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The tragedy of the commons as a voting game

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9. The Tragedy of the Commons as a Voting Game

Luc Bovens

1. Introduction

Tragedies of the commons are often associated with n-person Prisoner's Dilemmas.[1] This is indeed in line with Garrett Hardin's presentation of the story of the herdsmen whose cattle overgraze a commons in his seminal article "The Tragedy of the Commons" (1968) and Robyn Dawes's analysis of this story (1975). Bryan Skyrms (2001) argues that public goods problems often have the structure of n-person Assurance Games whereas Hugh Ward and Michael Taylor (1982) and Taylor (1987; 1990) argue that they often have the structure of n-person Games of Chicken. These three games can also be found as models of public goods problems in Dixit and Skeath (1999: 362–7). Elinor Ostrom (1990: 2–3) reminds us that the tragedy of the commons has a long history that predates Hardin starting with Aristotle's *Politics*. I will present three classical tragedies of the commons presented in Aristotle, Mahanarayan, who is narrating a little known 16th century Indian source, and Hume. These classical authors include four types of explanations of why tragedies ensue in their stories, viz. the Expectation-of-Sufficient-Cooperation Explanation, The Too-Many-Players Explanation, the Lack-of-Trust Explanation, and the Private-Benefits Explanation. I present the Voting Game as a model for public goods problems, discuss its history, and show that these explanations as well as the stories themselves

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align more closely with Voting Games than with Prisoner's Dilemmas, Games of Chicken, and Assurance Games.

2. The Tragedy of the Commons and the *n*-person Prisoner's Dilemma

Hardin (1968) argues that, just like herdsmen add cattle to a commons up to the point that is beyond its carrying capacity, the human population is expanding beyond the earth's carrying capacity. In both cases, there is an individual benefit in adding one animal or one human offspring, but the costs to the collective exceed the benefits to the individual:

> The tragedy of the commons develops in this way. Picture a pasture open to all. It is to be expected that each herdsman will try to keep as many cattle as possible on the commons. Such an arrangement may work reasonably satisfactorily for centuries because tribal wars, poaching, and disease keep the numbers of both man and beast well below the carrying capacity of the land. Finally, however, comes the day of reckoning, that is, the day when the long-desired goal of social stability becomes a reality. At this point, the inherent logic of the commons remorselessly generates tragedy.
>
> As a rational being, each herdsman seeks to maximize his gain. Explicitly or implicitly, more or less consciously, he asks, "What is the utility *to me* of adding one more animal to my herd?" This utility has one negative and one positive component.
>
> 1) The positive component is a function of the increment of one animal. Since the herdsman receives all the proceeds from the sale of the additional animal, the positive utility is nearly +1.
>
> 2) The negative component is a function of the additional overgrazing created by one more animal. Since, however, the effects of overgrazing are shared by all the herdsmen, the negative utility for any particular decision-making herdsman is only a fraction of –1.

Adding together the component partial utilities, the rational herdsman concludes that the only sensible course for him to pursue is to add another animal to his herd. And another; and another ... But this is the conclusion reached by each and every rational herdsman sharing a commons. Therein is the tragedy. Each man is locked into a system that compels him to increase his herd without limit—in a world that is limited. Ruin is the destination toward which all men rush, each pursuing his own best interest in a society that believes in the freedom of the commons. Freedom in a commons brings ruin to all. (Hardin 1968: 1244)

Let us construct an *n*-person Prisoner's Dilemma. Suppose that each player derives the same payoff from cooperating and the same payoff from defecting when *j* other players cooperate and ($n - 1 - j$) other players defect. In Figure 1, we plot the payoff from cooperating—*U[Cooperation]*—and the payoff from defecting—*U[Defection]*—as a function of the proportion of ($n - 1$) players defecting.

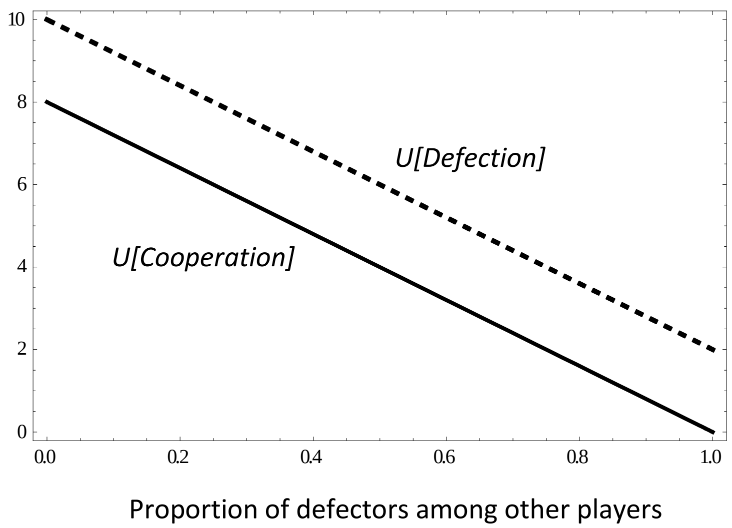

Figure 1. *N*-person Prisoner's Dilemma

No matter what strategy the other players are playing, it is better for an individually rational player to defect than to cooperate. Hence, there is a single equilibrium of All Defect in this game. But each person's payoff in the equilibrium profile is lower than each person's payoff in many other profiles, e.g. in the profile in which all cooperate, but also in some profiles with mixed cooperation and defection. Hence, the profile in which each person plays her individually rational strategy is strongly suboptimal. That is, there are profiles whose payoffs are strictly preferred by all to the payoffs in the equilibrium profile. This brings us to the core lesson of the Prisoner's Dilemma, viz. individual rationality does not lead to collective rationality. If there were a single center of control we would certainly not choose the equilibrium profile, since it is strongly suboptimal relative to some other profiles.

We could fill in the story of the herdsmen so that it is accurately represented by Figure 1. Defectors are herdsmen who each send an animal to the commons to feed. Cooperators refrain from doing so—e.g. they barn feed their animal. And their respective payoffs are precisely as is laid out in the graph. To do so, we need to make one assumption which is slightly unnatural, viz. that there is a negative externality of defection (commons feeding) on cooperation (barn feeding). This may be the case, but it is more natural to think of barn feeding as having a fixed payoff. If we assume that there is a fixed payoff, then there are three possibilities, as illustrated in Figure 2:

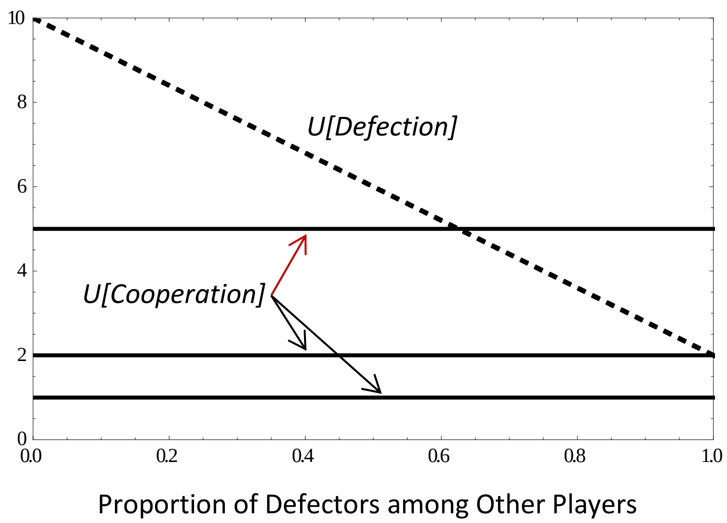

Fig 2. Hardin's Herdsmen with Fixed Payoff for Barn Feeding

(1) The individual payoffs of the All Defect equilibrium are greater than the payoff of barn feeding (bottom horizontal line), in which case the equilibrium profile is not strongly suboptimal relative to any other profile. Hence we no longer have a Prisoner's Dilemma.
(2) The individual payoffs of the All Defect equilibrium equal the payoff of barn feeding (middle horizontal line), in which case the equilibrium profile is not strongly suboptimal relative to any other profile (though it is weakly suboptimal relative to all other profiles.) This is a degenerate Prisoner's Dilemma.
(3) The individual payoffs of the All Defect profile are lower than the payoff of barn feeding (top horizontal line), in which case this profile is no longer an equilibrium. Hence we no longer have a Prisoner's Dilemma but rather a Game of Chicken, as will become clear below.

A more natural interpretation of the game in Figure 1 is littering. However many other people litter (Defect), each person prefers to litter rather than carry their empty beer cans back home (Cooperate). But with each additional item of litter, the town becomes less and less pleasant for litterer and non-litterer alike. This example is actually a clearer example of a tragedy of the commons that fits the mold of an *n*-person Prisoner’s Dilemma.

3. The Assurance Game

Skyrms (2001) takes the following quote from Rousseau’s *A Discourse on Inequality*:

> If it was a matter of hunting a deer, everyone well realized that he must remain faithful to his post; but if a hare happened to pass within reach of one of them, we cannot doubt that he would have gone off in pursuit of it without scruple...

Following Rousseau, an Assurance Game is also named a “Stag Hunt”. In this game, it is beneficial to cooperate assuming that others cooperate as well. However, if I cannot be assured that others will cooperate then it is better to defect. That is, if I can trust that others will hunt deer with me, then I am better off hunting deer with them; But if they do not stay by their station and decide to lash out and hunt hare, then I am better off abandoning my station as well and hunt hare.

We present the n-person version of the Assurance Game in Figure 3. In this game, it is better to cooperate when there is a low proportion of defectors and it is better to defect when there is a high proportion of defectors. The game has two equilibria, viz. All Cooperate and All Defect. The payoff to each in the All Cooperate equilibrium is higher than in the All Defect equilibrium.

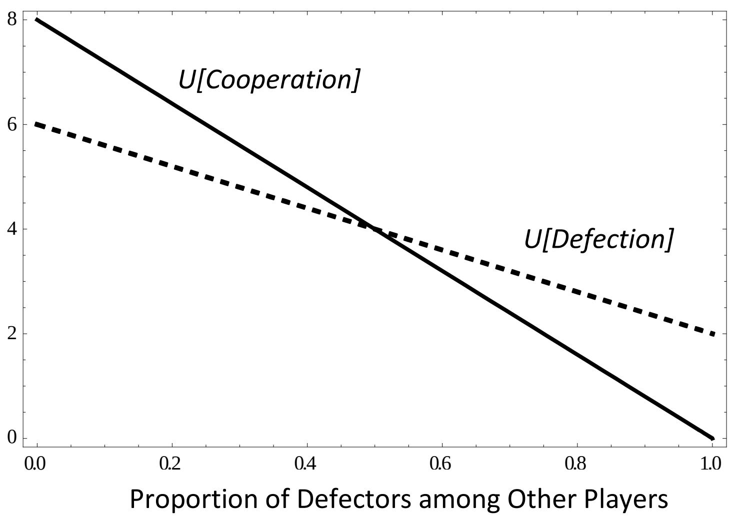

Figure 3. Assurance Game.

To interpret this in terms of a tragedy of the commons, we could stipulate that people can either choose to jointly work a commons with the promise of a large harvest if all participate or work some small garden plots by themselves with the sure outcome of a small harvest. If we cannot be assured that others will cooperate then the commons will remain untapped. In this case the tragedy of the commons comes about due to neglect rather than over-usage, due to lack of care rather than to depletion. In reality tragedy will often ensue due to a combination of usage and lack of proper care—i.e. due to unsustainable usage.

Why might the tragedy, i.e. the All Defect equilibrium, come about in the Assurance Game? It is not because the players choose individually rational strategies as in the Prisoner's Dilemma, since it is not the case that no matter what other players do, it is better to defect. If a sufficient number of other players cooperate, then it is better to cooperate as well. The only reason why one might be hesitant choosing cooperation is because of a lack of trust—the players may not trust that others will choose to cooperate. In the *n*-person Assurance Game, if we cannot trust a sufficient number of people to choose cooperation, then we should just resort to defecting. And we may not trust them to do so, because, as in the Stag Hunt, we may believe that they will be blinded by short term advantage or there may just be a lack of trust in society at large.

4. The Game of Chicken

In the Game of Chicken two cars drive straight at each other forcing the other one to swerve. If one drives straight and the other swerves then the one who drives straight wins and the one who swerves loses. If they both swerve then it's a tie. And clearly, if they both drive straight then tragedy ensues. Let defection be driving straight and cooperation be swerving. Then the best response to defection is cooperation and the best response to cooperation is defection. In the *n*-person Game of Chicken (Figure 4), at low levels of defection, defection has a higher payoff than cooperation. At high levels of defection, cooperation has a higher payoff than defection.

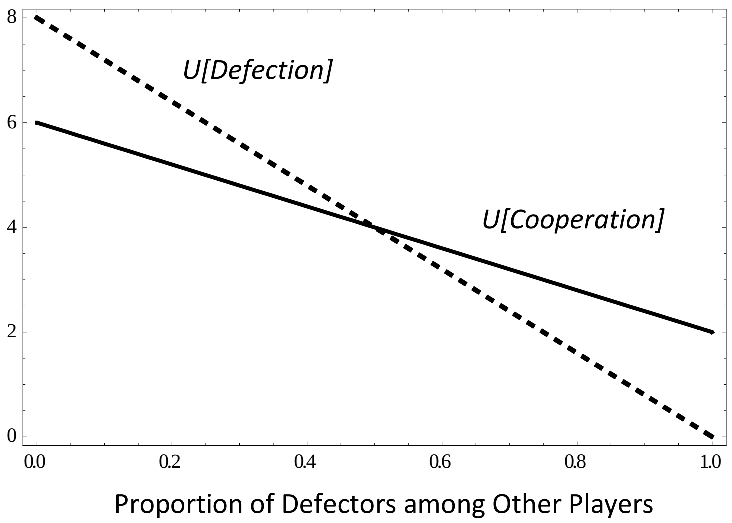

Figure 4. Game of Chicken with Depletion of the Commons

It is easy to make this game into an interpretation of a tragedy of the commons. In *Fair Division and Collective Welfare*, Hervé Moulin (2004: 171) presents a case of overgrazing a commons with a fixed payoff for the cooperative strategy of barn feeding. We extend the number of players from Hardin's case. For low proportions of defectors (i.e. commons users), the same holds as in the Prisoner's Dilemma: Defecting (commons feeding) trumps cooperating (barn feeding). But for high proportions of defectors, the commons is so depleted that one would incur a loss by bringing one's animal to the commons. Hence cooperation (barn feeding) trumps defection (commons feeding). So by simply extending the set of players, we can turn an *n*-person Prisoner's Dilemma representation of an overgrazing case into a Game of Chicken representation. Case (iii) in section 2, represented in Figure 2, with a fixed barn feeding payoff higher than the payoff of All Defect also fits this mold.

If we envision an *n*-person Game of Chicken with cars racing at each other from various sides, then defection (going straight) is only rational at a *very* high level of cooperation (swerving)—viz. when everyone else swerves—assuming that swerving avoids a collision. As soon as one other person goes straight, swerving is the rational response. So the intersection point between the line for defection and cooperation is between 0 and 1 defector. But this is just an artefact of the story. In the commons example, defecting (commons feeding) remains rational at high and mid-range levels of cooperation (barn feeding). We can put the intersection point between the line for defection and cooperation at any point in the graph and tell a fitting story.

In an *n*-person Games of Chicken, the equilibria in pure strategies are the profiles at the intersection point of the lines for cooperation and defection (at least for the continuous case). In low-level defection profiles, i.e. profiles to the left of the intersection point, it is better for a cooperator to unilaterally deviate to defection. In high-level defection profiles, i.e. profiles to the right of the intersection point, it is better for a defector to unilaterally deviate to cooperation. Only the profiles at the intersection point are equilibria.

How does tragedy ensue? There are two reasons. First, the equilibria are not welcome equilibria. In Moulin's example of the overgrazing of the commons, the intersection point is the point at which the commons are so depleted that it makes no difference whether the next herder brings his animal to the commons or barn feeds. Let the intersection point be at $m < n$ players. If we truncate the game to m players, then we can see that the game is like the *n*-person Prisoner's Dilemma as in Hardin's tragedy of the commons. (I have represented the truncated game in Figure 5 with $m = .4n$, i.e. if there are, say, 100 herdsmen in the original game, then there are 40 herdsmen in the truncated game.) In the non-truncated Game of Chicken, if the slope of the cooperation line is negative (as it is in Figure 4) then the outcome of all cooperate is strongly Pareto-superior to the outcome of any of the equilibria in pure strategies located at the intersection point.

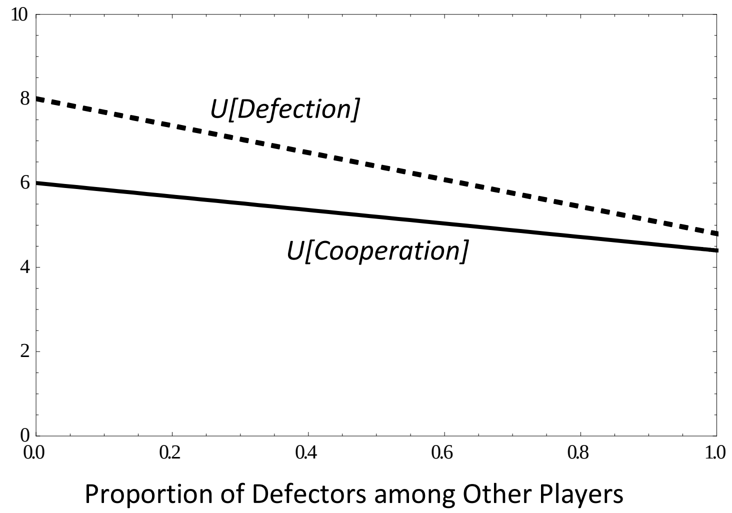

Figure 5. Prisoner's Dilemma as a Truncation of the Game of Chicken in Figure 4

Second, there is a coordination problem. When the herdsmen have to choose independently to bring their animal to the commons, they cannot coordinate who will and will not send their animal to the commons. They have no ground to prefer one of the equilibria in pure strategies over another. If they resort to an equilibrium in randomized strategies, i.e. they all choose to send their animal to the commons with a particular probability, then we may be lucky and end up with a number of commons feeders that is lower than the depletion point, but we may also be unlucky and end up with a number of commons feeders that is higher than the depletion point. Similarly, in the actual Game of Chicken, our luck runs out when all defect, i.e. go straight.

5. Three-Person Games

Let us consider three-person games for the three types of games that can represent tragedies of the commons. Start with the three-person Prisoner's Dilemma in Table 1. There is a persistent preference for defection over cooperation (D $\succ$ C) in the Prisoner's Dilemma, whether none, one or both of the other two people are defecting. The equilibrium is at All Defect. To construct an Assurance Game, we can turn around one preference, viz. C $\succ$ D when none of the other players are defecting. The equilibria are All Cooperate and All Defect. To construct a Game of Chicken, we can turn around one preference, viz. C $\succ$ D when all of the other players are defecting. The equilibria are Two Defect and One Cooperate.[2]

This construction of three-person games invites the following question: So what if we turn around the central preference, viz. C $\succ$ D when just one of the other players is defecting, all other things equal? This would turn the game into a Voting Game. Suppose that we have three players who would all vote in favor of a proposal if they would bother to show up and they need to muster just two votes to defeat the opposition. To cooperate is to show up for the vote and to defect is to stay at home. Then C $\succ$ D when there is exactly one other voter who shows up and D $\succ$ C otherwise—if nobody bothered showing up, then my vote would not suffice anyway, and, if two other people showed up, then my vote is not needed anymore. The equilibria in pure strategies in this game are Two Cooperate & One Defect or All Defect.

[2] We can also construct an Assurance Game by turning around two preferences, viz. C $\succ$ D when none or one of the other players is defecting. This would not affect the equilibria. And we can also construct a Game of Chicken by turning around two preferences, viz. C $\succ$ D when one or both other players are defecting. Then the equilibria are two players cooperating and one defecting. In terms of the graphs in Figures 3 and 4, we are just shifting the intersection points.

	Other players			
	CC	CD DC	DD	Equilibria in Pure Strategies
Prisoner's Dilemma	D $\succ$ C	D $\succ$ C	D $\succ$ C	DDD
Assurance Game	C $\succ$ D	D $\succ$ C	D $\succ$ C	CCC, DDD
Game of Chicken	D $\succ$ C	D $\succ$ C	C $\succ$ D	DDC, DCD, CDD
Voting Game	D $\succ$ C	C $\succ$ D	D $\succ$ C	DCC, CDC, CCD, DDD

Table 1. Variations of the Three-Person Game

I have shown earlier that by truncating the set of players in a Game of Chicken it can be turned into a Prisoner's Dilemma. Similarly, by truncating the set of players in a Voting Game it can be turned into an Assurance Game. If we take out the third voter, then nobody can shirk if they would like to see the social good realized. Now there are only two equilibria, viz. the optimal one in which all show up to vote and the suboptimal one in which nobody shows up. This will become important in our discussion of classical tragedies of the commons in the next section.

We can construct a representation of a voting game in terms of the n-person game with the payoffs of defection and cooperation as a function of the proportion of other players defecting. If the game is an actual vote then there will be exactly one pivotal point at which cooperation beats defection. E.g. suppose that there are 11 voters who would all vote yes if they were to come to the polls and we need to bring out 7 yes votes to win the vote. Then cooperation is preferred to defection by a player just in case there are precisely 6 of the 10 other voters who bothered to show up for the vote, i.e. if the proportion of defectors among the other players is .40. (Figure 6)

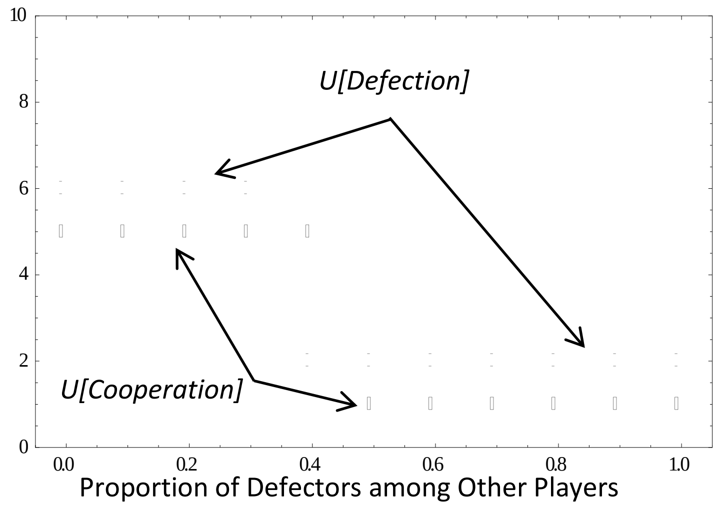

Figure 6. Voting Game with Single Pivotal Player.

We can also construct a generalized version in which defection trumps cooperation when there are few cooperators and when there are many cooperators, while there is a critical zone in the middle in which it is worthwhile contributing to the social good. The decreasing slopes indicate that the quality of the social good declines or that there is a decreasing chance that the social good will be realized as the defection rate goes up. The linearity assumption is just for simplicity. (Figure 7)

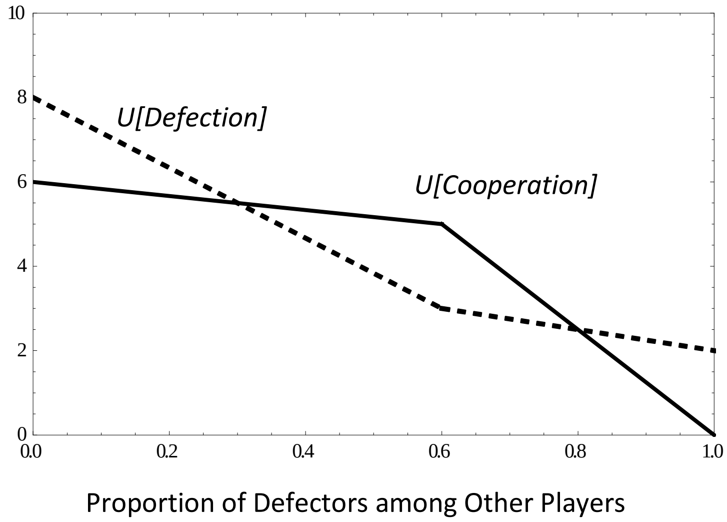

Figure 7. Voting Game with Critical Zone

Voting games do come with an air of tragedy. If the players can coordinate who will show up to vote, then all is well. They will play an equilibrium strategy with a pivotal player and the social good will be realized. But coordination is precisely the problem. When they cannot coordinate, then they may slide into the All Defect equilibrium and not show up for the vote. We also see the same pattern when voters cast a protest vote for a candidate whom they do not really want to vote in place. They trust that there will be a sufficient number of voters for an acceptable mainstream candidate. But without coordination, it may happen that there will not be a sufficient number and the protest candidate may gain unwanted traction.

There is a history of using games that have the structure of the Voting Game as an analysis of collective action problems. I will survey and discuss this literature before turning to classical tragedies of the commons.

6. Historical Background of the Voting Game

The name "Voting Game" is inspired by the Simple Voting Game in Moshé Machover and Dan Felsenthal, Def. 2.1.1. in (1998: 11) which in turn builds on L.S. Shapley and Martin Shubik (1954). A Simple Voting Game is a set of voters and a non-empty set of subsets of the voters which form winning coalitions and every superset of a winning coalition is also a winning coalition. E.g. under majority rule, the set of winning coalitions of the set of voters {a, b, c} is {{a, b}, {b, c}, {c, d}, {a, b, c}}. This Simple Voting Game induces a Voting Game as I understand it: if you belong to a minimal winning coalition, i.e. a winning coalition which is such that no proper subset of this winning coalition is itself a winning coalition, then you prefer to cast a vote rather than not to cast a vote; otherwise, you prefer not to cast a vote rather than cast a vote.

The Voting Game has the same structure as the game G3 in Taylor and Ward (1982: 356), adapted in Taylor (1987: 41), which is a model for the provision of lumpy public goods (i.e. step goods or threshold goods). Taylor and Ward (1982: 360) and Taylor (1987: 44-45) apply this game to the context of voting. Ward (1990) illustrates the game by adapting the story in Jerome K. Jerome's novel *Three Men in a Boat (To Say Nothing of the Dog)* (1889): Rowing is worthwhile if exactly one other person rows—if none row, then a single rower will make the boat circle and if the two others row, then there is no need to row. In Skyrms (2004: 117–22) the game is called "Three-in-a-Boat".

Jean Hampton (1987) also provides a game that has the same structure as the Voting Game to analyze Hume's case of Draining the Meadows. She refers to Norman Frohlich *et al.* (1975) and Norman Frohlich and Joe Oppenheimer (1970) as seminal work showing that collective action problems need not have the structure of *n*-person Prisoner's Dilemmas.

Frohlich and Oppenheimer (1970) introduce the notion of a lumpy public good and provide the by now classical example of building a bridge (1970: 109). They discuss the rationality of making donations for the provision of lumpy and non-lumpy public goods. For lumpy goods, it is rational to donate if and only if there is a chance that the donation will make a difference to the provision of the good, i.e. when the donation might be pivotal. The presentation of the games in terms of payoffs for defection and cooperation as a function of the proportion of defectors (or rather, of cooperators) goes back to Thomas Schelling (1973). Frohlich *et* *al.* (1975: 326–7) use this Schelling model (1973) to present a game in which the payoff of defection is greater at low and high level of cooperation whereas the payoff of cooperation is greater at intermediate levels of cooperation, as it is in the Voting Game. They indicate that voting is an extreme case of this game in which cooperation only pays off when there is a level of cooperation that makes a player's vote pivotal and present it as an example of a lumpy good.

Taylor and Ward classify G3, i.e. the Voting Game, as a variant of a three-person Game of Chicken. Their argument is that in G3, as in the Game of Chicken, we have an incentive to pre-commit to defection. Now this is true in the Game of Chicken. As I indicated in f. 1, we can construe two Games of Chicken in the three-person game, depending on the point at which there is a preference switch. If the game has the structure of the Game of Chicken in our Table 3, then two players have an incentive to pre-commit to defection, forcing the third to cooperate. If we turn around the preference from D $\succ$ C to C $\succ$ D 'in the column of other players playing DC and CD, then we still have a Game of Chicken. In this game, the first (but not the second) player still has an incentive to pre-commit to defection. The same holds, argue Taylor and Ward, in G3, i.e. in the Voting Game.

But this is not straightforward. If one person pre-commits to defection in the Voting Game, then what is left is an Assurance Game for the remaining players. And this game has an optimal equilibrium (Both Cooperate) and a suboptimal equilibrium (Both Defect). Hence, it is not guaranteed that a pre-commitment to defection will lead to the optimal equilibrium. If there is a high level of distrust both remaining players will choose defection. Taylor will respond that rationality prescribes that players choose the optimal equilibrium in the Assurance Game (1987: 19), but it is not clear that rationality alone is sufficient to secure trust.

Hampton analyzes a three-person variant of Hume's Draining the Meadows as a game that has the same preference structure as the Voting Game, as I will do below. But she disagrees with Taylor and Ward that this game is a game of Chicken (1987: 254 f. 7) and classifies it as a Battle of the Sexes. Her argument is that the Voting Game is a game like the Battle of Sexes in that we need to coordinate on an equilibrium—viz. we need to determine which two people are going to conduct the work. One might say that there is indeed a resemblance in this respect.

But there is a crucial point of dissimilarity. In the two-person Battle of Sexes, there are only two equilibria and there is a coordination problem. Hampton (1986: 151) presents a three-person Battle of Sexes to analyze the leadership selection problem in Hobbes. There are only three equilibria in this game and there is a coordination problem. Unlike in Hampton's three-person Battle of Sexes, there is a fourth suboptimal equilibrium in the Voting Game, viz. All Defect, aside from the three equilibria which pose a coordination problem. Games with different equilibria should not be classified under the same name.

One may quibble about how to delineate the precise extension of different types of games in game-theory. But following the Schelling model I do not see how the Voting Game could reasonably be placed in the same category as a Game of Chicken or a Battle of the Sexes. I take it to be an essential feature of the Game of Chicken and the Battle of the Sexes in the two-person game that defection is the rational response to cooperation and that cooperation is the rational response to defection. This generalizes, for the n-person game, to Defection being the rational response to All Others Cooperate and Cooperation being the rational response to All Others Defect. This feature is missing from the Voting Game and hence it warrants a separate classification.

7. Three Classical Tragedies of the Commons

We will now look at three classical sources that can readily be interpreted as tragedies of the commons. The oldest one is Aristotle's comment in the *Politics* about common ownership. Then there is a 16th century Indian story relating an exchange between the Mogul Emperor Akbar and his advisor Birbar. Finally there is Hume's famous story in the *Treatise* of the commoners who are charged with draining the meadows.

These stories long predate the advent of game theory and they can be modelled in different ways. But it is of interest to look at them closely and to assess what the authors considered to be the cause of the tragedy – i.e. what they take to be the reason why the social good is not being realized. Their diagnoses include various observations. Many of these observations are more readily interpreted in terms of a Voting Game, rather than a Prisoner's Dilemma, an Assurance Game, or a Game of Chicken.

We start with Aristotle's passage in the *Politics*. Ostrom (1990: 2) only quotes a short excerpt: "[W]hat is common to the greatest number has the least care bestowed upon it. Everyone thinks chiefly of his own, hardly at all of the common interest." What is the context of this quote? In the *Politics*, Bk II, 1261a-1262b, Aristotle discusses Socrates' suggestion that a community (of men) should own all things in common, including women and children. He discusses "a variety of difficulties" with "all of the citizens [having] their wives in common" (1261a). One of these objections is in the passage that Ostrom is quoting. Here is the complete passage:

> And there is another objection to the proposal. For that which is common to the greatest number has the least care bestowed upon it. *Everyone thinks chiefly of his own, hardly at all of the common interest*; and only when he is himself concerned as an individual. For besides other considerations, *everybody is more inclined to neglect the duty which he expects another to fulfil*; as in families *many attendants are often less useful than a few*. Each citizen will have a thousand sons who will not be his sons individually but anybody will be equally the son of anybody, and will therefore be neglected by all alike. (1261b) [emphasis added]

Aristotle seems to be worried in this passage not so much about the well-being of the women, but rather of their male offspring. He also mentions that the Upper-Libyans are rumored to have their wives in common but they divide the children among the potential fathers on ground of the resemblance that the children bear to these potential fathers. (1261a)

Aristotle provides a number of reasons why cooperation is foregone under common ownership regimes in this passage.

(A.i) Common owners only attend to their private interests and not to the common interest, and,

(A.ii) They expect others to cooperate so that the social good will be realized without their cooperation.

The passage on the ideal number of attendants in families suggests the following generalization:

(A.iii) Defection is more like to occur when there are many people rather than few people expected to procure the social good.

Rohit Parikh (2009) points to a 16th century Indian story from the Birbar jest-books as recorded in the 19th century by Mahanarayan and analyzes it as an *n*-person Prisoner's Dilemma. Since the passage is not well-known, I quote it in full:

> One day Akbar Badshah said something to Birbar and asked for an answer. Birbar gave the very same reply that was in the king's own mind. Hearing this, the king said, "This is just what I was thinking also." Birbar said, "Lord and Guide, this is a case of 'a hundred wise men, one opinion' (…) The king said, "This proverb is indeed well-known." Then Birbar petitioned, "Refuge of the World, if you are so inclined, please test this matter." The king replied, "Very good." The moment he heard this, Birbar sent for a hundred wise men from the

> city. And the men came into the king's presence that night. Showing them an empty well, Birbar said, "His Majesty orders that at once every man will bring one bucket full of milk and pour it in this well." The moment they heard the royal order, every one reflected that where there were ninety-nine buckets of milk, how could one bucket of water be detected? Each one brought only water and poured it in. Birbar showed it to the king. The king said to them all, "What were you thinking, to disobey my order? Tell the truth, or I'll treat you harshly!" Every one of them said with folded hands, "Refuge of the World, whether you kill us or spare us, the thought came into this slave's mind that *where there were ninety-nine buckets of milk, how could one bucket of water be detected*?" Hearing this from the lips of all of them, the king said to Birbar, "What I'd heard with my ears, I've now seen before my eyes: "a hundred wise men, one opinion'!" (Mahanarayan: 1888: 13–4. Emphasis added.)

The wise men say that they figured that one bucket of water could not be detected when mixed with 99 buckets of milk. Hence, they explain their defection in the following way:

(M.i) They expected all others to cooperate and hence, there was no need for them to cooperate.

Finally, let us turn to Hume who discusses the problem of draining a commonly owned meadow in the section entitled "Of the Origin of Government" in the *Treatise,* Bk Three, Part II, Section VII:

> Two neighbours may agree to drain a meadow, which they possess in common; because it is easy for them to know each others mind; and each must perceive that the immediate consequence of his failing in his part, is, the abandoning of the whole project. But it is very difficult, and indeed impossible, that a thousand persons should agree in such action; it being difficult for them to concert so

complicated a design, and still more difficult for them to execute it; while each seeks a pretext to free himself of the trouble and expence, and would lay the whole burden on others. Political society easily remedies both inconveniences.

We find the same theme in Hume as in Aristotle:

(H.i) It's easier to improve a commons with a few people than with many people.

His reason for this is as follows:

(H.ii) With fewer people, a single defection would be the end of the collective project.

He offers three problems when there are more rather than fewer people, viz.

(H.iii) "know[ing] each other's mind",

(H.iv) "concert[ing] so complicated a design", and,

(H.v) "execution", that is, of preventing that each other player would "seek a pretext to free himself of the trouble and expense" and "lay the whole burden on others."

8. The Classical Tragedies as Voting Games

These three classical stories all display tragedies of the commons. It concerns the lack of care for commonly owned property or for a resource that, on king's orders, needs independent attention from multiple people. Each of these stories can be construed as a Voting Game. In Mahanarayan's story, when there are few buckets of milk or many buckets of milk in the well, it makes no difference to the prospect of punishment whether one pours milk or water; But there is a critical zone between too few and too many in which one bucket of milk may reasonably make a difference between incurring the wrath of the King or not. In Hume's story, one can envision a critical zone in which cooperation is worthwhile whereas defection is a rational response when there are too few cooperators to carry out the project and too many so that one can safely shirk. And a similar interpretation can be proffered for Aristotle's attendants in domestic service.

But would this be the most plausible interpretation? To assess this, we need to look carefully at the explanations of what gets us into the tragedy in these classical sources. I distinguish between four types of explanations, viz. the Expectation-of-Sufficient-Cooperation Explanation, the Too-Many-Players Explanation, the Lack-of-Trust Explanation, and the Private-Benefits Explanation.

i. The Expectation-of-Sufficient-Cooperation Explanation

Both Aristotle (A.ii) and Mahanarayan (M.i) mention that each person expects that the social good be realized through the cooperation of others and hence that their cooperation is not needed.

This is not consistent with a Prisoner's Dilemma since there is simply no reason to assume that anyone would cooperate in a Prisoner's Dilemma. It is not consistent either with a Game of Chicken: We also expect massive defection with an extended set of players in a depletion of the commons. Nor is it consistent with an Assurance Game. In an Assurance Game, we might indeed expect cooperation considering that the optimal equilibrium is salient. But if we expect cooperation in an Assurance Game, then there is precisely reason for us to cooperate and there is no reason to defect. Hence, the Expectation-of-Sufficient-Cooperation Explanation is not consistent with a Prisoner's Dilemma, a Game of Chicken or an Assurance Game.

However, it is consistent with a Voting Game. This is precisely what people say to explain their lack of participation in democratic procedures: They expected a sufficient number of people to show up for the vote and decided that they themselves did not need to show up. In a Voting Game, there are indeed multiple equilibria that have a mix of cooperation and defection. If the players cannot coordinate their actions and make any binding agreements, then they may indeed find themselves in the predicament that too many of the players come to expect that there is enough cooperation and that their cooperation is not needed. And consequently the situation deteriorates into the equilibrium of All Defect.

ii. The Too-Many-Players Explanation

Both Aristotle (A.iii) and Hume (H.i and H.ii) mention that the problem of defection is particularly acute when there are more rather than fewer players. If we read this passage in terms of bargaining theory or cooperative game-theory—i.e. how can rational players come to an agreement about which solution in the optimal set they will settle on—then it is consistent with all games. The greater the number, the harder it is to reach an agreement due to conflicting conceptions of fairness, the desire to hold out, etc.

But we should bracket bargaining theory in our interpretation of this quote. Bargaining theory enters in when Hume's "political society" enters in. The claim here is that, before political society enters in, before we sit down to settle on a collectively rational solution, the problem of defection is more acute when there are many rather than fewer individually rational persons.

So what happens when we contract the set of players in the game from many to fewer players? In a Prisoner's Dilemma, individually rational people will simply defect whether there are more or fewer players. In a Game of Chicken, contracting the set of players would do no more than take out the cooperators and we would be left with a Prisoner's Dilemma with only defectors. So neither the Prisoner's Dilemma nor the Game of Chicken can provide an interpretation of the Too-Many-Players Explanation.

But now pay close attention to (H.ii)—with fewer people, a single defection would be the end of the collective project, Hume says. It is very natural to read this passage in terms of pivotality. With fewer people, it is more likely that each person is pivotal in carrying out the social project, whereas with more people, it is more likely that some people are dispensable. So with fewer players, it is more likely that we have an Assurance Game in which each player is pivotal or in which cooperation is at least worth each player's while. We have shown how an Assurance Game can be construed as a truncated Voting Game. If we take out the surplus of players in a Voting Game for whom cooperation is no longer beneficial when enough people cooperate, then a Voting Game becomes an Assurance Game. With few neighbors to drain the meadows in Hume's example, nobody can shirk and the project will come to completion. We can read Aristotle in a similar manner: With fewer attendants in a family, there is a better chance that the domestic work will be done well.

So this is once again an argument in favor of interpreting these classical tragedies as Voting Games. When Aristotle and Hume are insisting that fewer do better than more, we can plausibly say that they see tragedy ensue when the game is a Voting Game and that the situation can be improved by truncating it into an Assurance Game with a single salient optimal equilibrium.

iii. The Lack-of-Trust Explanation

Hume mentions problems of "knowing the minds of others" (H.iii) and of "execution", of others not finding "a pretext" to shirk (H.v).

If we stipulate that the players have common knowledge of rationality, then they do know the minds of others in a Prisoner's Dilemma. They know that all will defect in the Prisoner's Dilemma and so the Prisoner's Dilemma is not a good interpretation of Hume's quote. In a Game of Chicken that reflects a tragedy of the Commons, the expectation is also that many will defect—so many that the commons will be depleted. So also a Game of Chicken is not a good interpretation of Hume's quote.

Skyrms (2001: 2) suggests an Assurance Game as an interpretation of Hume's story of draining the meadows. Now it is indeed the case that there are two equilibria, viz. All Cooperate and All Defect. So one might say that we cannot know the minds of others and be confident what equilibrium they will play. And indeed, even if we agree to cooperate, then there still is a problem of "execution", of others not finding "a pretext" to shirk (H.v). So there is indeed an issue of trust: Just as in the Stag Hunt, the farmers might ask, How can I be confident that others will not be blinded by their short term interests and attend to their private business rather than show up to drain the meadows?

So admittedly an Assurance Game would be defensible in the face of the Trust Argument. The only reservation against this interpretation is that if the collective benefit is sufficiently large and the private benefit sufficiently small, then the optimal equilibrium should be sufficiently salient for the players to have at least some reason to expect that others will cooperate—they have some grounds to know the mind of others and to trust that others won't shirk.

However, knowing the mind of others and shirking are more unsurmountable problems in a Voting Game. There are multiple optimal equilibria and not knowing the minds of others, players do not know whether they are pivotal voters. Furthermore, it is all too easy to succumb to wishful thinking and create a pretext that there are likely to be a sufficient number of cooperators. Or, it is all too easy to succumb to defeatism and say that there are bound to be too many defectors. In an Assurance Game, trust may be the problem, but at least the saliency of the optimal equilibrium would provide some reason for trust, whereas in a Voting Game the structure of the game is an open invitation for distrust.

iv. The Private-Benefits Explanation

Aristotle explains the lack of care for commonly owned property on grounds of the fact that our agency is determined by our private benefits and not by the collective good (A.i).

I admit that there is a very natural way of reading this passage on the background of a Prisoner's Dilemma: Individual rationality is what leads to the sub-optimal equilibrium of All Defect in the Prisoner's Dilemma, whereas if our agency were determined by collective benefits we would not get caught there. It would also be consistent with a Game of Chicken when we extend the set of players beyond the point of profitable commons use, leading to a sub-optimal equilibrium of Many Defect.

However, there is also a reading of this passage that is consistent with an Assurance Game or a Voting Game. We might say that the preferences that enter into the definition of the game reflect benefits derived from the collective project and from personal projects. The benefits from the collective project are greater than from the individual project. However, we may attach more weight to the smaller benefits from the personal project than the larger benefits from the social project in our deliberation and our agency may be determined by these differential weights. This reading could be placed in the context of an Assurance Game: The smaller benefits from hunting hare have more weight in a player's deliberations than the larger benefits of hunting deer. Similarly, we can also place this reading in the context of a Voting Game: The smaller comforts of not showing up for the vote have more weight than the larger benefit we would derive from the social project of voting the proposal in place. We could invoke the Private-Benefits argument as a kind of myopia that drives us away from the optimal equilibrium towards the suboptimal equilibrium in an Assurance Game. It also drives us away from the optimal equilibria through the lack of coordination, self-deception and defeatism into the suboptimal equilibrium in a Voting Game. But granted, this last explanation is not decisive and is open to multiple interpretations which can be accommodated by all four games.

9. Conclusion

Tragedies of the Commons can plausibly be analyzed as Prisoner's Dilemmas, Assurance Games, Games of Chickens and Voting Games. Voting Games can trace their ancestry to the voting power literature on the one hand and to the literature on the provision of lumpy public goods on the other hand. I have looked at three classical sources (Aristotle, Mahanarayan and Hume) that cite different reasons of why tragedy ensues. These reasons can be captured in four explanations, viz. the Expectation-of-Sufficient-Cooperation Explanation, the Too-Many-Players Explanation, the Trust Explanation, and the Private-Benefits Explanation. The Voting Game is a particularly suitable candidate to analyze these classical tragedies. Not all these explanation point univocally to a Voting Game, but the balance of reasons is on the side of a Voting Game interpretation, rather than Prisoner's Dilemma, Game of Chicken or Assurance Game interpretations.

References

Aristotle, *Politics*, Bk Two.

H. Rackham translation.

http://www.perseus.tufts.edu/hopper/text?doc=Aristot.+Pol.+1.1252a&redirect=true

and Jowett translation.

http://ebooks.adelaide.edu.au/a/aristotle/a8po/book2.html

Dawes, R.M. 1975. "Formal Models of Dilemmas in Social Decision Making." In *Human Judgment and Decision Processes: Formal and Mathematical Approaches*, ed. M.F. Kaplan and S. Schwartz, New York: Academic Press, pp. 87-108.

Dixit, A. and Skeath, S. 1999. *Games of Strategy*. Norton.

Frohlich N. and Oppenheimer, J.A. 1970 I Get by with a Little Help from my Friends Source: *World Politics* 23 (1), 104-120

Frohlich, N., Hunt, T., Oppenheimer, J. and Wagner, R.H.(1975) Individual Contributions for Collective Goods: Alternative Models. *The Journal of Conflict Resolution*, 19 (2), 310-329

Hampton, J. (1986) *Hobbes and the Social Contract Tradition.* Cambridge University Press.

Hampton, J. (1987). Free-Rider Problems in the Production of Collective Goods. *Economics and Philosophy*, 3, 245-273

Hardin, G. 'The Tragedy of the Commons'. *Science* 13 December 1968: Vol. 162 no. 3859, 1243-1248 http://www.sciencemag.org/content/162/3859/1243.full

Hume, D. *A Treatise of Human Nature*. http://www.gutenberg.org/files/4705/4705-h/4705-h.htm#link2H_4_0094

Felsenthal, D.S. and Machover, M. 1998. *Voting Theory and Practice. Theory and Practice, Problems and Paradoxes.* Elgar.

Jerome, Jerome K. *Three Men in a Boat (To Say Nothing of the Dog)* Bristol: Arrowsmith 1889.

Mahanarayan 1888. *Urdu jest book: Lata'if-e Akbar, Hissah pahli: Birbar namah,* Delhi: Matba` Jauhar-e Hind [India Office Library 1144 VT]. Translated from the Urdu by Frances W. Pritchett.
http://www.columbia.edu/itc/mealac/pritchett/00litlinks/txt_akbar_birbal_fwp.html

Moulin, H. 2004. *Fair Division and Collective Welfare*. MIT Press.

Ostrom, E. 1990. *Governing the Commons.* Cambridge University Press.

Parikh, R. 2009. Knowledge, Games and Tales from the East. In: Ramaswamy Ramanujam and Sundar Sarukkai (eds.) *Logic and Its Applications, Third Indian Conference, ICLA 2009, Chennai, India, January 7-11, 2009. Proceedings*. Springer: Lecture Notes in Computer Science. 65-76.

Rousseau, J. (1984) *A Discourse on Inequality*. Trans. M. Cranston. New York: Penguin Books.

Schelling, T.C.(1973) Hockey Helmets, Concealed Weapons, and Daylight Saving: A Study of Binary Choices with Externalities. *The Journal of Conflict Resolution*, 17 (3), 381-428

Shapley, L.S. and Shubik, M. (1954)A Method for Evaluating the Distribution of Power in a Committee System. *The American Political Science Review*, 48 (3), 787-792.

Skyrms, B. 2001. The Stag Hunt. Pacific Division of the American Philosophical Association. http://www.socsci.uci.edu/~bskyrms/bio/papers/StagHunt.pdf

Skyrms, B. 2004 *The Stag Hunt and the Evolution of the Social Structure*. Cambridge University Press.

Taylor, M. and Ward, H. (1982) Chickens, Whales and Lumpy Goods: Alternative Models of Public Goods Provision. *Political Studies*, 30 (3), 350-70.

Taylor, M. *The Possibility of Cooperation*. Cambridge University Press 1987.

Ward, H. (1990) Three Men in a Boat, Two Must Row: An Analysis of the Three-Person Chicken Pregame. *The Journal of Conflict Resolution*, 34(3) (Sep. 1990) pp. 371-400.

Taylor, M. 1990. "Cooperation and Rationality: Notes on the Collective Action Problem and its Solution." In: K. S. Cooks and M. Levi (eds.) *The Limits of Rationality*. University of Chicago Press: London. 222–40.