PSA 2004
***Symposium:* The Semantic View of Theories, Scientific Structuralism and Structural Realism**

A Minimal Construal of Scientific Structuralism

Katherine Brading and Elaine Landry

The focus of this paper is the recent revival of interest in structuralist approaches to science and, in particular, the structural realist position in philosophy of science[1]. The challenge facing scientific structuralists is three-fold: i) to characterize scientific theories in 'structural' terms, and to use this characterization ii) to establish a theory-world connection[2] (including an explanation of applicability) and iii) to address the relationship of 'structural continuity' between predecessor and successor theories. Our aim is to appeal to the notion of shared structure between models to reconsider all of these challenges, and, in so doing, to classify the varieties of scientific structuralism and to offer a 'minimal' construal that is best viewed from a methodological stance.

1 Structuralism in Mathematics

Since much of what is taken as distinctive of scientific structuralism is tied to mathematical structuralism, we begin first with a brief description of what we mean by this. We take mathematical structuralism to be the following philosophical position: the subject matter of mathematics is structured systems[3] and their morphology, so that

- mathematical ‘objects’ are nothing but ‘positions in structured systems’, and
- mathematical theories aim to describe such objects and systems by their shared structure, i.e., by their being instances of the same kind of structure.

For example, the theory of natural numbers, as framed by[4] the Peano axioms, describes the various concrete[5] systems that have a Natural-Number structure. These structured systems are, for example, the von Neumann ordinals, the Zermelo numerals, and so forth; they are *models* (in the Tarskian sense of the term[6]) of the Natural-Number structure. The ‘objects’ that the theory of natural numbers talks about are then the positions in the various models. For example, the von Neumann ordinal ‘2’ is a position in the model ‘von Neumann ordinals’; the Zermelo numeral ‘2’ is a position in the model ‘Zermelo numerals’; and the theory of natural numbers describes the number ‘2’ in terms of the shared structure of these, and other, models that have the same kind of structure. If all models that exemplify this structure are isomorphic, then the Natural-Number structure and its morphology are said to present its *kinds of* objects[7], i.e., are said to determine its ‘objects’ only ‘up to isomorphism’.

As explained by Benacerraf [1965], mathematical structuralism implies that there are no natural numbers as particular objects, i.e., as existing things whose ‘essence’ or ‘nature’ can be individuated independently of the role they play in a structured system of a given kind. This is because the relevant criterion of individuation, viz., Leibniz’s Principle of the Identity of Indiscernibles, does not hold. For example, in one system of the natural numbers the property $2 \in 4$ holds for the natural number ‘2’ while in another it does not.

Yet clearly, since the systems are isomorphic, we want to say that we are talking about the same natural number '2'. In our terminology, we express this by saying that we are talking about '2' as a *kind of* object. More generally, we say that there are only mathematical 'objects' as kinds of objects, i.e., that there are 'objects' that can be individuated only up to isomorphism as positions in a structured system of a given kind. Thus, taking 'structured system' to mean 'model'[8], we say that a mathematical theory, while framed by its axioms[9], can be *characterized* by its models, and that the kinds of objects that the theory talks about can be characterized by their being positions in models that have the same kind of structure.

In the next section we use this admittedly brief sketch of mathematical structuralism as a starting point for elucidating what might be meant by scientific structuralism. There are two points, important for the contrast between mathematical and scientific structuralism, that enable us to further clarify our subsequent description of scientific structuralism. First, in physical theorizing it is important to keep clear the *semantic* distinction between kinds of objects and particular objects. As noted above, in mathematics this distinction is not possible; mathematical objects *are* kinds of objects rather than particular objects. This is what it means to say that mathematical 'objects' are characterized only by what can be said of their shared structure. When we speak about the natural number '2' we do not intend to refer to (or mean) any particular instance of the natural number 2. For example, we do not intend to refer to the ordinal '2'; rather, we are speaking about any kind of object that has the appropriate kind of structure.

A second difference is that in physical theorizing we also need the *ontological* distinction between theoretical objects and their physical realization. Thus, we need to maintain a level of description in which a physical theory can *talk about* electrons, as theoretical objects, without its having to *be about* electrons, as objects that are physically realized in the world. To talk about electrons (or unicorns) is not thereby to bring them into existence as physical objects. Again, in mathematics there is no such distinction; for a sentence to be about an object is for a sentence to talk about an object. For example, for a theory to be about the natural number 2 it is sufficient for it to talk (in a coherent manner[10]) about the natural number 2. Thus, for a mathematical object, 'to be', as Quine [1980] explains, is to be a value in the range of a bound variable.

We rely on the terminology of 'presentation' versus 'representation' to express these important distinctions. At the semantic level, we say that in mathematics the kinds of objects that the theory talks about are *presented* via the shared structure holding between the mathematical models. For example, the '2' of the von Neumann ordinals and the '2' of the Zermelo numerals are presented as the same kind of object, i.e., as the natural number 2, because the models in which they are positions have the same structure. Likewise for physical theories – theoretical objects, as kinds of physical objects, may be *presented* via the shared structure holding between the theoretical models. However, at the ontological level, a physical theory, insofar as it is successful[11], must also *represent* particular physical objects and/or phenomena and not merely present kinds of physical objects. In what follows we shall have more to say about the implications of keeping issues of semantics/presentation separate from issues of ontology/representation.

2 Structuralism in Science: An Analogy with the Mathematical

What is scientific structuralism? That is, in what sense can we claim that "science is the search for structure"[12]? Analogous to mathematical structuralism, one might say that, *minimally*, scientific structuralism is the view that the subject matter of science is structured systems and their morphology, so that

- scientific 'objects' are presented as nothing but 'positions in structured systems', and
- scientific theories aim to describe such objects and such systems by their same or shared structure, i.e. by their being instances of the same kind of structure.

If, once again, we replace the term 'structured system' with 'model', and recall that 'objects' are *kinds of* objects, then scientific structuralism can be described as the position that a scientific theory may be characterized by the collection of its theoretical models and that the kinds of objects that the theory talks about can be characterized as being 'positions in a theoretical model'.

Since the current scientific structuralist debates are (for the most part) framed within the 'semantic view of theories', our investigation will also be set within that framework. For our present purposes, the most important differences between the syntactic and semantic views arise through consideration of both the structure of scientific theories and the theory-world connection.

On the syntactic view of theories, a theory *is* an uninterpreted, or partially interpreted, axiom system *plus* correspondence rules, or co-ordinating definitions, that mediate so as to provide for the theory-world connection, e.g., that are used to provide a bridge between theoretical sentences and observational sentences. The semantic view of theories rejects the need for, and/or possibility of, correspondence rules and instead uses models (again, in the Tarskian sense of the term) to provide an *unmediated* theory-world connection. One may[13] then forgo the need for a precise axiomatization of the theory in favor of making precise the sense of model, so that a theory, even if framed by axioms, *is characterized by* a collection of its models.[14]

According to a more radical version of the semantic view, a scientific theory need not be axiomatized, or even axiomatizable; instead all emphasis is to be placed on models. A theory *is* a collection of models. Thus, to establish a theory-world connection we need only connect its models to the world. Since such models can clearly no longer be understood in the purely Tarskian sense, i.e., since one gives up any description of the theory in terms of axioms or sets of sentences and, thereby, forgoes taking ‘model’ to mean an interpretation that satisfies a set of sentences, this raises questions about what is meant by ‘model’ on this more radical view (see Jones [2005]). To side-step these questions, some have turned to characterize a theory more broadly as a *family of structures*[15], wherein a model is a type of structure. The issue of what, precisely, is meant on this approach by the terms ‘model’ or ‘structure’ is one we will not go into at this point.

Regardless of whether one adopts a syntactic or semantic view of scientific theories, the lesson that we believe scientific structuralists ought to draw from the analogy with mathematical structuralism can be summarized as follows: the ‘objects’ of a scientific theory are *kinds of* physical objects (rather than *particular* physical objects) and they are *presented* (rather than *represented*) by considering the shared structure of the models of the theory[16].

3 Applications of Shared Structure

In our consideration of mathematical structuralism, we have seen that the notion of ‘shared structure’ between models of a given theory can be appealed to in presenting the kinds of objects that the theory talks about. We have seen too that according to the semantic view of scientific theories, theories (regardless of how, or whether, they are formally framed) are to be *characterized as* a collection of models that *share the same kind of structure*. This application of shared structure speaks to the first challenge facing the scientific structuralist, viz., to characterize scientific theories in ‘structural’ terms, and so we are now in a position to reconsider the remaining challenges by relying on the notion of shared structure to account for *the uses* of this characterization.

The first use is the attempt to capture the theory-world connection by appealing to the relationship of shared[17] structure between the theory and the phenomena. For example, as Suppes has pointed out ([1960]; [1962]), scientific theorizing consists of “a hierarchy of theories and their models” (Suppes [1962], p. 255) that bridge the gap between the high level theory and the lower level phenomena that the theory is intended to be about. There is a theory, characterized by the collection of its models, associated with each layer (e.g., there is a high level theory, a theory of the experiment, a theory of the data) so that the relationship of shared structure between each layer (e.g., between the theory and the data) can be formally analyzed and experimentally evaluated[18]. So arranged, the *formal analysis* (by model-theoretic methods) of the relationship between the theory and the phenomena aims to close the gaps between the levels, for example, the gap between the high level theory and the theory of the data, by appealing to isomorphisms[19] to formally express the claim that their models have the same structure.

It is important to note that data models, for Suppes, are models in the Tarskian sense – they are models of *a theory of* data. As such, data models are far removed from 'mere descriptions of what is observed', i.e., from what we might call 'the phenomena'[20]. As Suppes notes, "the precise definition of models of data for any given experiment requires that there be a theory of data in the sense of the experimental procedure, as well as in the ordinary sense of the empirical theory of the phenomena being studied". (Suppes [1962, p. 253) Thus, two things are required to connect the high level theory to the phenomena: an experimental *theory of the data* and an empirical *theory of the phenomena*.

Suppes ([1960], [1962] and [1967]) details the evaluative criteria of those theories (theories of experimental design and of *ceteris paribus* conditions) that go into the construction of the experimental theory of the data. But, he is clear that, since there are no models (in the Tarskian sense) of these theories, one can formally characterize the experimental theory of the data only by the collection of its data models; and so one's formal analysis must *begin with* models of data. To then connect the data to the phenomena one must establish that their models have the same structure. But without an (empirical) theory of the phenomena, one cannot speak of the structure of the phenomena, i.e., one cannot characterize the structure of the phenomena in terms of the shared structure of its models. Suppes, however, is silent on the issue of why we should suppose that models of data have the same structure as the phenomena.

It is here, then, that we are presented with three options: i) from a *methodological* stance, we may forgo talk of the structure of the phenomena and simply begin with structured data, i.e., with data models; ii) from an *empirical* stance we may say that what structures the phenomena into data models is the high level theory; and finally, iii) from a *realist* stance we may say that what structures the phenomena is the world.[21] Regardless of one's stance, it should be clear that without a theory of the phenomena one cannot formalize the treatment of the structure of the phenomena in terms of data models alone, and so one cannot use the semantic view's account of shared structure between models to immediately close the gap between the theory and the phenomena, and thereby to establish a theory-world connection.

Data models, then, represent a significant cut-off point in our formal analysis; below the level of data models we require more than comparisons of shared structure between models to relate the levels of the hierarchy to one another. In recognition of this we separate the scientific structuralist's second challenge (to establish a theory-world connection) into two components: a) to give an account of *applicability* in terms of the shared structure between models of the theory and data models wherein models of the theory *present* the kinds of objects that the data models are intended to talk about so that their 'objects' have the same kind of structure, and b) to give an account of *representation* in terms of the shared structure between data models and the phenomena so that the phenomena that the theory is about are appropriately structured (by the theory or by the world).

The second use of the characterization of a scientific theory as a collection of its models is the appeal to the notion of shared structure to reconsider the relationship between predecessor and successor theories. This relationship is of crucial interest to structural realists in their attempt to overcome the so-called 'pessimistic meta-induction' argument and, in so doing, to make way for a modified version of the 'no miracles' argument. The 'pessimistic meta-induction' argument relies upon the existence of radical ontological discontinuities between predecessor and successor theories, and the strategy for overcoming the associated pessimism, as proposed by Worrall [1989], depends on the claim that the discontinuity at the ontological level is nonetheless accompanied by overall continuity at the structural level.

In support of the assertion of 'structural continuity' between predecessor and successor theories, Worrall points out that, for example, the mathematical equations of the Newtonian theory of gravitation can, in a rough and ready way[22] be retrieved, in the appropriate limit, from Einstein's general theory of relativity. Continuity of structure (as expressed by the equations) is maintained despite the fact that the two theories disagree over such ontological issues as the nature of material bodies (e.g., the meaning of the term ‘mass’), whether or not material bodies act directly and instantaneously on one another at a distance, and whether or not space and time are themselves influenced by the presence of material bodies. The suggestion is that, by restricting ourselves to the relationship of shared structure between predecessor and successor theories, we are able to recover the needed continuity through radical theory change, and so are in a position to offer-up a structural realist version of the ‘no miracles’ argument.

For the structural realist, read now as a kind of scientific structuralist, the point of the above example is that this relationship can be expressed in terms of shared structure between the models (e.g., the solutions) of Einstein’s equations and those of the Newtonian theory of gravitation. More generally, characterizing a theory as a collection of models allows one to account for structural continuity by appealing to the shared structure between models of the predecessor theory and models of the successor theory.

Various attempts have been made to *formally* capture each of these three applications of shared structure by specifying a particular type of ‘structure’, and hence, a particular type of morphism that should hold between models as types of ‘structures’. For the first application, i.e., characterizing the structure of a scientific theory in terms of the shared structure of its models, Da Costa and French [1990], for example, appeal to partial isomorphisms between models, as types of partial structures, so that a scientific theory *is* "its class of (mathematical) models, regarded as the structures it makes available for modeling its domain". (Da Costa and French [1990], p. 259)

Accounts of the second application of shared structure, i.e., using the above characterization of theories to capture the relationship between (models of) the theory and (models of) the phenomena, have been offered in terms of:

a) isomorphisms between either models as Tarskian models (Suppes [1967]) or models as state-spaces interpreted by Beth semantics (van Fraassen [1970] and Suppe [1977]),

b) partial isomorphisms between models as partial structures (French and Da Costa [1990]), and

c) embeddability either of empirical substructures and structures as state-spaces (van Fraassen [1980] or of partial structures and simple pragmatic structures in a function-space (French [1999]).

Nevertheless, as we have seen explicitly in our investigation of Suppes, while capturing the relation between theory and data models may be formally tractable, there still remains a gap between the data models and the phenomena that cannot be bridged in the same 'formal' manner. This issue, fundamentally the issue of how data models represent the structure of the phenomena, should thus be pressed with respect to each of a) through c).

Finally, attempts to formally capture the relation of ‘structural continuity’ between predecessor and successor theories by appealing to the shared structure of their respective models (again, as types of ‘structures’) have been made in terms of:

a) homeomorphisms between types of lattice structures (Da Costa, Bueno, French [1997]),
b) partial isomorphisms between partial structures in a function-space (French [1999]), and
c) partial homomorphisms between partial structures (French [2000]).

What remains open for discussion here, and what underlies the structural realism debates, is the question of whether the kind of structure that is retained is theoretical (mathematical) or phenomenological[23]; do we read the appropriate kind of structure from the theory or from the world? Forgoing this question for the moment, in each case, what the success of the three applications relies upon is an attempt to make *formally precise* the notion of ‘shared structure’. It is thought that “without a formal framework for explicating this concept of ‘structure-similarity’ it remains vague, just as Giere’s notion of similarity between models does…” (French [2000], p. 114). What all of these ‘formal’ attempts have in common, then, is that they seek to specify the type of shared structure at work in terms of some *specific type* of morphism between models as some *specific type* of ‘structure’, e.g., in terms of isomorphism, embeddability, partial isomorphism, homomorphism between Tarskian models, state-spaces, partial structures, etc.

This is not our approach. We wish to distinguish between what shared structure *is* (what specific type of structure and/or type of morphism *is* the appropriate one[24]) and what the presence of shared structure *tells us* (what the appropriate kind of structure is *for the task at hand*), and to place our focus on the latter. We say simply that two models share structure if there exists a morphism between them that preserves the 'appropriate kind' of structure, regardless of our having to specify this kind as a precise type of morphism. The 'appropriate kind' of structure depends on which of the three applications of the notion of shared structure is being appealed to, and also on the details of the particular task at hand. Thus, and this is where our emphasis is distinct, what shared structure tells us cannot be ascertained simply by looking at the types of 'structures' (or types of morphisms): the proof of the efficacy of appeals to shared structure is in the pudding, not in the recipe.

Given our more modest approach, we draw the following general conclusions concerning the appeal to shared structure for each challenge faced by scientific structuralists.

(1) By characterizing a scientific theory as a collection of models, shared structure between the theoretical models of a theory tells us what *kinds of objects* the theory talks about.

For example, given Newton's laws of motion and his law of universal gravitation, we can solve the generic two-body problem. These solutions are models of the theory, and they prescribe all and only the possible paths for Newtonian inertial-gravitational objects in two-body motion. In doing so, they thereby present the kind of object that the theory talks about, viz., a ‘Newtonian inertial-gravitational’ object.[25]

Turning next to consider an example from quantum theory, French ([2000], p. 107) writes that for Weyl, the shared group-structure of quantum theoretical models tells us how the kinds of objects that quantum theory talks about are to be structured so as to satisfy a global form of the Heisenberg commutation relations (see French [2000], p. 107). This speaks to Weyl’s ‘foundationalist’ programme (see Mackey [1993]).

French’s reconstruction of this programme suggests that it was by appeal to a particular type of structure and type of morphism that the gap between the group theoretic and the quantum theoretic systems was bridged. However, as he himself writes, it was “the reciprocity between the permutation and linear groups” that acted “as ‘the guiding principle’ of [Weyl’s] work and also as a ‘bridge’ within group theory” so that “[t]he application of group theory to quantum physics depends on the existence of this bridge between structures within the former” (French [2000], p. 109). What is doing the work here is ‘shared structure’, and *not* a specific *type* of shared structure. Thus, even noting the fact that both group theory and quantum mechanics were in a state of flux, the example does not speak to either the claim that “the partial structures programme provides the appropriate formalization of this feature [or openness]” or the claim that “what we have in this case is the *partial* importation of mathematical structures into the physical realm which suggests that the appropriate formal characterization of this relation is by means of a partial homomorphism” (French [2000], p. 110)

(2) In arranging a scientific theory as a hierarchy of models, shared structure between models at different levels tells us about the *applicability* of the models at one level to those at another; it can thus tell us, for example, about the applicability of theoretical models to data models.

French ([2000], p. 107), for example, discusses Wigner's use of the shared group-structure of quantum theoretical models and quantum data models for determining how the 'data' are to be structured so as to satisfy the fundamental symmetry principles. This use speaks to Wigner's 'phenomenological' programme (again, see Mackey [1993]). Again, it was *not*, as French suggests it was, by appeal to a type of structure or type of morphism that the analogy between atomic and nuclear systems became useful for accounting for the shared group-structure of atomic phenomena: it was because the shared Lie-group-structure between models of atomic systems and models of nuclear systems supplied an effective analogy for representing the laws of atomic phenomena in terms of symmetry principles.

French ([2000], p. 111) writes, however, that the analogy allowed one to see that "[t]he decomposition of the Hilbert space for a nucleon into proton and neutron subspaces is analogous to the decomposition of the corresponding Hilbert space for the spin of an electron… Indeed the relevant groups have isomorphic Lie algebras". But, even noting the fact that idealizations are needed to make this analogy work, this example does not speak to either the claim that "[t]hese kinds of idealizing moves can be represented via partial isomorphisms holding between the partial structures" (French [2000], p. 112) or the claim that "[t]his incomplete analogy between atomic and nuclear structure can be straightforwardly represented in terms of partial structures" (French [2000], p. 112). It is the appeal to the appropriate kind of shared structure, e.g., Lie-group structure, that is doing the work in the example that French gives, and no further analysis in terms of a specific *type* of structure or morphism is needed to ground the application of this analogy.

(3) When it comes to considering the relationship between predecessor and successor theories, shared structure between models of the theory can be used to tell us about the continuity of structure across theory change.

Here we take as our example Newtonian versus Special Relativistic mechanics, and in order to take the simplest possible case we consider inertial motion in these theories. In other words, we compare the inertial structure of Galilean spacetime to that of Minkowski spacetime. Both Newtonian and Special Relativistic mechanics satisfy the principle of relativity, and this implies that for each theory the coordinate transformations between inertial frames must form a group. In the first case we have the Galilean group, and, in the second, the inhomogeneous Lorentz group; both the Galilean and Lorentz groups of transformations being permutations of R^4. The relationship of shared structure between Newtonian mechanics and Special Relativity obtains when specific limiting conditions[26] are imposed within Special Relativity. Under these conditions, the Lorentz transformations reduce to the Galilean transformations and so the two theories share the same group-structure.

So far we have concerned ourselves only with those cases in which shared structure between models presents us with the *kinds of* objects that the theory *talks about*. Importantly distinct is the claim that shared structure gives us the particular objects that the theory *is about*, i.e., the claim that the theory *represents particular objects* rather than merely *presents kinds of objects*. We now turn to take-up the question of how a scientific theory is used to represent, that is, used to establish a 'theory-world connection'.

4 Beyond the Mathematical Analogy: From Presentation to Representation

In this section we consider, at last, the challenge of establishing how theories connect to the world. Viewing this challenge in light of our semantic structuralist characterization of a theory, the connection can be broken down into two main components: connecting theoretical models to data models, and connecting data models to the phenomena. We argue that while the first connection can be accounted for solely in terms of *presentation* of shared structure, the second demands the addition of something more.

In the spirit of Suppes, we again consider a hierarchy consisting of 'the phenomena'[27] at the bottom, the high level theory at the top, and various other levels in between. We say that the layer above 'the phenomena' is the experimental 'data' (for example, points on paper representing values arrived at by experiments), which we distinguish from the 'data models' (for example, points on paper with a curve drawn through them representing structured data). In other words, in plotting our 'data' we present our experimental results in a mathematically structured space, and then in constructing a data model we add further structure to the data by relating 'the points' to each other such that the relevant relations between them can be expressed in a mathematical manner.[28] Finally, above the data models we have the entire theoretical hierarchy, each layer being characterized by the models of the associated theory. (Forthcoming diagram.)

Returning to our initial query of how theoretical models connect to data models and how data models connect with the phenomena, we have already seen that the first question is straightforwardly answered by appeal to the notion of shared structure. That is, a theoretical model *applies* to a data model just in case, as explained in Section 3, they share the appropriate kind of structure and so can be said to *talk about* the same kinds of objects. To answer the second question, however, we need an account of *representation*: we need an account of how a physical theory comes to *be about* particular objects. Appeals to shared structure are not enough for this purpose.

Recall that data models are the lowest level at which we have a theory and its models, i.e., a theory of the data and its data models. Data models, then, can be taken as *truth-makers* in the Tarskian sense, but if they are to *be about* the phenomena they must also function as *representations* (see Jones [2005]). Recall, too, that the high level theory presents the kinds of objects, so if it is to be connected to the phenomena via data models, then one requires an account of *how* it represents the particular objects that the theory is purportedly about. Consequently, to establish a theory-world connection, it is necessary to go further than characterizing a theory as a collection of Tarskian models that presents the kinds of objects that the theory talks about.[29]

To move from presentation to representation, and so to move from Quine's semantic 'is' to an ontological 'is'[30], one needs something more than a minimal scientific structuralism. The question of the reality of particular physical objects and/or the truth of physical propositions cannot be settled semantically, i.e., cannot be settled merely by appeal to a Tarskian notion of a model and/or a Tarskian notion of truth: it depends crucially on some extra-semantic process whereby the connection between *what we say* and *what there is* is both established and justified. This is what we mean when we say that an account of representation[31] is required. The term 'model' in science is, of course, replete with connotations of representation and the temptation in the past has perhaps been for the semantic view of theories, with its use of Tarskian models (which, to repeat, are truth-makers and *not* representations), to piggy-back on this required representational role. In our view this is not acceptable: if the semantic view of theories is to do better than the syntactic view in tackling the problem of the theory-world connection, then it owes us an account of how its models (Tarskian or otherwise) gain their representational significance[32]. Indeed, as we will now see, it is the differences in how representation is treated that lead to the different varieties of scientific structuralism.

What we call *minimal* structuralism is committed only to the claim that the kinds of objects that a theory talks about are presented through the shared structure of its theoretical models and that the theory applies to the phenomena just in case the theoretical models and the data models share the same kind of structure. No ontological commitment – nothing about the nature, individuality or modality of particular objects – is entailed. Viewed methodologically, to establish the connection between the theoretical and data models, minimal structuralism considers only the appropriateness of the kind of structure and owes us no story connecting data models to the phenomena. In adopting a *methodological stance,* we forgo talk of 'the structure of the phenomena' and simply begin with data models. We notice that our theoretical models are appropriately structured (present objects of the appropriate kind) and shared structure is what does the work connecting our data models up through the hierarchy to the theoretical models, and so we suggest the methodological strategy of seeking out, exploring and exploiting the notion of the appropriate kind of shared structure, both up and down the hierarchy, and sideways[33] across both different and successive theories.

There are various ways of going beyond this methodologically viewed minimal structuralism, depending, in part, on how one wishes to make the theory-world connection. That is, depending on how one chooses to close the gap between the data models and the phenomena, a theory that presents us with the appropriate kinds of objects can also be claimed to *represent* (the structure of) physical objects in the world. Recall that we offered two alternatives to our methodological stance: from an *empirical stance*, one may hold that what structures the phenomena is the high-level theory, whereas from a *realist stance* one may hold that what structures the phenomena is the world. Such additional stances are all very well and good, but if we are to be motivated to move beyond the more modest methodological stance we need reasons. In particular, if we are to adopt either the empiricist or the realist alternative, we need a justification for the claim that data models share the same structure as the phenomena and, as a result, that the former can be taken as representations of the latter.

Adopting a empiricist stance, van Fraassen, as a ‘structural empiricist’, suggests that we simply *identify* the phenomena with the data models:

> the data model … is, as it were, a secondary phenomenon created in the laboratory that becomes the primary phenomenon to be saved by the theory. (van Fraassen [2002], p.252)

In this way, the step from presentation to representation is made almost trivially: the data models act as the ‘phenomena to be saved’ and so all we need to connect the theory to data models *qua* ‘the phenomena’ is a guarantee of their shared structure. van Fraassen makes this connection by using embeddability as a guarantee of the shared structure between theoretical models and ‘the phenomena’, maintaining that

> certain parts of the [theoretical] models [are] to be identified as empirical substructures, and these [are] the candidates for representation of the observable phenomena which science can confront within our experience. (van Fraassen [1989], p. 227)

This empiricist version of scientific structuralism avoids the question of why it should be assumed that the phenomena is represented by data models by simply collapsing any distinction between the two and so offers no justification for why such an identification should be presumed possible.

We think it is necessary, for any attempt which aims to move beyond a methodological stance, to provide an account of what allows us, in the first place, to make the identification between the phenomena and data models[34]. One such account, which stands mid-way between the empiricist and realist option, might arise from some form of structurally read neo-Kantianism, whereby the very process of representation (e.g., the synthetic unity of apperception) itself structures the phenomena so that the act of representation itself explains the possibility (indeed, the necessity) of identifying, in terms of their shared structure, the data models and the phenomena.

Structural realists, such as French and Ladyman, who adopt a realist stance and so presume that the world structures the phenomena, invoke the ‘no miracles’ argument to explain the necessity of identifying the structure of data models and the structure of the phenomena; it is used to argue that if there was no shared structure between the (data models of the) theory and the world (the phenomena) the success of science would be a miracle. Thus, while no detailed account of how the data models come to share structure with the phenomena is given, the possibility (again, necessity) of making the identification is itself justified by appeal to at least an argument.[35]

Structural realism, insofar as it identifies the structure of data models and the structure of the phenomena, is in all its forms, committed to the claim that the kinds of objects presented by our theory accurately represent the structure of particular objects of which ‘the world’ is claimed to consist. The forms of structural realism differ in just how far this representation is claimed to take us. The epistemological structural realist says that, with respect to the particular objects, all that can be known is that they are instances of the structural kinds given by our theories; all that can be *known* is their structure[36]. They remain open to the possibility, however, that the particular objects in the world have other properties that are not represented by the theory. Ontological structural realism can be understood as rejecting this last claim and asserting that the particular objects in the world have no properties beyond those that make them instances of certain structural kinds; all there *is* is structure[37].

Ladyman, however, is developing an alternative form of ontological structural realism which he terms ‘modal structural realism’.[38] Adopting this *modal stance*, one may say that the ‘structural kinds’ specify only the modal properties associated with what it is for a particular object to be an instance of that kind. To explain what might be meant here we again take an example from mathematics: it may be said that while $2\in4$ is a possible property of the natural numbers it is not a structural, i.e., a necessary, property because $2\in4$ is not true in all models that have a Natural-Number structure. Modal structural realism is, therefore, at once both more modest and more ambitious than other varieties of structural realism. Unlike the standard ontological version, it does not aim to capture *all* the properties of particular physical objects, but it does aim to capture their *necessary* properties. The modal properties transfer, via shared structure, to the particular instances of the kind, thus representing the modal relations between particulars.

Once again, what we seem to be missing is an account of why this representation works, e.g., an account of why the structural properties of kinds of objects can be identified with the necessary properties of particular objects. Indeed, as with standard structural realism, the claim that structural properties play a representational role at all is justified entirely by appeal to the ‘no miracles argument’. As minimal scientific structuralists, we eschew this representational role; we accept that if (models of) scientific theories *present* us with kinds of objects, then all that can be known of objects, as instances of those kinds, is their structure. But, in adopting a methodological stance, we remain open to the possibility (epistemic, ontic or modal) that particular objects may have properties that are not structured by how we present them.

5 Conclusion

We have made use of an analogy with mathematical structuralism in order to characterize what we call minimal scientific structuralism. On this account:

- A theory is characterized by the collection of its models, and the kinds of objects that the theory talks about are presented through the shared structure of those models.
- The applicability of the high level theory to the low level data is expressed in terms of the shared structure between their models.
- A relationship of structural continuity between predecessor and successor theories is expressed in terms of the shared structure between the models of the two theories.

No further analyses are needed for meeting the challenges facing the minimal scientific structuralist – by appealing to the shared structure of models we can characterize scientific theories in 'structural' terms, and use this characterization to explain the role of models in accounts of applicability, and to address the relationship of 'structural continuity' between predecessor and successor theories. In particular, we need no analyses in terms of specific types of morphisms or specific types of 'structure'. To account, however, for the connection between the theory and the world one must move past minimal scientific structuralism; here the issue of representation becomes crucial and so more than a methodological stance must be adopted. Just how such representation is to be accomplished and what justification we might give for believing that it is, is what divides scientific structuralism into its different varieties.

The empirical stance, taken by van Fraassen, simply asserts the identity of the data models and the phenomena. The neo-Kantian option, as one might reconstruct it from the writings of, say, Poinçare, has yet to be worked out in any informative way. And finally, the realist stance, adopted by the structural realist, offers only the ‘no miracles’ argument as evidence for the claim that the structure of the data models is shared by the structure of the phenomena. In any case, neither the framework of the semantic view of theories nor the appeal to shared structure alone offers the scientific structuralist a quick route to representation.

As things stand, without the needed justification, we advocate adopting a methodological stance towards a minimal construal of both scientific structuralism and structural realism; we embrace the strategy of seeking out, exploring and exploiting the notion of *the appropriate kind of shared structure*, both up and down the hierarchy, and sideways across models of the same, different, and successive theories. Of course, to account for the success of scientific representation, one can chose to take whatever additional stance one likes, but a stance itself is not a justification.

References:

Bokulich, A., [2003], "Horizontal Models: From Bakers to Cats", *Philosophy of Science*, Vol. 70, 3, pp. 609-627.

Cartwright, N., [1983], *How the Laws of Physics Lie*, Oxford University Press, Oxford.

— [2000], *The Dappled World: A Study of the Boundaries of Science*, Cambridge University Press, Cambridge.

Cartwright, N., Shomar, T., and Suarez, M., [1995], "The Toolbox of Science", in Herfel, W., Krajewski., W., Niiniluoto, I., and Wojcicki, R., (eds.), *Theories and Models in Scientific Progress*, Rodopoi, Amsterdam, pp. 137-149.

Da Costa, N.N.A., and Chauqui, R., [1988], "On Suppes' Set Theoretical Predicates", Erkenntnis, Vol. 29, pp. 95-112.

Da Costa, N.C.A., and French [1990], "The Model-Theoretic Approach in the Philosophy of Science", Philosophy of Science, 57, pp. 248-265.

Da Costa, N.C.A., Bueno, O., and French, S., [1997], "Suppes Predicates for Space-Time", Synthese, Vol. 112, pp. 271-279.

Dummett, M., [1991], *FREGE Philosophy of Mathematics*, Harvard University Press, Massachusetts.

Fraassen, van, B., [1970], "On the Extension of Beth's Semantics of Physical Theories", *Philosophy of Science*, 37, pp. 325-339.

— [1971], *Formal Semantics and Logic* , Macmillan, New York.

— [1972], "A Formal Approach to the Philosophy of Science", in Colodny, R.G., (ed.), *Paradigms and Paradoxes*, Pittsburgh University Press, Pittsburgh, pp. 303-366.

— [1980], *The Scientific Image*, Oxford University Press, Oxford.

— [1987], "The Semantic Approach to Scientific Theories" in Nersessian, N.J., (ed.) *The Process of Science*, Martinus Nijhoff, Dordrecht.

— [1989], *Laws and Symmetry*, Oxford University Press, Oxford.

— [1997] "Structure and perspective: philosophical perplexity and paradox", in Dalla Chiara, M.L., et al. (eds.) *Logic and Scientific Methods*, Kluwer, Dordrecht, pp. 511-530

— [2002], *The Empirical Stance*, Yale University Press, New Haven.

French, S., [1999], "Theories, Models and Structures: Thirty Years On", in *Philosophy of Science, Supplement: Proceedings of PSA* 1998.

— [1999], "Models and Mathematics in Physics: The Role of Group Theory", in J. Butterfield and C. Pagonis (eds.), *From Physics to Philosophy*, Cambridge University Press, Cambridge, pp. 187-207.

— [2000], "The Reasonable Effectiveness of Mathematics: Partial Structures and the Application of Group Theory to Physics", Synthese 125, pp. 103-120.

French, S. and Ladyman, J., [1999], "Reinflating the Model-Theoretic Approach", *International Studies in the Philosophy of Science* 13, pp. 99-117

Giere, R. N., [1985], "Constructive Realism", in P. Churchland and C. Hooker (eds.), *Images of Science*, University of Chicago Press, Chicago, pp. 75-98.

— [1988], *Explaining Science*, University of Chicago Press, Chicago.

— [1995], "Viewing Science", in D. Hull, H. Forbes and R. H. Buriam (eds.), *Philosophy of Science*, Vol. 2, pp. 3-16.

Gödel., K, [1947], “What is Cantor’s continuum problem?”, revised and expanded version in Benacerraf, P., and Putnam, H., (eds.), Philosophy of Mathematics, 2nd Ed., [1983], Cambridge University Press, New York, pp. 470-485.

Jones, M., [2005], “Models, the Semantic View, and Scientific Representation”, PSA 2004.

Ladyman, J., [1998], “What is Structural Realism”, *Studies in the History and Philosophy of Science*, Vol., 29, No., 3, pp. 409-424.

Landry, E., [1999], “Category Theory: The Language of Mathematics”, *Philosophy of Science* 66 (Proceedings): S14-S27.

— [2001], “Logicism, Structuralism and Objectivity”, *Topoi*: Special Issue – Mathematical Practice, Volume 20, pp. 79-95.

Landry E., and Marquis, J-P., “Categories in Context: Historical, Foundational and Philosophical”, *Philosophia Mathematica*, Volume, 13 (1), pp. xx

Lloyd, E., [1988], *The Structure and Confirmation of Evolutionary Theory*, Princeton University Press, Princeton

Mackey, G. W., [1993], ‘The Mathematical Papers’, in A. S. Wightman (ed.), *The Collected Works of Eugene Paul Wigner*, Part A, The Scientific Papers, Vol. 1, Springer-Verlag, pp. 241-90.

Morgan, M.S., and Morrison, M., (eds.), [1999], *Models as Mediators*, Cambridge University Press, Cambridge.

Niiniluoto, I., [1999], *Critical Scientific Realism*, Oxford University Press, Oxford.

Psillos, S., [1995], “Is Structural Realism the Best of Both Worlds”, *Dialectica* 49, pp. 15-46.

— [1999], *Scientific Realism: How Science Tracks Truth*, Routledge, London.

— [2001], "Is Structural Realism Possible?", *Philosophy of Science,* (Supplement) Vol. 68, pp. 13-24

— [2005], "*The* Structure, the *Whole* Structure and Nothing *but* the Structure?", PSA 2004.

Quine, W.V.O., [1980], From a Logical Point of View, 2nd Ed., Harvard University Press, Cambridge.

Redhead, M. L. G., [1980], "Models in Physics", *British Journal for the Philosophy of Science*, 31, pp. 145-163.

— [1987], *Incompleteness, Nonlocality and Realism*, Oxford University Press, Oxford.

Suarez., M, [2003], "Scientific Representation: Against Similarity and Isomorphism", *International Studies in the Philosophy of Science*, Vol.17, 3, pp. 225-244.

— [2005], "An Inferential Conception of Scientific Representation", *Philosophy of Science*, (Supplement), *forthcoming*.

Saunders, [1993], "To what physics corresponds" in *Correspondence, Invariance, and Heuristics; Essays in Honour of Heinz Post*, S. French and H. Kaminga, (eds.), Kluwer, pp. 295-326.

Sneed, J.D., [1981], *The Logical Structure of Mathematical Physics*, Reidel, Dordrecht.

Stegmüller, W., [1979], *The Structuralist View of Theories: A Possible Analogue of the Bourbaki Programme in Physical Science*, Springer, Berlin.

Suppe, F., [1974], *The Structure of Scientific Theories*, University of Illinois Press, Chicago.

Suppes, P., [1957], *Introduction to Logic*, van Nosotrand, New York

— [1960], “A Comparison of the Meaning and Uses of Models in Mathematics and the Empirical Sciences”, *Synthese*, Vol. 12, 1960.

— [1962], ‘Models of Data”, Logic Methodology and Philosophy of Science, Stanford.

— “What is a Scientific Theory”, in Morgenbesser, S. (ed.), *Philosophy of Science Today*, Basic Book, New York, pp. 55-67.

— [1969], *Studies in the Methodology and Foundations of Science*, Reidel, New York.

Worrall, J., [1989], “Structural Realism: The Best of Both Worlds”, *Dialectica* 43, pp. 99-124.

Endnotes:

---

[1] Discussions of structuralism in the philosophy of science literature have, quite naturally, centered on those sciences, like physics, that are formulated in mathematical terms; ours will do the same.

[2] For example, supporters of syntactic view sought to use their characterization of a scientific theory, as a partially interpreted calculus together with correspondence rules, to establish a theory-world connection in terms of their shared 'logical structure'. Those advancing the semantic view, as set-out by, say, Suppes, wherein a theory is a collection of (Tarskian) models, sought to likewise use their characterization to establish a theory-world connection but in terms their shared 'set-structure', i.e., in terms of the isomorphisms between their respective models.

[3] Note that we have changed the slogan of mathematical structuralism from 'mathematics is about *structures* and their morphology' to 'mathematics is about *structured systems* and their morphology'. This shift reflects our view that the aim of the structuralist is to account for the shared structure of *systems*, as opposed to answering such questions as: "What is a structure?" or "What kind/type of system is constitutive of what a structure is?". We thus adopt an *in re*, as opposed to an *ante rem*, interpretation of both mathematical and scientific structuralism. (See Landry and Marquis [2005] for more on this distinction.)

[4] Our use of the term 'frame' is intended to accord with Hilbert's claim that "… it is certainly obvious that every theory is only a scaffolding or schema of concepts together with their necessary relations to one another, and that the basic elements can be though of in any way one likes. …. One only needs to apply a reversible one-one transformation and lay it down that the axioms shall be correspondingly the same for all transformed things". (Hilbert [1899], pp. 40-41) And too our use of the term 'system' is to be understood in light of Hilbert's use of the term 'axiom system', as implemented in his *Grundlagen*. Bernays best sums this use as follows: "[we understand] the assertions (theorems) of the axiomatized theory in a hypothetical sense, that is, as holding true for any interpretation… for which the axioms are satisfied. Thus, an axiom system is regarded not as a system of statements about a subject matter but as a system of conditions for what might be called a relational structure". (Bernays [1967], p. 497)

[5] The use of the term 'concrete' is meant to indicate that there are two levels of mathematical structuralism. One that considers *concrete* mathematical systems (e.g., models) as instances (e.g., interpretations) of the same *kind* of structured system and one that considers *abstract* mathematical systems as instances of the same *type* of structured system. In the philosophy of mathematics literature the former is known as model-structuralism, while the latter is know as pure or abstract-structuralism. (See Hale [1996]) For example, at the concrete level, one could, as explained above, consider some set-structured systems as instances of the same kind (as isomorphic models) of the Natural-Number structure. At the abstract level, in contrast, one could consider, *à la* Bourbaki, all group-structured systems as instances of the same type, for example, as instances of the same type of set-structured system. For our purposes, we will limit our focus to concrete structured systems, or simply, to models. (See Landry and Marquis [2005] for a detailed discussion of the levels, varieties and interpretations of mathematical structuralism.)

[6] That is, a model of a set of sentences (where 'a set of sentences' is understood as 'a system of conditions' in Bernays' sense above) is an interpretation of a formalized language for which that set of sentences is true.

[7] In reference, then, to Benacerraf [1965], mathematical structuralism at this concrete level implies that there are no numbers as particular objects, i.e., as independently existing things whose 'essence' can be individuated independently of the role they play in a structured system of a given kind. There are, in our terminology, only numbers as *kinds of* objects, i.e., there are 'objects' that can be individuated, only up to isomorphism, as positions in a given kind of structured system.

[8] See Dummett [1991], p. 295 for a discussion of the problems of taking either 'structured system' or 'model' to mean 'structure'. While Dummett's analysis here is, in some sense, helpful, it confuses abstract and pure accounts of structuralism (again, see Hale [1996]), i.e., it confuses accounts that presume that abstract structures as themselves kinds of objects must be presented as positions in (higher) types of abstract structured systems with accounts that presume that abstract structures, as independently existing objects, must be "made up" of abstractly considered concrete kinds of objects, like sets. As already explained, structured systems *qua* models can be used to account for the shared structure of a *concrete kind* of structured system, e.g., for the shared structure of the elements and/or properties of natural numbers *qua* set-structured systems. However, at an abstract level, structured systems *qua* schematic types can also be

used to account for the shared structure of *abstract kinds* of structured systems. (See Landry and Marquis [2005] for a discussion of these issues and for what is meant by considering a structured system, at an abstract level, as a schematic type.)

[9] To say that the axioms provide a framework is not intended to be read as the 'formalist' claim that a theory ought to be viewed syntactically as an empty form or an uninterpreted calculus devoid of content. One could equally interpret this semantically; one could say that, though framed by its axioms, a theory is characterized by the collection of all its isomorphic models, i.e., by the collection of all its concrete structured systems that have the same kind of structure. So, for the mathematical structuralist, there need be no sharp divide between the syntactic and semantic view of theories. One must thus keep distinct the structuralist claim, that a mathematical theory is about a *kind of* object from both the formalist claim that a mathematical theory is about contentless form and the essentialist/Fregean claim that a mathematical theory is about independently existing (particular) objects. (See also Benacerraf's [1965], pp. 285-292, distinction between the formalist, the Fregean, and what he calls the 'formist'.)

[10] Note that these "conditions of coherence" need not be formally specified, e.g., they need not be analysed, as Hilbert desired, in terms of a logical notion of consistency, or an in terms of an axiomatic notion of coherence (Shapiro [1997]), but rather, may be taken as more in line with Gödel [1947], p. 477, who forgoes their formal analysis in terms of, say, "intrinsically necessary" in favor of more informal notions like success, fruitfulness, simplicity, etc.

[11] To remain agnostic about whether and/or how such representations need 'save the phenomena' or 'get a hold on reality', i.e., to remain agnostic about whether and/or how theories, to be successful, need be 'empirically adequate' or 'true', we leave the notion of success as unanalyzed.

[12] See Da Costa, Bueno and French, [1997]

[13] We say *may* forego such axiomatization since not all accounts of the semantic view do this. For example, both Suppes [1957; 1960; 1962], and Da Costa, French [1990] presume that a theory, even if characterized by its models, must be framed by a set-theoretic axiomatization/predicate. Yet, for the purposes of accounting for applicability, Suppes claims that "[t]he important distinction that we shall need is that a theory is a linguistic entity consisting of a set of sentences and models are non-linguistic entities in which the theory is satisfied (an exact definition of theories is also not necessary for our uses here)". (Suppes, [1960], p. 290)

[14] Clearly, the syntactic view of theories also admits talk of models; the crucial difference is that, for advocates of the syntactic approach, a theory *is* an axiomatic system (or set of sentences) *plus* correspondence rules and so its models (interpretations that make the set of sentences true) have only a mediate role to play in establishing the theory-world connection; even if models are used to interpret theoretical (and, perhaps observational) sentences, there still remains a gap between the types of sentences and so correspondence rules are needed to fill this gap.

[15] For accounts of a scientific theory as a family of structures see van Fraassen [1980] and French, [1999; 2000].

[16] The use of such a structuralist view of a mathematical theory for presenting scientific theories is well expressed by Weyl [1949], pp. 25-27: "…[a mathematical theory as] an axiom system is a logical mold of possible sciences… A science can determine its domain of investigation up to an isomorphic mapping. In particular it remains quite indifferent as to the "essence" of its objects…". It is this point that speaks against Psillos' [1995; 2001; 2005] position that a structuralist (or structural realist) cannot separate structure/form from nature/content. It is not that the structuralist is committed to *denying* that objects have an essence/a nature/content, it is that he, when presenting them mathematically, remains *indifferent* to such and so places his focus on their shared kind of structure. It is, in part, this point that we intend to capture by claiming that an object so presented is a kind of object and not a particular.

[17] Note that, to allow for talk of surplus structure, in the sense of Redhead [1980], we have changed to talking about shared structure. That is, while models of a mathematical theory have the same kind of structure, models of a physical theory, in so far as they are 'mathematical models' (again in the sense of Redhead), might have more structure than the kind needed to apply to the objects that the physical theory is intended to talk about. So the mathematical and physical models share a kind of structure in that the latter is embeddable in the former.

[18] See Suppes [1962] for a description of the "criteria of evaluation" and, particularly, see p. 259 for a list of the "typical problems" associated with each.

[19] See Suppes' [1967], p. 59 claim that "[t]he definition of isomorphism of models in the given context makes the intuitive idea of *same structure* precise". In an earlier work [1960] Suppes also states that once the [empirical] theory is axiomatized within a standard set-theoretical framework, the mathematical methods of using "representation theorems" and "embedding theorems" to capture facts about isomorphisms can be extended from mathematics to the empirical sciences.

[20] Van Fraassen makes a similar distinction, viz., "the point long emphasized by Patrick Suppes that the theory is not confronted with the raw data [again, what we call the phenomena] but with models of the data, and the construction of [the theory of] these data models is a sophisticated and creative process". (van Fraassen [1989], p.229) However, he then collapses this distinction by claiming that models of data are "the dress in which the debutante phenomena make their debut" (Ibid.). This identification of data models and phenomena will be further considered in section 4.

[21] See French [2000], p. 116-117 for his distinction between the empiricist and realist stance.

[22] See Worrall [1989], p. 148 for a further discussion of how Newtonian and Einsteinian gravitational theories directly contradict each other in important respects.

[23] See Da Costa, Bueno, French, [1997], p. 276

[24] Da Costa, Chuaqui [1988], Da Costa, French [1990] and Da Costa, Bueno, French, [1997], all seek to provide such accounts; they seek to provide an abstract analysis of what a structure is, and insofar as a theory is a family of structure, to thereby provide an abstract analysis of what theory is, by analyzing both in set-theoretic terms. This, as explicitly claimed in Da Costa, French [1990], so as to be in-line with both Bourbaki's set-theoretic account of mathematical structuralism wherein a kind of structure is a type of set-structured system and Suppes' [1957; 1960; 1962] slogan that "To axiomatize a theory is to define a set-theoretical predicate" (p. 249).

[25] This is a simplification. Using solutions to the the two-body problem to characterize the kind 'Newtonian inertial-gravitational' is problematic because the two-body problem is a special case, the *n*-body problem not being soluble in general. This does not, however, affect the principle by which the shared structure of solutions is related to kinds of objects.

[26] That is, we require that the relative speed between the two frames of reference is much less than the speed of light, and that the distance between the two observers, each at rest with respect to their respective coordinate systems, is small.

[27] We will have more to say shortly on the differences of what is meant by 'the phenomena'.

[28] Of course, a great deal of both theoretical and experimental work needs to be done in order to arrive at the data models, but this is not what we are concerned with here. Nor is our concern with the particular strategies of how we 'structure' the data (e.g.,. bottom-up, top-down, or even boot-strapping strategies). While no doubt (as Cartwright, Suarez, Giere, etc., have pointed out) such strategic investigations are necessary for the practical problem of constructing the hierarchy, we can nevertheless, once this hierarchy is so constructed, place the theoretical hierarchy above the data models and so consider the connection between a theory and the data models.

[29] To appreciate the same point, though expressed differently, see Giere's [1995] discussion of why Tarskian semantics is not appropriate for the representational role of models of physical theories. Note, too that even though Ladyman [1998] accepts that the semantic view is the most appropriate frame for the structural realist position, he agrees with Giere that Tarskian semantics cannot do the job of closing the gap between the theory and the world. As well, while Ladyman favors the use of the notion of partial isomorphism over Giere's notion of similarity to account for the shared structure of theoretical and data models, he further agrees with Giere that "that once the semantic approach is adopted the crucial issue is whether or not theoretical models tell us about *modalities*" (Ladyman [1998], p. 416.). We will have more to say about Ladyman's modal stance in Section 4.

[30] By the ontological 'is' we do not mean the metaphysical 'is' that ranges over the noumena; we are happy for this 'is' to range over just the phenomena, and too we allow for the phenomena be observable or unobservable. That is, we take no stand on the Kantian realism/idealism debate or the realism/constructive empiricism/instrumentalism debates.

[31] Taking this challenge to close the gap between theories and the world as being met by an account of truth (as opposed to being met by an account of representation) that would serve to fill-out the Tarskian notion of truth and so lend itself to the realist-empiricist debate, Da Costa and French [1990], p. 251 note various approaches that appeal to additional aspects of truth that can then be used to close this gap, e.g., Putnam's warranted assertability criterion, Fine's NOA, and their own pragmatic position. In contrast to such truth-

seeking approaches, examples of representational approaches include Giere [1988], van Fraassen [1989], and Hugh's [1999] supposition that 'theoretical hypotheses' provide the needed representational bridge by asserting some sort of correspondence (e.g., similarity or isomorphism) between the physical system under investigation and some part of at least one of the theoretical models. Such representational approaches also include accounts, like Cartwright et al, [1995], which seek to begin with phenomenological models and build-up representational relations by, for example, foregoing notions of similarity or isomorphism (see Suarez [2003]), and instead considering the inference patterns amongst these and theoretical models (see Suarez [2005]).

[32] Indeed, one could argue that at least the logical positivists saw the need for such even if their notion of cognitive significance was not up to the task.

[33] See Bokulich's [2004] for an excellent account of how analyses of horizontal models, i.e., models that are developed by way of analogy with models of a neighbouring theory (p. 623), are just as significant for picking out the appropriate kind of structure as are models of the theory and/or models of the data.

[34] van Fraassen's perspectival 'I' may be seen as providing the underpinning for such an account but how and/or why this relates to a structural account of science is left unanswered. That is, even if 'first-person' philosophy is called for, it must be explained how/why this then allows us to *identify* phenomena with data models; what notion of identity is called into play here?

[35] In contrast, in the neo-Kantian case, shared structure between the data models and the phenomena is a *consequence* of the representation being so constructed; it is not the basis upon which representation is achieved.

[36] Epistemic structural realism is more in line with the neo-Kantian option; it says nothing about the way the world is structured, but rather concerns itself with how we come to know its structure, i.e., it says we come to know the world by the structure of the phenomena.

[37] Notice, then, that while structuralism (which is about kinds of objects) does not imply relationalism (which is about particular objects, and claims that they are entirely characterized by their relations to one another), ontological structural realism (as characterized above) is committed to both.

[38] See Ladyman's [1998], p. 418 claim that "the abstract mathematical structures it [the theoretical parts of a theory] employs … must have some grip on reality. It is clear that the 'grip on reality' in question must go beyond a correct description of the actual phenomena to the representation of modal relations between them". Further detail of Ladyman's modal account was provided at the *Structuralism in Physics* workshop, in Florence 2003. See also Saunders [1993], p. 320, who suggestively remarks that "it has long been apparent that no workable account of nomological necessity can be made out at the level of unstructured particulars (except in the context of an unfathomable and antiquated notion of the 'rule of law')".