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Aristotle on the Unity of Change: Five *Reductio* Arguments in *Physics* viii 8

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Although the stated purpose of *Physics* viii 8 is to prove that only circular locomotion is infinitely continuous, it is generally recognized that a major sub-theme of the chapter has to do with the unity of change and centers on Zeno's dichotomy paradox. According to one influential account of this sub-theme, Aristotle returns to the dichotomy paradox in *Physics* viii 8, primarily to engage in a defensive maneuver. In *Physics* vi, while focused on the infinite divisibility of change instead of its identity conditions, Aristotle left open the possibility that occurrences that are 'one change' could have infinitely many parts that are also 'one change'.[1] By *Physics* viii 8, however, Zeno has brought Aristotle to realize that if this possibility is admitted, then what one chooses to call 'one change' is to a large extent arbitrary. But this Aristotle cannot countenance, because his entire theory of change is built upon the concept of a change as a thing uniquely definable as the passage from a particular state to a particular state. In *Physics* viii 8, then, Aristotle seeks to avoid this result by 'refining' the definition of 'one change' so that 'one change' can no longer have parts that are also 'one change' and by invoking the metaphysical machinery of the act-potency distinction to give a positive characterization of the difference between change parts and change wholes.[2]

According to Michael White, Aristotle 'refines' his definition of 'one change' in *Physics* viii 8 by strengthening the criteria of *Physics* v 4; criteria, which, White is correct to point out, do nothing to prevent this result on their own.[3] According to White, this 'refinement' consists in adding, to the criteria of *Physics* v 4 (i.e., the criteria that 'one change' must be in a continuous time, have a single subject throughout, and proceed throughout from a terminus of the same species to a contrary terminus of the same species), the additional condition that an occurrence that is 'one change' must be bracketed by periods of rest and contain no periods of rest. This condition is never stated as such in *Physics* viii 8, but it can be inferred from the doctrine, explicitly invoked in Aristotle's discussion

[1] For instance, at *Physics* vi 6.237a18-19, Aristotle says οὐ μόνον δὲ τὸ μεταβάλλον ἀνάγκη μεταβεβληκέναι, ἀλλὰ καὶ τὸ μεταβεβληκὸς ἀνάγκη μεταβάλλειν πρότερον, which seems to count change parts as changes themselves.

[2] This, interpretation is originally due to Broadie 1982, 131-158, but was later taken up and elaborated by White 1992, 102-6.

[3] White 1992, 105. Since this is a point that Broadie 1982, 151 misses, I shall focus primarily on White's account as the most viable version of the interpretation I am now discussing.

of the dichotomy paradox at 263a4-b8 and proved earlier at 262a17-b8, that if a moving thing uses a state as the end of one change and the beginning of another, it must come to rest in that state.

There is general agreement, however, that the proof of this doctrine at 262a17-b8 is marred by a serious and puzzling philosophical mistake,[4] since Aristotle seems to imply that there can be a first instant of having departed from a given state in the course of continuous change. If this is indeed a mistake, it is a serious one because it undermines Aristotle's response to Zeno, and, therefore, undermines Aristotle's entire theory of change. For if Aristotle has not, in fact, proved that what one chooses to call 'one change' cannot be entirely arbitrary, then Zeno is free to deny any sense to the concept of a change as a thing uniquely definable as the passage from a particular state to a particular state. This mistake is puzzling because it seems to betray confusion regarding a matter that Aristotle was quite clear about in *Physics* vi 5, viz., that there is no first instant of having departed from a given state in the course of continuous change.

I believe that if one feels puzzlement, here, it should be taken as a strong indication that one does not fully understand Aristotle's argument. Even if it is conceded that Aristotle can sometimes make obvious mistakes, it cannot be supposed that he assumes a claim in *Physics* viii 8 that he cogently and thoroughly refutes in a text that was, by all accounts, written earlier. I will demonstrate that the argument at 262a17-b8 has, indeed, been misunderstood because insufficient attention has been paid to the fact that it is a *reductio ad absurdum*, and that the absurdities it produces derive from an infinite fracturing of change. Now, of course, the infinite fracturing of change is precisely what Aristotle wants to avoid. The idea of this argument, however, seems to be that if we grant an assumption that implies this infinite fracturing of change, and logical contradictions can, in turn, be derived from it, then we are entitled to infer the contradictory of the assumption in question, i.e., to infer that what one chooses to call 'one change' *cannot* be entirely arbitrary, as well as to infer the impossibility of the infinite fracturing of change that follows from it. I will argue that not only the *reductio* argument at 262a17-b8 (let us call this '*reductio* (a)'), but also three other *reductio* arguments in *Physics* viii 8 can be interpreted as granting this assumption for the purpose of refuting it, *even though this assumption, and the infinite fracturing of change it entails, is not explicitly stated in these other arguments*: the indirect proof at 262b22-263a3 of the claim that a reversal in the direction of a rectilinear locomotion requires the moving thing to come to rest ('*reductio* (b)'), the indirect proof at 263b26-264a4 of the claim that the time of a change cannot be divided into indivisibles ('*reductio* (c)'), and finally, the indirect proof at 264b1-6 of the claim that a reversal in the direction of an alteration requires the moving thing to come to rest ('*reductio* (d)').

Thus, I will argue that while an infinite fracturing of change is on the face of

[4] See Sorabji 1976, 174; White 1992, 54-58. Cf. Bostock 1972, 42, and Charlton 1995, 139 that also claim that Aristotle made a mistake here.

reductio (a), it can be plausibly and profitably read into *reductios* (b)-(d) as well; plausibly because of close structural similarities that these arguments bear to *reductio* (a); profitably because doing so gives us a unified way to understand these arguments that is a better textual fit than taking them to be fixing Aristotle's criteria for the unity of change. I say this, in part, because the textual case for the latter interpretation is weak. The only explicit mention of criteria for the unity of change in *Physics* viii 8 is in another indirect proof, distinct from the ones that I have mentioned, of the same proposition proved in *reductio* (b), but that proceeds by an entirely different line of argument that involves no infinite fracturing of change and, therefore, seems to have nothing to do with Zeno's dichotomy paradox. I will analyze this argument ('*reductio* (e)'), to show how criteria for the unity of change *do* enter into the arguments of *Physics* viii 8, but it will be a different role from the one suggested by Broadie and White.

Reductio (a)

The key elements of Aristotle's explicit response to Zeno at 263a4-b8 are the claims that the parts of a change exist potentially, and not actually, that change parts exist actually only if their limits exist actually, and these exist actually only if they mark off kinetic discontinuities. As Aristotle puts it, the limits of a change exist actually when the moving thing uses a state as the end of one change and the beginning of another, and at 263a22-23 he invokes the result of his 'previous discussion' that using a state in this way creates a discontinuity. By the 'previous discussion', he clearly means *reductio* (a), where this result is first proved. Thus, while *reductio* (a) ultimately plays a role in establishing that only circular motion is infinitely continuous, the back-reference to it at 263a22-23 suggests that it also supplies a key premise used to address Zeno's dichotomy paradox. The core of *reductio* (a) is in the following passage:

> A is in locomotion and comes to a stop at B and again undergoes locomotion towards C. But when it undergoes locomotion continuously, A can neither have come to be at point B nor have departed <from it>, but it can only be there at an instant, not in any time interval, except that of which the instant is a dividing point, (a30) namely, the whole time interval. (If anyone will suppose that it has come to be there and has departed, A will always be coming to a stop when it is undergoing locomotion, for (262b1) A cannot have come to be at B and have departed at the same time. Therefore <it must have done so> at a different point of time. Therefore what is in between will be a time interval. And so, A will be at rest at B. And likewise at the other points too, since the same argument holds for them all. (b5) But when A, the thing undergoing locomotion, uses B, the middle, as both an end and a beginning, A must stop because it makes <the point> two, just as if one were to do so in thought.) But it has departed from point A as its starting point and has

> come to be at C when it finishes <its motion> and stops.[5] (*Physics* viii 8.262a26-b8)

The *reductio* argument, here, can be paraphrased as follows: Assume that (1) an object A moves continuously (262a28) from point E to point C along a spatial path containing the point B.[6] Assume, for *reductio ad absurdum*, that (2) A has come to be at point B and has departed from point B, but does not rest at point B. From this, somehow infer that (3) A is always coming to a stop when it is undergoing locomotion and somehow infer that (4) A has come to be at point B and has departed from point B at the same time. From the impossibility of (4) A having come to be at point B and having departed from point B at the same time, infer that (5) A has come to be at point B and has departed from point B at different times. From this, infer that (6) there is a time interval between when A has come to be at point B and when A has departed from point B. From this, infer that (7) A rests at point B, which contradicts the initial assumption (2) that A does not rest at point B. So infer the negation of the initial assumption [i.e., (2)], viz., (8) if A has come to be at point B and has departed from point B, it rests at point B. Q.E.D.

Richard Sorabji and Michael White have correctly pointed out that in order to infer directly from proposition (6) to proposition (7), one needs to assume that there is a first instant of having departed from point B (see n4, above). But Aristotle is quite clear in *Physics* vi 5.236a7ff. that in a continuous change, there is no first instant of having changed, and since having departed follows upon having changed (235b6ff.), then there is no first instant of having departed. So according to *Physics* vi 5, there is, indeed, a time interval between the instant when A has come to be at point B and every instant when A has departed from point B, but since every instant of having departed from point B is preceded by another instant of having departed from point B, it does not follow that A ever rests at point B. So Aristotle appears to have made a mistake, and fairly obvious one that he should have known better than to make.

I will establish, in due course, that there is no mistake, here,[7] but for the moment I would like to point out two things: First, since this argument is a *reductio ad absurdum*, one need not attribute all of its premises to Aristotle, because, inevitably, some of them will be adopted dialectically, for purpose of refutation. Second, propositions (5), (6), and (7) are logically otiose, since, once Aristotle has inferred proposition (4), which implies that A is both at B and away from B at the same time, he could have inferred from the impossibility of this directly to the conclusion (8), which is the contradictory of (2), the proposition that is supposed to have generated this impossibility. It was, therefore, not necessary for Aristotle

[5] The translations of Aristotle's *Physics* viii 8 as well as the commentary by Simplicius on this chapter are, with minor modifications, from McKirahan 2001. Unless otherwise noted, other translations of Aristotle are from Barnes 1995.

[6] The origin is not named, but it is convenient to call it 'point E'.

[7] And for this reason, I will not consider the ingenious attempts of White 1992, 54-58 to make Aristotle's 'mistake' out to be more subtle than it appears to be.

to infer directly from proposition (6) to proposition (7). From this, it follows either that Aristotle's 'mistake', as some have interpreted it, was sadly unnecessary, or that he had other reasons for assuming that there is a first instant when A has departed from point B. I will argue for the latter interpretation.

Little attention has been given by commentators to proposition (3), the claim, which Aristotle arrears to derive from proposition (2), that A 'will always be coming to a stop when it is undergoing locomotion'. This is a pity, I think, because it is the key to understanding *reductios* (a)-(d). What this proposition amounts to can be deduced as follows: The claim that A 'will always be coming to a stop when it is undergoing locomotion' apparently means that A will be coming to a stop at every now within the period of its motion. Since, to every now within the period of a locomotion, there corresponds a spatial point in the locomotion's path,[8] the claim that A will be coming to a stop at every now within the period of its motion also means that A will be coming to a stop at every spatial point along the locomotion's path. The point of the preamble to *reductio* (a) at 262a19-26 seems to be that A comes to a stop at point B and then moves on just in case it uses point B as the destination of one motion and the origin of another that follows it. Always coming to a stop in the course of a locomotion, then, amounts to using each of the infinitely many positions along the path of the motion as the destination of one motion and the origin of another that follows it. If A does this, however, the locomotion that we have implicitly assumed to be one must be viewed as many, and, indeed, infinitely many. For A to be always coming to a stop while it is undergoing locomotion, then, is for A to be undergoing infinitely many locomotions; as many locomotions as there are points along the motion's path. Aristotle's claim, then, is that if we allow that A can come to be at and depart from point B without resting, then A's motion will be divided at every stage.

But how does it follow from the fact that A uses *point B* as the destination of one motion and the origin of another without resting, that A *must* do so at *every* point along its path? Simplicius, *In Phys*. 1283,17 suggests that since B is an arbitrarily selected point along A's path, then there is no reason why the same result should not follow for any other point along the path. While true enough, this observation only gets us the result that A can use *any* point as the destination of one motion and the origin of another without resting, not that A can do so at *every* point, much less that it *must* do so at every point. Aristotle's rationale, I suspect, is that since A's motion is assumed to be continuous (262a28), i.e., since there are no pauses, breaks in the path of the change, changes in direction, or changes of subject, if A is allowed to use point B as the destination of one motion and the origin of another without resting, then there is no basis for maintaining a distinction between using a point as the boundary of successive motions and merely passing through it. And if this is the case, since A is obliged to pass

[8] See *Physics* vi 2, as well as Aristotle's claim, in *Physics* iv 11, that 'time follows change' and that 'change follows magnitude', since, as this is most often interpreted, this implies a one-one correspondence between the punctual elements in each of these continua.

through all of the points on the path of its journey, then A is also obliged to use each of these points as a terminus. If the example had envisaged A resting at point B, or another subject D taking over at point B, as in a relay race, so that the subject is not the same, or the motion changing direction at point B, one could adduce these features of A's motion as reasons for treating point B as a kinetic terminus in preference to other points along A's path. But if these things are ruled out *ex hypothesi*, and one still insists that point B is the boundary of successive motions, there is no reason not to view every point that A traverses as the boundary of successive motions. Thus, the Zenonian assumption, implicit in the dichotomy paradox, that what one chooses to call 'one change' is entirely arbitrary, leads, in the case of a continuous motion, to the result that the motion is divided at every stage.

In the light of this, the otherwise puzzling puzzle that immediately follows *reductio* (a) becomes readily intelligible:

> Therefore this is the response that should be given to the puzzle as well. For there is the following puzzle: (b10) if line E is equal to line F and A undergoes locomotion continuously from the extremity towards C, and if A is at point B at the same time as D is undergoing locomotion from the extremity F towards G uniformly and with the same speed as A, then D will have come to G before A does to C. For what sets out and departs (b15) earlier must arrive earlier. Therefore A has not come to be at B and departed from it at the same time, which is why it is delayed. For if <it has come to be and departed> at the same time, it will not be delayed, but <if this is to happen> it will have to come to a stop. Therefore we must not suppose that when A came to be at B, D was simultaneously moving from the extremity F (for if A will have come to be at B, (b20) its departure will also occur, but not at the same time); rather, it was there at a cut of time and not in a time interval. (*Physics* viii 8.262b8-22)

The puzzle envisages two motions by subjects A and D that are perfectly analogous (i.e., both are continuous, are over the same distance, and have the same speed, etc.) except that A is said to pass through a mid-point B on its way to its destination. And assuming that using a point as the boundary of successive motions is the same as merely passing through it, the poser of the puzzle purports to convict Aristotle of a philosophical howler by attributing to him the view that by merely passing through point B, A will arrive at its destination later than D arrives at its destination, even though their motions are otherwise perfectly analogous. But, according to Aristotle, it is not merely passing through point B that delays A's passage, but coming to be at point B and departing from point B, so either A merely passes through point B, in which case, the delay (and the absurdity) does not result, or A comes to be at point B and then departs from point B, and, therefore must rest, in which case the two motions are not perfectly analogous because one is continuous while the other is not.[9] Hence, the solution of the puzzle consists in the alleged fact that if using a point as the boundary of successive motions is not the same as merely passing through it, then if a moving thing uses a point as the boundary of successive motions, the motion it undergoes must be discontinuous.[10] Aristotle's solution to Zeno's dichotomy paradox is exactly analogous, insofar as he argues that since there *is* a difference between using a point as the boundary of successive motions and merely passing through it, then a motion that uses a point as the boundary of successive motions must be discontinuous at that point (263a4-b8). It is implicit in Zeno's paradox, as well as in the present puzzle, that there is no such difference, and indeed, we might be sympathetic with such a denial, since, indeed, a distinction between coming to be at point B and departing from point B without resting and merely passing through point B seems, at first sight, like a distinction without a difference. But then Aristotle will claim that what we take to be continuous, and intuitively unified motions must really be divided at every stage, and this, ultimately, will have unacceptable consequences.

This is because, if intuitively unified motions must really be divided at every stage, then, according to Aristotle, we are entitled to make certain inferences about what the motion, the time of the motion, and the path of the motion are divisible into and composed of. Aristotle argues at *Physics* vi 1.232a18-22 that any division of a continuous locomotion will also divide the time of the motion as well as the path. And it is clear from the arguments in *Physics* vi 2 that in such a division, each of these continua will be divided at *corresponding* points, so that the division of a continuous locomotion at every stage entails the division of the path of the motion at every point as well as the time of the motion at every now.[11] In *Generation and Corruption* i 2 Aristotle concedes to the atomists that if a

[9] Aristotle says 'we must not hold that when A came to be at B [or was at B], D was at the same time in motion from the extremity of F'. Rather, we would need to say either that A was at point B but did not come to be at point B, or that D paused while A was at rest at point B for the motions to correspond to one another in all respects (i.e., to have the same distance and same speed) except for the fact that A passes through point B.

[10] This is the contrapositive of the claim just invoked to justify the inference from the fact that a moving thing undergoes a continuous motion and uses a point along its path as the boundary of successive motions, to the fact that it must do so at every point along its path, i.e., the claim that if the antecedent of this conditional is satisfied, then using a point as the boundary of successive motions is the same as merely passing through it. Aristotle argues that since the consequent is false, then the antecedent is impossible. Thus, Simplicius claims that Aristotle 'brings up a puzzle that is directed against [the view that if A comes to be and departs from B, it must rest at B] and he solves the puzzle by making determinate what was assumed indeterminately in it' (*In Phys*. 1284,21-2) i.e., the false claim 'that what is at something also comes to be at it and departs <from it>' (*In Phys*. 1285,10), which I take to be tantamount to the claim that using a point as the boundary of successive motions is that same as merely passing through it. (Simplicius suggests that this is assumed for A but not for D, but Aristotle does not name a point on F-G that corresponds to point B that is only occupied 'at a cut'. Rather, this claim is left out of the account entirely, and I think this is the point of the objection, viz., that merely naming point B on path E-C could not plausibly slow down A in its transit of this line.)

[11] This will also follow from Aristotle's claim, in *Physics* iv 11, that 'time follows change' and that 'change follows magnitude'. See n8 above.

magnitude were divided everywhere, it would be divided into points. The rationale given is that if a magnitude were divided everywhere, then when the division is finished only indivisibles could be left since divisibles would contain places that are still undivided (316a23ff.). If we assume that Aristotle is making a similar dialectical concession in *Physics* vi 1 and viii 8, then the division of the locomotion at every stage, the path of the locomotion at every point, and the time of the locomotion at every now, will divide the locomotion into instantaneous motions (κινηµάτα, as Aristotle calls them), the path of the locomotion into points, and the time of the locomotion into nows. And since, as Aristotle says at *Physics* vi 1.231b10-11, each continuum 'is divisible into the parts of which it is composed', if a locomotion were divided at every stage, the locomotion itself would be composed of κινηµάτα, the path of the motion would be composed of points, and the time of the locomotion would be composed of nows.

This, I believe, gives us a new way to understand *reductio* (a) since, if the foregoing is true, then there is a fairly straightforward route to the disjunctive conclusion that either there is a first instant of having left point B, or A has come to be at point B and has left point B at the same time. And if it ultimately follows from the assumption (which Aristotle adopts for the purpose of refutation in *reductio* (a)) that A can come to be at and depart from point B without resting, that there is a first instant of having left point B, we need not suppose that Aristotle has forgotten his arguments in *Physics* vi 5 that this is impossible.

In *Physics* vi 1 Aristotle refutes the claim that a continuum can be composed of indivisibles by assuming that the components of a continuum can stand in only three relations, viz., being continuous with each other, being contiguous with each other, and being successive, and arguing that indivisibles can stand in none of these relations.[12] He invokes the definitions of *Physics* v 3 for these terms: two things are successive if and only if one follows the other in sequence and there is nothing of the same kind in between, two things are contiguous with each other if an only if they are successive and in contact (i.e., have limits that coincide), and two things are continuous with each other if and only if they are contiguous with each other and their limits that coincide are one. So if a continuum were composed of indivisibles, Aristotle argues, then these indivisibles would need to be either continuous with each other, contiguous with each other, or successive. A continuum cannot be composed of indivisibles that are continuous with each other because indivisibles have no limits distinct from any other part of them to form a unity (231a24-7). So if a continuum were composed of indivisibles, these indivisibles would need to be either successive or contiguous with each other. If a continuum were composed of indivisibles that are successive, then these indivisibles would follow one another in sequence with no additional indivisibles in between (231b6-10). This means that there would be a unique first indivisible

[12] In the opening sentence of *Physics* vi 1, Aristotle claims that if the definitions of 'continuous', 'contiguous', and 'successive' in *Physics* v 3 are correct, then it is impossible for a continuum to consist of indivisibles. This impossibility can only follow if the definitions in *Physics* v 3 exhaust the field of possibilities of the ways in which the constituents of a continuum can be related.

that is distinct from and after each indivisible in the continuum. Yet, if a continuum were composed of indivisibles that are contiguous with each other, then these indivisibles would need to be in contact. But since indivisibles can only be in contact whole to whole, then they would coincide and fail to be distinct (231a29-b6). Since contradictions result from both of these disjuncts, and the disjunction exhausts the remaining possibilities for the ways in which components of a continuum can be related, Aristotle concludes that a continuum cannot be composed of indivisibles.

We can derive analogous absurdities from the assumption that the path of A's motion is composed of points: According to the argument of *Physics* vi 1, if the path of A's motion were composed of points, then these points would need to be either contiguous with each other or successive. If the latter, then there would be a unique first point after point B that is distinct from point B at which it is first true to say that A has departed from point B. Since there is always a first instant of having reached any point, there will be two distinct instants such that one will be a unique first instant of having reached point B and the other will be a unique first instant of having reached the first point after point B, and, therefore, a unique first instant of having departed from point B. Yet, if the path of A's motion were composed of points that are contiguous with one another, then these points would need to be in contact. But since points can only be in contact whole to whole, then point B, and any point contiguous with it would coincide and fail to be distinct. If A departs from point B by moving to a contiguous point, since this point coincides with point B, A will both have come to be at point B and have departed from point B at the same time. Thus, if the path of A's motion were divided into points, either there is a unique first instant of having departed from point B, or A has come to be at point B and has departed from point B at the same time.

Our initial interpretation of *reductio* (a), then, should be revised so that Aristotle infers from the claim that (3) A is always coming to a stop when it is undergoing locomotion, the disjunction that either (4) A has come to be at point B and has departed from point B at the same time *or* (4.1) there is a first instant of having left B. And since (4) directly entails the contradiction that A is both at B and away from B at the same time, Aristotle can infer the truth of proposition (4.1) by disjunctive syllogism. But, of course, proposition (4.1) is not true, in Aristotle's view. It is, however, a valid consequence of proposition (2), which is the target of Aristotle's *reductio*, and it is a proposition that contradicts proposition (2) because it implies that A has rested at point B after all. The offending propositions (4) and (4.1) follow from proposition (2) because proposition (2) implies that what we choose to call 'one change' is entirely arbitrary, and this implies proposition (3), which, in turn, implies propositions (4) and (4.1). It is for this reason that the argument can be taken as an attack on the Zenonian assumption that what we choose to call 'one change' is entirely arbitrary.

A lot hinges, in this proof, on the assumption, implicit in Aristotle's refutation of the notion that a continuum can be composed of indivisibles in *Physics* vi 1, that if, *per impossibile*, a continuum were resolved into indivisibles, these indivisibles would need to be discretely ordered. This follows from the assumption that there are only three ways in which the components of a continuum can be related, each of which having the succession of the components built into their definitions. But why should the possibilities for the ways in which points composing a magnitude can be related be restricted to relations producing discrete orders? If there is a 'mistake' in *reductio* (a), this is where it lies: Aristotle is willing to grant, *per impossibile*, that a continuum may be divided into an infinitely numerous point set, but he refuses or fails to countenance the idea that this point set might be densely ordered. This is because, if every line is divisible into an infinite, dense sequence of points, and every sub-segment of a line is likewise divisible, then every point set constituting a line will have infinitely many subsets, each having infinite cardinalities. But, as Sorabji 1983, 212 and 323 points out,[13] the following passage from *Physics* iii 5 evidently argues that this is incoherent:

> It is plain, too, that the infinite cannot be an actual thing and a substance and principle. For any part of it that is taken will be infinite, if it has parts; for to be infinite and the infinite are the same, if it is a substance and not predicated of a subject. Hence it will be either indivisible or divisible into infinites. But the same thing cannot be many infinites. (Yet just as part of air is air, so a part of the infinite would be infinite, if it is supposed to be a substance and principle.) Therefore the infinite must be without parts and indivisible. But this cannot be true of what is infinite in fulfillment; for it must be a definite quantity. (204a20-29)

This highlights the fact, noted so often by commentators, that the distinction between Aristotle's concepts of continuous and discrete is not the same as ours. Aristotle's distinction is not a distinction between dense and discrete orders respectively. It is a contrast between relations, both of which produce discrete orders, but where the *relata* of one bears an additional metaphysical relation of having unified limits.

Reductio (b)

As I said at the outset, the advertised purpose of *Physics* viii 8 is to prove, by

[13] Bostock 1988, 261 has objected to Sorabji's interpretation of the passage just quoted, arguing that in it, Aristotle seeks to rule out only infinitely extended substances with infinitely extended parts, not infinite sets with infinite subsets. But even, so, there is evidence for a broad construal of the doctrine at *Physics* vi 7.238a9-11, since, there, he applies it to times. In this passage, the assumption that infinite times cannot have infinite times as subsets seems to be required in Aristotle's proof that a finite distance cannot be traversed in an infinite time. Aristotle's argument implies that if the whole of a distance is traversed in an infinite time, no proper part of this distance can be traversed in an infinite time. (Cf. White 1992, 63 and Ross 1923, 100, which argues that Aristotle takes division everywhere to resolve a continuum into a finite number of points. I claim that he takes division everywhere to resolve a continuum into an infinite number of points, each subset of which is finite.)

disjunctive syllogism, that only circular motion is infinitely continuous. Aristotle does this by first assuming that there must be some motion that is infinitely continuous, and that there are only three types of motion, viz., circular, rectilinear, and a mixture of rectilinear and circular (261b27-31). He then sets about eliminating rectilinear motion as a possibility as follows, assuming that mixed rectilinear and circular motion will be eliminated with it, because if either of the unmixed motions are not continuous, then neither will any motion composed of them: Presumably because of the finite size of the universe, if a given object G undergoes rectilinear locomotion, then the locomotion that it undergoes is finite (Simplicius argues for this as an implicit assumption at *In Phys*. 1278,15). So if G's locomotion is finite, sooner or later it must turn back in the direction from whence it came. But if G turns back, then G has come to be at a limit of its motion and has ceased to be at a limit of its motion, and if G has come to be at a limit of its motion and has ceased to be at a limit of its motion (alternatively, if G has used the limit both as the *terminus ad quem* of a prior motion and the *terminus a quo* of a subsequent motion), then it must come to rest at this limit. If G rests at a limit of its motion, however, then G's motion is not continuous. So if a given object G undergoes rectilinear locomotion, then its motion is not continuous. *Reductio* (b) effects an indirect proof of the key premise that if G has come to be at a limit of its motion and has ceased to be at a limit D of its motion, then it must come to rest:

> If G is undergoing locomotion towards D and then turns back and undergoes locomotion downwards again, it has used the extremity D as an end (b25) and a beginning—the one point as two. This is why it must have stopped. It has not come to be at D and departed from D at the same time, for in that case it would simultaneously be and not be there at the same instant. But in fact we must not apply the solution of the previous puzzle. We cannot say that G is at D at (b30) a cut of time, but that it has not come to be or departed <from D>. For it must reach an endpoint that exists actually, not potentially. Now although the points in the middle exist potentially, this one does so actually; it is an end when considered from below (263a1) and a beginning when considered from above. And therefore it is related in the same way to the motions. Therefore a thing that turns back on a straight line must stop. Therefore there cannot be continuous eternal motion on a straight line. (262b23-263a3)

Simplicius, *In Phys*. 1281,4ff. assumes that what is proved in this *reductio* is logically entailed by the proposition proved in *reductio* (a) because he takes the proposition proved in *reductio* (a) to be the following: For every point on the path of A's continuous locomotion, including its *terminus a quo* E and *terminus ad quem* C, if A has come to be at this point and has ceased to be at this point, then A rests at this point. A fairly obvious problem with this is that it makes *reductio* (b) otiose. If the proposition proved in *reductio* (a) directly entailed the proposition proved in *reductio* (b), then there would be no reason to undertake the latter *reductio*, because the proposition that it proves can be *directly* inferred from the proposition proved in *reductio* (a). Nothing in the text, however, requires Simplicius' reading, and, in fact, it clashes with the way I have interpreted *reductio* (a). In *reductio* (a), the absurd result that either there is a first instant of having left point B, or A has come to be at point B and has left point B at the same time followed from the division of A's motion at every point, and the division of A's motion at every point followed, in part, from the assumed homogeneity of A's motion, which in *reductio* (a), forced us to conclude that if A could come to be at and depart from point B without resting, it must do the same at every point along its path. But the heterogeneity of G's motion in the present case forces no such conclusion upon us. Since the movements to point D and from point D are motions of different species, we can adduce this feature of the change as a reason for treating point D as a kinetic terminus while denying this status to all of the other states that G passes through while approaching and receding from point D.

In the light of this, I will instead take *reductio* (a) to prove a slightly different proposition than what Simplicius suggests, viz., that for every point between, *but not including* points E and C on the path of A's continuous locomotion from point E to point C, if A has come to be at this point and has ceased to be at this point, then A rests at this point. On this view, the proposition proved in *reductio* (a) does not directly entail the proposition proved in *reductio* (b). Rather, *reductio* (b) produces the same absurd result as *reductio* (a) because, once it is admitted that G can arrive at and depart from the point of reversal D without stopping as envisaged in *reductio* (b), it must also be admitted that A can arrive at and depart from any other point without stopping as in *reductio* (a). In other words, once we allow that this arriving and departing without resting can occur at the end or beginning of a change, and given the premises of Zeno's dichotomy paradox, there is no reason to deny, without being entirely arbitrary, that it can occur in the course of a change as well. So though not explicitly stated, the same infinite fracturing of motion that we found in *reductio* (a), and which ultimately followed from the Zenonian assumption that what we choose to call 'one change' is entirely arbitrary, is assumed in *reductio* (b), and, here, it produces the same absurd results.

Thus, the structure of *reductio* (b) is closely analogous to the structure of *reductio* (a), except that whereas object A moves continuously from point E to point C along a spatial path containing the point B, object G moves continuously to *and from* point D along a spatial path *limited at one end* by point D. And whereas in *reductio* (a), Aristotle infers from the assumption (2) that A has come to be at point B and has departed from point B without resting to the claim that A is always coming to a stop when it is undergoing locomotion, in *reductio* (b) Aristotle infers from the assumption (2) that G has come to be at point D and has departed from point D without resting *first* that G can come to be at and depart from *any* point in its motion to and from D without resting, and *then* that (3) G is always coming to a stop when it is undergoing locomotion. The rest of the argument is exactly analogous.

Reductio (c)

Another indirect proof, in which the infinite fracturing of change is implicit, and which I also propose to read as an attack on Zeno's assumption that what we choose to call 'one change' is entirely arbitrary, is found in the following passage. I will call this '*reductio* (c)':

> If anything that is after previously not being, must come to be a thing that is, and is not when it is coming to be, then the time interval cannot be divided into indivisible times. For if D was becoming pale at A, but it has simultaneously come to be and is <pale> at another indivisible (b30) time, B, which is contiguous <with A>—if at A it was coming to be <pale>, it was not <pale>, and at B it is <pale>—then between <A and B> there must be some process of coming to be, and consequently (264a1) a time interval in which it was coming to be. The same argument does not hold for those who deny that there are indivisibles. Instead, it has come to be and is <pale> at the last point of the very time interval in which it was coming to be <pale>, and nothing is contiguous with or successive to this, whereas indivisible times are successive. (263b26-264a6)

This argument can be paraphrased as follows: If a time period A when D is undergoing a process of coming to be pale were divisible into indivisible nows,[14] then these nows would be successive.[15] If the nows in time period A were successive, then there would be a last now in period A (call it α_n), in which it is not yet pale, that is immediately succeeded by a first now B in which D is pale, B being the instant at which the process of coming to be pale is completed. If the process is completed at B, and D is not yet pale at α_n, then the process of coming to be pale must have extended beyond α_n to B, but if this is so, then there must have been a time interval between α_n and B, because, *ex hypothesi*, D was undergoing a process of coming to be pale, and every part of a process is temporally extended.[16] So α_n and B are not successive after all and the time period when a

[14] Simplicius vacillates between interpreting A as a period composed of indivisibles (1297,22-23) and an indivisible itself (1297,30). Philoponus, *ad loc*, recasts the problem, substituting indivisibles A and B for A, and indivisible C for B, so that what Aristotle calls A is composed of A and B. At any rate, Aristotle's claim that D was coming to be (ἐγίγνετο, imperfect tense, 263b28-29) pale in A, in the light of his doctrine that there is no motion at an instant, would seem to require A to be a time period.

[15] Aristotle talks indifferently, in this passage, of nows being contiguous or successive, but he seems to mean the technical sense of 'successive' throughout, since the puzzle assumes that 'contiguous' times A and B are distinct. If A and B were 'contiguous' in the technical sense, then the last now of A would not be distinct from B, and Aristotle's conclusions would not follow.

[16] Simplicius suggests a way of strengthening the argument by heading off various strategies of

process is occurring cannot be divided into indivisible nows. But if A is not composed of indivisibles, then there is a first instant in which D is pale but no last instant in which D is not pale, since the first moment of being pale limits the period of coming to be pale and of being not-pale. Q.E.D.

I claimed that *reductio* (a) argued from the assumption that A can come to be at point B and depart from point B without resting to the result that the path of A's locomotion is divided at every point, and, therefore, is divided into and composed of points. Since the time of a motion is co-divided with the magnitude, we can construct an analogous argument from the same assumption to the effect that the time of A's locomotion is divided at every now, and, therefore, is divided into and composed of nows. In *reductio* (c), Aristotle leaves it to the reader to construct this analogous argument about D, and proceeds directly to demonstrate the absurdity of the notion that the time of a motion can be composed of nows. If the time of a motion were composed of indivisible nows, Aristotle argues, these nows would need to be successive, but if this were so, an absurdity would result. Unlike *reductio* (a), however, in which the absurdity involved the existence of a *first* instant away from a given state in a continuous change, in this argument the absurdity involves the existence of a *last* instant away from a *terminus ad quem*.

Another feature of this argument that distinguishes it from *reductios* (a) and (b) is that the change it describes is cast in terms of contradictories (τὰ κατὰ ἀντίφασιν) rather than contraries. We know that contradictories like pale and not-pale, and being and not-being (*Phys*. vi 10.241a29-30, *Meta*. 1008a8-9) admit no *tertium quid* (*Phys*. v 3.227a9-10, *Meta*. 1011b23-24, 1057a34, 1069a3-4), and it seems natural to suppose that, for this reason, a change between pale and not-pale, and indeed any change between contradictories, will be instantaneous. Aristotle's usual method of describing instantaneous change is to say that something has come to be, or has come to be F without ever being in the process of coming to be, or coming to be F.[17] But *reductio* (c) explicitly envisages D *coming to be* (ἐγίγνετο, imperfect tense) pale over a period of time[18] (263b26-27,

escaping Aristotle's conclusion that there must be a time interval between α_n and B: Instead of claiming that α_n is succeeded by a first now B in which D is pale, B being the instant at which the process of coming to be pale is completed, according to Simplicius, Aristotle should have said that α_n is succeeded by a now B in which D is *already* pale (*In Phys*. 1297,26-27; 1297,39-1298,3). Since, *ex hypothesi*, D is undergoing a process of coming to be pale, then that process must be completed at B or at some earlier time. If the process is completed at B, then Aristotle's result follows. But with Simplicius' formulation, one could refute the suggestion that the process was completed at some earlier time than B. If someone claims that the process was completed at the limit of α_n, then Aristotle could reply that this is impossible because α_n has no limit. One could also refute the claim that the process was completed at an earlier now than B (call it β_{n-1}), and that it was *this* now that was contiguous with α_n, on the grounds that D would be pale at β_{n-1} and not pale at α_n, and the same problem would arise again.

[17] Individual substantial forms are able to have come to be without ever coming to be, and the same is true of surfaces, lines, contacts, changes, and states of the soul, as well as relations and partless things like points. See, e.g., *Phys*. vii 3.247b1-248a9, viii 6.258b17-18, *Cael*. 280b6-7, b15-16, b21-23, b26-27, *Meta*. 1002a32-34, 1027a29-30, 1039b26, 1043b15-16, 1044b21, 28, 1088a34-35.

[18] Thus, as Simplicius points out, the argument does not apply to things that are and are not with-

264a2-5), and adds that D is not-pale while it is coming to be pale (263b30). Now if D is coming to be pale *throughout* A (i.e., D does not rest in any part of A), is not-pale throughout A, and there is no *tertium quid* between being pale and being not-pale, it is not immediately obvious what progression of states D is passing through while in period A to justify saying that it was coming to be pale throughout this time. That there must be such a progression of states follows from the assumption that D is becoming pale throughout period A, so that it contains no periods of rest, and from Aristotle's definition of rest such that 'a thing is at rest if its condition in whole and in part is uniform now and before' (*Phys.* vi 3.234b5-7).[19] Moreover, since D is not at rest at any time during A, since Aristotle defines rest as he does, and since *reductio* (c) proves that the nows within A must be densely ordered rather than discretely ordered,[20] then the progression of states D is passing through during A must also be densely ordered, otherwise, A will contain a period of rest.[21]

There are, I think, three possibilities for extricating Aristotle from this difficulty: (1) The first is to suppose that while D is not-pale but coming to be pale, some process is going on that, while not identical to the alteration, is causally related to it in a way that would allow us to say D is coming to be pale because it is going on. (2) The second is to suppose while D is not-pale but coming to be pale, it is coming to be paler by degrees, passing through a dense sequence of degrees of pallor.[22] On this interpretation, 'not-pale' is just a name for any degree of pallor that is not maximally pale. (3) The third option is to suppose that while D is not-pale but coming to be pale, it is coming to be pale piecemeal. On this interpretation, 'not-pale' is just a name for any state in which some portion of D is not pale.[23]

Both Sorabji and Broadie choose option (1), but while Broadie tentatively sug-

out coming to be or passing away (*In Phys.* 1297,20).

[19] In other words, the mere passage of time cannot count as a 'coming to be'.

[20] By 'densely ordered' I mean, here, what White 1992, 22 calls 'distributive density': Indivisibles, in the sense of actual divisions in a continuum, are densely ordered in this way if and only if for any two actual divisions that one makes, it is always possible to make another actual division between these two actual divisions in the continuum.

[21] We can deduce this consequence as follows: If the instants in period A are densely ordered and the series of states that D passes through are not, then between every two instants *u* and *w* there is another instant *v*, but it is not the case that between every two incompatible states *x* and *z*, there is another state *y* that is incompatible with both *x* and *z*, and, therefore, it is the case that there are at least two incompatible states *x* and *z* that D passes through, between which there is no state *y*. Since states *x* and *y* are incompatible, they must hold at different times *u* and *w*, but if this is the case, there is some time *v* between *u* and *w* at which D must either be in state *x* or state *z*. If D is in state *x*, then D was at rest between instant *u* and instant *v*. If D is in state *z*, then D was at rest between instant *v* and instant *w*. Hence, if the instants in period of A are densely ordered and the series of states that D passes through are not, then A will contain a period of rest.

[22] See, e.g., *Cat.* 10b26-30: 'Qualifications admit of a more and a less; for one thing is called more pale or less pale than another, and more just than another. Moreover, it itself sustains increase (for what is pale can still become paler)—not in all cases though, but in most.' Cf. *Phys.* v 2.226b1-9.

[23] This option is discussed at *Phys.* v 6.230b32-231a1, vi 4.234b10-20; vi 9.240a16-29; vi 10.240b21-31.

gests that the causally related process may be something like an external agent preparing to paint D, Sorabji claims to find an example of such a process in *De sensu* 6. Recognizing that this process needs to be continuous, Sorabji 1976, 172n23 focuses on how this might be the case while resulting in a discontinuous switch to complete pallor from some shade of pallor that is discriminably different from complete pallor. The answer, he claims, is Aristotle's concept of non-intrinsic continuity (τὸ δὲ μὴ καθ' αὑτὸ συνεχὲς) mentioned at 445b28. A change in color, or in this case, pallor, from one discriminable shade to the next in a discontinuous sequence of discriminable shades, may be non-intrinsically continuous, claims Sorabji 1983, 411, because it is 'produced by a continuous change in the proportions of earth, air, fire, and water in a body'. The idea, presumably, is that if D is, for instance, a bucket of paint that is one discriminable shade away from being completely pale, by continuously varying the mixture of pigments, a painter can cause the paint to alter to the next discriminable degree of pallor, viz., complete pallor. Variations in pallor less than this increment 'are not perceptible except by being part of the whole variation (ὅτι ἐν τῷ ὅλῳ), by which Aristotle probably means that they only *contribute* to the whole variation' (446a18).

The main problem with this approach, as I see it, is that it makes *reductio* (c) an argument about perceptible alteration rather than about alteration as such, and Aristotle gives no indication that he intends such a restriction. In any event, interpretative charity should prevent us from supposing that Aristotle believes, in general, that perceptible change exhausts the field of change. We cannot, for instance, infer from the fact that we cannot see the length of a shadow changing over the course of a minute, that the length of the shadow is not changing during that minute.[24] Nor is there textual warrant to attribute to Aristotle the more restricted assumption that perceptible alteration exhausts the field of alteration. Certainly, colors like shades of pallor are defined by Aristotle as 'perceptibles', but it follows from nothing Aristotle says that in order for a shade of pallor to be perceptible, the difference between it and every other shade of pallor must also be perceptible, which is what would be required for every change of pallor to be a perceptible change of pallor. Part of Sorabji's motivation, here, is to accommodate the fact that at 445b21-22, Aristotle says that 'τὰ εἴδη of color, taste, sound, and other perceptibles are limited', which he takes to mean that the set of actually *discriminable* shades of color is limited.[25] But this does nothing to rule out the

[24] Moreover, Aristotle's claim in *Physics* vi 6 that whatever has changed was changing earlier, and whatever was changing earlier, had changed before that, *ad infinitum*, effectively commits him to the existence of changes that are so small that they cannot actually be seen, even in principle.

[25] Many commentators will want to rule out option (2) because they take 445b21-22 to amount to the stronger claim that the set of *perceptible* shades of color is limited. Against this view, Murphy 2008, 194n18 has pointed out that τὰ τῶν χρωμάτων εἴδη are said to be only seven in number at *De sensu* 442a19-25. Since this is an implausibly small number to be the perceptible shades of color or even the discriminable shades of color, he concludes that by εἶδος, Aristotle must mean a species of color under which many shades fall. Still, many commentators will, no doubt, adduce *Physics* vi 5.236a36-b18 as independent evidence that the path of a qualitative change is not infinitely divisible.

possibility that while our painter is continuously varying the mixture of pigments in D, and before D alters to the next actually discriminable shade, D is passing through an infinite series of *perceptible* shades of pallor that differ by degrees that are only *potentially* perceptible. This, however, is perfectly consistent with option (2), and option (2) does not restrict the scope of *reductio* (c) to actually perceptible alteration.

The notion that there are very small potentially perceptible differences in magnitude and quality, and, therefore, very small potentially perceptible changes between both actually perceptible magnitudes and actually perceptible qualities, can be found in *De sensu* 6: Aristotle says that though a ten-thousand part in a grain of millet is too small to see by itself, it is nonetheless perceptible potentially, and indirectly (μὴ καθ' αὑτό) because the size of the whole grain of millet that includes it is actually perceptible, otherwise we would have a perceptible thing (the grain of millet) composed of imperceptibles, which is absurd. It follows from this that if a millet seed grew by a ten-thousandth part, the change in size would be potentially perceptible, but not actually perceptible because the increment of change would be potentially perceptible. Aristotle gives every indication that an analogous account applies to alteration, though his example, here, is sound instead of color.[26] Aristotle argues that though the sound in a quarter-tone within a whole strain is too small to hear by itself, that sound is nonetheless

But what this text actually says is that 'that into which' something alters, like pallor, is indivisible, and that 'only in qualitative change is there anything indivisible in its own right'. Murphy argues convincingly, that 'that into which' something alters is the *terminus ad quem* of an alteration, and not the path, but even if this were not conceded, the concluding sentence of this passage says only that only qualitative changes have something indivisible in its own right, not that *all and* only qualitative changes do. So the orthodoxy that Aristotle does not countenance continuous qualitative change seems not to be justified by the passages usually adduced in its favor. Murphy, moreover, finds a passage in *De sensu* 3 that countenances an example very much like the one I propose for option (2), in which a painter causes D to pass through an infinite series of perceptible shades of pallor by continuously varying the mixture of pigments in D: 'Colors will thus, too be many in number on account of the fact that the ingredients may be combined with one another in a multitude of ratios' (440b18-20). Murphy points out that not only does Aristotle make the color of paint a function of the proportion of the colors mixed, here, but he makes the *explananda* the many shades of color and the *explanans* the many ratios of mixing. 'If the same shade could be produced by many ratios', argues Murphy 2008, 192-193, 'then that there are many ratios need not entail that there are many shades. Thus, since there are infinitely many (and indeed a dense ordering of) ratios of the form *n*:*m*, we should expect that there is also a dense ordering of shades of color'.

[26] E.g., after the initial passage from 445b4-20 that raises questions about the perceptibility of very small magnitudes Aristotle says: 'The solution of these questions will bring with it also the answer to the question why the εἴδη of color, taste, sound, and other αἰσθητῶν are limited' (445b20-22). Then he introduces the distinction between the potentially and actually perceptible and gives two examples that are clearly meant to be analogous, one involving magnitude and one involving quality: 'It is owing to this difference [i.e., the difference between potential and actual perceptibility] that we do not see its ten-thousandth part in a grain of millet, although sight has embraced the whole grain within its scope; *and it is owing to this, too*, that the sound contained in a quarter-tone escapes notice, and yet one hears the whole strain, inasmuch as it is a continuum; but the interval between the extreme sounds escapes the ear' (445b31-446a4). White 1992, 128-130 also points out that Aristotle proposes an analogy between magnitude and quality in this passage rather than a contrast.

audible potentially, and indirectly (μὴ καθ' αὑτό) because the whole strain that includes it is actually audible, otherwise we would have an audible thing (the whole strain) composed of things that are not audible, which is absurd. It follows from this that if the pitch of a sound increased by a quarter-tone, the change in pitch would be potentially audible, but not actually audible because the increment of change would be potentially audible. Aristotle even says that the whole strain (not the lyre string, as in Sorabji's version), like the magnitude, is continuous (446a2), so that the point is to assimilate qualitative change to quantitative change with respect to continuity, not to contrast them.

This certainly seems to leave the door open for continuous alteration, and given the absence of any compelling reason to rule it out, and the fact that Sorabji's version of option (1) unduly restricts the scope of *reductio* (c), option (2) seems to be preferable to option (1).[27] But we have yet to consider option (3), which supposes that while D is becoming pale from being not-pale, it is becoming pale piecemeal, and is doing so continuously because its subject is infinitely divisible. Sorabji rejects option (3) because Aristotle says that D is not-pale during period A (263b30), and he thinks, presumably, that this is inconsistent with D being partially pale. Sorabji takes this view, I assume, because if Aristotle had meant that D was only partially pale in A, claiming that D was not-pale in A would be a misleading way of expressing it. The same objection would appear to apply to option (2), which assumes that D was becoming paler by degrees rather than by parts throughout A. But this consideration alone is not persuasive enough to prefer option (1) to either options (2) or (3), given the fact that *De sensu* 6 seems to support option (2) instead of option (1), and since there are many texts that support option (3) (see Sorabji 1976, 172n23). It seems reasonable, then, to suppose that Aristotle could have meant, in *reductio* (c), *either* that D is coming to be completely pale by degrees throughout A as in option (2) *or* is coming to be completely pale piecemeal throughout A as in option (3), and that when he describes this as 'coming to be pale from being not-pale', he is choosing an abstract way to describe one or another of these continuous changes.[28]

[27] A final objection to option (2), due to Broadie 1982, 157, is easily dispensed with: Noting that, in the immediately preceding section (263b9-26), Aristotle talks of D changing from pale to not-pale as well as from not-pale to pale, Broadie objects that D's change from pale to not-pale 'cannot consist in a graduated passage through shades between white and some final color. For the object cannot be white and an intermediate color at the same time, whereas it is white and in process at the same time. As soon as it comes to some shade S, however "close", that we are willing to regard as distinct from white, then at that moment we are bound to say that the process to not-white is over'. But when Aristotle says that the pale of D is perishing (imperfect) through the whole of A (ἐφθείρετο <τὸ> λευκὸν ἐν τῷ A παντί, 263b22), there is no reason why we cannot imagine this as a process of coming to be less and less pale since we know from *Categories* 10b26-30 that pallor comes in degrees. And something less pale is, nonetheless pale. As long as the transition to not-pale is preceded by a *period* in which D is pale but becoming not-pale, D's change can be recast as a continuous change.

[28] There is, moreover, independent evidence that contradictory terms like 'not-pale' are supposed to have precisely this abstractive function for Aristotle. In *Prior Analytics* i 46, Aristotle says that each contradictory term 'not-F' (οὐ F) has an instance of a privation that can also be denoted by 'un-F' (ἀ-F) underlying it (51b25-28). Since privations come in degrees, the function of the term

Reductio (d)

The fourth indirect proof that I will analyze, which I will call '*reductio* (d)', is found at 264b1-6. Like *reductio* (c), it is also cast in terms of the contradictories 'pale' and 'not-pale':

> Further, the following argument is more proper <to the subject> than the previous ones. The not-pale has perished and has become pale at the same time. Therefore, if the alteration to pale and from pale is continuous and does not remain stationary for any time interval, (b5) the not-pale has perished and has become pale and has become not pale at the same time. For the time of the three will be the same. (264b1-6)

This passage is particularly compressed and requires significant filling out under any interpretation. But it is clear, at any rate, that the argument is meant to reproduce the results of *reductio* (b) in the case of an alteration: Just as in *reductio* (b) we have a reversal in the direction of a change without a pause, and just as in *reductio* (b), the absurdity results that a thing has arrived at and has departed from a particular state at the same time. I will argue that, given this similarity in structure, it is not only reasonable but also fruitful to fill this argument out by assuming that the same infinite fracturing of change found in *reductio* (a) and implied in *reductio* (b) is also implied here, and that this infinite fracturing of change ultimately stems from Zeno's assumption that what we choose to call 'one change' is entirely arbitrary.

At first sight, though, one might think that the most natural way to read this passage is to assume that it turns on the discontinuity of contradictory change, and that no infinite fracturing of a continuous change is involved. Bostock in Bostock and Waterfield 1996, 296, for instance, claims that Aristotle's implied conclusion that D—'D' is the name given to the moving thing in the immediately preceding discussion (263b16)—rests for a time when it changes direction follows only if pale and not-pale are strict contradictories and that the change between them is discontinuous, otherwise, instead of resting, D might be moving through a *tertium quid* between the time when it has come to be pale from being not-pale, and then has come to be not-pale again (i.e., the time period that must exist due to the impossibility of D having come to be pale and having come to be not-pale at the same time). But then Bostock complains that the argument, being

'not-F' seems to be to refer indiscriminately to a range of instances of 'un-F', as Aristotle implies at *Physics* i 4.188b3-6: 'Pale (λευκόν) comes from not-pale (οὐ λευκοῦ)—and not from any not-pale (ἐκ παντός), but from dark or some intermediate (ἐκ μέλανος ἢ τῶν μεταξύ).' Aristotle tells us elsewhere that the other colors are μεταξύ dark and pale in the sense of being 'mixtures' of darkness and pallor ('Crimson, violet, leek-green, and deep blue, come between pale and black, and from these all others are derived by mixture', *De sensu* 442a23-25, cf. *Cat*. 12a17-19 and *Meta*. 1053b31). Since dark is the privation of pale (*Phys*. iii 1. 201a5-6, *Sens*. 442a26), this 'mixture' of darkness with pallor can only mean the deprivation of pallor. Since pallor comes in degrees (*Cat*. 10b26-30), the idea seems to be that the visible spectrum reduces to a range of privations of pallor, with dark, as the complete privation of pallor at one end, complete pallor at the other, and the other colors arrayed in between as partial privations.

only about instantaneous change, ends up being more limited in its application than *reductio* (b), and this is a problem, presumably, because Aristotle claimed earlier, at 264a8, that the arguments from then on were supposed to be more λογικός and general than the arguments that went before. Bostock also complains that the argument is, in any event, undermined by Aristotle's arguments in *Physics* vi 6 that all genuine changes are continuous, including even changes between contradictories, since these arguments imply that every discontinuous change between contradictories can be redescribed as a continuous change between contraries (cf. Bostock 1995, 199, and Murphy 2008, 215). Obviously, if the changes from not-pale to pale and then back to not-pale, as Bostock interprets them, can be redescribed as continuous changes between contraries, then Aristotle's desired conclusion does not follow on the way Bostock interprets the argument.

Now Bostock, it seems, must take the phrase 'and does not remain stationary for any time interval' at 264b4 to be epexegetical in the sentence 'the alteration to pale and from pale is continuous and does not remain stationary for any time interval', in which case, 'continuous' just means that there is no period of rest between when D comes to be pale from being not-pale and then comes to be not pale again. But this phrase can also be taken as a genuine conjunction, so that the medium in which the alteration takes place is supposed to be continuous. If we consider Aristotle's arguments in *Physics* vi 6 that all such alterations are continuous, and the fact that we have just seen, only 27 lines earlier, a contradictory change that probably *is* supposed to be redescribable as a continuous change between contraries, then it seems at least textually plausible to take Aristotle's argument in this way. Moreover, if we take Aristotle's argument in this way, then it can be seen as generating the same absurdities associated with the infinite fracturing of change that arose in *reductios* (a) and (b) from Zeno's assumption that what we call 'one change' is entirely arbitrary.[29] A benefit of reading *reductio* (d)

[29] The way that *reductio* (d) generates these absurdities, under the suggested interpretation, is as follows: If, as in options (2) and (3) of interpreting *reductio* (c), we assume that when Aristotle says 'pale' he means 'completely pale' and that when he says 'not-pale' he means 'not completely pale' either in the sense of possessing some definite but unspecified degree of pallor that is not maximally pale (option (2)) or having some definite but unspecified proportion of its surface that is not pale (option (3)), *reductio* (d) can be recast as follows: Assume that (1) an object D undergoes a continuous alteration to being completely pale from being not completely pale, and then to being not completely pale from being completely pale. Assume, for *reductio ad absurdum*, that (2) D has come to be completely pale and has ceased to be completely pale, but does not rest while being completely pale. From this, infer that D can come to be at and cease to be at any of the stages of its continuous alteration to and from being completely pale without resting. From this, infer that (3) D is always coming to a stop when it is undergoing alteration and from this infer that either (4) D has come to be completely pale and not completely pale at the same time or (4.1) there is a first instant of having ceased to be completely pale. From the impossibility of (4) D having come to be completely pale and not completely pale at the same time infer by disjunctive syllogism that (4.1) there is a first instant of having ceased to be completely pale, and infer directly [i.e., from the impossibility of (4)] that (5) D has come to be completely pale and has ceased to be completely pale at different times. From this, infer that (6) there is a time interval between when D has come to be completely pale and when D has

in this way is that it brings out an assumption that was merely in the background of *reductios* (a) and (b), viz., that the infinite fracturing of change envisaged there makes the departures as well as the arrivals from the intermediate points changes in their own right. At *Physics* vi 10.240b34-241a1 Aristotle claims that a motion consisting of κινημάτα is a motion consisting of arrivals. But if a locomotion is composed of κινημάτα and the path of the locomotion is composed of discretely ordered points, then it is also composed of departures, because the departure from any given point will be identical to the arrival at the 'next' point, even if, absurdly, the 'next' point is not distinct from the one it follows. So under one description, a motion consisting of κινημάτα is a motion consisting of arrivals, but under another description, it is a motion consisting of departures.[30] In *reductio* (d), this fact is brought to the fore by the abstractive function of the term 'not-pale'. Whereas in *reductio* (b), G was said to have both departed from D and undergone motion in a contrary direction, here, our object is merely said to have become not completely pale from being completely pale, which would appear to cast the departure from being completely pale as a motion with its own *terminus ad quem*. Under normal circumstances, we would ignore this implication because we assume that the real *terminus ad quem* cannot be simply a departure from being completely pale, but some definite degree of darkness. But if the motion is indeed composed of κινημάτα, then the implication is exactly right: The departure from being completely pale simply *is* the motion to the first state of being not completely pale, and this departure is a distinct motion with its own *terminus ad quem*.

Thus, the fact that D's departure from being completely pale is a change in its own right ultimately follows from the infinite fracturing of change implicit in granting the possibility of treating any stage of a continuous change as a terminus. But if D's departure from being completely pale is a change in its own right, then it follows directly from this that there is a first instant of being not completely pale. At *Physics* vi 5.235b30-236a7, Aristotle argues for the claim that there is always a first instant in the *terminus ad quem* of a change, and at *Physics* viii 8.263b9-26, for the corollary that the instant dividing a period of coming to be F from the period of being F that follows it must be allocated to the period of being F.[31] Or rather, as Bostock 1995, 196 points out, at *Physics* vi 5.235b30-236a7 Aristotle argues that 'if there is a first time in which a thing has changed, then that time must be an instant'. Bostock characterizes the antecedent, here, as

ceased to be completely pale. From this, infer that (7) D rests while being completely pale, which contradicts the initial assumption (2) that D does not rest while being completely pale. So infer the negation of the initial assumption [i.e., (2)], viz., (8) if D has come to be completely pale and has ceased to be completely pale, it rests while being completely pale. Q.E.D.

[30] Cf. Broadie 1983, 134 and 143 that claims that Zeno argued for a reductive account of motion similar to the modern at-at conception of motion insofar as Zeno and the proponents of the at-at conception both reduce motion to a set of instantaneous parts.

[31] Sorabji 1976, 172 and Broadie 1982, 155ff. point out that what is envisaged by Aristotle at 263b15-26 is a temporally extended process of coming to be not-pale. Sorabji cites 263b21-22, where Aristotle says εἰ ἐγίγνετο οὐ λευκὸν καὶ ἐφθείρετο <τὸ> λευκὸν ἐν τῷ Α παντί.

a perfectly reasonable, but by no means mandatory assumption if change is thought to proceed from some *terminus a quo* to some *terminus ad quem* in a period of time.

Broadie 1983, 137-139, however, argues that the antecedent is mandatory, and, indeed, follows from Aristotle's definition of change in *Physics* iii 1. Since Aristotle defines change in terms of a single potentiality for a single as-yet non-actual actuality, as an incomplete actuality to realize some terminus, and as something that has the 'directional form' of an 'F-wardness' throughout, then the logic of 'completion' requires the end of the change, when the terminus is reached, when the moving thing 'has become F' and 'is F', to be punctual, and the first such time at which the moving thing is F.[32] This is confirmed by Aristotle's claim at *Physics* vi 5.236a7ff. (cf. 241b2ff.), that a first instant of being in the *terminus ad quem* 'exists and is real' because 'it is possible for change to be completed and there is an end to change'.

Reductio (a) Again

With this insight in hand, we can now shed new light on *reductio* (a). Aristotle says in *Physics* vi 6.237a18-19 that whatever has changed was changing earlier, and whatever was changing earlier, had changed before that, *ad infinitum*, so that every changing object has completed an infinite number of changes. Aristotle says at *Physics* vi 5.235b6ff. that either changing and departing and having moved and having departed are identical or the latter of each pair 'follows' the former. So every period of moving is also a period of departing and every instant of having moved is also an instant of having departed. From this we can deduce that whatever has departed was departing earlier, and whatever was departing earlier, had departed before that, *ad infinitum*, so that every departing object has completed an infinite number of departures. Aristotle also says, in his response to Zeno at *Physics* viii 8.263b6-9, that it is possible to have traversed an infinite number of half-distances accidentally,[33] since it is an accidental property of a magnitude to be an infinite number of half-distances.[34] Hence, an object A that has moved to point C from point B, as in *reductio* (a), has completed an infinite number of departures from point B accidentally. So if it can be assumed (as Aristotle clearly wishes to assume) that A is moving to point C and from point B throughout its transit from point B to point C, then each of these instances of having departed point B (i.e., having ceased to be at point B and having come to be

[32] Cf. Broadie 1983, 155 that argues that coming to be is logically over at the moment at which it has become and, therefore, is what it was coming to be.

[33] There is controversy whether this is a potential or actual infinity. Sorabji 1987, 170 thinks that this is an unwelcome implication for Aristotle. Alexander thinks it is a potential infinity and that the word 'potential' must be supplied (*ad loc apud Simplicium*, *In Phys*. 1291,35ff.). Simplicius suggests that, instead of carelessly omitting the word 'potential', Aristotle is using a new sense of the word 'accidental', which is equivalent to 'potential' (*In Phys*. 1292,35).

[34] Analogously, Simplicius argues at *In Phys*. 1282,30-1283,5 that a moving thing is incidentally at all of the intermediate points because being F at an instant is incidental to being F over a period.

not at point B) are accidental consequences of having moved to point C. But if one denies the existence of a unified motion from point B to point C, there is nothing of which any of these instances of having departed from point B are an accidental consequence. So if, A departs from point B, as it surely must if it continues to move, then its departure must be a motion in its own right, and if the departure from point B is a motion in its own right, it must conform to the definition of motion in *Physics* iii 1-2.

In summary, a local motion has the accidental description of being a sequence of arrivals and departures.[35] *Physics* v 2 defines an accidental change as a change that is dependent upon another change for its existence, e.g., relational change and change in action and passion are accidental because they depend upon the occurrence of a change with respect to some other category. On this account, departing from the *terminus a quo* is an accidental change because it depends for its existence on the existence of some temporally extended change from some *terminus a quo* to some *terminus ad quem*. The absurdities in *reductios* (a)-(d) arise from treating this as an essential description. Who would treat a sequence of arrivals and departures as an essential description of local motion? Anyone who assumes that what we count as 'one change' when we are talking about a temporally extended occurrence is entirely arbitrary, viz., Zeno. This, finally, brings us to *reductio* (e) and Aristotle's only explicit discussion, in *Physics* viii 8, of criteria of unity for changes.

Reductio (e)

Reductio (e) is at 264a9-21, and it eliminates, like *reductio* (b), the possibility that there could be an infinitely continuous back and forth rectilinear motion, and, therefore, serves the same purpose as *reductio* (b) in the overarching proof of the chapter, by disjunctive syllogism, that only circular motion can be infinitely continuous. What is significant about *reductio* (e), though, is that it does this by an entirely different line of argument that involves no infinite fracturing of motion, and, therefore, seems to have nothing to do with Zeno's dichotomy paradox, and it invokes criteria for the unity of change, which is something that *reductios* (a)-(d) do not do.

Although *reductio* (e) itself is found about two thirds of the way through chapter 8, we find Aristotle setting the stage for it early on, at 261b31-262a12. Immediately after he sets up the disjunctive syllogism at the beginning of chapter 8 (261b27-31), Aristotle argues as follows: Presumably because of the finite size of the universe, if a given object A undergoes rectilinear locomotion, then the locomotion that it undergoes is finite. If A's locomotion is finite, sooner or later it must turn back in the direction from whence it came. But if A turns back, then A undergoes motions that are contrary, and therefore diverse in species. But, Aris-

[35] Cf. Broadie 1983, 143, where she claims that, according to Aristotle, motion over a period cannot be reduced to a series of arrivals and departures because 'arrival' cannot be understood without reference to a temporally extended motion.

totle points out, he has previously established that motions that are diverse in species cannot be one. So A's motion is not one. But then, instead of drawing the conclusion that since A's motion is not one, it cannot be continuous, Aristotle offers a consideration why A's motions to and from a point of reversal must be contrary, viz., they would arrest each other if they were simultaneous. Then he breaks off this line of argument entirely and proceeds to set up *reductio* (a), not returning to *reductio* (e) until much later in the chapter, when he writes:

> Everything that is in motion (a10) continuously and arrives at a certain thing in its locomotion, if it is not knocked out of its way by anything, was also undergoing locomotion to that thing before. For example, if it arrived at B, it was also undergoing locomotion to B—not <only> when it was near, but right from when it started its motion. For why now rather than earlier? And likewise for the other <kinds of motion>. So, then, when a thing that (a15) is in locomotion from A reaches C, it will come back to A, being in motion continuously. Therefore, when it is in locomotion from A towards C, it is then also in locomotion to A in respect of its motion from C, and so <it is undergoing> contrary <locomotions> at the same time. For the rectilinear <locomotions> are contrary. And at the same time it is changing from something in which it is not. Therefore, if this is impossible it must (a20) come to a stop at C. Therefore the motion is not single, for motion that is interrupted by a stationary state is not single. (264a9-21)

Reductio (e), in effect, finishes the line of argument just outlined by establishing that A's motion, since it is diverse in species, is not continuous. The argument may be summarized as follows: Suppose some motion is diverse in species, yet continuous, e.g., suppose A moves continuously to point C and turns back without resting. In this case, the limit of the motion to point C is identical to the limit of the motion from point C, so the two changes are continuous and, therefore, one. But a change has its goal from its inception, so one change will at once be directed to point C and from point C, which is impossible. So a motion cannot be diverse in species, and at the same time, continuous. So if A undergoes a motion that is diverse in species, then A's motion is not continuous.

We can discern, in the foregoing, two criteria for the unity of change at work, both of which can be found in *Physics* v 4. In the passage where Aristotle is setting the stage for *reductio* (e) (261b31-262a12), we have the familiar criteria of *Physics* v 4, viz., an occurrence that is 'one change' must be in a continuous time, have a single subject throughout, and proceed throughout from a terminus of the same species to a contrary terminus of the same species (cf. v 4.227b3-228a19). In *reductio* (e) itself, however, another criterion of unity is implicit that is also found in *Physics* v 4, viz., an occurrence is 'one change' if and only if it is continuous (τήν τε ἁπλῶς μίαν ἀνάγκη καὶ συνεχῆ εἶναι, εἴπερ πᾶσα διαιρετή, καὶ εἰ συνεχής, μίαν, 228a20-22). At 228a23-24, Aristotle says that two change parts are continuous with each other if and only if their limits are one. From this we can infer that a change is continuous if and only if the limits of any two parts into which it is divisible are one (Bostock 1995, 183 makes this inference), and we must assume this is the case if and only if the change is in a continuous time and in a continuous medium, and is undergone by a single subject throughout. Now these criteria overlap to the extent that they both require a continuous time and the same subject throughout, but they differ insofar as one of them requires uniformity of direction throughout the change, and the other requires a continuous medium. I will call the former a teleological criterion of unity, in that it requires that the change have one direction throughout.[36] I will call the latter a topological criterion of unity because it requires that the change have no gaps in it. What *reductio* (e) does, in effect, is to prove that these teleological and topological criteria of unity for motions, when deployed together, make it impossible for reversed rectilinear motion to be continuous. If the limits of two contrary motions are 'one', and the motions are, therefore, 'one' according to the topological criterion, this will create a motion that fails the teleological criterion by being a motion that, absurdly, tends in two directions at once.

Conclusion

Of course, none of this should impress Zeno, or anyone else who refuses to accept Aristotle's definition of 'one change'. And as Michael White, points out, none of this prevents 'one change' from having parts that are also 'one change' where the parts are all of the same species. According to White, this is the function of *reductio* (a), which achieves this by implying that, in addition to the teleological and topological criteria just mentioned, an occurrence that is 'one change' must be bracketed by periods of rest and contain no periods of rest. While it can, of course, be claimed that the result of *reductio* (a) is a new condition for a change being 'one change', *reductio* (a), as well as *reductios* (b)-(d) can equally, and perhaps more plausibly be seen as refutations of the assumption, implicit in Zeno's dichotomy paradox, that what one chooses to call 'one change' is entirely arbitrary. I say 'more plausibly' because the textual evidence for attributing the former intention to Aristotle in *Physics* viii 8 is weak. The only mention of criteria of unity in *Physics* viii 8 occurs 12 lines before *reductio* (a), but, as we just saw, the criteria, there, are invoked within an argument that has nothing to do with the infinite fracturing of change. And indeed, the criteria stated in these lines could not rule out 'one change' having parts that are also 'one change' because they are the weaker criteria of *Physics* v 4. Now White's position would presumably be that *reductio* (a) is adding a new condition to the list just cited from *Physics* v 4, but if this were what Aristotle took himself to be doing here, it

[36] At *Physics* viii 8.261b34-262a6 Aristotle gives an example of what he means by 'one in species': Since contrary motions, such as upward and downward locomotion are motions of different species (262a6), if a motion consisted of parts that are contrary motions, it would fail to be unified. So in the case of locomotion, every part of a motion must have the same direction, all are upward, or all are downward etc.

is certainly odd that he does not say so, especially if the addition were motivated by a realization of the inadequacy of the criteria set forth just a few lines earlier. Although the implicit effect of *reductio* (a) may be to add a new condition to this list, Aristotle certainly does not advertise this as his intention.

At any rate, Aristotle seems to have already settled, in *Physics* vii 1, upon an entirely different way to strengthen his definition of 'one change' in order to head off the possibility that 'one change' could have parts that are also 'one change'. At 242b31-42, Aristotle purports to summarize the necessary and sufficient conditions for a change to be one in *Physics* v 4, but states things in a slightly different way: A change is numerically one if and only if it proceeds from something numerically one and the same to something numerically one and the same in a period of time that is numerically one and the same. 'Numerically one and the same time' seems to mean, a 'continuous time' as in *Physics* v 4, but as for the rest, the idea seems to be that the moving thing is moving from one *terminus a quo* to one *terminus ad quem* and that each remains the same throughout the change. Thus, *Physics* vii 1 strengthens the teleological criterion by adding the requirement that at every stage of a unified change, not only is the mover moving in the same direction throughout the change, but it is also moving to and from a single pair of termini that remain the same throughout the change. What this does, essentially, is to ensure that any change satisfying the definition of 'one change' does not have parts that also satisfy the definition of 'one change', since if it did, it would be moving to and from termini that are not one and the same throughout.[37] I say that Aristotle *already* settled, in *Physics* viii 8, upon an entirely different way to strengthen his definition of 'one change', because *Physics* vii has been thought to have been written after *Physics* v and vi but before *Physics* viii,[38] and, indeed, has been taken as an earlier draft of *Physics* viii.[39] This, however, raises the question of why, if an appropriately strengthened criterion of unity has already been formulated in *Physics* vii 1, is it the unrevised criteria that appear in *Physics* viii 8? A clue, I think, lies in the odd structure of the text of *Physics* viii 7 and 8: The end of *Physics* viii 7, from 261a31 on, forms a summary of conclusions to follow, but only some of them. At 261b3-7 we get a preview of *reductio* (e), but no mention is made of *reductios* (a)-(d). Then, in chapter 8, where these proofs are laid out, Aristotle first starts laying the ground work for *reductio* (e) (261b31-262a17), but breaks off to pursue, in *reductios* (a) and (b), an alternate route to the conclusion that *reductio* (e) is supposed to prove

[37] Broadie 1982, 152 shows that it is a relatively trivial matter to demonstrate, with this new criterion, that 'one motion' cannot have parts that are also 'one motion'. (Though Broadie mistakenly reads this strengthened criterion into *Physics* v 4.227b20-9. See n3 above.)

[38] Ross 1936, 16 dates *Physics* vii after both *Physics* v and vi; after *Physics* v because of the references at 242b8 and b41-42 to *Physics* v 4 and 247b13 to *Physics* v 2, and after *Physics* vi because of the reference in the β text at 242a6 to *Physics* vi 10. There seem, however, to be no backward references in *Physics* viii to *Physics* vii. Ross follows Jaeger in taking *Physics* vii to be an earlier work than *Physics* viii.

[39] E.g., Simplicius, *In Phys*. 1036,3ff. seems to take *Physics* vii to be an earlier draft of *Physics* viii, the latter taking up the main points of the former and treating them in a more exact fashion.

(viz., that a reversal in the direction of a rectilinear locomotion requires the moving thing to come to rest), and then digresses on Zeno's dichotomy paradox (263a4-b8) before he finally gets around to finishing *reductio* (e) two thirds of the way through the chapter. The fact that the summary of arguments at the end of chapter 7 has no mention of *reductio* (b), the fact that *reductios* (b) and (e) prove the same result, and therefore play the same logical role in proving that only circular motion is infinitely continuous, and the fact that Aristotle starts with *reductio* (e) but then breaks off to pursue *reductio* (b) and a digression on Zeno's dichotomy paradox, would seem to suggest that *reductio* (e) is part of an earlier line of argument for the claim that only circular motion is infinitely continuous, and that *reductios* (a) through (d) and the return to Zeno at 263a4-b8 is, as Ross would say, an excrescence on the original plan of *Physics* viii 8, a digression that has an attack on Zeno as its purpose, rather than building and defending an Aristotelian theory of change.

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