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## AN IMPOSSIBILITY RESULT FOR COHERENCE RANKINGS

**ABSTRACT.** If we receive information from multiple independent and partially reliable information sources, then whether we are justified to believe these information items is affected by how reliable the sources are, by how well the information coheres with our background beliefs and by how internally coherent the information is. We consider the following question. Is coherence a *separable* determinant of our degree of belief, i.e. is it the case that the more coherent the new information is, the more justified we are in believing the new information, *ceteris paribus*? We show that if we consider sets of information items of any size (*Holism*), and if we assume that there exists a coherence *Ordering* over such sets and that coherence is a function of the probability distribution over the propositions in such sets (*Probabilism*), then *Separability* fails to hold.

### 1. INTRODUCTION

You hear someone mention at a party that there are large colonies of wild boar roaming the southern tip of Greenland. You read in the newspaper that the Japanese stock market will drop and that the unemployment rate in the US will rise. Should you believe this information? It depends. Spelling out the factors that it depends on and in what ways it depends on these factors is the challenge for a theory of justified belief. We should assess how *coherent* the new information is. Does it mesh well with what we believe about wild boars, climate conditions in Greenland, economic matters,...? Is there a connection between the Japanese stock market and the US unemployment rate? We should also ask ourselves how *reliable* the information source is given our background beliefs. What expertise does our information source have about the fauna of Greenland and of economic matters?

Justification is not an all or nothing matter, but rather a matter of degree: If the sources are independent, then the more coherent the information is *and* the more reliable the informers are, the more justified we are to accept the new information. This is clearly a truism, but put in this format the thesis raises an interesting question that has not received any attention amongst epistemologists, viz. are coherence and reliability *separable* factors in their impact on justified belief? Let us explain. Suppose that medical doctors routinely prescribe X-ine and Y-ine to improve some function or other. We are curious to know why X-ine and Y-ine are good for performance enhancement. The first thing we might wish to find out is whether the effectiveness of X-ine and Y-ine are separable. Is X-ine a good thing for performance enhancement, *ceteris paribus*? The *ceteris paribus* clause requires that we keep the dosage of Y-ine fixed. Is Y-ine a good thing for performance enhancement, *ceteris paribus*? The *ceteris paribus* clause requires that we keep the dosage of X-ine fixed. If the answer is twice yes then we have gained some headway on the route to understanding how the treatment works. There may still be some interaction effects between X-ine and Y-ine, but at least we can say that the treatment works is at least partly because X-ine is effective and because Y-ine is effective. The impact of X-ine and Y-ine are separable. But things are more interesting when X-ine is not effective, or is even counter effective, for certain dosages of Y-ine, or vice versa: then we cannot say that X-ine, respectively Y-ine, are effective by themselves. There are curious interaction effects between X-ine and Y-ine that need to be studied to understand the workings of X-ine and Y-ine.

Returning from our excursion into medicine, the question that will concern us is whether coherence and reliability are separable factors. We will take up one direction<sup>1</sup> of this question here, viz.

- (i) *Separability*. The more coherent the new information is, the more justified we are in believing the new information, *ceteris paribus*.

Following John Bender (1989, pp. 2–3), we distinguish between coherence as a relation between a single new item of information and my background beliefs as opposed to as a property of a set of new information items. The former represents an atomistic conception of coherence. For every particular new item of information, we ask how well it meshes with our background beliefs. The latter represents a holistic conception of coherence. For a set of new items of information, we ask how coherent this set is internally, i.e. how well the items mesh with each other.

## 2. THE ATOMISTIC CONCEPTION OF COHERENCE

The better a new item of information fits in with our background beliefs, the more coherent it is on the atomistic conception of coherence. We define a probabilistic measure of how coherent the information is

$$(1) \quad c(\{I\}) = \text{Prob}(I|K),$$

where  $I$  is the newly acquired item of information,  $\{I\}$  is the information set, and  $K$  are our background beliefs. Let  $\succcurlyeq$  be the relation of ‘... being equally or more coherent than...’. An information set  $\{I\} \succcurlyeq \{I'\}$  iff  $c(\{I\}) \geq c(\{I'\})$ . We let  $P(\cdot) = \text{Prob}(\cdot|K)$  for simplicity of representation.

There are many theoretical commitments packed into this appeal to a measure of coherence, viz.

- (ii) *Atomism.* The relation  $\succcurlyeq$  is defined over information singletons,
- (iii) *Ordering.*  $\succcurlyeq$  is an ordering, i.e. the relation  $\succcurlyeq$  is transitive and complete,  
and
- (iv) *Probabilism.* A coherence measure over information sets is a function of the probability distribution over the propositional variables whose positive values are the constituents of the information set.

In the case of a singleton set,  $I$  is the propositional variable, whose positive value is  $I$  and whose negative value is  $\neg I$ .

The process through which the information was obtained may be more or less reliable. Think of an information gathering process as a medical test. The reliability of a medical test can be assessed by means of the following two conditional probabilities:  $P(R|\neg I)$ , i.e. the conditional probability that the test will be positive ( $R$ ), given that you have the disease ( $I$ ), and  $P(R|I)$ , i.e. the conditional probability that the test will be positive given that you do not have the disease. A fully reliable test says of what is that it is and of what is not that it is not:  $P(R|I) = 1$  and  $P(R|\neg I) = 0$ . An entirely worthless test is a test that is no better than consulting some random device: the chance of getting a positive report is the same whether you have the disease or not, i.e.  $P(R|I) = P(R|\neg I)$ . We are interested in partially reliable tests, i.e. tests for which  $P(R|I) > P(R|\neg I) > 0$ . Such tests can be located on the continuum between both fully reliable tests and entirely worthless tests. Hence, for partially reliable tests, the *likelihood ratio*<sup>2</sup>

$$(2) \quad x = \frac{P(R|\neg I)}{P(R|I)} \in (0, 1).$$

We assign a degree of reliability

$$(3) \quad r(\{I\}) := 1 - x$$

to the information that you have the disease as provided by the test. Similarly, let us suppose that our background information permits us to assign a degree of reliability  $r(\{I\})$  to the information as provided by a partially reliable source.<sup>3</sup>

It follows from Bayes Theorem that our degree of confidence in the newly acquired information is

$$(4) \quad P^*(I) = P(I|R) = \frac{1}{(1 - r(\{I\})/c(\{I\}) + r(\{I\}))}.$$

Our definition of a coherence measure satisfies *Atomism*, *Probabilism*, and *Ordering*. It is easy to see that *Separability* holds. From (4), it is clear that whatever the value of  $r(\{I\})$ , raising

the value of  $c(\{I\})$  lowers the value of the denominator and hence raises our degree of confidence that I. We conclude that

**Theorem 1.** *Separability, Atomism, Ordering and Probabilism are consistent.*

### 3. THE HOLISTIC CONCEPTION OF COHERENCE

Suppose that I receive multiple information items from different sources. Some information sets hang together better, fit together better, mesh better than others,... This permits us to state the following commitment:

- (v) *Holism.* The relation  $\succcurlyeq$  is defined over non-empty information sets.

We have defined *Holism* and *Atomism* so that *Atomism* entails *Holism*, but not vice versa. Nothing hinges on this. We could also have defined both theses so that they are mutually exclusive. The atomistic relation  $\succcurlyeq$  would then be defined over singletons and the holistic relation  $\succcurlyeq$  over  $n$ -tuples for  $n \geq 2$ . We will now show that *Separability*, *Holism*, *Ordering* and *Probabilism* are an inconsistent quadruple.

To say that an information set contains multiple items of information is to say that we have received the information from multiple sources. Furthermore, these multiple sources should not act in unison, since then the situation is indistinguishable from having received the information from a single source. To determine the degree of confidence that the information is true we stipulate in our model that the information gathering processes are independent.

In *Hume's Abject Failure*, John Earman (2000, pp. 56–9) develops a model to determine the probability that at least one witness report of a miracle is true when the reports come from independent witnesses. We develop a parallel model, but are interested not in the posterior probability that the disjunction of the witness reports is true, but rather in the posterior probability that the conjunction of the witness reports

is true (see also Bovens and Olsson, 2000, p. 690 and pp. 696–70 and 2002, pp. 143–4).

Suppose that there are  $n$  independent and partially reliable sources and each source  $i$  informs us of a proposition  $I_i$ , for  $i = 1, \dots, n$ , so that the information set is  $\{I_1, \dots, I_n\}$ . Let us name  $I_i$  a *fact variable* and  $R_i$  a *report variable*.  $R_i$  can take on two values, viz.  $R_i$ , i.e. after consultation with the proper source, there is a report to the effect that  $I_i$  is the case, and  $(R_i$  i.e. after consultation with the proper source, there is no report to the effect that  $I_i$  is the case. We construct a joint probability distribution  $P$  over  $I_1, \dots, I_n, R_1, \dots, R_n$  satisfying the constraint that the sources are independent and partially reliable.

We model the independence of the sources by stipulating that  $P$  respects the following conditional independences:

$$(5) \quad R_i \perp\!\!\!\perp I_1, R_1, \dots, I_{i-1}, R_{i-1}, I_{i+1}, R_{i+1}, \dots, I_n, R_n | I_i \\ \text{for } i = 1, \dots, n$$

or, in words,  $R_i$  is probabilistically independent of  $I_1, R_1, \dots, I_{i-1}, R_{i-1}, I_{i+1}, R_{i+1}, \dots, I_n, R_n$ , given  $I_i$ , for  $i = 1, \dots, n$ . There are two aspects to this characterization of independent witnesses, viz.  $I_i$  screens off  $R_i$  from all other fact variables  $I_j$  and from all other report variables  $R_j$ . The reports of independent witnesses are determined by whether the facts they report on hold or not. They may not always assess things correctly, but they are not influenced in their reports by whether other facts hold, or by whether there are reports to the effect that other reports hold. We also assume that all witnesses are equally reliable, i.e.  $r(\{I_i\}) = r$  for all witnesses  $i = 1, \dots, n$ .

It can be shown<sup>4</sup> that, given the constraints on  $P$ ,

$$(6) \quad P^*(I_1, \dots, I_n) = P(I_1, \dots, I_n | R_1, \dots, R_n) = \frac{a_0}{\sum_{i=0}^n a_i (1-r)^i},$$

where  $a_i$  is the probability that exactly  $i$  propositions are false. Note that  $\sum_{i=0}^n a_i = 1$ . For example, for an information triple containing the propositions  $I_1, I_2$ , and  $I_3$ ,  $a_2 = P(I_1, \neg I_2, \neg I_3) + P(\neg I_1, I_2, \neg I_3) + P(I_1, \neg I_2, \neg I_3)$ . The vector  $\langle a_0, \dots, a_n \rangle$  is the *weight vector* of the probability distribution  $P$  defined

over the propositional variables  $I_1, \dots, I_n$ . Throughout this paper we assume that  $a_0 \neq 0$ .

The obvious question is: What is a proper measure of coherence  $c(\{I_1, \dots, I_n\})$ ? Luckily, we do not *need* to answer this question to show that *Separability* fails, if *Holism*, *Ordering*, and *Probabilism* hold. By *Probabilism*,  $c(\{I_1, \dots, I_n\})$  must be a function of the probability distribution. Now let us take a probability distribution  $P$  over an information pair  $\{I_1, I_2\}$  and a probability distribution  $P'$  over an information pair  $\{I_1', I_2'\}$ . We construct the weight vectors  $\langle a_0, a_1, a_2 \rangle$  and  $\langle a_0', a_1', a_2' \rangle$ . Let  $\langle a_0, a_1, a_2 \rangle = \langle 0.20, 0.70, 0.10 \rangle$  and  $\langle a_0', a_1', a_2' \rangle = \langle 0.10, 0.10, 0.80 \rangle$ . We calculate our degrees of confidence in the new information by means of the expression in (6) when  $r$  is 0.90:

$$(7) \quad P^*(I_1', I_2') \approx 0.85 > 0.74 \approx P^*(I_1, I_2)$$

and when  $r = 0.50$ :

$$(8) \quad P^*(I_1', I_2') \approx 0.29 < 0.35 \approx P^*(I_1, I_2)$$

By *Ordering*, either  $\{I_1, I_2\}$  is more or equally coherent than  $\{I_1', I_2'\}$ , or vice versa. Suppose the former is true. Then *Separability* fails, i.e. it is false that the more coherent the new information is, the more justified we are to believe that the new information is true, *ceteris paribus*, since for  $r = 0.90$ , this fails to hold. Suppose the latter is true. Then *Separability* fails as well, since for  $r = 0.50$  it fails to hold. Hence, without even having defined a probabilistic measure of coherence for information pairs, we can conclude that *Separability* fails.

We anticipate the following rejoinder. There are other features of the probability distribution than coherence that also determine our degree of confidence that the new information is true. One might suggest that it is not only the coherence of the new information, but also how *expected* the new information is given our background knowledge.<sup>5</sup> The obvious measure of expectancy is  $a_0$ , i.e. the prior joint probability of the new information. This measure indicates how expected the new information is in its totality relative to our background

knowledge. So the rejoinder goes as follows. In assessing whether coherence is separable, the *ceteris paribus* clause should cover not only the reliability of the witnesses but also the expectancy of the information set.

This defense could indeed be made successful if we restrict our attention to information *pairs*. To see this, let *Dualism*, like *Atomism*, be a special case of *Holism*:

(v) *Dualism*. The relation  $\succcurlyeq$  is defined over information pairs.

For all existing measures of coherence, the following claim holds true. If two information pairs have the same expectancy, then the coherence of the information pair is a negative function of  $a_1$ . Given this rather innocent assumption, we show that

**Theorem 2.** *Separability, Dualism, Ordering and Probabilism are consistent.*

To respect *Dualism*, we compare two information pairs  $\{I_1, I_2\}$  and  $\{I_1', I_2'\}$ .  $\{I_1, I_2\}$  has the corresponding weight vector  $\langle a_0, a_1, a_2 \rangle$  and  $\{I_1', I_2'\}$  has the corresponding weight vector  $\langle a_0', a_1', a_2' \rangle$ . By *Probabilism*, a coherence measure is a function of the probability distribution over the propositional variables whose positive values are the propositions in the information set. Certainly, we can define a coherence measure that respects *Ordering*. It is easy to show that for all existing coherence measures that respect *Dualism*, *Probabilism*, and *Ordering*, the information pair  $S$  is more or equally coherent than an information pair  $S'$  with equal expectancy if and only if  $a_1' \geq a_1$ . It follows from (6) that

$$(9) \quad P^*(I_1, I_2) = \frac{a_0}{a_0 + (1 - a_0)x^2 + a_1(x - x^2)}$$

*Separability* holds if we let the *ceteris paribus* clause cover both the reliability and the expectancy. If we keep the reliability of the witnesses and the expectancy of the information set fixed, i.e. we keep  $r$  (and hence  $x$ ) fixed and let  $a_0 = a_0'$ , then more coherent information pairs, i.e. information pairs with a lower value of  $a_1$  in their weight vectors, warrant higher degrees of confidence.



However, let us return to the general case, substituting *Holism* for *Dualism* again. Even if we assume that the *ceteris paribus* clause covers both the reliability and the expectancy, *Separability* still fails for larger information sets. Let us take a probability distribution  $P$  over an information triple  $\{I_1, I_2, I_3\}$  and a probability distribution  $P'$  over an information triple  $\{I_1', I_2', I_3'\}$  with the associated weight vectors  $\langle a_0, \dots, a_3 \rangle$  and  $\langle a_0', \dots, a_3' \rangle$ . Let  $\langle a_0, \dots, a_3 \rangle = \langle 0.05, 0.30, 0.10, 0.55 \rangle$  and  $\langle a_0', \dots, a_3' \rangle = \langle 0.05, 0.20, 0.70, 0.05 \rangle$ .

Now let us calculate our degrees of confidence in the new information by means of the expression in (6) when  $r$  is 0.90:

$$(10) \quad P^*(I_1', I_2', I_3') \approx 0.65 > 0.61 \approx P^*(I_1, I_2, I_3)$$

and when  $r$  is 0.50:

$$(11) \quad P^*(I_1', I_2', I_3') \approx 0.15 < 0.17 \approx P^*(I_1, I_2, I_3)$$

Notice that the expectancy of the information, as measured by  $a_0$ , is fixed between both information sets. Whatever coherence ordering we impose on these information triples, *Separability* fails. It is false that the more coherent the information set, the greater our degree of confidence that the information is true, *ceteris paribus*, even if we let the *ceteris paribus* clause cover the reliability and the expectancy of the new information. If we assume that  $\{I_1, I_2, I_3\}$  is more coherent than  $\{I_1', I_2', I_3'\}$ , then *Separability* fails when we set  $r = 0.90$ . If we assume that  $\{I_1', I_2', I_3'\}$  is more coherent than  $\{I_1, I_2, I_3\}$ , then *Separability* fails when we set  $r = 0.50$ . Hence, either way, *Separability* fails.<sup>6</sup> Hence, we have shown that

**Theorem 3.** *Separability, Holism, Ordering and Probabilism are inconsistent.*

This theorem holds even if we let the *ceteris paribus* clause in *Separability* cover both reliability and expectancy.

We provide the following generalization of this counter example for  $n > 3$ . It follows from (6) that

$$(12) \quad P^*(I_1, \dots, I_n) = \frac{a_0}{a_0 + (1 - a_0)x^n + \sum_{i=1}^{n-1} a_i(x^i - x^n)}$$

Take an information  $n$ -tuple  $\{I_1, \dots, I_n\}$  with weight vector  $\langle a_0, a_1, \dots, a_n \rangle$ . We show that there is an information  $n$ -tuple  $\{I_1', \dots, I_n'\}$  with weight vector  $\langle a_0, a_1', \dots, a_n' \rangle$  so that *Separability*, *Holism*, *Probabilism* and *Ordering* are inconsistent. Note that  $P^*(I_1, \dots, I_n) = P^*(I_1', \dots, I_n')$  if and only if

$$(13) \quad \Delta := \sum_{i=1}^{n-1} (a_i' - a_i)(x^i - x^n) = 0$$

For some  $k$  such that  $2 \leq k \leq n-2$  and  $a_k \neq 0$ , let

$$(14) \quad \begin{aligned} a_{k-1}' &= a_{k-1} + \delta \\ a_k' &= a_k - 3\delta \\ a_{k+1}' &= a_{k+1} + 2\delta \end{aligned}$$

with  $0 < \delta < a_k/3$  and let  $a_j' = a_j$  for all other components. We can then calculate that

$$(15) \quad \Delta = \delta x^{k-1}(x - 1/2)(x - 1).$$

Hence,  $P^*(I_1, \dots, I_n) = P^*(I_1', \dots, I_n')$  for (and only for)  $x = 0$ ,  $1/2$ , and  $1$ , and so for (and only for)  $r = 0$ ,  $1/2$ , and  $1$ . Furthermore, since  $P^*$  is a continuous function of  $r$ ,  $P^*(I_1, \dots, I_n) < P^*(I_1', \dots, I_n')$  for  $r \in (0, 1/2)$  and  $P^*(I_1, \dots, I_n) > P^*(I_1', \dots, I_n')$  for  $r \in (1/2, 1)$ . This algorithm is just one way to obtain counter examples to the claim that *Holism*, *Probabilism*, *Separability*, and *Ordering* are consistent. There are many ways of doing so.

#### 4. DISCUSSION

So what can be given up in the inconsistent quadruple of *Probabilism*, *Separability*, *Holism* and *Ordering*? We will argue that giving up any of these principles is at least *prima facie* unappealing. The set up of our argument mirrors Arrow's impossibility result for social welfare rankings (Arrow, 1963). Our result has a similar paradoxical flavour. Just as there cannot be a social welfare ranking that satisfies four minimally reasonable conditions, there cannot be a

coherence ranking that satisfies four seemingly plausible principles.

#### 4.1. *Probabilism*

The motivation for probabilism is the following aspiration. If we have justified background beliefs concerning the reliability of our sources and the chances that certain propositions might or might not be true, then the formal calculus of probability theory can be invoked to determine whether we are justified to believe the information that we have received from independent witnesses. One of the determinants of whether we are justified to believe new information seems to be the coherence of the new information. Hence the natural question to ask is how we can give a probabilistic interpretation of the notion of coherence. There is an early attempt to provide such an interpretation in C.I. Lewis (1946, p. 338) and two recent attempts can be found in Shogenji (1999) and Fitelson (2003). Our own theory is presented in Bovens and Hartmann (2003b). Anti-Bayesian epistemologists may take our result to be evidence that what Bayesians are after is pie in the sky, but *we* would certainly like to see how much of the project can be salvaged.

#### 4.2. *Holism*

No contradiction occurs as long as we conceive of coherence in an atomistic fashion. One might suggest that our result enjoins us to think about whether we are justified to believe new items of information in a piecemeal fashion. For every new item of information, we should ask how well it coheres with our background beliefs taken by itself and how reliable the informers are, and decide on the basis hereof whether to believe the new information item. But this is to give up on an important intuition that C.I. Lewis (1946, p. 346) and Bonjour (1985, p. 148) capitalize on. We are often presented with information items such that we would not be willing to believe the new items of information if considered one by one,

but as a whole we are willing to believe them due to their internal coherence.

#### 4.3. *Separability*

There are some notorious cases of non-separability in the philosophical literature. For instance, Kant argued in the *Foundations of the Metaphysics of Morals* (1990, p. 9) that, although it is better to have both a good will and, say, smarts, it is actually worse to be smart when the good will is absent (Oddie, 2001a and 2001b). But in cases where separability fails, the explanatory demand is higher. It is not sufficient to say that the good will and smarts are conducive to value in agency. The good will has a special status for Kant as a virtuous trait and is radically different from other virtuous traits such as smarts. We know that highly coherent and highly expected information from highly reliable sources induces a high degree of confidence that the information is true. In the absence of a special explanation, there is a presumption that the more coherent the information is, the more confident we may be that the information is true, *ceteris paribus*. A special explanation can defeat this presumption, but we cannot see how a special explanation would go in this case. One might question what should be subsumed under the *ceteris paribus* clause. It is certainly a reasonable move to include the expectancy of the new information. But we have shown that this move does not bring us an inch further once we move from information pairs to information triples. One could of course try to continue this line of defense by arguing that the determinants of our degree of confidence are the reliability of the new sources, the expectancy of the new information, the coherence of the new information, *and some other probabilistic feature*. This strategy might work, but in the absence of a concrete proposal, there is little to argue against. Alternatively, one can parse the coherence of an information set into multiple separable components. This is the escape route from the impossibility result that we explore in Bovens and Hartmann (2005).

#### 4.4. *Ordering*

We believe that this is indeed the weakest link in the quadruple. Suppose that we are confronted with two information sets. Is it always meaningful to make a judgment to the effect that one information set is more coherent than the other? Of course there are cases in which we just lack the probabilistic information. But let us restrict ourselves here to cases where the complete probabilistic information is present. For instance, suppose that the information sets contain medical data and that the probabilistic relations between symptoms and diseases are well known. Sometimes, comparative judgment of coherence between two information sets are uncontroversial. For instance, suppose that one information set ascribes symptoms to a patient that all point in the direction of a single disease, while another information set ascribes symptoms that rarely coincide. But sometimes, there may be complex relations of positive and negative relevance between the items of information in the information sets and it may not make much sense to say that one information set is more coherent than the other. Once we give up on *Ordering* there is no reason to believe that we can construct the kinds of counter examples that we have presented earlier. It might well be the case that we have picked information sets and their associated vectors so that it would make no sense to impose an ordering over such pairs. We explore this particular escape route from the impossibility result in Bovens and Hartmann (2003a and 2003b, pp. 28–55).

#### NOTES

<sup>1</sup> We will not take up the other direction of this question, viz. is it true that the more reliable the sources are, the more justified we are in believing the new information *ceteris paribus*, here. Following Bovens and Hartmann (2003a, pp. 28–88), it can be shown that two information sets are equally coherent if and only if they are characterized by the same *weight vector* (2003a, p. 17), which is introduced below. From equation (2.3) (2003a, p. 31), we can then read off that the posterior joint probability of the propositions contained in equally coherent information sets is greater when the information is provided by more reliable sources.

<sup>2</sup> One needs to be careful when talking about the likelihood ratio in Bayesian confirmation theory. Sometimes the likelihood ratio is defined as in (2) (e.g. in Howson and Urbach 1993, p. 29), sometimes as the reciprocal of the formula in (5) (e.g. in Pearl 1988, p. 34).

<sup>3</sup> Our model of a partially reliable source matches interpretation (ii) of ‘dubious information-gathering processes’ in Bovens and Olsson (2000, p. 698).

<sup>4</sup> The proof is straightforward: Apply Bayes Theorem to the right-hand side of (7); simplify on grounds of the conditional independences in (6), divide the numerator and denominator by  $P(R_i|I_i)^n$  and substitute in the measure  $r$  and the parameters  $a_i$  for  $i=1, \dots, n$ , as defined underneath.

<sup>5</sup> Another way to conceive of this is that the expectancy of the information items is the coherence of the *conjunction* of the new information items on the atomistic conception of coherence.  $a_0$  is a measure of how well the information, taken as a whole, fits in with our background beliefs. (*Cf.* Bovens and Hartmann (2005)).

<sup>6</sup> The reader who is familiar with the literature on separability may wish to know whether it is weak or strong separability that is in question. (See Broome, 1991, pp. 60–89.) As long as we consider only two determinants of our degree of confidence, viz. reliability and coherence, weak separability and strong separability are coextensive. When we are considering three determinants of our degrees of confidence, viz. reliability, coherence and expectancy, weak separability and strong separability are no longer coextensive. We have shown that weak separability fails. Since strong separability entails weak separability, strong separability fails as well. In Bovens and Hartmann (2005) we argue that weak but not strong separability holds for the components of a coherence vector.

<sup>7</sup> Our research was supported by the Alexander von Humboldt Foundation, the Federal Ministry of Education and Research and the Program for the Investment in the Future (ZIP) of the German Government through a Sofja Kovalevskaja Award.

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