Mathematical Application and the No Confirmation Thesis

by Kenneth Boyce

Abstract: Some proponents of the indispensability argument for mathematical realism maintain that the empirical evidence that confirms our best scientific theories and explanations also confirms their pure mathematical components. I show that the falsity of this view follows from three highly plausible theses, two of which concern the nature of mathematical application and the other the nature of empirical confirmation. The first is that the background mathematical theories suitable for use in science are conservative in the sense outlined by Hartry Field. The second is that the empirical relevance of mathematical statements suitable for use in science is mediated by their non-mathematical consequences. The third is that statements receive additional empirical confirmation only by way of generating additional empirical expectations. Since each of these is a thesis we have good reason to endorse, my argument poses a challenge to anyone who argues that science affords empirical grounds for mathematical realism.

1. Introduction

Some proponents of the indispensability argument for mathematical realism maintain that the empirical evidence that confirms our best scientific theories and explanations also confirms their pure mathematical components.[1] Against this view are those who argue for one reason or another that the pure mathematical statements implied by background mathematical theories suitable for use in science are immune from empirical confirmation. Proponents of this "no confirmation thesis" include (among others) nominalistic scientific realists who deny that the mathematical components of our best scientific theories are literally true,[2] mathematical

[1] See (Quine 1981a, 1981b, 1951, 1948) and (Putnam 1979a, 1979b) for classical sources of inspiration for this argument. For some prominent contemporary versions see (Baker 2016, 2009, 2005) and (Colyvan 2006, 2001).

[2] (Leng 2010), (Melia 2008)

rationalists who hold that our knowledge of pure mathematical truths is entirely non-empirical,[3] and some quietists who maintain there is no fact about whether mathematical entities exist.[4]

Arguments in favor of the no confirmation thesis commonly appeal either to the premiss that mathematical entities are non-spatiotemporal and causally inert (if they exist at all) or to the premiss that pure mathematical statements are not subject to empirical tests in the same way as other scientific claims. Those who appeal to the former premiss argue that because that is so statements solely about mathematical entities fail to generate any empirical expectations and are thereby not empirically confirmed.[5] Those who appeal to the latter premiss argue that the pure mathematical components of our scientific theories and explanations are not empirically tested so as to be subject to empirical disconfirmation and for that reason are also not subject to empirical confirmation.[6]

It is far from evident however that either of these arguments succeeds. As Mark Colyvan (2006: 233-34) points out, the general principle that we cannot have empirical evidence for statements about things that are causally isolated from us is false, since we have empirical evidence for the existence of objects outside our light cone. Alan Baker (2003) argues, furthermore, that if mathematical theories do in fact play an indispensable role in science, then it is at best unclear whether mathematical objects fail to make any difference in how things go with the observable world. It is also not clear, furthermore, that pure mathematical statements are immune from empirical disconfirmation. Colyvan (2001: 123-24) suggests, for instance, that if we found mathematics is dispensable to our most well confirmed scientific theories, that would

[3] (Frege 1950), (Hale and Wright 2001), (Marcus 2015)
[4] (Balaguer 1998), (Yablo 2009)
[5] (Balaguer 1998: 132-36), (Leng 2010), (Vineberg 1996).
[6] (Maddy 1997: 138-43), (Parsons 1983: 195-97), (Sober 1993).

be an empirical reason to deny that the background mathematical theories used in science are true (since in that case we could think of them as purely instrumental).

In this paper I develop a novel argument for the no confirmation thesis that relies on neither of the above considerations. Rather, I show that the no confirmation thesis follows from two highly plausible theses concerning the nature of mathematical application in conjunction with another highly plausible thesis regarding the nature of empirical confirmation. The first of these is that the background mathematical theories suitable for use in science are conservative in the sense outlined by Hartry Field (1980, 1992) (i.e. that their conjunction with any non-mathematical statements has all and only those non-mathematical implications had by the original non-mathematical statements). The second is that the empirical relevance of mathematical statements suitable for use in science is mediated by their non-mathematical consequences. The third is that a given statement receives additional empirical confirmation relative to a given body of background information and auxiliary assumptions only if it generates empirical expectations that are not generated by that body of background information and auxiliary assumptions alone.

Field (1980, 1992) has mounted powerful formal arguments in favor of the view that the background mathematical theories which are *in fact* used in science are conservative. And as I further explain below, there are additional reasons to endorse the normative thesis that any theory *suitable* for use in science has that property. The second thesis is not only intuitively plausible in its own right, but (as I also explain) supported by some of the same considerations as the first. The third thesis can seem nearly self-evident; statements do not receive empirical support beyond what they already have unless they generate additional empirical expectations that can be vindicated. Each of these theses is independently motivated and one that proponents of the indispensability argument have at least some reason to endorse. Yet it can be shown that they jointly imply the no confirmation thesis. I offer a proof of this result after discussing each thesis in more detail.

2. Concerning the Claim that Mathematics is Conservative

Let's begin with the thesis that the background mathematical theories suitable for use in science are conservative. This thesis can be stated more precisely as follows:

> *Conservation:* For any background mathematical theory, M_T*, that is suitable for use in science, the conjunction of M_T* with any body of non-mathematical statements, N*, has as logical consequences all and only the same non-mathematical statements as does N*.[7]

This thesis is plausible independently of any denial of mathematical realism. As Field (1980: 12-13) points out, it naturally accords with the widely held view among mathematical realists that the background mathematical theories used in science are both necessarily true and knowable a priori.[8] It is also a widely held view, furthermore, that all of the mathematics required for science can be embedded within a standard sort of set theory.[9] And Field *proves* that one such theory (namely, Zermelo-Fraenkel set theory with choice, modified to include urelements) is conservative.[10]

Beyond the descriptive issue of whether the background mathematical theories actually used in science are conservative, however, lies the normative issue of what it takes for such a theory to be *suitable* for that use. Scientists use mathematics as a means of facilitating

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[7] I stipulate that part of what it is for a statement to be "non-mathematical" is for it to be "agnostic" concerning whether there are mathematical objects in the sense specified by Field (1980: 11-12).

[8] It should be noted, however, that that while the claim that the background mathematical theories used in science are conservative naturally accords with this view, it does not obviously follow from it, for reasons pointed out by Melia (2006: 205).

[9] Although see (Melia 2006: 205-208) for some challenges to the claim that all of the mathematics used in science can be so embedded.

[10] Here I gloss over certain technical niceties, covered by Field (1980: 16-19), which are not germane to the present discussion.

discoveries about the concrete world evidenced by empirical considerations. In order to be suited for this purpose background mathematical theories should not prejudice the results in advance by logically precluding certain non-mathematical statements prior to empirical investigation. But mathematical theories that are not conservative do just that. As Field (1980: 13) puts it "*Good* mathematics *is* conservative; a discovery that accepted mathematics isn't conservative would be a discovery that it isn't good."

3. Concerning the Empirical Relevance of Mathematics

This brings us to the second of the two theses mentioned in the introduction, the thesis that the empirical relevance of mathematical statements suitable for use in science is mediated by their non-mathematical consequences. That is, for any background mathematical theory suitable for use in science, the pure and mixed mathematical statements associated with that theory generate additional empirical expectations by way of having non-mathematical consequences (in conjunction with the pertinent background mathematical theory, non-mathematical background information, and auxiliary assumptions) that are not had by the relevant body of non-mathematical background information and auxiliary assumptions alone. This thesis may be stated more precisely as follows:

> *Mediated Relevance:* For any background mathematical theory suitable for use in science, M_T*, any mathematical statement, M*, that is either implied by M_T* or by M_T* in conjunction with various non-mathematical statements, and any body of non-mathematical background information and auxiliary assumptions, B*, if M* generates empirical expectations relative to B* that are not generated by B* alone, then there is some non-mathematical statement that is logically implied by M*&M_T*&B* that is not logically implied by B*.

This thesis might appear trivial if empirical expectations are conceived of as non-mathematical statements, and generating empirical expectations is conceived of in terms of having those statements as logical consequences. But I take it that a hypothesis in conjunction with a relevant body of background information and auxiliary assumptions can generate empirical expectations without having them as logical consequences. One might, for example, expect to find droppings in the cupboard given the hypothesis that it is being frequented by a mouse, in spite of the fact that this hypothesis together with one's background information and auxiliary assumptions does not logically imply that is what one will find. Nevertheless the above thesis is supported by a variety of considerations.

One such consideration is that it is intuitively plausible that a given mathematical statement generates new empirical expectations only when (if true) it provides additional information about the non-mathematical world (where providing information is conceived in terms of eliminating various logical possibilities). One might object to the claim that this supports Mediated Relevance by arguing that sometimes mathematics can provide such information by way of helping us uncover various non-mathematical implications of our background information and auxiliary assumptions. But in that case the mathematical statements in question function merely as a tool of logical discovery and need not be regarded as true in order to play that role. So in that case it is not the presumed truth of mathematical statements that generate the relevant empirical expectations but rather only that of the non-mathematical background information and auxiliary assumptions.

Another point in favor of Mediated Relevance is that it is supported by the same kinds of considerations that support Conservation when the latter is taken as a normative thesis. One plausible criterion of suitability for the scientific use of a background mathematical theory is that it is free of empirical bias. A mathematical theory's being free of such bias is compatible with the various *mixed* mathematical hypotheses we might entertain by making use of that theory generating new empirical expectations relative to our non-mathematical background information and auxiliary assumptions, but not with the theory's generating such expectations relative to our non-mathematical information and auxiliary assumptions all on its own.

In addition to these general considerations also lies the fact that Mediated Relevance accords with specific ways in which mathematics is often thought either to generate empirical predictions or to play a role in scientific explanation. Mathematics is often thought to play a role in generating empirical predictions for example by way of the logical inferences it permits to various non-mathematical statements. When mathematical statements merely facilitate inferences to the logical consequences of our non-mathematical background information and auxiliary assumptions, they function merely as tools of logical discovery, and the assumption they are true plays no essential role. In order for the assumption that various mathematical statements are true to generate new empirical expectations in this way, it must be that they (together with the relevant background mathematical theory and body of non-mathematical information and auxiliary assumptions) have additional non-mathematical consequences. So any case in which mathematical statements generate additional empirical expectations in this manner will be one that accords with Mediated Relevance.

A similar point holds with respect to some of the ways it has been suggested that mathematics plays a role in scientific explanation. Since scientific explanations target known phenomena they often do not involve the generation of new empirical predictions. There is nevertheless an intimate relationship between scientific explanation and the generation of empirical expectations, because scientific explanations often work by exhibiting how certain claims render various empirical phenomena expectable relative to some body of auxiliary assumptions and background information that does not include the fact that those phenomena occur.

Alan Baker (2016, 2009, 2005) famously argues for example that number theory plays an indispensable explanatory role in accounting for the prime-numbered year lifecycles of certain species of North American cicada. It plays that role furthermore by virtue of the fact that the relevant empirical explananda can be derived from number-theoretic statements in conjunction with a relevant body of empirical background information and auxiliary assumptions. In any such case, if on the one hand, the explananda in question turn out to be logical consequences of the pertinent non-mathematical body background information and auxiliary assumptions alone, then that body generates the relevant empirical expectations on its own, and so the case vacuously accords with Mediated Relevance. If on the other hand, the empirical explananda do not follow from that body alone, then the case is a non-trivial instance of Mediated Relevance (provided at least that statements reporting empirical phenomena are either non-mathematical statements in their own right or mixed mathematical statements that imply such).

Aidan Lyon argues that when mathematics plays an explanatory role in science it does so by way of providing what Frank Jackson and Philip Pettit (1990) call "program explanations," where a program explanation "is one that cites a property or entity that, although not causally efficacious, ensures the instantiation of a causally efficacious property or entity that is an actual cause of the explanandum" (Lyon 2012: 566). It is evident that if a given mathematical explanation serves as a program explanation then that will be because various mathematical statements together with various non-mathematical assumptions and background information implies certain logically contingent non-mathematical statements that render the phenomena in question expectable relative to those assumptions and background information. So any mathematical explanation of this sort will also provide an instance of Mediated Relevance.

Since these are just specific examples of how mathematics is alleged to play a role in generating empirical expectations within science and may not be exhaustive, the fact that Mediated Relevance is instanced when mathematics is applied in these ways does not entail that the corresponding universal generalization is true. Even so, the more we find that the various ways mathematics is used in science accord with Mediated Relevance, the more plausible Mediated Relevance becomes in the absence of counterexamples. Another important consequence of there being a wide array of such cases (including several examples that figure prominently in versions of the indispensability argument) is that a variant of the proof given in Section 5 could also be used to support the conclusion that pure mathematical statements suitable for use in science receive no empirical confirmation in all those cases in which Mediated Relevance holds (even if the universal generalization fails). And this result might be enough to undermine any extant version of the indispensability argument that claims to show that the background mathematical theories used in science receive empirical support.

4. Concerning the Nature of Empirical Support

The thesis that a given statement receives additional empirical confirmation relative to a given body of background information and auxiliary assumptions only if it generates additional empirical expectations may be stated more precisely as follows:

> *Empirical Expectation:* For any statement, P* and any body of background information and auxiliary assumptions, B*, P* receives additional empirical confirmation relative to B* only if P* generates some empirical expectations relative to B* that are not generated by B* alone.

I do not believe that much needs to be said in favor of this thesis. Statements receive empirical support by virtue of the vindication of the empirical expectations they generate. Statements that do not generate additional empirical expectations do not receive additional empirical support.

One might object that cases in which hypotheses receive additional empirical support by way of figuring into scientific explanations of previously known phenomena, without generating new empirical predictions, afford potential counterexamples. But I take it that there are no such cases. As noted above, scientific explanations often work by way of rendering empirical phenomena expectable relative to some body of background information and auxiliary assumptions that do not include the fact that the empirical phenomena to be explained occurs. In those cases, the explanations in question do generate new empirical expectations relative to *that* body of information and auxiliary assumptions, and thereby receive additional empirical support relative to *it*. But if such explanations do not generate any new empirical expectations relative to the larger body of non-mathematical information and auxiliary assumptions we actually have, then they do not receive any *additional* empirical confirmation. Rather what often happens in this sort of case is that we are led to increase our rational confidence in an explanatory hypothesis through the discovery of relations of empirical support that already hold.

It should also be noted that Empirical Expectation is a *qualitative* principle that pertains to whether further empirical information provides *additional* confirmation for a certain claim. That is, it pertains to *incremental confirmation* rather than to absolute confirmation and *says nothing about the degree* to which empirical confirmation occurs. Such a qualitative principle is all that is needed for present purposes. If there are empirical grounds for believing any of the background mathematical theories suitable for use in science, then it must be that at least one of them is incrementally confirmed by additional empirical information, relative to *some* body of background information and auxiliary assumptions (one that contains no empirical content for example). Even so, it is important to attend to these limitations, lest some (especially those inclined to scientific realism) be tempted to believe there are counterexamples to Empirical Expectation where none exist.

Empirical Expectation is compatible, for instance, with its being the case that one theory receives a greater degree of empirical support than another, in spite of the fact that both are empirically equivalent. Perhaps one theory constitutes a more eloquent and parsimonious explanation of the data, for example, and so is better confirmed by it. Empirical Expectation is also compatible with there being cases in which a scientific theory is not better confirmed than a potential rival and yet still confirmed well enough to merit acceptance. A certain sort of scientific anti-realist might propose, for instance, that we can avoid commitments to concrete unobservables by replacing each of our standard scientific theories with the claim that it is empirically adequate, or perhaps with the relevant application of Craig's elimination theorem. Since the resulting theories are logically weaker than the originals, a scientific anti-realist might argue, they must be at least as well confirmed on our total evidence. Even so, a scientific realist might respond, it is consistent with Empirical Expectation that in these cases our empirical evidence favors the adoption of *both* theories, rather than merely the logically weaker of the two.

5. A Proof

It can be shown that the theses discussed in the previous three sections jointly imply the no confirmation thesis, where the latter may be stated as follows:

> *No Confirmation:* For any background mathematical theory suitable for use in science, M_T*, any mathematical claim, M*, that is implied by M_T*, and any body of non-

mathematical information and auxiliary assumptions, B*, it is not the case that M* receives additional empirical confirmation relative to B*.

Here is a proof of the advertised result:

> Assume that Conservation, Mediated Relevance and Empirical Expectation are true. Assume for reduction that No Confirmation is false. Suppose M_T is a background mathematical theory suitable for use in science, M a mathematical claim that is implied by M_T, B a body of non-mathematical background information and auxiliary assumptions, and that M receives additional empirical confirmation relative to B (note that it follows from the falsity of No Confirmation that there is such a case). It follows from Empirical Expectation that M generates some empirical expectations relative to B that are not generated by B alone. It follows from Mediated Relevance that there is some non-mathematical statement that is logically implied by M&M_T&B but not by B. Since M_T implies M, it follows that there is some non-mathematical statement that is logically implied by M_T&B but not by B. Since B is a body of non-mathematical statements, however, it follows from Conservation that there is no non-mathematical statement that is logically implied by M_T&B but not by B. Contradiction!

Since Conservation, Mediated Relevance, and Empirical Expectation are each claims we have good reason to believe, the above proof affords a challenge to anyone who maintains that the use of mathematics within science affords empirical grounds for mathematical realism.[11]

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