is the case with other would-be deductions of [self]-constituting metaphysical universality, this one may be (re)formulated as a (literally) *self-referential paradox*: (cf. 3.44–3.45 above), that



29 Each 'reasonable being' 'must' be able to will that [each 'reasonable being' 'must' be able to will that [[...]]].

The indefinite 'implosion' of "[[...]]"'s suggests a fault-line in Kant's attempts to assimilate a practical *Verstandeswelt* (*mundus intelligibilis*, or conceptual world; cf., e.g., *Gr*, 458 and *KdpV*, 74) to the *Noumena* of his theoretical or speculative philosophy.

For the *characteristic* of the latter in the *Dialectic* is their conceptual *underdetermination* and openness to *plurality* of interpretation, at least "*in spekulativer Absicht*" ("in speculative intent"). (Compare the Antinomies, and Kant's incisive analyses of traditional 'proofs' of the existence of god).

But this underdetermination and hermeneutic plurality contrasts sharply with the 'regulative' 'necessity' and unicity Kant postulated for his 'realm of ends'—in (Gr, 445), for example, when he wrote that

30 Wer also Sittlichkeit für Etwas und nicht für eine chimärische Idee ohne Wahrheit hält, muß das angeführte Prinzip derselben zugleich einräumen.

Who(soever) considers morality to be something, and not a chimerical idea without truth, must concede the principles we have advanced.

As a would-be "reasonable being", I would certainly "consider morality to be something". But I would also consider the *dichotomy* Kant imposes in this passage a groundless *petitio* of the doctrine I called '*semantic monism*': denial, in the case at hand, that

31 there could be any *alternative* to 'existence' of 'ultimate' 'speculative', 'practical' and 'teleological' interpretations and other "truths"), except utter inconsistency ("eine chimärische Idee").

Such begged dichotomies and 'first-cause'-analogies underlie the ambiguities that arise from Kant's "ascription problem", mentioned earlier, and in particular the notoriously uneven success of Kant's "examples" at *Gr*, pp. 422–424 and 429–430 mentioned earlier.

But they also suggest why there is something deeply, genuinely and recurrently *correct* as well as '*problematic*' in Kant's '*transcendental*' acknowledgment, at (p. 419), that

32 es durch kein Beispiel, mithin empirisch auszumachen sei, ob es überall irgend einen dergleichen Imperativ gebe....

it is not to be made out through any sort of example, and thus empirically, whether there is anywhere any imperative of the kind.

Since *every* moral dilemma we are likely to face comes trailing clouds of background conditions and boundary-assumptions, it seems to me eminently 'reasonable' (in Kant's own sense) to

33 abandon Kant's *semantic monism*, in the passage from *Gr*, 445 quoted above, and interpret its implicit '*if-then*' assertion ("Whoever considers...must concede...."),



as the *hypothetical* (rather than *categorical*) *meta-imperative* it seems in fact to be.

Once one *does* this, however, it also seems 'reasonable' to grant him that

- 34 reflective—and ultimately reflexive and [self]-determining—contemplative activity and respect for such activity in all other "reasonable Beings" are the recurrent—but 'liminal' and essentially incomplete 'grounds' of moral action, in keeping with
- 35 Aristotle's observations of children's desires (*orexeis*) to 'know' and praises of *theoria* (or *energeia theoretike*) as the 'highest' forms of thought, namely "thought about thought", and
- 36 Kant's injunction to 'displace ourselves' ("sich hineinversetzen") into "intelligible" realms of ends (in his sense of "intelligible"), conceived as an
- 37 *empathetic* imperative to 'displace ourselves', however inadequately, into the sufferings and aspirations of other "reasonable beings" (Gustave Gilbert may have been right about the origins of "evil").

I have already argued (in 27 and 28 above)

- 38 that respect for sentience, in all its gradations—construed as inchoate energeiai theoretikai—is the mark of Kant's elusive 'good will', in all its gradations, and
- 39 that provisional *respect* (Kant's *Achtung*) for the emergence and possible presence of such *theoria* in *others* is an inherent constituent of that regulative ground, and basis for a more modest and heuristic categorical imperative.

This is the essence, for me, of Kant's alternative formulation of the imperative, as respect for the generative capacities of '[self]-legislating' *theoria* in other 'reasonable beings', to be treated as *ends* (not only *in* [themselves], but to [themselves]), rather than (mere) 'means'.

What we should enjoin ourselves to "respect", in other words—in a form of collective 'self-legislation'—are 'innocent' (non-'harming') forms of Aristotle's *desire*.

This respect, in a sense, might also be an appropriate dialectical/skeptical *elenchos*—"*in praktischer Absicht*"—of Descartes' rhetorical paranoid worries in Meditation II about the "hats and clothes under which might lie automata" ("*pileos et vestes sub quibus latere possent automata*", AT VII 32).

Who knows? But more to the point, I believe: what do 'we' know about [ourselves], that entitles us to judge that other potential 'reasonable beings'—Kant's phrase—are not up to 'our' standards?

Kant's explicitly latitudinarian view of the distribution of possible 'reasonableness' (cf. *Gr*, 408 and 426) got this right, I think. And John Searle, in his "Chinese Room" parable—a fallacious '*EA*-argument' which exactly reverses Descartes' 'worry' about the hats and clothes on springs—got it thoroughly and dogmatically wrong.

I believe, at any rate, that some of these insights may have inspired the "awe" (*Ehrfurcht*) Kant struggled to express, in his famous vision of the vault above him and moral law within him.

For it is this processive "awe" *itself* that moves us: that we cherish in our children; that we want to foster and protect in each other, and in ourselves. The vault and the



'law' are only placeholders for it, provisional emblems for an elusive and forever underdetermined *Erhabenheit* (sublimity) in Kant's 'theoretical' vision—and ours.

6 Reflective inquiry and "Systeme von Zwecken"

At *KdrV*, B 672, Kant characterised a "*System*" as "*eine gewisse kollektive Einheit zum Ziele der Verstandeshandlungen*", and he sketched suggestively 'metatheoretical' and 'hierarchical' roles for such "*Systeme*" in *KdrV*, B 692, quoted above.

'Experience', on the account I have offered, 'is' (or may pragmatically be viewed as) a indefinitely ramified *'System'* of such *'Systeme'*, which 'individuate' and 'objectify' in graduated ways as they 'ascend'.

'Stages' in such metatheoretic hierarchies, moreover, have an interesting sort of 'distributive' or 'kollektive Einheit', for

1 *all* Stone spaces (topological spaces of semantic interpretations of essentially incomplete first-order theories) are *topologically and measure-theoretically 'isomorphic'* (cf. 1.3 and 7.29–7.30 below).

What *distinguishes* particular theories from other particular theories in this hierarchical myriad are their (metatheoretically 'intended') formal *vocabularies*, of course; but also, and more deeply,

- 2 their patterns of syntactical and semantic interpretation, which order and interrelate them; and
- 3 *their* (*recursion-theoretic*) *complexities*, which effectively determine the 'isomorphisms'.

In this section, I will argue

- 4 that Kant tacitly acknowledged in the third *Critique* that he had offered *an artificially isolated and experimentally 'controlled' structure of 'experience'*, which could not (for example) even determine its allegedly fixed (Newtonian) boundary-conditions. (It was known, for example, in the late eighteenth century that Newton's analyses—which construed the planets as point-masses—could not cope with their mutual interactions, much less account for observed perturbations in their actual orbits.);
- 5 that Kant saw that some sort of *metatheoretic ascent* or *Regressus* might present itself if the 'observational' base of his "Anschauungen" turned out not to 'determine' more 'heuristic' and nuanced aspects of scientific inquiry (the indefinite 'complexities' of 'the organic', for example); and finally,
- 6 that he saw that *underdetermination* of that 'intuitive' observational base might also be reflected in *iterations of KdrV*, B 692's *conceptual register-shifts*; and that such iterations might '*open*' the first *Critique*'s rigid two-stage architectonic, and blur or relativise its allegedly sharp 'transcendental' demarcation of "*Verstand*" from "*Vernunft*".

One aspect of this recognition might be the oddly enhanced role given in the third *Critique* to "reflectierende *Urteilskraft*" ("reflective judgment"), characterised as a form of mediation ("Mittelglied", KdU, iv) between 'understanding' and 'reason'.



In the first and third *Critiques*, this *Vermögen* assumed the interestingly 'metatheoretic' role of "*Überlegung*" ("reflection" or "deliberation"; literally, "overlaying"):

"Bewußtsein des Verhältnisses gegebener Vorstellungen zu unseren verschiedenen Erkenntnisquellen" ("awareness of the relation of given representations to our various sources of knowledge", KdrV, B 316); and

"Unterscheidung der Erkenntniskraft, wozu die gegebenen Begriffe gehören" (KdrV, B 317) ("discrimination of the capacity for knowledge to which the various concepts belong"), [welche] "auf die Gegenstände selbst geht" ("[which] applies to the objects themselves", KdrV, B 319).

In this 'capacity', "reflective Urteilskraft" effectively

- 7 assigned orders or levels to (what might be called) 'object'- as well as 'conceptformation' ([welche] "auf die Gegenstände ...geht"); and in so doing, it
- 8 ambiguated (and transcended') the otherwise rigorously closed architectonic of 'reason' in the first and second *Critiques*, in which "*Reflexion*" appeared only in the guise of an "*Amphibolie*" (cf. *KdrV*, B 316–349).

In metalogical analyses, counterparts of such (potentially higher-intentional) acts of 'discernment'—'the' local distinction between theory and metatheory, for example—function as type-or register-raisers which ride the wavefronts of metatheoretical hierarchies, and locally 'transcend' what they 'discern'.

It is plausible, therefore, that such a metaphysically 'skeptical' (or at least inquisitive) *Vermögen* might have given Kant the conceptual instrument he needed to absorb and buffer the 'awareness' I attributed to him in 4–6 above.

Similar preoccupations might also have led him to

- 9 give greater prominence to the incomplete, underdetermined and (in my analogy) type-straddling notion of a "*System*" which 'merely' regulated emerging boundaries of 'experiences' (cf., once again, *KdrV*, B 672 and *KdrV*, B 692); and
- 10 postulate evolving (and perhaps ramifying) forms of "Reflexion" about 'natural' and 'organic' Systeme of "Zwecke" and "Zweckmäβigkeit", which would (re)inform and (re)interpret extensions of the Analytic's scientifically as well as 'teleologically' inadequate Anschauungen and Begriffe.

And this, in turn, might have led Kant—or someone troubled by the equivocal ways in which he used words like "Vollständigkeit" in the Analytic and Dialectic—to

- 11 consider more latitudinarian (and more 'counterfactual'-supporting) notions of 'causality',
- 12 acknowledge at least tacitly that the 'closure' of the *Aesthetic* and *Analytic* doesn't 'work', and
- 13 explore 'merely regulative 'principles' and forms of 'causality' in 'heuristically' adequate notions of *Zweckmäßigkeit* (glossed at *KdU*, B344 as "*die Gesetzmäßigkeit des Zufälligen*" ("the lawlikeness of the contingent"; cf. 18 and 19 below) *ohne* (a final, uniform) '*Zweck*' (purpose).



Kant's characterisation of "Zweckmäßigkeit" as a 'merely regulative' "Gesetzmäßigkeit des Zufälligen", just cited, also raises methodological questions about the absence of 'counterfactual support' for 'lawlike' assertions in the KdrV):

- 14 do claims that 'real' lawlikeness (*Gesetzmäßigkeit*) only holds for systems which are 'necessary' in the sense that they are deterministic not beg the question of the Analytic's 'underlying' ('Laplacian') determinism?

 And if they do,
- 15 what are we to make of (ostensibly) 'metaphysical' (as opposed to 'merely' mathematical) assertions that 'real' 'experiential' causation—construed as a 'physical' form of alethic modal entailment—'supports counterfactuals', if there are no counterfactuals to support?

The problems raised by these queries are oddly cognate to Hume's failure to distinguish between 'factual' (indicative) and 'counterfactual-supporting' (subjunctive) implications in his 'definitions' of "Cause" in the first Enquiry (Hume 1975, §60, 76). But they are hardly confined to the metaphysics of Hume and Kant.

For they persist in cognate problems for *any* 'causal' metaphysics in which we *do not* (and, I believe, *cannot*) know how to 'choose' '*ultimate*' metatheoretic 'boundary-conditions' for 'virtual' modal semantics of 'worlds' and 'accessibility'-relations which interpret their '*causes*'.

Prompted by (what I have read as) Kant's tacit acknowledgment that 'mediation' by 'reflective judgment' might iterate 'the' Anschauung/Begriff/Idee-hierarchy, I have therefore sought to

- 16 *relativise* that hierarchy;
- 17 'open' its rigid three-level Architektonik;
- 18 *iterate* such relativisations in ramified patterns of *metatheoretical ascent*; and finally
- 19 appeal to such patterns to formulate 'conceptual' interpretations of ('theoretical', 'practical' and 'reflective') 'experience' within shifting horizons of metatheoretic complexity.

Metalogical studies of such hierarchies suggest that essentially incomplete and underdetermined *Systeme* and their hierarchies may 'regulate' emerging boundaries of experiences, and that notions of *Zweckmäßigkeit* may 'inform' and (re)interpret such regulative systems and their notions of 'causality'.

Within various stages of such 'systems', for example, one can 'rationally' as well as 'reasonably' assimilate *Endzwecke* or *causae finales* to provisional forms of metalogical *explanation* or *interpretation*, but accept that no 'ultimate' 'finality' for them will ever come into view.

Such 'principles' and forms of 'causality', moreover, would be ('merely) "heuristic", "reflective", "regulative" and "problematic". In analogy with certain aspects of Kant's usage, they would be

20 "heuristisch", in that they ('merely') serve "'den' "besonderen Gesetzen der Natur nachzuforschen ohne über 'die' Natur hinaus 'den' Grund 'der' Möglichkeit



derselben zu suchen" ("inquire into 'the' particular laws of nature ...without searching beyond nature [to inquire] about its ground", KdU, B355).

(The single quotation marks in the quotation are obviously mine: we tend to forget that the original senses of "physis" and "natura" were processive as well as 'organic'.) They would be

21 "reflektiv" in that they seek to discern theories' orders of complexity and relative interpretability, but do not define or determine 'ultimately' 'canonical' interpretations for them.

(Kant's (1974) "reflective Urteilskraft" sought 'only' to 'subsume' its intentional objects, whereas the 'Vermögen' he called "understanding" 'determined' them; cf., e.g., KdU, B 311–313 and B 365).

They would be ('merely')

22 "regulative" in that they are "nur ...Regel[n], welche ...einen Regressus gebiete[n], dem es niemals erlaubt ist, bei einem Schlechthinunbedingten stehen zu bleiben" ("only rule[s] which prescribe a regress in which one is never permitted to halt at an utterly unconditioned [limit]"; KdrV, B 536–537).

(Notice that Kant retained in this context Hume's distinction between "Geset-zlichkeit" and what Wittgenstein later called "Regelbefolgung"...)

And such 'principles', 'concepts' and 'judgments' would finally be

- 23 "problematic" in that
 - (a) "man das Bejahen oder Verneinen [derer] als bloβ möglich (beliebig)
 annimmt" ("one takes the affirmation or denial [of them] as possible
 (optional)", KdrV, B100);
 and
 - (b) they are "...Begrenzung[en] gegebener Begriffe" [welche] "mit anderen Erkenntnissen zusammenhäng[en], dessen objektive Realität ...auf keine Weise erkannt werden kann" ("boundaries of given concepts" [which] "fit together with other forms of knowledge whose objective reality cannot in any way be known", KdrV, B 310).

(Cf. also the remark that

23(c) "[i]ns Innere der Natur dringt Beobachtung und Zergliederung der Erscheinungen, und man kann nicht wissen, wie weit dieses mit der Zeit gehen werde" ("observation and distinction of appearances penetrate into the interior of nature, and one cannot know how far this might go [on] over time", KdU, B334)).

Some definitions and characterisations may clarify other aspects of the relativisations I have sketched. In the context of 16–19 above, for example,

- 24 a "system" may be any finitely (or finitarily) 'overviewable' substructure of an eventually first-order metatheoretic hierarchy;
- 25 an act or instance of "*reflection*" (or "*reflective judgment*") 'locally' identifies a given system's relative 'position' in such a metatheoretic hierarchy (as above; cf. once again *KdrV*, B316ff);



- 26 an "end" or "purpose" ("Zweck") (cf. 5.20) is once again a (syntactically or semantically) interpreted *object-theoretic assertion* (the German 'Sachverhalt') at some stage in such a hierarchy;
- 27 a (regulative) "law" (or 'merely' 'zweckmäßige' "Maxime") is a modal implication which asserts that certain 'means' or hypothetically contributory 'causes' jointly 'entail' certain 'ends' in a metatheoretic environment in which we do not and cannot know what ranges of 'counterfactual' alternatives the implication might 'support'.

At a number of points in the third *Critique*, Kant argued for the claim that cognitive agents have a right to 'impute' ("*ansinnen*") certain attributes or judgments to others, and such "imputation" is not confined to aesthetic judgments.

The more common noun-usage "Ansinnen" has many senses in Kant's and other German-speakers' usage ("demand", "request" "expectation" "imposition" and "point of view" among them), and

- 28 an assertion that a given '*reflective judgment*' of mine is "*ansinnbar*" (an adjectival form Kant does not use) might therefore be construed as
- 29 a 'liminal' assertion that whatever 'interprets' 'my experience' (in ways which may not be fully knowable to 'me', or even expressible in 'my' language) 'should' interpret 'my' 'reflective judgment' or expression of that 'experience' accordingly.

Such '*interpretability*'—which might be construed, as Kant suggests, as a form of *communicability*—would assure a form of *relative consistency*:

30 if 'the world' 'makes sense', so do 'I', or at least 'my' expression or reflection of a given aesthetic or 'practical' aspect of it. (Or so 'I' think).

Methodologically, at least, this metalogical reading of the 'subjektive Algemeingültigkeit' of Kantian aesthetic judgments seems to me defensible.

More relevantly, it may clarify the conceptual *asymmetry* between *aesthetic* "Ansinnbarkeit" and moral "solidarity". For the former demands that others 'appreciate' what I 'appreciate', and the other that I try to 'comprehend' the capacity to 'appreciate' in others. The former may or may not be a 'just' demand, but it is literally 'egocentric' and 'self-absorbed' in ways the latter is not.

Be that as it may, the principal burden of this section has been to make two closely related points:

- 31 that what is genuinely 'regulative' of 'inquiry' are attempts to consider, critique and appreciate "Zwecke" ("purposes"); and
- 32 that *zetetic inquiry* is not only regulative of 'experience'. It is also a form of generative *skepsis*, and a form—and perhaps prototype—of "*Zweckmäßigkeit ohne Zweck*".

Distributively, one might also characterise such 'experiential' systems and their 'dynamics' as graduated forms of "*Erfahrungsmäßigkeit ohne* (*schlechthinnige*) *Erfahrung*", and construe them as heuristic 'regulative ideals' of such 'experience' (or '*expérience*(*s*)').

Since uniform quasi-Kantian 'preconditions' of such 'experience' would not, in particular, be object-theoretically definable at any stage of theoretical inquiry, the



underdetermination of such 'systems' boundary-conditions and 'counterfactual' environments might also be

- 33 compared with the 'confusion' Leibniz attributed to attempts to understand 'notiones concretae', and
- 34 interpreted as a *margin of the infinite* (*or at least indefinite*) *in the finite*, or at least the conceptually finitary.

'Origins', 'preconditions' and semantically 'intended' interpretations are objecttheoretically invisible in all the gradations of metatheoretical hierarchies they 'condition' and 'regulate'.

We cannot, for example, define or measure 'all' 'conceivable...'/'conceptual...' definition or measurement. For the only way to remove the "'..."'s is to *relativise* one's inquiry to particular expressible theories in which one can begin to make sense of 'conception', 'definition' and 'measurement'. (Notice the parallels between these notions and mathematical 'boundary-conditions' and physical 'experimental preparation'.)

If the 'cogito'-like responses Descartes, Berkeley and Kant offered to Sect. 3's criterial hierarchies 'worked', in the presuppositionless ways their authors apparently intended, it would be to witness the *in*coherence of the (meta)theory in which they are formulated.

For 'last' clauses in the stoic/skeptical dialogue which gave rise to 'cogito'-like arguments will always reinstate skeptical doubt of skeptical doubt, in order to reinstate minimal stoic consistency-demands.

Fixed-point arguments, for example, would have little 'constitutive' force for 'the' *unicity* of 'experience', but quite a bit of 'regulative' value as 'local' guidance for avoidance of certain conspicuous forms of 'transcendental illusion'.

If one could, for example, 'define' 'transcendence' in one of Sect. 2's 'intelligible' theories, one could also formulate a 'diagonal' assertion which would be equivalent to [its own] 'transcendence'.

But this would undermine the 'definition': for no syntactical interpretation of such a sentence would accurately 'define' ('the' 'preconditions' of) [its own] 'immanence'. If Einstein had been 'right', therefore, that

35 'the' world is 'complete' (a word he often used in his debates with Niels Bohr) and epistemically accessible (since 'der Alte', like Descartes' 'god', does not 'deceive' us),

he would also (on good Gödelian grounds) be 'wrong',

36 for 'it' would either be as incomplete as 'we' are, or as epistemically inaccessible to 'itself' as 'it' is to 'us'.

One the evidence of such arguments, it might be a 'transcendental illusion' that 'experience' has a 'constitutive' "Grenzbestimmung", and 'reasonable beings' would be more 'reasonable' to the extent they understand this.

This predicament might be regulative of Kant's 'Schicksal der Vernunft' as well as 'expérience' and 'compréhension', in both French senses of these words. But it might also be one of reason's cradle-gifts: a source of

37 skeptical self-awareness that 'ultimate' self-knowledge is little more than a semantic paradox; and of



38 'creativity' in that no such 'ultimate' Übersicht bounds 'reasonable beings' conceptual event-horizons.

To see that this might be so, consider two recurrent arguments of the sort canvassed earlier:

- 39 a 'neo-Kantian' one that some stage of metatheoretic ascent is closable as 'experience'; and
- 40 a 'Peircean' one that 'the' *entire ascent* is linear and unique, and *this 'course of inquiry*' is 'experience'.

In this context, all I have done is propose a *nonlinear alternative to Peircean 'limit(s)*', in which convergence is plural and there is no 'ultimate' hierarchy-internal 'overview'.

That 'intelligible' theories cannot 'intend' their own interpretation and that hierarchies of them are indefinitely iterable is a metalogical theorem.

That ramification of such hierarchies is limitless is a regulative ideal of experiential *openness* as well as contemplative "*Erhabenheit*" ("sublimity"), in ways I will try to explore in the next section.

That 'we' can 'go on' in such hierarchies is an article of 'reasonable faith', sustained by 'liminal' postulations of consistency and 'intentional' interpretation which hover indefinitely at the margins of what they define.

7 Reflective inquiry and Kant's two "Unermeßlichkeiten"

At the end of the *Kritik der praktischen Vernunft*, Kant (1974) wrote the following resonant sentence, later inscribed on his cenotaph in Königsberg:

Zwei Dinge erfüllen das Gemüt mit immer neuer und zunehmender Bewunderung und Ehrfurcht, je öfter und anhaltender sich das Nachdenken damit beschäftigt: der bestirnte Himmel über mir, und das moralische Gesetz in mir. (*KdpV*, p. 288)

Two things fill the sensibility with ever new and growing awe, the more often and more persistently reflection occupies itself with them: the starred firmament above me, and the moral law within me.

To me at least, these lines suggest that for all his 'critical' acceptance of Newtonian "Mechanism", Kant saw something comparably "erhaben" ("sublime") in 'regulative ideals' of the macrocosmos 'above' us and microcosmos 'within' us.

Prompted by this interpretation, I will appeal in this section to 'liminal' as well as limitless aspects of the foregoing sections' interpretation of 'experience' to

- 1 argue for an evenhandedly 'regulative' interpretation of both these ideals" and
- 2 compare Kant's two "*Unermeßlichkeiten*" ("immeasurabilities"; cf. 7 below) with metalogical and metaphysical counterparts of Pascal's two *infinités* or *abîmes*".

Recall (or observe) first that Kant considered "Achtung" ("respect") and "Ehrfurcht" ("awe") intentional attitudes appropriate to (inadequate) attempts to contemplate two



closely interrelated limiting ideals: "Pflicht" ("duty"), and "das Erhabene" ("the sublime").

"Achtung", for example, in the third Critique was the

- 3 "sense of the inadequacy of our capacity to attainment of an idea which obligates us" ("Gefühl der Unangemessenheit unseres Vermögens zur Erreichung einer Idee, die für uns Gesetz ist") (KdU, B 96).
 - When that "respect" is engendered by contemplation of
- 4 "the intellectual, in itself purposeful [and] (morally) good" ("das intellektuelle, an sich selbst zweckmäßige (das Moralisch-)Gute", cf. KDU, B 120), then regard for this ideal is also "erhaben" ("sublime").

For it is

- 5 "not pleasure, but self-estimation (of the humanity in us), which raises us above our need [for pleasure]" ("kein Vergnügen ist, sondern eine Selbstschätzung (der Menschheit in uns), die uns über das Bedürfnis desselben erhebt") (KdU, B 228), and its view or contemplation
- 6 "gives us just access to ...the idea of a great **system** of natural ends" ("[uns] zu der Idee eines großen **Systems** der Zwecke der Natur ...berechtigt'), which we
- 7 "love as well as contemplate for its **immeasurability**, and find ourselves ennobled in that contemplation." ("lieben, sowie ihrer Unermeßlichkeit wegen mit Achtung betrachten und uns selbst in dieser Betrachtung veredelt fühlen...") (KdU, B303, emphasis mine).

Such "Liebe", finally, engenders a 'higher' and more 'autonomous' form of 'humanist' respect, namely

8 "Ehrfurcht", or "respect for a ruler ... which lies within us, [and therefore] awakens a sense of the sublime in our own [self-]determination which inspires us more than anything beautiful" ("Achtung ... gegen seinen Gebieter, [der] ... in uns selbst liegt, [und daher] ein Gefühl des Erhabenen unserer eigenen Bestimmung erweckt, was uns mehr hinreißt als alles Schöne") (Die Religion innerhalb der Grenzen der bloßen Vernunft, VI, 23-24fn).

In what follows, I will assimilate

9 Kant's first "Unermeßlichkeit" ("der bestirnte Himmel über mir") to the immeasurably ramifying *metatheoretic "Systeme*" of 'intelligible' theories evoked above in Sects. 2 through 4,

and

10 his second "Unermeβlichkeit" ("das moralische Gesetz in mir") to the correlatively immeasurable and ramifying 'hypotheoretic' "Systeme" of variably encoded forms of awareness or consciousness which may be expressed 'within' them,

respect for which I offered earlier as less prescriptive but (in my view at least) no less "erhabene" Alternativen to the various forms of Kant's categorical imperative.

These assimilations will carry with them cognate metalogical interpretations of two other "Ideen" in Kant's writings:



- 11 'physical' "Organism" (or "Organisation"), invoked in several contexts in the third Critique; and
- 12 '*liminal*' 'systematic' "Horizonte", mentioned twice in the first Critique, and at considerably greater length in Kant's lectures on "Logik".
- "Physik" (or "das Physische"), was emphatically not what we would call "(Newtonian) physics", but
- 13 ein Doctrinal System empirischer Erken[n]tnis (nicht ein empirisches System denn der Begriff von einem solchen enthält einen Wiederspruch), a doctrinal system of empirical knowledge (not an empirical system, for the concept of such a thing contains a contradiction).

As the parenthetical remark suggests, such a 'higher-order' *System* might require '*critical*' discernment of '*empirical*' theory from '*transcendental*' metatheory to avert (semantic) paradox ("*einen Wiederspruch*"), and have

14 zweyerley Objecte: (1) was überhaupt Gegenstand der Erfahrung ist (2) dessen Möglichkeit selbst nicht anders als durch Erfahrung erkennbar ist, wovon also die Wirklichkeit vor der Möglichkeit nothwendig vorhergeht die also nicht a priori erkannt werden kann (KW 2003, XXII, pp. 398–99). two sorts of objects: (1) whatever is an object of experience (2) [something] whose very possibility is not knowable other than through experience, the reality of which necessarily precedes therefore its possibility[, and] therefore which cannot be known a priori.

Quasi-Leibnizian 'notiones completae', for example, might 'regulate' Newtonian 'Mechanism''s reference-frames and boundary conditions, as well as instances of what Kant called "Organism", or "das Organische" (terms which do not appear in the third Critique, where he replaced them with more 'dynamic' and 'processive' usages such "organisieren" and "Organisation").

In his deconstruction of Swedenborgian mysticism, for example (*Traüme eines Geistersehers*, KW, XVIII, p. 13), Kant observed in passing that "*das physische ist nicht pneumatisch, sondern organisch*", and he remarked in other non-'critical' texts that

- 15 [d]ie dynamische Erklarungsart ist entweder mechanisch durch Werkzeuge die selbst bewegender Kräfte zu ihrer Existenz bedürfen und wenn sie ihrer Natur nach Zwecke ihrer Bildung voraussetzen [o]rganisch vorgestellt werden. (KW, XXI, p. 233)
 - the *dynamic mode of explanation* is either *mechanical*, by means of instruments which themselves require moving forces for their existence, [or] if they are represented *organically* in accordance with their nature [and] presuppose [aims or] ends for their formation.

and formulated a

16 ...Definition eines organischen Körpers ...daß er ein Körper ist dessen jeder Theil um des anderen willen (wechselseitig als Zweck und zugleich als Mittel) da ist.— Man sieht leicht daß dies eine bloße Idee ist der a priori die Realität (d.i. daß es ein solches Ding geben könne) nicht gesichert ist.



Man kann die *Erklärung dieser Fiction* auch anders stellen: Er ist ein Körper an welchem die innere Form des Ganzen vor dem Begriffe der Composition aller seiner Theile ... in Ansehung ihrer gesammten bewegenden Kräfte vorhergeht (also *Zweck* und *Mittel zugleich* ist). (*KW*, XXI, p. 210)

...definition of an organic body ...that it is a body whose every part is there for the sake of [every] other (reciprocally as end and means).—One sees readily that this is a mere idea, whose reality (that is that such a thing could exist) is not a priori ensured. One can also put the explanation of this fiction differently: it is a body for which the inner form of the whole precedes the concept of the composition of all of its parts ...in view of their collective moving forces ([and] therefore is at once end and means).

(It may be worth observation that more precise analogues of Kant's struggle to 'define' 'the organic' may be found in mathematical-physical notions of an "interacting field").

Freed, in short, from the narrow confines of his 'transcendental' rationales for Newton's differential equations, Kant acknowledged that his "*Physik*"s 'merely regulative' entailments might lead into 'rational' but indefinitely complex realms of *Fiction(en)* and *bloße Ideen* (cf. Newton's notorious remark that "*hypotheses non fingo*").

One might, for example, 'transcend' (KW, XXI, p. 233) deterministic 'heteronomy' with appeals to prototypes of 'complexity', 'emergence' and 'self-organisation', if such "bloße Ideen" were made (meta)mathematically precise in reasonably predictive ways.

'Skeptical' arguments, I believe, suggest that they can, and my modification of Galileo's famous dictum to suggest in 0.1 above that 'the book of philosophy' might (and perhaps should) be written in the language(s) of metamathematics was partly animated by this conviction.

It was also guided by a belief that if such programmatic conjectures turn out to have regulative value, the fate of Carnap's *Aufbau* suggests that

- 17 they will have to be adaptive and provisional rather than prescriptive, and that
- 18 those who make them would be well advised to acknowledge their *essential incompleteness* and recurrently '*liminal*' *metamathematical margins*.

For it is quite conceivable that

19 interrelations between 'the' 'book of philosophy' and 'language of (meta)mathematics' might *themselves* form 'systems' of 'intelligible' theories whose metametamathematical properties *evolve* and *ramify* in ways we cannot anticipate, much less dictate.

Still—as the Danish saying goes, "blind chickens find also a grain" ("blinde høns finder også et korn"), and an allegedly 'heuristic' proposal should try to 'find' ("heurein") something from time to time.

Here is a possible "korn". In one of 4.10's hierarchically organised "systems", answers to queries about *object-theoretic consequences* might serve as 'means' to more complex metatheoretic 'ends', and conversely.

Finitary 'intelligible systems' and their insights might therefore 'evolve' in such hierarchies—in which higher-order metatheoretic 'ends' in one 'systematic' context



became *object-theoretic* 'means' in another—and stages of such evolution might be construed as

20 [d]as Materiale[,] in so fern es nur *problematisch gedacht* und eine Tendenz enthält es sich assertorisch als gegeben vorzustellen (Organisch, Unorganisch). (*KW*, XXII, p. 480)

[t]he material[,] in so far as it is only *thought problematically* and possesses a tendency to represent itself as given assertorily (Organic, Inorganic).

All of this, once again, would remain 'merely regulative', 'physically' underdetermined and metamathematically incomplete ...ad indefinitum. But such underdetermination might at least be informative, as well as compatible with several of Kant's more 'skeptical' (or at least 'merely regulative') remarks—that

- 21 [d]ie Mathematik wird durch Philos. indirect begründet ...(*KW*, XXII, p. 78), [m]athematics is indirectly grounded through philos[ophy], for example; or that
- 22 ([d]er Begriff von organisi[e]rten Körpern gehört zum Fortschreiten im System der Wa[h]rnehmungen des Subjects das sich selbst affici[e]rt) (KW, XXII, p. 398); [t]he concept of organised bodies belongs to what is processive in the system of perceptions of the subject which [ap]perceives itself; or that
- 23 [m]an fängt da nicht von Objecten an sondern von dem **System** der Möglichkeit sein eigenes denkendes Subject zu constitui[e]ren und ist selbst Urheber seiner Denkkraft (KW, XXI, p. 79).
 - [o]ne does not begin from objects[,] but from the **system** of the possibility to constitute one's own thinking subject and [one] is the originator of one's capacity for thought;
 - or even that
- 24 die zweyte [organische Ordnung der Natur, d.i. die Form derselben nach Regeln] ist auf einer Idee gegründet, die des einzelnen sich als Werkzeug zu einer Einrichtung bedient, die aus den einzelnen Naturdingen nach allgemeinen Gesetzen nicht entsprungen wäre.) (KW, XVII, p. 418)
 - the second [organic order of nature, i.e. the form of it in accordance with rules] is based on an idea, which makes use of the particular as [an] instrument for an arrangement which would not [otherwise] have arisen from the separate things of nature in accordance with general laws.

My appeals to metatheoretical underdetermination might also be compatible with Kant's rare but carefully formulated (if 'merely regulative') remarks about '*Horizonte*' in the first *Critique*:

25 Der Inbegriff aller möglichen Gegenstände für unsere Erkenntnis scheint uns eine ebene Fläche zu sein, die ihren scheinbaren Horizont hat, nämlich das, was den ganzen Umfang derselben befaßt, und von uns der Vernunftbegriff der unbedingten Totalität genannt worden. Empirisch denselben zu erreichen, ist unmöglich, und nach einem gewissen Prinzip ihn a priori zu bestimmen, dazu sind alle Versuche vergeblich gewesen. (KdrV, B787)

The aggregate of all possible objects for our knowledge seems to us to be a plane



surface, which has its apparent [or specious] horizon, namely that which includes the entire extent of that knowledge, and has been called by us the *concept-of-reason* of unconditioned totality. To reach this [horizon] empirically is impossible, and [as for efforts] to determine [or define] it in accordance with a particular principle a priori, all [such] efforts have been in vain.

I have also argued that 'der' Vernunftbegriff 'der' unbedingten 'Erfahrung' is no more unique or "a priori zu bestimmen" than 'der' Vernunftbegriff 'der' unbedingten 'Totalität'—an assimilation which also seems to me compatible with Kant's remark (KdrV, B686–687, slightly abridged below) that

26 Man kann sich die systematische Einheit unter den drei logischen Principien auf folgende Art sinnlich machen. Man kann einen jeden Begriff als einen Punkt ansehen, der als der Standpunkt eines Zuschauers seinen Horizont hat, d.i. eine Menge von Dingen, die aus demselben können vorgestellt und gleichsam überschauet werden. Innerhalb diesem Horizonte muß eine Menge von Punkten ins Unendliche angegeben werden können, deren jeder wiederum seinen engeren Gesichtskreis hat; ...und der logische Horizont besteht nur aus kleineren Horizonten ..., nicht aber aus Punkten, die keinen Umfang haben (Individuen). Aber zu verschiedenen Horizonten, d.i.... die aus eben so viel Begriffen bestimmt werden, läßt sich ein gemeinschaftlicher Horizont, daraus man sie insgesamt als aus einem Mittelpunkte überschauet, gezogen denken, ...bis endlich ...der allgemeine und wahre Horizont ...aus dem Standpunkte des höchsten Begriffs bestimmt wird und alle Mannigfaltigkeit als ...unter sich befaßt.

One can make *systematic unity* [more] graphic [sensory] under the three logical principles in the following way. One can see an arbitrary concept as a point which as the *standpoint* of a viewer has its *horizon*, i.e., a set of things which can be represented from it and surveyed, as it were. Within this horizon, *one must be able to posit a set of points in infinitum*, *each of which has its more limited field of vision*, ...and the logical horizon consists only of smaller horizons, ...not of points, which have no extension (individuals). But to different horizons, ...which are determined from different [corresponding] concepts, a common horizon, from which one can survey them all as [if] from a center, can be traced out in thought, ...until finally ...the general and true horizon is determined ...from the standpoint of the highest concept, and the manifold comprehended ...under it.

Many other interesting if problematic things could be said about Kant's "Sinnlich-machung" in the metalogical framework of this essay.

In the ellipses of this passage, for example, he interpreted "Horizonte" as "Gattungen, Arten und Unterarten"..., whose most straightforward metalogical counterparts would be

- 27 (consistent and recursively defined) "types" or imbricated collections of unary predicates in the language of a given "intelligible" theory T.

 One could, moreover, define
- 28 **binary**, *n-ary* and '*nullary*' '*types*'' as well, and identify the latter with *consistent*, *recursively axiomatisable theories* which extend **T**. For each such type, one could also define



29 a 'canonical' *boolean algebra* (called its 'Lindenbaum algebra'), as well as corresponding (topological) '*Stone spaces*'—mentioned earlier in 1.3 and 6.1–6.3—in which extensions of the given type determine *closed subspaces*.

'Complete' counterparts of such 'types', finally, could also be defined (as elements of the aforementioned 'Stone spaces'), but only in appropriate metatheories, and there would be uncountably many of them.

The elements of such spaces—metalogical analogues of Kant's "points, which have no extension"—would be "durchgängig bestimmt", in Kant's language ("notiones completae", in Leibniz'), but 'almost all' of them would fail to be 'intelligible', in the metalogical sense sketched earlier.

One might try to assimilate Kant's "allgemeine(r) und wahre(r) Horizont" to an entire 'Stone space' of such 'points', and such spaces are indeed 'universal' in various topological and metatheoretic senses.

But they are also isomorphic to spaces of *infinite sequences of 'random' coin tosses*, an observation which suggests that such 'Venunftideen' would degrade rather than embody informational 'truth'.

For it is also known that *every* "*intelligible*" theory has the same Stone space, whose 'generality' and 'canonicity' would therefore render its 'reference-frame' useless for discrimination of one 'intelligible' theory from another.

To me at least, such observations call to mind two well-known predicaments in the history of philosophy:

- 30 the immanent white noise of Spinoza's (1967) transcendent deus sive natura; and
- 31 Kant's remark that "concepts" without "intuitions" are "blind" (in the sense that every intelligible theory may be said to furnish its own class of "intuitions").

But it also suggests that a form of "Erhabenheit" may be found in

32 limitless 'universes' of 'intelligible' theories' interpretations and capacities for 'autological' [self]-reference,

and that this 'sublimity' may be even more 'disproportionate' than Kant's "bestirnter Himmel" and the "two abysses" evoked in one of the most resonant passages of Pascal's (1963) Pensées:

33 Qui se considérera ...[I]es deux abîmes de l' infini et du néant, il tremblera dans la vue de ses merveilles, et je crois que sa curiosité se changeant en admiration, il sera plus disposé à les contempler en silence qu'à les rechercher avec présomption.... Quand on est instruit, on comprend que la nature ayant gravé son image ...dans toutes choses, elles tiennent presque toutes de sa double infinité: c'est ainsi que nous voyons que toutes les sciences sont infinies en l'étendue de leurs recherches.... Toutes choses sont sorties du néant et portées jusqu'a l'infini.... causées et causantes, aidées et aidantes, médiatement et immédiatement,



Voilà notre état véritable. C'est ce qui nous rend incapables de savoir certainement et d'ignorer absolument.

Anyone who considers ...the two abysses of the infinite and the void, will tremble in the sight of its marvels, and I believe that—curiosity changing into admiration—will be more disposed to contemplate them in silence than explore them with presumption.... When one is learned, one comprehends that since nature has graven its image in ...all things, almost all of them have something of this double infinity: in this fashion we see that all forms of knowledge are infinite in the range[s] of their inquiries.... All things have emerged from the void and are carried into the infinite, ...[and are] caused and causing, aided and aiding, mediately and immediately, Therein is our veritable condition. This is what renders us unable to know with certainty, and be ignorant absolutely.

In the essay's final section, I will

- 34 argue that the 'underdetermination' of ramified 'experiential' hierarchies—a fragile and inadequate form of 'freedom'—has the dignity of one of its 'preconditions'; and
- 35 compare the "abîmes" of its endless 'ramifications' with the deltas and estuaries of Norman MacLean's "river", in which faint traces might linger long after we cease to 'exist'.

8 Reflective inquiry as a form of skeptical "Theoria"

In an unpublished essay called "Skeptical *Theoria* as a Regulative Ideal", I

- 1 compared phenomenal *underdetermination* to a 'transcendental' precondition of an activity I called 'skeptical *theoria*' (a '*via negativa*'), and
- 2 assimilated such such theoria (or 'via contemplativa') to the "reine Selbsttätigkeit" ("pure self-actuation", Gr, 452) which Kant called Freiheit.

I also observed that

3 the ancient skeptics attached a *regulative* (or '*contemplative*') value they found significant to such *epoche and the search for it*.

That 'regulative' value, I also suggested (cf. 1.9 and 5.15 above), is Aristotle's 'desire', not for 'knowledge', but for quasi-Kantian forms of 'disinterested' inquiry.

For 'pyrrhonist' and 'academic' skeptics alike believed ('undogmatically', of course) that it conferred a kind of peace of mind they called "ataraxia" (roughly "equanimity", literally 'non-perturbation'), a word one also finds in Stoic and Epicurean writings.

Also of interest, I believe, in the context of this essay's metalogical analogies, is the 'problematic' 'reflexivity' of such 'ataractic' self-attribution and 'practical' "self-actuation". For both introduce—as Kant observed—an implicit 'critical' (or 'metatheoretical') distinction between a 'self' and what calms or 'actuates' it.

In "Skeptical *Theoria*" (1997) as well as (Boos 1987), I also outlined a related analogy between skeptical *epoche* about 'phenomenally' undecidable assertions, and



(some of) the responses Immanuel Kant advocated to 'transcendent', 'antinomial' assertions of 'pure reason', which are, by definition, undecidable within 'the' Bereich der Erscheinungen.

I drew inspiration for this, in part, from Kant's own high valuation of the antinomies, despite his disparaging remarks about skepticism in the "*Transcendental Methodology*" (cf. *KdrV*, B784–B797), but also from the evidence canvassed in foregoing sections that

- 4 significant *hermeneutic* and *alethic* notions such as interpretability, definability, and (a fortiori) 'truth' may indeed be Kantian '*Grenzideen*', rather than Kantian '*Begriffe*'—in meta*logic and* in meta*physics*; or at least that
- 5 such hermeneutic and alethic notions are *relational* notions: *conjectural and schematic templates* which our finite intellects are fortunate enough to be able to project for heuristic investigation and dialectical and practical inquiry.

'Disquotational' 'truth' of a theory, for example—Tarski's "Convention T"—is a jejune second-order template. Nothing substantive emerges from it until we *refine* it to the *relational* meatheoretic notion of 'truth in a structure'.

Even *then*, moreover, 'intelligible' first-order theories will propose *infinitely many* distinct candidates for such structures, which may be examined in infinitely many branching candidates for appropriate *metatheories*.

'Speculatively' as well as 'practically', therefore, attempts to formulate [self]-grounding 'intelligible' theories of the sort Descartes, Berkeley and Kant sought cannot escape a kind of *complementarity* between the intentional *finitude* of our conceptual horizons, and *lack of closure* of the concept-formation(s) we undertake within them.

For 'intelligible' theories face a choice between two '*complementary*' alternatives: *undecidability* and *inexpressibility*. This (literal) *dilemma*—a simple consequence, of Gödel's insights into [internal] diagonalisation—may be posed as follows.

Such a theory—of 'the self', say; or 'the world'; or Anselm's formula 'that than which nothing greater can be conceived' (Kant's "psychologische, kosmologische und theologische Ideen")—must itself be incomplete. It can formulate, but never prove, [its own] existence (consistency/interpretability).

By simple contraposition, then:

A complete limiting (meta)theory—not only of Kant's three *Ideen*, but also of 'the' distinctions Kant tries to draw between *speculative* and *practical*, *Phaenomena* and *Noumena*, *Begriffe* and *Ideen*, *Vernunft* and *reflektierende Urteilskraft*—would not be 'intelligible' and could not, *a fortiori*, *formulate* or *express* (encode) [its own] existence.

To put a mildly provocative point on it:

6 if a 'canonical' intelligible theory **T** of 'experience' (or 'the god of the philosophers'; or 'der heilige Wille'; or) were provably 'complete' in some tacitly metatheoretic faithful extension **U** of **T** (i.e., if the extension **U** decided every assertion in the language of **T**), such a 'proof' would bear witness to the inconsistency of **U** as well as **T**.

This recurrent *complementarity*—between *intelligibility* (or *expressibility*) and putative (metatheoretic) *universality*—seems to me to take many forms in the his-



tory of philosophy, as I've tried to suggest in Sect. 3's examinations of the arguments of Descartes, Berkeley and Kant.

More precisely, I have argued that we cannot ensure, or even canonically *express*, certain ideals and *Grenzideen*, with anything like the constitutive *Vollständigkeit* and semantic unicity that most classical metaphysicians (and most analytic philosophers) have demanded of (and casually attributed to) 'truth'.

I have also tried to suggest

- 7 that such semantic *Vollständigkeit* (completeness) is what is really at issue—as Kant saw—in classical metaphysical examinations of the [self], and of 'the god of the philosophers' and Kant's three *Ideen*; and
 - More controversially, I have argued
- 8 that one can trace a number of analogies and between other metaphysicians' 'probative' arguments made on behalf of these *Ideen* and Kant's tacit appeals to 'transcendental' design-arguments and the 'archimedean' leverage he attributed to his 'critical' distinctions and 'transcendental deductions'; and
- 9 that the "durchgängige Bestimmung" Kant claimed to have 'deduced' for 'the' 'constitutive' structure of 'experience' may be assimilated to the indeterminate "Vollständigkeit der Bedingungen" he (cogently) characterised in the Dialectic as a kind of underlying master-Vernunftidee.

In defense of the 'locality' of 'critical' theory/metatheory-distinctions, I have also argued that we have no 'ultimate' decision-procedure which would enable us once and for all to discern, in our 'theoretical' and metatheoretical inquiries, proof from [proof] (existence from [existence]; consistency from [consistency];).

Particular *theories*, for example—or at least their more feasible axiomatic approximations—*may* be 'locally' immanent. But their '*intended interpretations*'— even 'the' 'intended' interpretations of theories as 'obvious' as Peano arithmetic (an axiomatisation of 'the' natural numbers)—are 'locally' but recurrently *transzendent*.

Such 'intended interpretations' are surely 'abstractions' in some sense, if anything is. But such abstracta—which are not to be identified with the theories themselves, for this quickly gives rise to semantic paradoxes—require even more 'abstract' metatheories in which their 'intentions' can be grounded.

Even then, moreover, they are (I would suggest once again) either

- 10 theory-relative placeholders, which ride indefinite regresses of ascents of metatheoretic 'forms'; *or*
- 11 'noumenal' notions which are inexpressible in [themselves] (the echo of Kant's usage is once again deliberate).

On the processive account of 'experience' I have offered, the former are problematic but recurrently 'immanent'. The latter, by contrast, promote the notion of an '*intended interpretation*' to the level of its conceptual incompetence.

For nothing can (*provably*) '*ground*' or '*interpret*' what it cannot *express*—an observation which often seems to me to have eluded metaphysicians and philosophers of language who engage in relationally unqualified talk about 'truth'.

Correlatively, attempts to 'ground' (end) metatheoretic ascent in [self] -interpretation—allegedly secured in the sort of 'last' metatheory considered in Sects. 2 and 3—seem to me



12 parade-examples of what Kant called "transzendent(al)er Schein" (cf. KdrV, B352 ff.), on a more or less equal basis with attempts to ground 'the god of the philosophers' in [self]-causation, [self]-origination, [self]-organisation, or [self]-design.

Indeed, Kant's attempts to 'intend' and secure a canonical extratheoretic interpretation of a *collective* (as well as subreptively uniform) [self], first, would seem to parallel rather closely Descartes' attempt to do this for a '*psychological*' [self], along the lines outlined above in Sects. 2 and 3.

More generally, 'secular' efforts to find *last*, 'perfect', self-realising metatheories for such (putatively) 'universal' notions aspire to 'complete' [themselves], in ways which recall *ontological*, 'cosmological', 'physico-theological' and other 'regress'-ending attempts to beg 'theological' notions of ultimate 'design'.

Methodologically, for example,

13 claims that 'all' of mathematical physics (say)—or 'all' 'real' knowledge of a language, for that matter—can be subsumed in some elusively 'intended' or 'intuitive' interpretation of 'physics'—or what it 'really' 'is' to 'speak a language'—

seem to me closely cognate to

14 Pascal's baffling embrace of Jansenist dogma; Spinoza's admirably stoic (but conceptually ill-posed) assertions about 'deus sive natura'; Leibniz' more sympathetic (but equally ill-posed) 'principe de la raison suffisante'; and Kant's transzendentale Verstandeseinheit, whose 'deduction' he acknowledged would have to be a Geschäft der Vernunft in KdrV, B692.

For such 'global' and 'universal' interpretations are all theory-marginal, in the sense that they would have to be 'intended' and re-'intended' in ever-wider metatheories. But the more 'global' one's theoretical aspirations—and more 'holist' one's 'contemplation' or theoria—the more open to metatheoretic ascent and emergent reinterpretation(s) such aspirations and theoria will have to be.

Such tensions already appeared in Gaunilo's shrewd suggestion that limiting cases of the 'conception' and 'perfection' Anselm wanted us to 'conceive' and mentally 'perfect' might either be 'trivial', or at least imperfect ('the most perfect island') or inconceivable.

They also recurred more strikingly in Hume's (or "Philo"'s) brilliantly formulated suggestions in the quotation from the *Dialogues Concerning Natural Religion* which began this essay—that 'human' notions of [design] might provide very dubious templates for theological extrapolation.

In Kant's case, I have argued, there is also a sense in which he offered 'cogito'-like 'transcendental arguments' as attempts

- 15 to furnish (or beg) a *collective* counterpart of Descartes' *individual* (but allegedly generic) '*existence*'-proof, and
- 16 derive the 'existence of the external world' (as well as a resolution of Descartes' 'substantial-union'-problem) from the alleged exhaustiveness and universality of such a 'complete' and 'necessarily possible' collective 'intentionality'.

But if what is *intentionally* 'possible' is collapsed in such ways to what is *experientially* 'real'—a not entirely distortive thumbnail-sketch of Kant's 'critical' rationale



for Berkeleyan and Humean proposals—another variant of the recurrent dilemma I've tried to canvass presents itself: *either*

- 17 one opens 'experience' to embrace indefinite hierarchies of provisional forms of semantic paradox; or
- 18 one acknowledges that 'experience' is an inexpressible and almost neoplatonic 'ideal', and its alleged 'closure' a *mystical* as well as *unintelligible* '*ultimate*' *semantic paradox*.

One might construe this dichotomy as a sort of conceptual 'zero-one-law', or counterpart of Niels Bohr's "komplementarität": if it is not the case that 'everything intelligible' is 'problematic', what 'ensures' us that 'it' is 'unintelligible'?

Put in yet another form: we seem to *want* (or in our more judicious moods, 'postulate') '*complete*' knowledge and '*complete*' (unique) interpretation (cf. Pascal's acknowledgment that "*le silence éternel de ces espaces infinis m'effraie*").

But all we can 'intelligibly' (or 'reasonably') attain is *incomplete* 'understanding', conjectural as well as plurally interpretable clarification, and fleeting forms of *a-lētheia* ('non-ignorance'), rather than 'ultimate' 'truth'.

Postulations of 'complete' knowledge, 'objective' truth and 'unique' interpretation, like postulations of 'ultimate' subjective 'enlightenment', may be little more than thinly veiled projections of intellectual vanity and vicarious aspirations to metaphysical immortality.

So much for what might be called the *via negativa* of skeptical *theoria* ("*contemplation*"), a dialectical interpellation which emerges—I have argued—from fairly straightforward forms of metalogical analysis.

Its *analogon*—partially and tantalisingly reflected in the remarks of writers such as Nicholas of Cusa—is the skeptical *via contemplativa* evoked in 1.9, at length in Sect. 5, *passim* in Sect. 7 and more briefly in 8.1–8.3 above.

By way of transition to such a 'way' or 'path', consider first the following remarks and conjectures about the differences between 'skepticism' and 'mysticism'.

- 19 We tend to see more 'simply' 'organised' systems as 'instruments' for 'us' (or at least for Kant's "*empirisches ich*").
- 20 We are more reluctant (quite understandably) to regard 'ourselves' as mere 'means' or 'instruments' for 'others', whether or not they are more 'complex' than 'we' are.
- 21 We tend finally to beg the 'existence' and 'uniqueness' of 'gods', 'theodicies' and ineffably complex secular 'designs' as *Wunschvorstellungen* of what might 'confirm' or at least '(over)see' 'us'.

"Intellectual vanity" aside, there seems to me a *prima facie* tension between 20 and 21, just above, thrown into sharper relief by Hume's thumbnail critique of anthropomorphic 'idealism' ("What particular privilege has this little Agitation of the Brain...") in the *Dialogues Concerning Natural Religion* (1976), cited earlier.

I have already argued, in effect, that such a tension defines that work's principle (and proto-'critical') *disputatio*—

22 whether the 'deist' "Cleanthes" can define a tenable 'critical' middle-ground between the 'skeptic' "Philo"'s deconstructive counterexamples, and the quasi-



'mystical' commonplaces of "Demea", a Christian straw-man who stands in for the much more searching 'fideist' musings of Blaise Pascal—

and tried to offer suggestive metalogical evidence that "he" ("Cleanthes") cannot. To see why I find this 'evidence' persuasive, consider first my suggestion that

- 23 "Demea"'s modest rhapsodies are indeed little more than faint echoes of much more eloquent passages from the *Pensées*, one of which I have already quoted, but
- 24 Pascal's evocations of our "disproportion" play a propaedeutic role in his larger dogmatic argument, as (at least arguably) did comparable invocations of ineffable infinities in the *Enneads* of Plotinus and *Ethica* of Spinoza.

In response to worthy defenders of these and other forms of 'semantic monism' (animated by a "monistic pathos", in Lovejoy's words, 1965), I have endeavored to

- 25 interpret skeptical "*criterial problems*" and "*lapse[s] into infinit[ies]*" as generative structures worth further study;
- 26 argue that 'ultimate' metaphysical 'unities' 'deduced'—by Spinoza, among others—from begged dichotomic premises are metalogically 'unintelligible"; and
- 27 offer 'locally transcendental' interpretations of a "pluralistic pathos" in which ramifying 'intentional' hierarchies reflect Kant's "moral law within us", and ramifying 'intensional' hierarchies reflect Kant's 'starred heaven(s) above us".

This then is the skeptical counterpart of the dogmatic 'via contemplativa', mentioned earlier: a 'reflective' 'pathos' of limitless 'critical' hierarchies for

28 Demea's (and Pascal's) *metalogically untenable fideism*, and Cleanthes' (and Kant's) *metalogically untenable "middle way*".

One indication of the latter's 'untenability' may be found in Kant's 'critically' calibrated claims to have to 'deduced' the 'necessary existence' and unique interpretability of 'experience' from its 'possibility'.

For such formulations have appeared almost word for word in certain modal logicians' claims to 'formalise' Anselm's 'proof' of the 'necessary existence' and unique interpretability of 'god' from its 'conceivability'.

In §90 of the third *Critique*, moreover (which bears the interesting title "*Von der Art des Fürwahrhaltens in einem teleologischen Beweis des Dasein Gottes*"; "On the Manner of Holding-to-Be-True in a Teleological Proof of the Existence of God"), Kant struggled in 1790 to formulate

29 "analogical" notions of "proof" which might apply to 'teleological' insights, desires for 'higher' forms of justice and other 'merely regulative' aspects of 'reason"s 'fate'.

In this essay, I have tried to propose

30 'analogical' notions of 'proof' in limitless metatheoretic and hypotheoretic hierarchies as 'intelligible' media for the expression of 'reason's 'fate', as well as Kant's 'secular humanist' ideals.



In accordance with the 'via negativa' sketched earlier, much may be processively 'hidden' and 'uncovered' in such 'experiential' hierarchies (contra Wittgenstein), in the sense that

- 31 no 'intelligible' theoretical '*horizon*' is or will ever be '*comprehensive*', for example (much less 'define' its own '*azimuth*');
- 32 'merely regulative' postulation of certain forms and probabilistic extensions of metamathematical deduction are all the 'inductive confirmation' we should anticipate; and
- 33 Galileo's mathematical "book" and its 'intelligible' metamathematical extensions are 'physically' as well as 'metaphysically' 'precise' to the extent they are conceptually (and generatively) 'heuristic' and 'intelligible'.

In keeping with such *Leitmotive*, I have also argued that Kant's 'critical' attempts to discern 'concepts' from 'ideas of reason' led him to a valuable (and 'heuristically' generative) insight:

- 34 that such methodological *clarifications* and 'synthetic' *realisations* of otherwise uninterpreted *syntactical* evidence are—'*locally*', but '*inherently*'—*semantic* and *metatheoretic* rather than '*immanent*'; and
- 35 that his tacit acknowledgments in the *Dialektic* and third *Critique* that such metatheoretical undertakings might be—'locally' but recurrently—'transzendent' suggest two more skeptical, 'neo-Kantian' conjectures: that
- 36 the very 'intelligibility' of ('our') 'experience' renders its 'realisations' (its "Verwirklichungen" or "Vergegenständlichungen") provisional, plurally interpretable and inherently incomplete and 'merely regulative'; and finally that
- 37 attempts to analyse and argue more precisely about such 'realisations' may lead us into hierarchies—also 'local', but deeply heuristic as well as 'merely' regulative—of 'intended (re)interpretations' of 'ourselves' as well as our 'experiences'.

'Local' and 'heuristic' notions of 'truth', 'completeness' and '(intended) interpretation', after all—like their 'absolute' dogmatic counterparts—are closely interrelated with (equally 'local' and 'heuristic') notions of 'design'.

And these, in turn, yield 'critical' rationales for 'essentially incomplete' notions of 'knowledge', 'structure' and 'understanding'—'inductive understanding', for example, that one 'knows' (provisionally, and in certain feasibly characterisable situations) 'how to go on'.

Like 'design', 'truth' and the other epistemic and metaphysical ideals just cited make sense for a given theory, in a particular metatheory. 'Contemplatively' invaluable, they become delusive—and 'dogmatic' as well as uninformative—only when they too are raised to the level of their incompetence in vain efforts to 'diagonalise' them over 'all' theories in 'all' metatheories.

(We tend to lose sight, once again, of the facts that a 'proof' was originally a 'test' (a 'probatio'), and 'truth', in English, derived from a common germanic verb which meant to '(en)trust' or 'believe'.)

I have also tried to argue at various points that



- 38 iterated 'zetetic' recourse to metatheoretic clarification and refinement may be heuristically '*instrumental*', where
- 39 '*instruments*' are ('locally') object-theoretic '*means*' to more 'systematic' but counterfactual as well as potentially heuristic ('locally') metatheoretic '*ends*'.

In particular, such 'means' or 'instruments' are object-theoretic with respect to what (conceptually or metatheoretically) 'instrumentalises' them, and 'experience' 'organises' 'means' and 'ends' in metalogical hierarchies of 'object' - and 'concept'-formation which ramify without end.

(In this sense, what I wish to offer is a form of skeptical as well as 'locally transcendental idealism', which hearkens back to Lucretius' 'alphabet', as well as the 'logica magna' Leibniz sought.)

In their efforts to explain 'physical' 'experience', for example, mathematicians and mathematical-physicists—Galileo's successors—resorted to more and more elaborate *metatheories* to 'define' (or posit 'existence' of) object-theoretic 'instruments' as 'means' to more and more subtle (and conceptually 'elegant') 'physical' ends.

Mindful of this, I have tried to argue that mathematically literate metaphysicians and epistemologists—Kant's successors—should *search* ('zetetically') for more elaborate 'intelligible' *metatheories* to 'define' (or posit 'existence' of) object-theoretic 'instruments' as 'means' to subtler (and more conceptually 'elegant') 'intended' interpretations of

- 40 'cognitive' ends which clarify, at least provisionally, 'liminal' notions of 'identity', 'awareness' and 'individual consciousness', and
- 41 'ethical' and 'deontological' ends which clarify, even more provisionally, collectively 'liminal' notions of 'equity', 'dignity', 'equanimity' and 'collective moral sensibility'.

If these analogies are tenable, one might also characterise

- 42 Hume's and Kant's 'idealist' efforts to clarify 'physical' patterns in terms of 'cognitive' counterparts as an attempt to clarify subtilia in subtiliora, and
- 43 Kant's "Primat des Praktischen" as a conjecture that 'ethical' and 'deontological' ends or aims may be 'deeper' and more conceptually complex than their 'physical' and 'cognitive' counterparts.

Kant's 'counterfactual' and 'nonconstructive' uses of "Regel' and "regulativ", for example, were as ambiguously suggestive as Hume's ambiguous attempts to define "Cause" in §60 of the first Enquiry. Perhaps it is 'regulative' of broader forms of 'experience' that we can 'nonconstructively' 'anticipate' schematic patterns we cannot 'determinately' 'understand'?

Graduated forms of 'consciousness', in particular, might be attributed to Sect. 6's 'systems' (or more accurately to metastable patterns of such systems' evolution in appropriate Stone spaces), to the extent they

- 44 range '*freely*' over '*internal*' interpretations of [themselves], and '*external*' interpretations of [what might be *beyond* themselves]; where
- 45 'freedom'—like its 'physical' counterparts—is an inherently contextual and relational as well as graduated notion:



46 a system is '*free*' to the extent the range and complexity of its responses exceed that of any 'internal' or 'external' initial and boundary-conditions it may 'take into account' in formulation of such responses.

Admittedly, all these conjectures and analogies are 'impressionistic', and conceptually (much) more inchoate and rudimentary than I would wish them to be.

But I believe they may be heuristically useful, and more particularly, that the *indefinite extendibility* of *metatheoretic 'ascent*' and the *non-'well-foundedness*' of *hypotheoretic 'descent*' are *dual aspects of intelligibility* which '*mirror*' each other in heuristically useful ways.

'Hypotheoretic' attributions of grades of 'intention', 'cognition' and 'autonomy', for example, may 'mirror' 'metatheoretic' attributions of 'interpretation', 'verification' and 'experimental isolation'. ("The way up and the way down are one and the same.")

'Reflective inquiry', in this context, is inquiry in localisable stages of such hierarchies, and in their indefinitely iterable intentional ascents and intensional descents. As 'we' 'see' 'simpler' conative and cognitive structures, so may more 'complex' conative and cognitive structures 'see' 'us'. ("Whatever ye do to the least of these....")

Consider, by way of partial summary, the following assertions and (rhetorical) questions.

Kant's 'transzendentale Methode' was arguably an attempt to make a certain sort of 'sufficient ('synthetic a priori') reason' a fixed point for [itself].

One might construe the arguments of this essay as an attempt to make metalogical '*inquiry*' be an (endlessly transitional, but conceptually persistent) *fixed point for* [*itself*].

Contrary to Kant's express views in the *Analytic*, the more "rein" and "a priori" a theory (or "Erkenntnis") is, the more (metalogically) incomplete it is, and (therefore) the less "vollständig" it can be. Might this be a form of 'practical' as well conceptual 'complementarity'?

The "Vollständigkeit" Kant attributed to "experience" was "constitutive", but that of ideas-of reason ('merely') "regulative". Might recognition that both forms of inquiry and concept-formation are 'essentially' incomplete, be a mark of John Keats' 'negative capability' (cited above), as well as the Würde (dignity) Kant attributed to 'reasonable beings'?

These questions are "rhetorical" in the sense that I find 'good reasons' to *believe* (but cannot 'prove') that the 'contemplative' answer to each is "yes".

For it seems a mark of stoic epistemic *virtue* and *awareness* of our boundless *limitations* to continue to inquire and observe, knowing that 'the' path(s) branch in liminally unobservable directions. It may even be a mark of some sort of epistemic or conceptual '*existentialism*' (in which the 'essence' of 'essential incompleteness' '*precedes*' '*existence*').

Indeed, an awareness of 'reason's fate' and its Sisyphean nature may be part of that 'fate', as well as a regulative ideal of Kant's "reasonable beings". But it might also (as I suggested earlier) be one of their more 'enlightened' cradle-gifts. (To paraphrase Beckett: "I 'must' go on. I can't go on. I 'will' (to) go on...")



Aristotle's 'desire to know', for example, is not simply 'sensory'. It is also the desire to 'value' and to 'cherish' (This is a kind of 'cognition'). And a broader and more 'reflective' as well as 'practical' form of this is the desire to 'value' and 'cherish' what can 'value' and 'cherish'.

To 'know' is to recognise what (apparently) 'is'. To 'value' is to recognise (or 'contemplate') what may be, or might have been.

To form a 'purpose' and 'evaluate' alternative 'means' to attain it is to form a (usually counterfactual, and in all cases metatheoretic) 'plan' in keeping with such 'recognition' or 'contemplation'.

Such a plan, finally, is (*practically*) *good* to the extent it '*values*' the (indeterminate) plurality and range of *other* 'reasonable beings' '*plans*'.

What is 'just' is what 'reaches out' (the original semantic sense of Aristotle's "desire"), by analogy with what is 'felt within' (the 'golden rule'). This is 'regulative' of equity and other 'practically' significant forms of empathy and sympathy.

An absence of both in a 'cognitive system' is *not 'irrational*'. But it *is* (metatheoretically) '*unreasonable*'. It marks the absence or suppression of a form of higher-order awareness.

The distinction between '*irrational*' and '*unreasonable*' is 'practical', metatheoretical, inherently '*liminal*' and '*merely regulative*' at every stage of 'experience'.

Refusal or inability to make this distinction is a dogmatic 'cognitive' deficiency or lack of 'sensibility' (or a deliberate attempt to dissemble, 'blunt' or 'coarsen' certain 'natural' forms of cognitive awareness).

There is also a fairly clear parallel between Hume's *de facto* assimilation of "*conceivable*" to "*consistent*", Kant's 'categorical imperative' and Kant's assimilation of "*gültig für uns*" = *allgemeingültig für jedermann*" ("valid for us" = "universally valid for everyone"; *Pr*, 1976, Sects. 18–19);

Both conflate "inconceivable to 'us" with "inconceivable relative to a(n allegedly normalised but subreptively begged) conception of 'us' (of what 'we' are). (For comparison, cf. also Kant's remarks about "nur komparative Allgemeingültigkeit"—"merely comparative universal validity", Sects. 18–19).

Such conflation may also be construed as a begged assertion that 'the' theory of ['the' world] is a *faithful* or *conservative extension* of 'our' theory of [the world], whatever *Dinge an sich* it may hypothetically adjoin.

If, moreover, 'the world's interpretation *were* faithful, as in the analysis of '*cogito*'-like arguments sketched earlier, it would assure a not-so-desirable form of *equiconsistency*:

47 'the world' would be consistent iff 'I' am (or in Kant's case, iff 'we' are).

A 'solipsist' metalogical consequence of this—that 'the world' would cease to 'exist' iff 'I' cease to 'exist'—would effectively nullify Kant's categorical imperative, as well as the 'skeptical' variant I proposed above:

48 to treat others *not as* (mere) *means*, *but as* (vulnerable as well as 'reasonable' beings in search of 'higher') ends in (and beyond) themselves.



Given also that 'strong' interpretations of 'us' and 'our' world-views are not 'faithful' in the metalogical sense outlined earlier, the more relevant epistemic as well as 'practical' question would *not* seem to me

49 whether 'absolute' (ethical or epistemic) norms, standards or criteria 'exist';

but rather—given the metalogical 'unintelligibility' of 'absolute' norms, standards and criteria in the senses outlined above—

50 whether (and when) it is *reasonable* for sentient beings to impose *their* norms, standards or criteria as such.

Along similar lines, it also seems to me that *dicta* that "everything is determined" would *not* imply that 'everything is forgivable' (on the model of "tout comprendre, c'est tout pardonner"), but rather that

- 51 'practical' notions of 'forgiveness' and 'sympathy' as well as 'responsibility' would have no meaning...; for
- 52 a 'free' being which refused to admit its actions might be *wrong* would be an inconsistent *cognitive* as well as ethical agent, in the sense that
- 53 nothing could be '*right*'—'practically' or ethically 'right'—to a being which could not admit—'wholeheartedly' admit—that it might be *wrong*.

A 'rational' being is also 'practical' as well as 'reasonable' to the extent it acknowledges that it is neither unique nor self-sufficient, and undertakes to seek to 'respect' cognate desires and forms of awareness in its (apparent) 'semblables'.

At every stage of 'reflective inquiry', this awareness is graduated, underdetermined and physically vulnerable. But its absence is a 'cognitive' deficiency all the same, in the sense that it closes itself to broader forms of semantic 'awareness'.

For zetetic inquiry is not only regulative of 'experience'. It is also, it seems to me, a 'liminal' form—and perhaps prototype—of Kant's "Zweckmäßigkeit ohne Zweck" and other 'nonconstructive' notions one might devise, such as

- 54 "Gefühlsmäßigkeit ohne (eindeutig determinierte) Gefühl(e)";
 - "Regelmäßigkeit ohne (eindeutig determinierte) Regel(n);
 - "Rechtmäßigkeit ohne (eindeutig determiniertes) Recht";
 - "Gesetzmäßigkeit ohne (eindeutig determiniertes) Gesetz";

To the extent such "-mäßigkeiten" urge respect for honest inquiry, they may express something like Kant's "purely good will" as well as Aristotle's 'desire to 'know'. And respect for them enjoins us to value two forces which through the green fuse drive the flower: 'desire' for 'understanding' and 'sympathy' 'within' us', and 'awe' for the 'starred firmament' 'above' us.

The 'desire' in particular, takes many forms: a desire to (continue to be part of) (conscious) 'experience'; a desire to 'understand' the diapason of 'experience' in ways which cannot be fulfilled; and a desire to seek, value and protect that fragile 'understanding' as the 'reason' of 'reasonable beings'.

Kant had a point, therefore, about the 'intentional' and 'self-referential' aspects of epistemic, 'practical', aesthetic, and 'reflective' 'autonomy'. But autonomous



'selves'—essentially by definition—have no 'ultimate' clarification of what it 'is' they are supposed to 'legislate', or 'ultimate' intelligible 'aim' to which everything else is 'reduced' or 'interpreted'.

For the self-referentiality of 'intelligible' 'Selbstgesetzgebung' and 'Selbstzwecke' must be incomplete and 'problematic', in the sense that they are assertible only in processively metatheoretic contexts which must provisionally be resolved in yet 'higher' 'coded' counterparts.

Any conceptual act which claimed to 'verify [its own] 'existence' (or "Reinheit", or 'experimental isolation', or)' would therefore undo the 'verification', by relativising the 'existence' (or "Reinheit", or 'experimental isolation', or).

'The' boundary line between Kantian 'reason' and 'experience' is also (in)determinate iff 'the' boundary between Kantian 'analytic' and 'synthetic' is (in)determinate iff 'the' boundaries between Kantian 'Spekulation' and 'Praxis' and 'Spekulation' and 'Reflexion' are (in)determinate.

The *more 'global'* one's theoretical aspirations, therefore—and the more '*holist'* one's theory or *theoria*—the *more open* to metatheoretic ascent and emergent reinterpretation(s) such aspirations and *theoria* will have to be.

The *broader* the *Rechtmäßigkeit* of *Deduktion* becomes, the less clear it is why it is *not* simply '*reflective*', '*regulative*' and '*problematisch*', and the more suggestive the thought that *such breadth is a 'good' thing*.

Skeptical 'theoria' ('contemplation') and the forms of ataraxia ('peace of mind) also have their own '*light*'-images—*skepsis* itself, for example, and *Horizonte* among them—which reflect (and reflect on) their more 'dogmatic' and 'hieratic' counterparts (Plato's *eidē*, Descartes' *intuitio*, neo-Platonic '*emanations*',).

Incomplete and inadequate as the skeptical images are, they are at least more 'open' to further 'clarification', for they are exquisitely 'relational', by definition as well as programmatic intent ('insight' against what 'backlight'? 'scrutiny' and 'enlightenment' 'relative' to what source? 'escape' in which 'direction', and toward which form(s) of greater 'illumination'?)

Unlike Plato's (literally 'linear') 'line' and Kant's 'two truths', such images also evoke a distributive and 'graduated' skeptical counterpart of Aristotle's 'theoretical' *noesis noēseos* (thought about thought), whose ramifications suggest to me a great tree of conceptual life.

The 'abstraction' characteristic of metalogical analysis is a counterpart of 'egalitarianism' in ethics, and 'experimental isolation' in 'natural philosophy'. (Cf. also Kant's "rein" and "a priori")

These views seem to me to reflect elusive egalitarian ideals of Kantian ethics which transcend his secular-pietist preoccupation with 'absolute' forms of 'duty', and enjoin us to 'respect' fleeting nuances of 'consciousness' which 'reasonable beings' intermittently enjoy, and may (or may not) endeavor to 'respect' in their fellows.

Such genuine egalitarianism is a 'merely regulative' ideal. And intelligible "Zwecke" 'exist' only in provisional stages or levels of Zweckmäßigkeit. But the very openness and incompleteness of such ideals may offer a glimpse into comparably 'open' aspects of moral eternity.



For whatever "particular privilege this little Agitation of the Brain" may have (in response to the opening remark from Hume's *Dialogues*), it is probably not particular to 'us'.

And complex 'intentional' "systems" of "inquiry" which "ramify" with what they "discover" may be 'regulative' of other (would-be) 'reasonable beings', which may pose or have posed similar questions, in the past, in the future, or "in another galaxy, long ago and far away".

What rides the wavefronts of 'for the sake of which' (so to speak) is ('reflective') 'consciousness', which flourishes as it inquires, in unfinished forms of 'eudaimonia' as well as 'theoria'. And it is about this we should be generous and egalitarian (the 'categorical imperative', like the 'golden rule', is a moral 'identity of indiscernibles' principle.)

Aspirations to understand deeper aspects of this ideal are 'desirable in themselves', and collectively, they afford a kind of scheme of provisional fixed-points for what I have called 'reflective inquiry'.

The conviction that we can make 'reflective' appeals to 'merely' regulative ideals of such 'schematic fixed points' is a regulative ideal 'in itself'. But it requires (as a 'precondition', so to speak) that we accept (in simple terms) that

55 it's *all 'problematic*', for *expressible* ('intelligible')'contemplation' must be *incomplete*.

(Compare James' first 'free' intentional 'act', to 'believe' in his 'freedom' to act.) The skeptic's first 'un(der)determined' intentional act is to 'believe' in skeptical 'un(der)determination'—as something 'known to reason alone' *and* 'known from (within) 'experience'.

None of this provides any 'final' resolution for the questions which eternally recur. *Which* "systems", for example? 'Consciousness of 'what'? Where will it 'end'?

To interrogate ourselves in this way is to pose *questions*, once again (such as Kant's *drei Fragen* in KdrV, B 832–833 and *Logik*, IX, A 25),

"die sie ...nicht beantworten kann, denn sie übersteigen alles Vermögen der menschlichen Vernunft" (*KdrV*, A VII, cited earlier) ("which one...cannot answer, for they exceed every capacity of human reason")....

The same recurrent tensions also evoked one of Pascal's deepest and most 'existential' (as well as 'secular') insights in the following well-known passage, elliptically quoted earlier, and perhaps the most eloquent evocation ever written of the sort of courage Kierkegaard later attributed (dismissively) to his 'knight of infinite resignation' (in the original Danish simply 'knight of infinity'—'Uendelighedens Ridder'):

La dernière démarche de la raison est de reconnaitre qu'il y a une infinité des choses qui la surpassent....

L'homme n'est qu'un roseau, le plus faible de la nature, mais c'est un roseau pensant.

Il ne faut pas que l'univers entier s'arme pour l'écraser; une vapeur, une goutte d'eau suffit pur le tuer. Mais quand l'univers l'écraserait, l'homme serait encore plus noble que ce qui le tue, puisqu'il sait qu'il meurt et l'avantage que l'univers a sur lui. L'univers n'en sait rien.



Toute notre dignité consiste donc en la pensée. C'est de là qu'il faut nous relever et non de l'espace et de la durée, que nous ne saurions remplir. Travaillons donc à bien penser: voilà le principe de la morale. (*Pensées*, 188 and 200)

The last step of reason is to recognise that there are an infinity of things that surpass it....

A human being is a reed, the weakest of nature, but it is a thinking reed. It isn't necessary for the whole universe to take arms to crush it; a vapor, a waterdrop is enough to kill it. But when the universe crushes it, the human being is still nobler than what kills it, for it knows that it is dying and the advantage the universe has over it.

The universe knows nothing.

All our dignity consists therefore in thought. It's from there we should take our orientation, and not from space and time, which we cannot fill. So let us work to think well: this is the principle of morals.

In the limitless contexts of reflective inquiry and skeptical 'experience' (with apologies to Leonard Woolf), the journey not the arrival matters.

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