Babylonian astronomy: a new understanding of column Φ

Schematic astronomy, old prediction rules, riddles, loose ends, and new ideas

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Abstract

The most discussed and mysterious column within the Babylonian astronomy is column Φ. It is closely connected to the lunar velocity and to the duration of the Saros. This paper presents new ideas for the development and interpretation of column Φ. It combines the excellent Goal-Year method (for the prediction of Lunar Six time intervals) with old ideas and practices from the "schematic astronomy". Inspired by the old "TU11" rule for prediction of times of lunar eclipses, it proposes that column Φ, in a similar way, used the sum of the Lunar Four to predict times of lunar eclipses as well as the duration of one, 6, and 12 months by means of what usually is called "R–S" schemes. It also explains fully the structure and development of such schemes, a fact that strongly supports the new interpretation of column Φ.

1 Preliminaries for readers not familiar with Babylonian astronomy

This paper makes frequent reference to the following sources for Babylonian astronomy. They are found in a procedure text, TU 11, and in two collections of cuneiform tablets, Enūma Anu Enlil and MUL.APIN.

Enūma Anu Enlil (henceforth **EAE**) is a compilation of astral omens and related material that has early roots, in part back to the third millennium BC. The most important tablet for this paper is **EAE 14**; it existed around 700 BC; but we do not know when it was composed. It is not an omen tablet but contains astronomical quantities approximated by numbers and written in schemes. **MUL.APIN** is a more advanced astronomical handbook but uses similar numerical schemes. It is more

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elaborate and comprehensive than EAE, and it was composed sometime between 1200 and 750 BC (Hunger and Steele 2019).

TU 11 is a procedure text written in Uruk towards the end of the third century BC and published as No. 11 in "Tablettes d'Uruk" (Thureau-Dangin 1922). The tablet is a copy of an older tablet, but we do not know when that was composed. Parallel tablets dating back to the fifth century BC have been found in the British Museum. TU 11 contains a variety of early prediction rules and methods for finding lunar events. Most methods were introduced by means of calculated examples. Some were based on EAE numbers; others give advice of how to make predictions based on observations (Brack-Bernsen and Hunger 2002). Two prediction rules are used extensively in this paper: the **TU 11 prediction method** for Lunar eclipses, which was probably developed around 650 BC, and the **Goal-Year method** which was in practice since 600 BC.

The **Babylonian** lunisolar **calendar** was regulated by astronomical events. Each day started at sunset, a new month began in the particular evening in which the new crescent became visible for the first time after the conjunction of sun and moon. The year had mostly 12 months, but around every third year a thirteen month was inserted, so that month I (Nisanu) always started near the spring equinox. Note that the day number within each month had information about the phase of the moon: its first quarter would occur around day 7, full moon around day 15, the second quarter around day 22, and the moon would be invisible on the last day, i.e. the day of conjunction between sun and moon. **Years** in the cuneiform texts are usually Babylonian calendar years. Beginning from the later fourth century BC, they were counted by numbers of the so-called Seleucid Era (**SE**). Year 0 SE started in the spring of 311 BC, and from then on the years were counted continuously.

In Mesopotamia, time intervals were measured in time degrees UŠ. **UŠ** is, in modern terms, a measure of how far the celestial sphere has rotated during the time interval in question. One whole revolution of the celestial sphere takes 24 h. Therefore, 24 h equal 360°, 1 h equals 15° = 15 UŠ, and 4 min equal 1 UŠ.

In EAE and MUL.APIN, time intervals smaller than a day were measured in bēru, UŠ, and NINDA, where 12 bēru = 24 h, 30 UŠ = 1 bēru, and 60 NINDA = 1 UŠ.

In both compendia, time intervals were also measured in mina and shekel, where 1 mina = 60 shekel and 1 shekel ≈ 1 UŠ.

The Babylonians used a **sexagesimal number system** for their astronomical calculations. This is a positional system with basis 60, just as our decimal system with has basis 10. We follow the convention of Neugebauer, which separates sexagesimal digits with commas except that a semicolon separates units from the sixtieths, so that, for example, 2.13; 20 equals $2 \times 60 + 13 + 20/60$.

In order to give an impression of the schematic astronomy, some of the numerical schemes are reproduced in Table 1 in a condensed and simple form which exclusively uses the time unit UŠ. Roman numbers represent the Babylonian month names, and the duration of day and night and the retardation of the moon are given for day 15 of each month, i.e. the day of full moon. The first four columns are a compilation of tables C and D from EAE. The last column gives the time shift calculated according to the TU 11 prediction rule for lunar eclipses.

Table 1 A condensed compilation of some of the numerical schemes from the schematic astronomy

Month day 15	Day length	Night length	Retardation of the moon = 1/15 night length	Time shift after 1 Saros = 1,0 + 1/3 night length
I	3,20	2,40	10; 40	1,53; 20
II	3,40	2,20	9; 20	1,46; 40
III	4,00	2,00	8; 00	1,40; 00
IV	3,40	2,20	9; 20	1,46; 40
V	3,20	2,40	10; 40	1,53; 20
VI	3,00	3,00	12; 00	2,00; 00
VII	2,40	3,20	13; 20	2,06; 40
VIII	2,20	3,40	14; 40	2,13; 20
IX	2,00	4,00	16; 00	2,20; 00
X	2,20	3,40	14; 40	2,13; 20
XI	2,40	3,20	13; 20	2,06; 40
XII	3,00	3,00	12; 00	2,00; 00

The duration of day and night is from EAE tables C and D. The daily retardation of the moon and the time shift of eclipses after 1 Saros are derived from the length of the night. All times are given the unit UŠ

Note that the numbers in each column differ from month to month (i.e. from line to line) by a constant difference, increasing or decreasing between fixed limiting values. Such numerical functions are called linear zigzag functions. The TU 11 prediction rule leads to the linear zigzag function X with a minimum of 1.40 UŠ and a maximum of 2.20 UŠ as shown in the last column.

Lunar Eclipses reoccur under similar conditions after one **Saros**.

The **Saros = 223 lunar months ≈ 18 Years** is an important period, known and used by the Babylonian astronomers at least since the seventh century BC.

A **Saros scheme** for Lunar eclipses is a matrix in which the Babylonians had written year and month of those full moons which were in danger of becoming eclipsed. Not all lunar eclipses in such a matrix would have been visible from Mesopotamia; therefore, we call henceforth all dates in the scheme **Eclipse Possibilities,** or **EPs**. Each column of the scheme covers one Saros = 18 years and has 38 lines with the 38 EPs of that Saros. There are 19 EPs with the moon close to the ascending node and 19 EPs near the descending node. Table 2 reproduces a small part of the whole scheme with 24 Saros cycles, as reconstructed by J. Steele (2000). Each line of the scheme lists a series of eclipses (or EPs) taking place at distances of 1 Saros.

On the edge of some Saros scheme fragments, a few numbers were written. The number N in front of a line gives the approximate time shift from one column to the next one, i.e. from one eclipse to the next one expected 1 Saros later.

An example may illustrate how such predictions work: if the beginning of an eclipse was observed at time T on 19 April year -712, then the eclipse expected to occur on 1 May -694 could be predicted to begin at time $T + 1.50$ UŠ. It turned out that the early Babylonian astronomers had developed a "simple 18-year function" for the prediction of times of coming eclipses. A numerical

|	EPs	Saros cycle 1	Saros cycle 2	Saros cycle 3
Table 2 A small part of the reconstructed Saros scheme for Lunar eclipses				
1,40	1	− 746 Feb 6	− 728 Feb 17	− 710 Feb 27
2,10	2	− 746 Aug 2	− 728 Aug 12	− 710 Aug 23
…	3	…	…	…
…	…	…	…	…
1,50	35	− 730 Apr 9	− 712 Apr 19	− 694 May 1
	36	− 730 Oct 2	− 712 Oct 13	− 694 Oct 24
	37	− 729 Mar 30	− 711 Apr 9	− 693 Apr 20
	38	− 729 Sep 22	− 711 Oct 2	− 693 Oct 13

zigzag function found on cuneiform tablets attests to such a practice. Also, the TU 11 method introduced above will lead to such a "18-year function": for the 19 EPs taking place near descending (or ascending) node, the time shift can be calculated by the TU 11 method. This leads to a linear zigzag function with the period 18 years, maximum 2,20 UŠ and minimum 1,40 UŠ. It is more precise but agrees well with the other numerical function (Brack-Bernsen and Steele 2005). This TU 11 Saros function is, of course, derived as a function of the month; however, it tabulates time shifts not month by month as our zigzag function X, but in steps of 12 or 11 months.

The most remarkable achievement of Babylonian astronomy consists of the **Lunar table texts.** They origin from the last three centuries BC. A system A table text combines skilfully the effects of all variables that determine the conditions at full or old/new moon. A typical table text will have 12 or 13 lines, one for each month within a year, and up to 13 or 19 columns recording the numerical functions necessary for the calculations. Neugebauer (1955) identified the columns of a system A table text by the following letters:

$$T, \Phi, B, C, E, \Psi, F, G, J, C', K, M, P$$

The astronomical significance of some of the columns is the following. In the first column, T is the independent variable, namely the year and month in the Seleucid era. Each line of the table contains in the following columns the astronomical functions for the month T. The next **column Φ is the main concern of this paper**. In the following columns, **B** is the position $\lambda_{\mathbb{C}}$ of the moon in the ecliptic, **C** is the duration of daylight, **E** is the latitude $\beta_{\mathbb{C}}$ of the moon, **F** is its velocity $v_{\mathbb{C}}$, **M** is the time of conjunction or opposition, and **P** is the time between rising or setting of sun and moon at full or new moon. The duration of the synodic month is calculated as a sum of two components: **G** is the contribution depending on the lunar velocity and **J** depending on the position of the moon in the ecliptic. Columns G and F are both intimately connected with column Φ.

Column Φ yields a linear zigzag function with maximum = 2,17; 4,48,53,20 UŠ, minimum = 1,57; 47,57,46,40 UŠ, and a period P$_\Phi$ slightly less than 14 months. If the Φ numbers are mapped in a coordinate system as in Fig. 1, the discrete Φ points will be placed equidistantly on a linear zigzag function, returning back after 6247
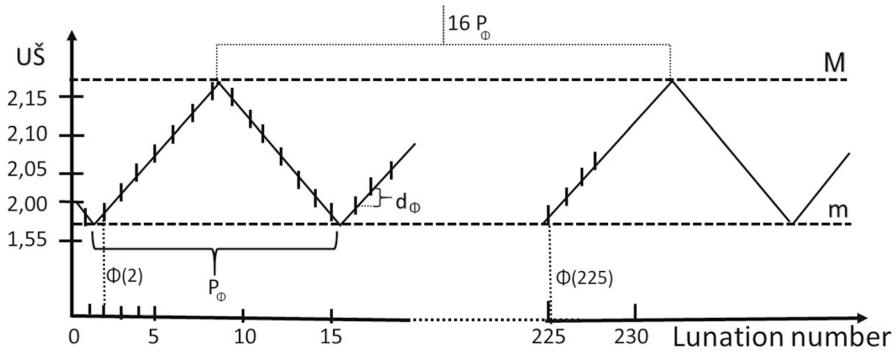
Fig. 1 Column Φ plotted versus the month (lunation) number L. The small vertical intervals represent successive numbers in column Φ, through which we have drawn straight lines. Half the numbers lie on ascending branches and the other half on descending branches

points and exactly 448 zigzags. Only after 6247 months = 448 Periods P_Φ, the numbers will repeat. 448 P_Φ = 6247 months $\cong$ 505 Years.

The period P_Φ of Φ equals that of F, i.e. P_F, which is the period of the lunar velocity measured once each month at full or new moon, respectively. Φ measures the time shift between lunar eclipses situated 1 Saros = 223 months apart. To be more precise, Φ gives the lunar contribution to the time shift, while the larger contribution depending on the solar velocity has not been found. There are many open questions to this enigmatic column Φ. Accordingly, there have been several attempts to interpret and reconstruct column Φ.

This paper combines old prediction methods and techniques and results in a new understanding of column Φ. Our new understanding of column Φ answers many questions about number schemes, and it gives advice for finding the lunar contribution to the duration of 1, 6, and 12 months, respectively. It gives a natural explanation of the development and structure of such schemes and calculations.

For the calculation schemes, the extrema of the zigzag function in Fig. 1 was cut off: at the top by 2,13; 20 UŠ and at the bottom at the values of 1; 58,31, 6, 40 UŠ. This truncated version $\bar{\Phi}$ of Φ, visualized like in Fig. 1, would have the peaks removed and replaced by horizontal lines at the levels of 2,13; 20 UŠ and 1; 58,31, 6, 40 UŠ, respectively. According to our hypothesis, the numerical linear zigzag function Φ is developed as a numerical fit to observed data. Maximum and minimum of those data are around 2,15 UŠ and 1,59 UŠ, respectively.

2 Introduction

The mathematical lunar table texts of system A start with the quite enigmatic column Φ.[1] It is connected to the duration of the Saros, an eclipse period of 223 synodic months $\cong$ 18 years. Neugebauer writes (1955, p. 44) that column Φ must describe a phenomenon very closely related to the lunar velocity, and in HAMA (1975, p. 505) he summarizes the result of joint research (with Aaboe, van der Waerden, Britton, Neugebauer, and Sachs) in the definition:

$$1 \text{ Saros} = 223 \text{ syn. M.} = 6585\text{d} + \Phi$$

counted from the end of the month for which Φ is tabulated. But this understanding does not solve the problem of how to derive Φ from observations, since systematic observations of eclipses situated at distances of 1, 2, 3, 4, 5, 6, Saroi would lead to a function varying in tune with the solar and not with the lunar velocity. Therefore, it is a challenge to reconstruct column Φ from Babylonian observations.[2] Britton (2009) and de Jong (2017) have given their answer to the challenge. This paper presents another answer: it combines the excellent Goal-Year method (for the prediction of Lunar Six time intervals) with older ideas and practices from the "schematic astronomy". And, inspired by the old "TU 11" rule for prediction of times of lunar eclipses, it shall present a new idea for the development and interpretation of column Φ. The proposed connection between Φ, the Goal-Year method, and the Lunar Six delivers a natural derivation of the Babylonian method for finding the duration of 1 and 12 months by means of what we here call "R–S schemes". It also explains fully the structure and development of such schemes, a fact that strongly supports our new interpretation of column Φ.

3 Column Φ and its functions

A lunar system A table text lists astronomical data for consecutive months in consecutive lines. The first column T in these texts lists dates: year number in the Seleucid era and name of month. All following columns list astronomical quantities, calculated for the month given in column T. Some tablets calculate conditions around new moon and others the same around full moon. The second column, Φ, plays a significant role in these texts. All full-moon texts are connectable, and hence datable, through column Φ. And all new-moon texts are in the same way connectable and datable, through column Φ_1.[3] But full- and new-moon texts are not connectable, which means that the numbers in Column Φ of a system A lunar tablet text indicate the type: either full- or new-moon calculations.

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[1] Column Φ lists time intervals measured in time degrees, UŠ, where 360 UŠ = 24 h, so that 1 h = 15 UŠ and 1 UŠ = 4 min.

[2] See Brack-Bernsen (1980, 1990).

[3] ϕ_1 is used by conjunction. It is derived from column Φ, also named Φ_2, which has its origin in observations around full moon.

Table 3 The structure of a R–S scheme for finding out how much the duration of a lunar month has changed from 1 month $(m-1)$ to the next (m)

Line number	R	S	$S-R=\Delta G(m)$	G
	$\Phi(m-1)$	$\Phi(m)$	$\Phi(m)-\Phi(m-1)$	$G(m)$
$k-1$	R_{k-1}	S_{k-1}	$R_{k-1}-S_{k-1}=\Delta G_{k-1}$	G_{k-1}
k	R_k	S_k	$R_k-S_k=\Delta G_k$	$G_k=G_{k-1}+\Delta G_{k-1}$
$K+1$				$G_{k+1}=G_k+\Delta G_k$

This change, ΔG_{k-1}, is found as the difference between the Φ values R_{k-1} and S_{k-1} in the line above. The duration of the month k, $G_k = G_{k-1} + \Delta G_{k-1}$, equals the duration of the month $k-1$ above plus the difference ΔG_{k-1}

Column Φ gives the variable lunar contribution to the length of the Saros. The Saros ($=223$ synodic months) is an eclipse period, which was known to the Babylonians at least since the seventh century BC. The numbers in column Φ can be placed on a linear zigzag function having up to 6 sexagesimal places.[4] They repeat only after $1,44,7 = 6247$ synodic months, which is more than 500 years. Column Φ is in a special way used to find the time of oppositions. The time differences between two consecutive oppositions, $\Delta^1 t$, were calculated as 29 days plus the sum of two components G, a contribution due to the varying lunar velocity, and J, a contribution due to the variable solar velocity.

$$\Delta^1 t = 29^d + G + J$$

The values of G were found from a particular scheme involving two different values R and S of Φ, where R and S measure the Saros length for two consecutive months, respectively. In his book on procedure texts (2012, p. 146) Ossendrijver summarizes how the interpolation scheme for computing G from Φ works. He comments: "*A remarkable and unique feature of lunar system A is that G is computed from Φ. It cannot be stressed enough that there is no obvious and compelling astronomical or mathematical reason for computing the duration of 1 ^{m} from the duration of 223 ^{m}*". He also described the use of the fully developed system, which is based on a truncated version of Φ. Here, we shall concentrate on the long linear parts of G and neglect the details around the extrema: Table 3 illustrates the structure of the R–S scheme. It was constructed as auxiliary table for finding G for an arbitrary month m by means of its Φ value, $\Phi(m)$. Similar schemes were constructed by the Babylonians for finding the lunar contributions W and Λ to the duration $\Delta^6 t$ and $\Delta^{12} t$ of 6 or 12 successive months, respectively. These schemes all have the same structure. The scheme did not list all Φ numbers, but only values of Φ in steps of 1 Saros, so that G in most cases was found through linear interpolation within the R–S scheme. An example may illustrate the functioning of the scheme: you want to find $G(m)$. Take the case where the value $\Phi(m)$ of month m is listed in column S, line k of the

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[4] The sexagesimal number system is a positional system with 60 as basis, just as our decimal system with has 10 as basis. I follow the convention of Neugebauer, so that, for example, 2,13;20 equals 2x60 + 13 + 20/60.

scheme. Then, you find the value $G(m) = G_k$ listed in the last column of the scheme. This value $G(m) = G_k$ for month m is found from the value G_{k-1} in the line above by adding the difference ΔG_{k-1} : $G_k = G_{k-1} + \Delta G_{k-1}$, so that the change in G from line k − 1 to line k of the scheme was the difference: $\Delta G_{k-1} = S_{k-1} - R_{k-1}$.

For all months, for which the value of Φ did not occur, the corresponding value of G was found through interpolation within the R–S scheme. Similarly, the difference in time between two oppositions (or conjunctions) 12 months apart, $\Delta^{12}t$, was found as an entire number of days plus a lunar contribution Λ and a solar contribution Υ:

$$\Delta^{12}t = 354^d + \Lambda + \Upsilon$$

The lunar contribution Λ was here, again, found from a scheme using the difference between two values R and S of Φ. But this time S was the Φ value of the month in question, while R was its value 12 month earlier. The trick behind such a procedure, using the R–S scheme to determine time intervals, is that it delivers a function with the same period P_Φ as Φ, but with another amplitude.

For more information about such number schemes for finding G and Λ functions, see Neugebauer (1975, pp. 505–513), Britton (2009) and Ossendrijver (2012, pp. 125 ff. and 145–155). Britton (2009) has presented a detailed study and a reconstruction of these schemes, using skilled manipulations of numbers in combination with modern algebra. (See pages 376–379 where the cuneiform tablets, BM 36699 + 51262 and BM 36737 for finding G, and BM 36311 for finding Λ and W, have been edited and reconstructed by Britton.)

We thus understand how to calculate times of oppositions as the Babylonian astronomers did it. We know how to use column Φ and the R–S schemes for finding the duration ($\Delta^1 t$, $\Delta^6 t$, and $\Delta^{12}t$) of 1, 6, and 12 months, but we still do not know how these numerical methods and schemes were developed. And we still do not know how column Φ was developed from the observations made by the Babylonian astronomers. It cannot have been found from observation of lunar eclipses situated 1 Saros apart: see Brack-Bernsen (1980), where it was investigated how the length of 1, 12, and 223 synodic months could be found as the sum of two functions, one giving the lunar contribution and the other giving the solar contribution.[5] The duration of a Saros can approximately be written as a constant plus two terms:

$$1\ \text{Saros} = \Delta^{223}t = \text{cons.} + A_\odot + B_\mathbb{C}$$

In case of the Saros, the solar term dominates, which means that one will get a function with the year as period (as $A_\odot$) by comparing durations of Saroi, starting at consecutive lunations. The period of $B_\mathbb{C}$ (i.e. that of Φ) was suppressed by the dominating term $A_\odot$. This means that column Φ could not have been found empirically from observation of times of eclipses.

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[5] At opposition, sun and moon have the same zodiacal longitude. Thereafter, the sun moves around 30°, while the moon, moving around 12 times as fast as the sun, passes through the whole zodiac plus the arc moved by the sun, until it again is at the same length as the sun. Clearly, both velocities—that of the sun and that of the moon—will determine the difference in length and time between two consecutive oppositions.
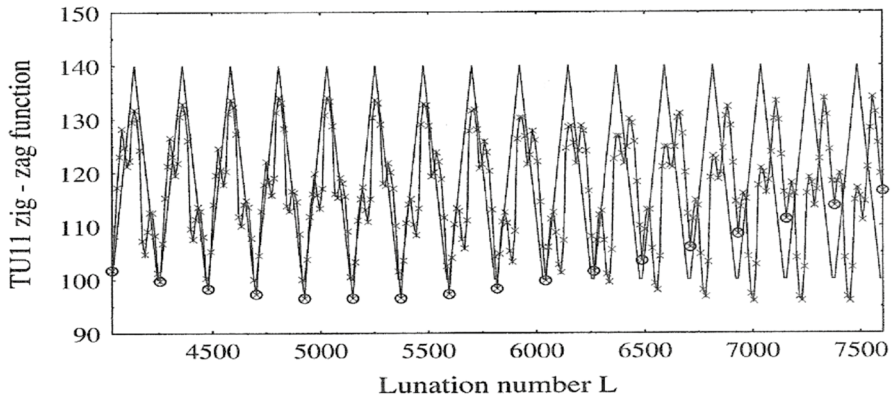
Fig. 2 For a series of eclipse possibilities at descending nodes from the Saros Cycle scheme, the Saros length is compared to the reconstructed TU 11 Saros length zigzag function. The comparison takes place over 16 consecutive Saros, starting with the very first EP of SC 5, 21 March 675 BC and ending with EP 1 on SC 21, 26 April 369 BC. The first EP of each SC is marked by a circle. As independent variable, we use L, i.e. the Goldstine numbers (1973) for the respective lunations. The Saros length, measured from consecutive EPs at descending node, is marked by crosses and connected by a thin line. The TU 11 zig-zag function is drawn with a thick line

Aaboe had in (1968) published several table texts with Φ occurring in R–S schemes for finding the lunar component of $\Delta^1 t$ and $\Delta^{12} t$. He raised similar questions in (1969, p. 16): *"It is clear that the remaining questions … are raised by Φ. The central role of Φ is obvious, and it is now apparent that Φ was in continuous use—in the strong sense that its values computed month by month connect the earliest to the latest texts—since times already before the System A schemes reached their final form. However, I am still at a loss to explain in a satisfactory manner how the amplitude of Φ can be derived from the sort of observations which were recorded by the Babylonians … And there is still the uncomfortable fact that Φ is found side by side with early and primitive solar models, while S^6 as constructed above depends on the fully developed System A solar scheme. We can only hope that the appearance of new texts will help us solve these problems"*.

The tight connection between Φ and the lunar velocity made it evident that one should look for Babylonian observations which contained information on the lunar velocity. The only possible observations that I found were some time intervals (regularly measured around full or new moon) between setting and rising of the sun and the full (or old/new) moon. There are six such time intervals, called the Lunar Six. Two intervals were measured around conjunction, and four intervals, the Lunar Four, were measured around opposition. These time intervals were observed and recorded regularly. Working with computer-simulated Lunar Six data, through trial and error, I finally found that the sum Σ of the Lunar Four oscillated in tune with the Saros and that the linear zigzag function $\hat{\Sigma}$ which approximated the sum Σ optimally, varied

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[6] S means here the proposed solar component of the duration of the Saros, as reconstructed by Aaboe (1969).

with the same amplitude, period, and phase as column Φ. Surprisingly, it turned out that $\Phi - 100°$ was a perfect fit to the observable curve Σ (see Brack-Bernsen 1990, Fig. 2). This has led to the hypothesis that Φ equals $\hat{\Sigma} + 100°$, where $\hat{\Sigma}$ is the linear zigzag function, approximating Σ, shifted by 100°. Note that there is an important difference between the zigzag functions which we find in the schematic astronomy, and those in the later ACT table texts. The former have integer periods, while the latter often have non-integer periods. This poses, however, not a severe problem, since we have indications how such a new type of numerical function may have been developed: namely by successive approximations, as exemplified in the atypical text E which starts with a period that is ½ too long and then corrects it. For further details, see "Appendix 2".

4 Attempts to reconstruct column Φ

Based on the fact that function $\hat{\Sigma}$ had the same period and amplitude as column Φ, I proposed that Column Phi had been derived from the sum of the Lunar Four: $\Phi = 100° + \hat{\Sigma}$, this being a pure empirical zigzag function, perhaps without connection to the duration of the Saros (see Brack-Bernsen 1990). The first thing I noticed, analysing computer-simulated Lunar Four data, was that the linear zigzag function $\hat{\Sigma}$ was in phase with the function F for the lunar velocity of system A, but it had twice the mean value of F.[7] In Brack-Bernsen and Schmidt (1994), the astronomical significance of Σ was analysed by means of spherical astronomy and spherical geometry. The problem of oblique ascension of ecliptic arcs was explained by modern means. We understood the significance of Σ and its connection to F on our premises; but at that time, we were aware of the fact that the Babylonians could neither have known our results nor understood our astronomical interpretation.

In the meantime, it has been demonstrated that the Lunar four were, indeed, observed and treated regularly since the seventh century BC, and we know that they were measured with a high precision, so that such data, indeed, could have been used for the construction of column Φ (see Brack-Bernsen 1997). In addition, we realized that the Babylonians had found a very elegant and efficient method for the prediction of Lunar Six time intervals, namely the Goal-Year method (see Brack-Bernsen 1999, and TU 11, Brack-Bernsen and Hunger 2002). We now understand the Babylonian argumentation, so that we know how and why they used the partial sums of the Lunar Four: $\check{S}\acute{U} + NA$ is the retardation of the setting moon measured at full moon from one morning to the next, and $ME + GE_6$ is the retardation of the rising full moon measured from one evening to the next. Therefore, their sum, $\Sigma = \check{S}\acute{U} + NA + ME + GE_6$ is the retardation of the moon measured over two days, and this was known and understood by the Babylonians. Σ approximates the movement of the moon relative to the sun over two days, which is a good measure for the lunar velocity measured over two days. This explains why F has exactly the same

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[7] $\hat{\Sigma}$ has the maximum 37;30 UŠ, minimum 17;30 UŠ, and mean value 27;30 UŠ. The mean value of F is 13;30 UŠ, which is very close to 13:45 = ½ times 27;30.

phase as Φ and roughly half its mean value. For more information on the Lunar Six and the Goal-Year method for the prediction of Lunar Six, see "Appendix 1".

The tablet TU 11 contains a variety of procedures, one of which is important in this connection: sections 9–12 give rules for the prediction of eclipses, situated 1 Saros apart, by means of calculated examples. Surprisingly, this method, which was based on the early schematic astronomy, was quite good (see Brack-Bernsen and Steele 2005).[8] Later, in this paper we shall see how the «know how» behind the Goal-Year method in connection with this schematic predicting rule can lead to a new understanding of column Φ. As a strong support of this new approach, I see the fact that it gives a natural explanation for the construction and structure of the R–S schemes: how they calculate the lunar components G, W, and Λ to the durations of 1, 6, and 12 months.

There have been several other approaches to reconstruct column Φ from Babylonian observations by Britton (2007, 2009) and de Jong (2017). Although there are good impulses and ideas in these papers, I am convinced that they do not give any final solution. One problem which I see, especially in Britton's reconstruction, is that too much modern mathematics and techniques have been used in the reconstructions—despite the fact that the Babylonian mathematicians had no kind of algebra. A reconstruction of Φ must be possible without the use of algebra and modern techniques.

Another problem which I see is, for instance, that in Britton's reconstruction of column Φ the solar year, with the solar velocity approximated by a step function, plays an important role. Britton postulates that the whole system A was created at once by one genius astronomer, while I think that column Φ was derived early, and that all the other columns were developed over time by several astronomers. One reason for this conviction is that we have an early text S (mentioned above in the quotation of Aaboe) in which values of Φ are given in a scheme where the movement of the sun was described by a more primitive mean-value system.[9] This seems to imply that column Φ was developed independently of the mathematical solar theory. For other questionable points in Britton's reconstruction, see Brack-Bernsen (2020, pp. 163, 164). John Steele (private communication) has drawn my attention to the fact that Text S may have been written as late as 300 BC. This text employs, for instance, abbreviations for the zodiacal signs which were only used after 370 BC. The zodiac was introduced around 450 BC. Text S could be a copy of an older text—which I tend to believe. This would explain the occurrence of the mean-value system for the solar movement—or it may be a newer text which analyses old astronomical data in connection with the fully developed function Φ. The important question in this connection is, if column Φ was developed before and independently of the step function for the solar movement or not. The hypothesis for the development of Φ,

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[8] During the collaboration of this paper, Steele and I decided to use the term "Schematic astronomy" for the early Babylonian astronomy, which are found as number schemes in EAE and MUL.APIN. We chose this name in order to signalize that we see these early mean-value schemes as the beginning of the Babylonian astronomy.

[9] See Text S in Britton (1989, pp. 30 and 31), and Aaboe et al. (1991). Text S comprises the cuneiform tablets BM 36850 and BM 36737.

presented in this paper, makes it independent of the model for the solar velocity; but the hypothesis does not exclude the possibility of a contemporaneous model for the solar velocity. Until now, it was taken for granted that column Φ was constructed under the assumption of a solar velocity of 30° per month, which is the case for the winter months when the sun is in the zodiacal sign from Virgo to the end of Pisces. If so, then it must be pointed out that it would be somewhat inconsistent if column Φ in text S were written next to a column where the solar movement is calculated just by using a mean value for its velocity.

After a conference in Jerusalem 2018, Steve Shnider started a discussion with me about column Φ. He wanted to understand why I was not satisfied with John Britton's reconstruction. (See Shnider 2017, where he gives a survey on the research on column Φ and then presents Britton's reconstruction in a good and comprehensive way.) It developed to become a long and fruitful discussion, where I could present and discuss my understanding and point of view on the developing Babylonian astronomy and Steve could question and argue. He ended up with a proposal how one can connect the Lunar Four to eclipse observations. I was pleased to see his proposal; but still I was a bit sceptical, since he used modern algebra. I want to thank Steve Shnider warmly for the fruitful discussion, since it let me come back to old ideas, which I shall present here.

5 The Lunar Six and the Goal-Year Method

The Goal-Year method (for details see "Appendix 1") is very elegant and was utilized over hundreds of years. A so-called Goal-Year text for a year Y collects all Lunar Six data from year Y-18, the year situated 1 Saros earlier than year Y. An early collection of such data can be found on the table Kambyses 400, collecting all Lunar Six from April 523 BC to April 522 BC (see Kugler 1907 SSB I, pp. 61–75). The Goal-Year tablets were compiled as sources for astronomical predictions for Babylonian years between 241 BC and 74 BC (the goal years being SE 79 to SE 256), see Hunger (2006). This implies that the Goal-Year predicting method had been used at least during 450 years. It connects Lunar Six data situated 1 Saros (1 Saros = 223 synodic months) apart. In order to predict the time $\check{S}\acute{U}_i$ from moonset to sunrise ($\check{S}\acute{U}$) in month i, one just has to take the value $\check{S}\acute{U}_{i-223}$ measured one Saros earlier and add a third of the daily retardation $(\check{S}\acute{U} + NA)_{i-223}$, of the setting moon:

$$\check{S}\acute{U}_i = \check{S}\acute{U}_{i-223} + 1/3(\check{S}\acute{U} + NA)_{i-223}$$

I am convinced that this excellent prediction method must have played an important role in the construction of the columns related to the lunar velocity. We find one important hint to the Goal-Year method in the mathematical astronomical texts of system A, namely that the value of Φ for month M gives the duration of the Saros connected to month M. The month M is the independent variable in a lunar system A table text. It is written in the first column T, and the corresponding value of Φ is written in the next column Φ. All other variables are given in the following columns of this line for month M. In HAMA (1975, p. 505) Neugebauer writes

"The fundamental relation which connects the function Φ with the eclipse cycle of 223 months rests on the definition

$$1 \text{ Saros} = 223 \text{ syn. M.} = 6585^d + \Phi$$

The relation is derived under the assumption that the solar velocity has the round value of $30^{o/m}$". This formulation gives the impression that the Babylonian Lunar theory was derived from theoretical assumptions, like our modern science. I find it more probable that the number schemes and numerical functions were found empirically, by fitting observed values of one observable by sequences of numbers having the same period and amplitude as the observable. Now the Φ number written next to month M in a system A lunar table gives the duration of the Saros starting (or ending) with month M. I see this fact as a "greeting from the Goal-Year method".[10] Remember that the time from sunrise to moonset (ŠÚ_i), in month i, was determined by data measured 1 Saros earlier. The change in visibility time of the moon between the two full moons i and $i-223$ was calculated as $1/3(\text{ŠÚ}+\text{NA})_{i-223}$, where $(\text{ŠÚ}+\text{NA})_{i-223}$ is the daily retardation of the setting full moon, measured 1 Saros earlier than month i. Therefore, I believe that column Φ somehow must have grown out of the Goal-Year method and was developed first, after which the whole system A was developed.

6 Schematic astronomy and the Goal-Year method

Already in the schematic astronomy, periodically varying phenomena were described by numerical functions approximating the period and amplitude of the phenomena in question. The schematic astronomy connected variables with the same period. In EAE XIV and MUL.APIN,[11] the Babylonian month M was used as an independent variable from which many other quantities were derived. For instance, the lengths of day and night given in a number scheme were organized by the Babylonian months. But also, the visibility time of the moon and its daily retardation were derived from the night length, and hence, they were functions of the month M. Let me illustrate how the daily retardation of the moon may have been found by the early astronomers. The time interval during which the moon was visible on the sky was derived from the length of the full-moon night in month M in question: at full moon, the schematic moon was visible the whole night, from sunset to sunrise. At new moon, the moon was invisible; therefore, within 15 days the visibility time of the moon changed from 0 to the whole night N(M). For the first 15 days of month M, the visibility time of the moon increased by N(M)/15 per day. This explains why the Babylonians approximated the daily retardation of the moon in month M by N(M)/15 (the night length at full moon in month M, divided by 15).

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[10] See "Appendix 1".

[11] See Al-Rawi and George (1991), Hunger and Pingree (1989), and Brack-Bernsen (2005).

We have later learned that also the duration of the Saros was derived from the length of the night (TU 11, pp. 80–85). The Saros equals a whole number of days plus approximately 8 h, ∼ 1/3 (day + night). In TU 11, sections 10 and 12, the shift in time $(t - t_0)$ between two lunar eclipses one Saros apart was calculated as $1 + 1/3$ (length of) night. We have called this method the TU 11 predicting method: let t_0 be the time of an observed lunar eclipse, and then the time t of the eclipse expected 1 Saros later was calculated as $t = t_0 + 1$ mina $+ 1/3$ night length, which leads to a linear zigzag function X with minimum 1,40; 00 UŠ $= 100°$ and maximum 2,20; 00 UŠ $= 140°$, since 1 mina equals 60 UŠ $= 1,0$ UŠ (see Preliminaries)

$$\text{TU 11 Predicting method:} \quad t = t_0 + 1, 0 + 1/3 \,(\text{length of}) \,\text{night},$$

(see Brack-Bernsen and Steele 2005). Note that $(t - t_0)$ just measures the shift in time between the old eclipse and the one a Saros later. The duration of a Saros, $\Delta^{223} t$, was implicitly approximated by

$$\Delta^{223}t = 6585 \text{ days} + 1, 0 + 1/3 \text{ night}.$$

If we identify the lunations by numbers, we can formulate the TU 11 rule as

$$t(i) = t(i - 223) + 1, 0 + 1/3 \text{ night}.$$

In our paper (2005), we also presented cuneiform tablets with similar number schemes that confirmed the predicting rule. The old numerical functions were adjusted to the Saros Cycle scheme, so that their zigzag functions tabulated the Saros time shift not month by month but in steps of 11 or 12 months. In that way, all 19 eclipse possibilities within a Saros, taking place near the same (ascending or descending) node, were included. Therefore, the linear zigzag function covered 18 years corresponding to one column (i.e. one Saros Cycle, SC) of the Saros scheme. In analogy to these number schemes, we had reconstructed the TU 11 zigzag function for the 19 eclipse possibilities within a Saros Cycle, and we could show that this linear zigzag function for the time from 675 BC to 522 BC approximates the Saros length surprisingly well. Therefore, we concluded that the Babylonians since the seventh century BC had a well-working scheme for the prediction of lunar eclipse times.

However, from 520 BC to 369 BC we see that there is an increasing shift between the real Saros length and the TU 11 function (see Fig. 2). The Babylonians may have noted the discrepancy and looked for corrections. Around the same time, the new and very elegant and precise Goal-Year predicting method must have been developed. We have strong arguments that the Goal-Year method was known at least from 550 BC, since table Kambyses 400 collects all Lunar Six for a whole year—many of which may not have been visible and hence must have been predicted. The quality of those Lunar Six data is so high that we must conclude that they were found by means of the Goal-Year method.[12]

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[12] Later, Huber and Steele (2007) have been able to date collections of Lunar Six data back to 643 BC.

Remember that this method utilizes the two different values ŠÚ+NA and ME+GE$_6$ for the daily retardation of the moon. In case of ŠÚ+NA, ŠÚ is the time from moonset to sunrise, and NA is the time from sunrise to moonset measured the next morning M. We see that ŠÚ+NA measures how much later the full moon sets on morning M than on the day before—it measures the daily retardation of the setting moon. Similarly, ME+GE$_6$ is the retardation of the rising full moon measured over 1 day. We have here a differentiation of the daily retardation of the moon: one measured at the western horizon and one measured at the eastern horizon. From the viewpoint of classical astronomy, we can say that the Babylonians had discovered the effects of the oblique ascension of ecliptic arcs. That was not the case in the schematic astronomy. EAE XIV, table D, had only one value for the daily retardation of the full moon, namely 1/15 night for the month in question. Several Babylonian model calculations, performed on the basis of the early schemes, neglect the difference between the ecliptic and the equator (see TU 11, section 19). This is not the case with the Goal-Year method: the time interval ŠÚ+NA is a measure for how fast the full moon passes the point of opposition, this effect being superimposed by the effects of the varying angle between the western horizon and the ecliptic at moonset (the effects of oblique ascension). Similarly, the time interval ME+GE$_6$ is determined by the momentary velocity of the full moon, but superimposed by the fact that the rising time of ecliptic arcs are essentially determined by the momentaneous angle between the ecliptic arc and the eastern horizon. Knowing the normal stars along the path of the moon, the Babylonians knew and could observe the changing positions (angles) of the moon path to the horizon. There are good reasons to suppose that the Babylonians had noticed the effects of oblique ascension and found a way to eliminate it: the dominating effect of oblique ascension can be largely reduced by adding the two time intervals ŠÚ+NA and ME+GE$_6$ resulting in the sum ŠÚ+NA+ME+GE$_6 = \Sigma$. When the velocity of the moon is high, Σ will be long, while it is low when the lunar velocity at the full moon in question is low. These effects were observable and could easily have been found by the Babylonian astronomers.

I propose now that the Babylonians must have realized that $\Sigma = $ ŠÚ+NA+ME+GE$_6$ could be a good value for the retardation of the moon, measured over two days around full moon. The TU 11 predicting method ($t(i) = t(i-223) + 1,0 + 1/3$ night) combined with this new knowledge may have led to the construction of a new numerical function for the prediction of times of eclipses involving the sum ŠÚ+NA+ME+GE$_6$ instead of the varying length of the night divided by 3. Remember that the daily retardation of the moon in EAE was derived from the length of the actual night: it was calculated as 1/15 night. Using our algebra, we would maybe calculate as follows: 1/3 night = 5 times 1/15 night = 5 times the daily retardation of the moon, and we know that the sum $\Sigma = $ ŠÚ+NA+ME+GE measures the retardation of the moon measured over two days. Therefore, we would replace 1/3 night by 5/2 (ŠÚ+NA+ME+GE$_6$) = 5/2 Σ, resulting in the new rule, where the time shift between the lunar eclipse in month $i-223$ and that taking place in month i one Saros later could be calculated as: $t(i) - t(i-223) = 1,0 + 5/2$ (ŠÚ+NA+ME+GE$_6$).

But the Babylonians had no algebra to calculate 1/3 night = 5/2 Σ. Maybe they could have figured it out; but we have no evidence for such a function. My proposal below is inspired from section 18 in TU11, where the goal-Year method is applied to data from the schematic astronomy—instead of using the observed 1/3(ŠÚ + NA), the text used 1/30 night length, which equals ½ and not 1/3 of the daily retardation. Here, I shall propose the opposite, to transfer a rule from the schematic astronomy and use observational data instead of schematic data. In both cases, the factor used for the variable not is the one we would expect, using our algebra for finding it. What I think that they have done is to replace the old eclipse predicting rule with a rule involving (ŠÚ + NA + ME + GE$_6$) instead of 5 times the daily lunar retardation = 1/3 night, ending up with a function varying in phase with Σ and with similar maximum as X (which is the linear zigzag function defined by the TU 11 eclipse predicting rule, see Preliminaries). When X is reproduced in form of the number scheme, we have a "linear zigzag function" with minimum 1,40 and maximum 2,20. We can approximate the sum Σ of the Lunar Four by the linear zigzag function $\hat{\Sigma}$. This function $\hat{\Sigma}$ varies roughly between 20 and 40 UŠ. If this function is added to the minimum, 1,40 of X, then we get a new zigzag function with a higher minimum but approximately the same maximum as X. Therefore, the new prediction rule could be something like: the time shift $t(\text{new}) - t(\text{old})$ between eclipses 1 Saros apart equals 1,40 UŠ + $\hat{\Sigma}$.[13] Such a numerical function would have approximately the same maximum 2,20, as the old function, but vary in phase with the lunar velocity, and it fits very well for all months during the winter. Here, I got back to my first intuitive proposal from 1990; but now with a better understanding and in a new interpretation: Φ is the (preliminary) Babylonian function for the duration of the Saros: $\Phi = \Delta^{223} t = (6585 \text{ days } +) 100° + \hat{\Sigma}$, or better formulated as a hypothesis: the time shift, Φ, between eclipses 1 Saros apart was taken to be 1,40 U Š + $\hat{\Sigma}$

$$\Phi = 1,40 \text{ UŠ} + \hat{\Sigma}$$

6.1 Cuneiform support for the arguments and methods proposed above

Now one could object that this is a speculative mixture of methods and that one should use 1,0 UŠ + 5/2 $\hat{\Sigma}$ and not 1,40 UŠ + $\hat{\Sigma}$. My answer would be to   refer to a peculiar prediction rule found in TU 11 section 18, where we find similar calculations: in this section 18, the time NA$_1$ from sunset to setting of the new crescent in a month II is found. The text tells us to go 18 years back to NA$_1$ of month II in the old year (i.e. the year 1 Saros earlier), and subtract a time difference which is derived from the length of the night: 1/30 night. Then to subtract this value from the old N A$_1$: NA$_1$ (new) = NA$_1$ (old) − 1/30 night. According to the schematic theory, 1/30 night equals the daily retardation of the moon divided by 2.

This procedure reminiscent of the Goal-Year method for finding NA$_1$:

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[13] Note that the sexagesimal number 1,40 equals our decimal 100.

$\qquad$ NA_1(new) $= NA_1$(old) $- 1/3$(daily retardation of the new crescent)

$\qquad$ Or in the other formulation: $(NA_1)_i = (NA_1)_{i-223} - 1/3(\text{ŠÚ} + NA)_{i-229}$

The Goal-Year method has utilized the fact that the daily retardation of the new moon in month $i-223$, which cannot be observed, is well approximated by the daily retardation of the full moon measured 6 months earlier in month $i - 229$. Section 18 of TU 11 seems to combine ideas from the Goal-Year method with old methods: the daily retardation of the moon is in section 18 derived from the length of the night, exactly as in EAE. The Goal-Year method subtracts $1/3$ of the daily retardation of the moon, whereas this method subtracts $1/30$ night which, according to the EAE theory equals $\frac{1}{2}$ and not $1/3$ of the daily retardation of the night. To us this is inconsistent. Here, again we have a hint of the fact that the Babylonians had not developed any kind of algebra.

One could also object that the length of the Saros, $\Delta^{223} t = 6585^d + \Phi$, where Φ equals $1,40 + \hat{\Sigma}$, cannot be verified by observation since the Saros varies in phase with the year. My point is that the proposed time shift $(1,40 + \hat{\Sigma})$ was derived from the old TU 11 rule by introducing the new successful observables. This new eclipse rule was introduced theoretically, not necessary based on observations (or eventually based exclusively on observations undertaken during the winter). And here, again, we have examples of old schematic astronomy producing functions and methods from their schemes: functions which do not reproduce celestial phenomena correctly. Hermann Hunger (2017) has interpreted a number scheme and shown that it gives the velocity of the sun as a function of the month. This velocity function is in accordance with the EAE schemes: the solar velocity is taken proportional to the length of the day, and the ratio of longest to shortest daylight is 2:1, exactly as in the schematic astronomy. But the constructed numerical function does not depict nature correctly. The variation of the solar velocity becomes too high, but much graver is the fact that the velocity is highest at summer solstice (SS). The apogee of the solar velocity in the Babylonian system A is astronomically correctly situated at Gemini 20°. This means that the sun is moving at its slowest velocity around SS. Seemingly, the Babylonian astronomers have used their theory to deduce other models and consequences of the number schemes. But such a practice is still a common method in modern science. Different models or theories are tested against each other by calculating and looking at consequences of the models and then compare to measurements. New ideas in one kind of theory are also tested in other approaches. I am convinced that the schematic astronomy was used by the Babylonians in a similar way, and sometimes, they ended up with functions that did not reproduce celestial phenomena correctly. Several texts confirm that the Babylonians performed such calculations. It just shows that a new theory is better—or needed. Therefore, I propose that at some time between 500 BC and 400 BC, the old eclipse predicting rule was replaced, and the numerical function $\Phi = 1,40 + \hat{\Sigma}$ **was taken to be the time shift between Lunar eclipses taking place 1 Saros apart.**[14]

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[14] Note that we are back to the intuitive proposal from 1990 since $\Phi = 1,40 \text{ UŠ} + \hat{\Sigma}$ equals $100° + \hat{\Sigma}$.

It is a bit too modern, or "leading", to say that the Babylonians here had a function for the duration of the Saros: 1 Saros = 6585 days + Φ. I call this formulation leading, because it implies that the Babylonians calculated length of the Saros to be 6585 1/3 day and used this knowledge to find the time of the new expected eclipse. In the new interpretation, it is just the time shift which was calculated. And that was all that they needed: the so-called Saros schemes collected all those months in which the full moon was in danger of being eclipsed. For those months, the Babylonian astronomers could determine the time of an expected lunar eclipse. But note that this time shift $\Phi = 1,40 + \hat{\Sigma}$ was not restricted to eclipse observations; it could (like the time shifts for Goal-Year predictions) be measured for consecutive months! If we now remark that the time of an eclipse is a good measure for the time of opposition, then we have here a function which could be used for all oppositions and not only for eclipse lunations. I therefore propose that the $\Phi = 1,40 + \hat{\Sigma}$ was the theoretical Babylonian function for the shift in time between oppositions 1 Saros apart. Note that this understanding of Φ makes it independent of the knowledge of a step function for the solar velocity.

7 Consequences of the new proposal that $\Phi = 1,40 + \hat{\Sigma}$ was the function for the time shift between oppositions 223 months apart

Only at lunar eclipses is it possible to observe the time of opposition with an acceptable accuracy—and hence to determine the duration of the Saros. But there are not many such observable events, and the question is how to connect them. With the new proposed term $\Phi = 1,40 + \hat{\Sigma}$ for the Saros length, it was possible to determine the time of opposition (or conjunction) for consecutive lunations, and not only for observable lunar eclipses. The Lunar Four, which had been observed and collected monthly over many hundred years, made it somehow possible to construct the linear zigzag function Φ, a function, which **for consecutive lunations** could predict the time shift between oppositions taking place 1 Saros apart. With the proposed understanding of Φ, the Babylonians would have had an instrument which could connect discrete eclipse lunation with all other lunations.

Let me try to illustrate the idea by means of the Goal-Year method and first point out how important it is to have continuous observations month by month**. The Goal-Year method can deliver a link between observations undertaken month by month and those undertaken Saros by Saros.** Having only disjunct eclipse observations at disposal, it is much harder to find regularities and understanding of dependencies, than if one has month by month observations.

We remember that the Goal-Year tables collected all Lunar Six data necessary for the prediction of Lunar Six data for the goal year Y. In the Diaries (Sachs and Hunger 1988–1996), as well as in the Goal-Year tables, we often find a Lunar Six sign together with its size measured in UŠ, but with the additional remark that it was not observed [NU PAP "I did not watch"]. Evidently, such data must have been predicted. Let us therefore imagine how the Babylonians could predict such data: on the Goal-Year tablet for the year in question, they had listed all the necessary lunar data

month by month. I shall first give an example using our modern notation to illustrate such a calculation, and afterwards, I shall show in words how easily such calculations and comparisons could be done by head.

Imagine that we have a table where the times ŠÚ_{n-1} and ŠÚ_n as well as NA_{n-1} and NA_n, observed for two consecutive full moons $n-1$ and n, were noted. The intervals ŠÚ_{n+222} and ŠÚ_{n+223} to be observed 1 Saros later could be found:

$$\text{ŠÚ}_{n+222} = \text{ŠÚ}_{n-1} + 1/3(\text{ŠÚ} + \text{NA})_{n-1}$$
$$\text{ŠÚ}_{n+223} = \text{ŠÚ}_n + 1/3(\text{ŠÚ} + \text{NA})_n$$

Subtracting the two equations gives

$$\left(\text{ŠÚ}_{n+223} - \text{ŠÚ}_{n+222}\right) = \left(\text{ŠÚ}_n - \text{ŠÚ}_{n-1}\right) + 1/3(\text{ŠÚ} + \text{NA})_n - 1/3(\text{ŠÚ} + \text{NA})_{n-1}$$
Or $\left(\text{ŠÚ}_{n+223} - \text{ŠÚ}_{n+222}\right) - \left(\text{ŠÚ}_n - \text{ŠÚ}_{n-1}\right) = 1/3(\text{ŠÚ} + \text{NA})_n - 1/3(\text{ŠÚ} + \text{NA})_{n-1}$

Let us denote the difference between two consecutive ŠÚ by $\Delta^1 \text{ŠÚ}$ and the difference between two ŠÚ situated one Saros apart by $\Delta^{223} \text{ŠÚ}$, then we have:

$$\Delta^1\text{ŠÚ}_{n+223} = \left(\text{ŠÚ}_{n+223} - \text{ŠÚ}_{n+222}\right) \text{ and } \Delta^1\text{ŠÚ}_n = \text{ŠÚ}_n - \text{ŠÚ}_{n-1} \text{ with}$$
$$\Delta^{223}\text{ŠÚ}_n = 1/3(\text{ŠÚ} + \text{NA})_n \text{ and } \Delta^{223}\text{ŠÚ}_{n-1} = 1/3(\text{ŠÚ} + \text{NA})_{n-1},$$

and we can formulate the equation above in the following way:

$$\Delta^1\text{ŠÚ}_{n+223} - \Delta^1\text{ŠÚ}_n = \Delta^{223}\text{ŠÚ}_n - \Delta^{223}\text{ŠÚ}_{n-1}$$

The change in $\Delta^1\text{ŠÚ}$ over one Saros: $\Delta^1 \text{ŠÚ}_{n+223} - \Delta^1 \text{ŠÚ}_n$ equals the monthly change in $\Delta^{223} \text{ŠÚ} = \Delta^{223} \text{ŠÚ}_n - \Delta^{223} \text{ŠÚ}_{n-1}$.

Note also that the prediction rules easily could deliver a way to update the ŠÚ of month $n+222$ to that of month $n+223$:

$$\text{ŠÚ}_{n+223} = \text{ŠÚ}_{n+222} + \left(\text{ŠÚ}_n - \text{ŠÚ}_{n-1}\right) + 1/3\left(\left(\text{ŠÚ} + \text{NA}\right)_n - \left(\text{ŠÚ} + \text{NA}\right)_{n-1}\right)$$
$$(1)$$

This means that ŠÚ_{n+223} now can be found also from ŠÚ_{n+222} by means of the data from the Goal-Year table for the actual year.[15] And similarly ŠÚ_{n+224} can be found from ŠÚ_{n+223}.

This may look a little complicated as I have used our algebra to present the calculations. Therefore, I shall now present these calculations in words. Imagine that a Babylonian astronomer is using a Goal-Year tablet for predicting Lunar Six for year Y. All data necessary for that purpose were collected on the Goal-Year tablet for year Y. In order to predict ŠÚ of month M in this new year Y, he finds the

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[15] The term "update" was introduced by Ossendrijver (2012), he used it for the procedures of the mathematical astronomy, where a series of consecutive phenomena of moon, sun or a planet were calculated: time and positions of one phenomenon were found or "**updated**" from one phenomenon to the next by applying additive or subtractive differences.

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value of ŠÚ in month M in his old year Y-18, and to this value he adds the time shift 1/3 (ŠÚ+NA) (M old) [= 1/3 (ŠÚ+NA)$_n$]. In this way, he finds ŠÚ (M new) [= ŠÚ$_{n+223}$]. Similarly, he can find ŠÚ of the next month M+1 in his new year Y by adding the time shift 1/3 (ŠÚ+NA) (M+1 old) to the value of ŠÚ (M+1 old). Now it does not require any algebra to conclude that the difference between ŠÚ in month M and M+1 in the new year Y equals the difference between the two old ŠÚ plus the difference of the time shifts: if, for example, the time shift in month M is 4° and that in month M+1 only 3°, then clearly, the difference between ŠÚ of month M and M+1 in the new year will become 1° smaller than it was in the old year. I think that this consequence of the Goal-Year calculations is important, since we have here a method which allows one to find Lunar Six times from their values in the month before. Maybe these considerations could be used for a new investigation of the atypical cuneiform text K, in which the Lunar Six intervals are updated from month to month by applying differences depending on the moon's zodiacal position.[16]

8 Goal-Year techniques and the development of the *R–S* schemes

Another consequence of our hypothesis for Φ is that we now can give an easy and natural explanation of how the number schemes for finding the change in 1 or 12 synodic months can be derived from the R and S versions of column Φ. We just have to transfer the Goal-Year method to the times T of opposition. Remember that the Goal-Year method finds a time interval, e.g. ŠÚ, from its value 1 Saros earlier, ŠÚ(old), by adding the time shift for the Saros in question to the observable time. Similarly, the time interval from sunset to opposition, T(new), in month M could be found from its value T(old) determined for the full moon 1 Saros earlier by the addition of the time shift Φ:

$$T(\text{new}) = T(\text{old}) + \Phi$$

Our hypothesis postulates that the Babylonians found the (sarosly) shift at times of oppositions as 1,40+Σ or as numbers in column Φ. In the example above, the time shift of ŠÚ was 1/3(ŠÚ+NA). The time shift Φ between oppositions situated 1 Saros apart simply uses 1,40+the sum Σ of all Lunar Four, for which the same kind of rules must be valid as for the partial sums (ŠÚ+NA) and (GE$_6$+ME). Note that the time shift implicitly gives the duration of the Saros between the old month and the new month. This means that Φ is a measure for the Saros beginning with month M(old) and ending with month M in the new year. We shall see that exactly as in the example above, expressed in Eq. (1), the time T(new) of an opposition in month M(new) can similarly be found from its value in the month before, i.e. from month (M-1)(new) using known values from the old year.

Let us look at the concrete calculation of times of consecutive oppositions: the time T(new) of an opposition in a new year Y could be found by adding the shift Φ to the time T(old) of the opposition which had taken place 1 Saros earlier. For a

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[16] See Neugebauer and Sachs (1969) and Ossendrijver (2012, pp. 116–120).

series of consecutive lunar oppositions …$O(-1)$, $O(0)$, $O(1)$, $O(2)$, $O(3)$,… we shall call the corresponding times after sunset: …$T(-1)$, $T(0)$,$T(1)$, $T(2)$, $T(3)$, …$T(n)$,… These times can determine the times of the oppositions taking place one Saros later, namely at the times …$T(-1+223)$, $T(223)$, $T(1+223)$, $T(2+223)$, $T(3+223)$, …$T(n+223)$ after sunset. The time shift for these oppositions then is the corresponding value of Φ, i.e. the shift in time between the oppositions n and $n+223$. We shall call this time shift $\Phi(n)$. Our hypothesis thus means that

$\Phi(n) = 1,40 + \hat{\Sigma}$ **is a measure for the duration of the Saros starting at month n and ending at month $n+223$.** According to this hypothesis, we find

$$T(n - 1 + 223) = T(n - 1) + \Phi(n - 1)$$
$$T(n + 223) = T(n) + \Phi(n)$$

The time shift $G(n+223)$ between two consecutive oppositions $n-1+223$ and $n+223$ can now be found:

$$G(n + 223) = T(n + 223) - T(n - 1 + 223) = T(n) - T(n - 1) + \Phi(n) - \Phi(n - 1)$$

Now the difference $T(n) - T(n-1)$ is the time shift, $G(n)$, between the two old successive oppositions $O(n-1)$ and $O(n)$. Therefore, we get:

$$G(n + 223) = G(n) + \Phi(n) - \Phi(n - 1) \qquad (2)$$

We have here found G (for the duration of $\Delta^1 t$ of month $n+223$) by its value $G(n)$ in the "old" year and the difference between two consecutive Saroi, exactly as in the R–S schemes. Similarly, $G(n)$ can be found from its value one Saros earlier, i.e. from $G(n-223)$ by adding the difference $\Phi(n-223) - \Phi(n-1-223)$ of the two Saroi ending at the oppositions n and $n-1$:

$$G(n) = G(n - 223) + \Phi(n - 223) - \Phi(n - 1 - 223) \qquad (2a)$$

Here lies the clue for the understanding of the R–S method. Remember that the R–S scheme listed values of Φ in steps of 1 Saros, just as in the equations above.

Formulas (2) and (2a) describe exactly the way to find the values of G by means of an R–S scheme, where G and the difference S–$R = \Delta G$ in one line was used to find G in the next line. The structure of such a table is shown in Table 3. It is repeated in Table 4, so that we can identify the elements with the notation in Eqs. (2) and (2a) above, where G and Φ (on the ascending branch) are written as functions of the lunation number n.

Let us imagine the case where the Φ values $\Phi(n) = S_k$ and $\Phi(n-1) = R_k$ (for a month n and the previous month $n-1$) were listed in line k of such an R–S scheme. Together with $G_k = G(n)$, these values were used for finding $G(n+223) = G(k+1)$ one Saros later, listed in the line k + 1 below, according to Eq. (2):

$$G(n + 223) = G(k + 1) = G(k) + (\Phi(n) - \Phi(n - 1)) = G(k) + (S_k - R_k) = G(k) + \Delta G_k$$

Table 4 The R–S schemes as in Table 3, but now we identify the months' numbers according to Eq. (2a) above

Line number ≈ opposition number	R $\Phi(m-1)$	S $\Phi(m)$	$S-R=\Delta G$ $\Phi(m)-\Phi(m-1)$	G $G(m)$
$k-1 \approx n-223$	R_{k-1}	S_{k-1}	$S_{k-1}-R_{k-1}=\Delta G_{k-1}$	G_{k-1}
$k \approx n$	$R_k=\Phi(n-1)$	$S_k=\Phi(n)$	$S_k-R_k=\Delta G_k$	$G_k=G_{k-1}+\Delta G_{k-1}$
$k+1 \approx n+223$				$G_{k+1}=G_k+\Delta G_k$

Note that the R–S schemes only tabulate Φ values in steps of 1 Saros: let R_k and S_k represent the values $\Phi(n-1)$ and $\Phi(n)$, and then R_{k-1} and S_{k-1} in the line above will list the Φ values of month $(n-1-223)$ and month $(n-223)$ which occurred 1 Saros earlier

Similarly, the value of G in line k is found from its value in the previous line $k-1$ (corresponding to lunation $n-223$, 1 Saros earlier) through the addition of $\Delta G_{k-1}=S_{k-1}-R_{k-1}=\Phi(n-223)-\Phi(n-1-223)$ (calculated in line $k-1$)

$$G_k = G_{k-1} \pm \Delta G_{k-1}$$

We can now rewrite this equation which describes the R–S scheme, knowing that the values of G, G_k, and G_{k-1} refer to two full moons n and $n-223$ taking place 1 Saros apart. We can identify the months involved and replace G_k by $G(n)$, G_{k-1} by $G(n-223)$, and ΔG_k by $\Phi(n)$–$\Phi(n-1)$, resulting in

$$G(n) = G(n-223) + \Phi(n) - \Phi(n-1)$$

and we are back to our Eq. (2), which obviously conforms with the R–S scheme.

The R–S scheme uses the fully developed function Φ. But note that the procedures and calculations described in eq. (2) could be performed "by hand" for each month for which the old values $T(n)$ and $T(n-1)$, and the durations $\Phi(n)$ and $\Phi(n-1)$ of the Saroi beginning with month n and $n-1$, respectively, were known. It is therefore natural to propose that the Babylonians in the beginning have used the values of $1,40+\Sigma$, found by observations, for such calculations. Later, they somehow constructed the linear zigzag function $\hat{\Sigma}$ which approximates the "observed" Σ optimally, leading to $\Phi=1,40+\hat{\Sigma}$. Using Φ, one could in principle calculate all G values "by hand". The Babylonians have obviously constructed the R–S schemes in order to facilitate the calculations. The structure of the scheme, for example, that there is 1 Saros from one line to the next and that the Φ values of two consecutive months are needed for finding G, is dictated by the procedure. Now we understand the astronomy and arithmetic behind the R–S schemes, but the end result is a bit tricky. We can see from the fully developed procedure that a lot of elegance, numerical tricks and technical finesses were required for the construction. Our scheme above is a simplified version of the final schemes. Some small tricks and approximations were used in order to save calculations and compensate for the fact that the period 6247 of Φ is not exactly a multiple of φ, which is the change in Φ value from one Saros to the next. In addition, the extrema of Φ were cut off, so that the maximum was truncated at the round value 2,13:20. This means that the numbers, which we find in the final version of the R–S number schemes, come from the truncated

version $\bar{\Phi}$ of Φ. I surmise that this special number 2,13; 20 was used as a name for the R–S schemes and not for the original, non-truncated version of Φ.[17] Lunar table texts of system A list numbers from the non-truncated Φ. These numbers were used for finding G by means of the refined scheme, but the principles behind the scheme remained the same as outlined above. In order to explain how the calculations for finding G from Φ work, I have [following Neugebauer (1975, p. 509 table 14)] presented all four columns R, S, S-R$=\Delta G$, and G, which are needed for the construction of such a scheme. In a developed final scheme, only S$=\Phi$ and G are necessary. It is therefore possible that the finished table for finding $G(n)$ from its value Φ (n) only had the two columns S and G. The structure is, however, explained better, when we also consider all columns R, S, ΔG, and G.

9 The "2,13,20" schemes for finding G from Φ by interpolation

Until now there has been a variety of reconstructions of the R–S schemes: in HAMA pp 508 and 509, Neugebauer has in table 14 given his reconstruction and explained how it works. Later (2009) in table 3.2 on pages 378 and 379, Britton presents his complete and reconstructed version of the R–S table for finding G. Here, he had also included newly found cuneiform texts with $\bar{\Phi}$ and G numbers. Britton's reconstructed table is a little different from that of Neugebauer, some numbers are different and Britton indicates behind the R and S numbers of $\bar{\Phi}$ if they are placed on a descending or ascending branch. In addition, he had introduced new names for column R and S, calling them I$=\Phi_{s,n}$ and II$=\Phi_{s,n+1}$, respectively. But the principle of the schemes is the same. I have followed the convention of Neugebauer and Britton, which finds G_k in line k from G_{k-1} and ΔG_{k-1} in line k-1 above: $G_k=G_{k-1}+\Delta G_{k-1}$. This convention assumes that the values of Φ in a lunar ACT text indicate the duration of the Saros **starting** during the month in question, since this Φ value is also used as index for finding the corresponding value of G for that month. However, if the Φ values in a ACT table text would give the duration of the Saros **ending** with the month n in question, then the R–S scheme must have been constructed on the basis of the slightly different formula $G_k=G_{k-1}+\Delta G_k$, using the difference S–R in line k, since the Saros ending in n in this case would be $\Phi(n)$ instead of $\Phi(n-223)$.[18] In any case, we now understand the basis for the construction of the R–S schemes, and we know how they were used: for a date—month n—in a lunar ACT table text, the time shift G was found in that line of the R–S scheme where $\Phi(n)$ was listed. This was, indeed, the way in which the Babylonians used the scheme. I quote Ossendrijver (2012), who on page 148 has translated an advice from a procedure text: *Opposite Φ_k, increasing, you put down G_k. [Whatever] (the amount) by which it*

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[17] It has been proposed that the number "2,13,20" was the Babylonian name for column Φ in the same way, as the number "18" was used as name for the Saros. Maybe "2,13,20" rather refers to the truncated version of Φ and thus to the R-S schemes.

[18] Many years ago, John Britton surmised that the values of Φ in ACT table texts listed the durations of the Saros ending with the month in question. If so, then the alternative formula ($G_k=G_{k-1}+\Delta G_k$) should be used. It is my hope that this new understanding will lead to a reconstruction of the R-S schemes, which all can agree on.

exceeds Φ_k, increasing, until Φ_{k+1}, [increasing,] you multiply by 3; 22,30, and what comes out for you, you multiply by $c_k\Phi$, and add to G_k, and put down.

The structure of our scheme must be correct, and we know the way in which it was used: when $\Phi(n)$ of a month n is listed in column S, line k, then $G(n)$ can be found in column G of the same line k. The quotation above also indicates how values of G for the months n, for which $\Phi(n)$ did not occur in column S, were found: by interpolation in the scheme between the lines k and k+1, where $\Phi_{(k)} < \Phi(n) < \Phi_{(k+1)}$. The interpolation was easy for all lunations on the linear part of G, since the difference between the lines, $\Phi_{(k+1)} - \Phi_{(k)}$, was always equal to the well-known number $\varphi = 0; 17,46,40$, whereby the number $0; 17,46,40 = \varphi$ is the difference between Φ values for two months situated 223 months apart.

Until now, we have been considering values of Φ situated on the ascending branch of the linear zigzag function $\Phi = 1; 40 + \hat{\Sigma}$. This led to the numerical function $\hat{G}$ described by Neugebauer in HAMA p. 487ff. and helped in understanding the procedure and its development. We know that sometimes function Φ was truncated around its maximum to the value 2,13; 20. The truncation around the minimum was at 1; 58,31, 6, 40. The new version $\bar{\Phi}$ of Φ was called "2,13,20" by the Babylonians. All arguments above are true for the long linear parts of the R–S schemes. Around the extrema of $\bar{\Phi}$, the same rules work for the "truncated values" of $\bar{\Phi}$. But the interpolation between the lines around the extrema was modified, namely in the cases where the difference between R and S was smaller than φ.

Ossendrijver (2012, pp. 145–150) has edited a series of procedure texts which give advice for calculations when G is found by interpolation—most of them concern the truncated and irregular parts of the scheme. He has compiled the separate computation rules into an auxiliary scheme (table 4.21). The scheme has 36 lines and lists the independent variable Φ in the first column. It starts with the truncated value 2,13; 20 and give in steps of 1 Saros the following Φ values, but also extreme values, outside the truncated area: the minimum m which occurs only in Φ_1 and also the maximum M value of Φ. Clearly, the advice for interpolation shows us that R–S schemes were meant to be used for calculating G for both full and new moon by means of the independent variables Φ or Φ_1.

10 The development of the schemes for finding Λ (and W) from Φ

In his two detailed studies in Babylonian Lunar Theory, Part I and II, Britton has given a sound analysis of data and useful reconstructions of numerical schemes. He had found and utilized connections and inner structures of the Babylonian system, some of which we have found above in a different way. In tables 2.4a and 3.1, Britton (2009, p. 372 ff.), in continuation of the work of Aaboe (1971), presents a scheme for finding W and Λ, the duration of 6 and 12 months, as a function of Φ. The last scheme is based on cuneiform tablets (BM 36311 and BM 36699) with numbers from small parts of Φ Λ schemes. We shall here see how such a table can be derived in the same way as the "2,13,20" scheme for G was developed.

Exactly as shown above, the hypothesis that $\Phi = 1,40 + \hat{\Sigma}$ will lead to a scheme for calculating the durations of 12 or 6 months, respectively. Here, the process

will only be shown for the time shift Λ between lunations 12 months apart. The same arguments will lead to W in the case of 6 months.

Note that our procedure described above, directly leads to the following equation:

$$G(n + 223) - G(n) = \Phi(n) - \Phi(n - 1) \tag{3}$$

The change in G over one Saros equals the monthly change in Φ. But note that this is a direct consequence of the way it is constructed. A similar rule is valid for the numerical functions Λ and Φ, and our hypothesis for Φ can again easily explain how schemes for finding Λ (and W) from Φ were developed:

Let $T(n)$ and $T(n-12)$ be the time from sunset to opposition for two lunations situated 12 months apart. Since the time shift, $\Phi(n)$ and $\Phi(n-12)$, from these old lunations to the full moons in month $n + 223$ and $n\text{-}12 + 223$ is known, we get:

$$T(n + 223) = T(n) + \Phi(n) \text{ and } T(n - 12 + 223) = T(n - 12) + \Phi(n - 12)$$

Therefore, the time shift Λ between two oppositions situated 12 months apart can be found from the values given for the corresponding full moons 1 Saros earlier:

$$\Lambda(n + 223) = T(n + 223) - T(n - 12 + 223) = T(n) - T(n - 12) + \Phi(n) - \Phi(n - 12)$$

Note that the difference $T(n)$–$T(n-12)$ equals the time shift $\Lambda(n)$ between the two lunations $n-12$ and n. Therefore, we get:

$$\Lambda(n + 223) = \Lambda(n) + \Phi(n) - \Phi(n - 12) \tag{4}$$

Here, again, we naturally get a rule that the Sarosly change in the duration of 12 months equals the change of Φ after 12 months:

$$\Lambda(n + 223) - \Lambda(n) = \Phi(n) - \Phi(n - 12)$$

Equation (4) shows how a R–S scheme for finding Λ from Φ must be constructed: it will again have **lines** with Φ—numbers of months situated 1 Saros apart, but the numbers of Φ in **column R and S** must correspond to lunations which have a distance of 12 months. The development of the schemes for finding Λ from Φ is outlined here. Similarly, the scheme for finding W from Φ can be derived from our hypothesis, and we would again naturally find the rule that the Sarosly change in the duration of 6 months equals the change of Φ after 6 months. These rules are a simple consequence of the proposed way to find times of oppositions by means of Φ and Goal-Year techniques. We note that such connections between numbers in the schemes (namely that the change in G over one Saros equals the monthly change in Φ, and similarly, the change in Λ over one Saros equals the change in Φ after 12 months) are just a consequence of the proposed derivation from our hypothesis. The Babylonians, of course, knew these rules which we also find written on cuneiform tablets.

Britton formulated a similar, but very general rule which he called the "Interval Rule" and which he proposes to be one of the theoretical bases for the construction of the general lunar theory.

I quote Britton (2009) p. 361:

"*Interval Rule*". An important element in the development of the theory is a general theorem, which I call the "Interval rule". This recognizes that if there are two intervals with a common starting point such that one contains A units of some variable sub-interval such as months, and the other B such units, then, generally and strictly, the change in A over B sub-intervals will equal the change in B over A sub-intervals, i.e. $D_B A = D_A B$.

The rule is correct, and Britton continues to prove it using our algebraic notation heavily. It is, of course, allowed to analyse and explain the old calculations and methods by means of modern science and its tools, e.g. algebra. But such methods should not be used to construct the Babylonian calculating schemes, and it is questionable if the Babylonian numerical astronomy was constructed on a solid theoretical basis like our sciences. In addition, it is not necessary at all to propose an abstract theoretical basis, since we have seen that the relevant cases of "interval rules" in this new approach are a simple consequence of the way times of oppositions were calculated. We have here a special rule for the time shift of 1, 6, and 12 months—and not a general and abstract interval rule.

Britton's reconstruction of column Φ is (to my taste) too technical, and it is heavily based on a proposed—and beautifully symmetric—function for the duration of 235 months. He emphasizes the fact that after 235 months $\cong$ 19 years, sun and moon will be at the same point of the ecliptic so that lunations which are 235 months apart are only influenced by the irregular lunar velocity and not by the solar velocity. This is correct in principle. But it is questionable if the Babylonians noticed and used it. In table 3.1 mentioned above, Britton adds the values of Φ to Λ in each line of the reconstructed scheme, and so he finds the shift in time after 235 months. But we have no textual evidence for such a scheme; the Babylonians just used the 19-year period to regulate intercalations in their calendar, but we have no evidence for its use for other astronomical purposes. We know, for sure, that the Babylonians knew and utilized the Saros extensively.

de Jong (2017) goes another way in his article "On the Origin of the Lunar and Solar Periods in Babylonian Astronomy". He starts out with a clearly written and very nice overview of the Babylonian functions and their astronomical basis—explained by our astronomy. Then, he focusses on the old 27-year Sirius period, which is attested in several early astronomical texts. The period of 27 years equals approximately 334 synodic months, and de Jong proposes that this relation, 27 (sidereal) Years $\cong$ 334 synodical months, was used to find the duration of the Saros in years. This means that the Sirius Year was taken to be what we call the tropical year. The Babylonians did at that time not differentiate between these two types of the year. Later, the 19-year calendar cycle = 235 synodic months was found and used for calendar regulations from around 500 BC and onward. Now, 54 Sirius years = 2×334 months = 668 months, and 3 Saroi = 3×223 months = 669 months. Therefore, de Jong suggests that the Babylonians used the connection: 3 Saros = 54 years + 1 month to find that 1 Saros = 18 years + 1/3 month. This equality is then used to determine period relations between the Saros and solar years, the first being: 37 Saroi = 667 years. By the stepwise subtraction of 3 Saroi − 1 month = 54

years, one gets other period relations. After 3 steps, one arrives at the relation: 505 years = 6247 months, which is the period of column Φ. Maybe the period was found in this manner and then used for constructing Φ. If so, then this early column Φ had, indeed, been composed from a theoretically constructed period relation. It must, however, still be investigated how the observed data of the Lunar Four came into play, since we also have to explain how the Babylonians' function Φ was constructed so that it oscillates with the same amplitude, period, and phase as our proposed function $1,40 + \hat{\Sigma}$. Another possibility, which I see for finding the period of 6247 months, is that the skilled Babylonian astronomers were analysing special chosen data of Σ, for example, in distances of a whole number of Saroi, namely those special months for which the sum Σ changed rapidly. In this way, they could have found how much the little change "φ pro Saros" had been added up after n Saroi. Such a knowledge could have been used, too, to find the period of Φ. Maybe the two approaches can be combined; at least they seem to support each other since they both build on early astronomy and old practices.

11 Babylonian practices, summary, and conclusion

Let me summarize and draw attention to the important points. The rule for prediction of opposition times $T(n+223) = T(n) + \Phi(n)$, as introduced above, makes it possible to determine the new opposition from an old one using the data written on a Goal-Year tablet alone. By calculating the time for consecutive months, the Babylonians may easily have found the way to determine $T(n+1)$ from $T(n)$ as described above. As consequence, the following rule emerges naturally: the difference $G(n+223)$ between two consecutive oppositions, $n+222$ and $n+223$, can now be found from Eq. (3)

$$G(n + 223) = G(n) + \Phi(n) - \Phi(n - 1)$$

Note that the new $G(n+223)$ is found from the old $G(n)$ plus the difference $\Phi(n) - \Phi(n-1)$ in duration of the two Saroi beginning in month $n-1$ and n. If one does not want to calculate month by month, but create a practical scheme for finding values of G, then such a scheme must acquire a structure like the R–S schemes with two versions of Φ at a distance of 1 (or similarly 12 months). The old difference S–R must be added to (or subtracted from) the old G value, and the result could practically be listed in the line below. We see that the whole structure of the schemes for the calculation of G and Λ comes out very naturally. It also explains why the Φ value attached to a month n measures the duration of the very Saros starting (or ending) at month n. The new understanding, namely that the numerical function $\Phi = 100 + \hat{\Sigma}$ was used as the shift in time between two oppositions taking place 1 Saros apart, explains technical details and answers many questions: not only does it explain why the calculation of G (the time shift between consecutive full moons) involves the previous Saros, but it also leads to the concepts behind the construction of the R–S schemes. And it only uses calculations, techniques, and arguments which were at the disposal of the Babylonians. I see all these connections or arguments as a strong

support for the proposal. Especially, the fact that the Saros from month n to month $n+223$ is used for finding $G(n+223)$ indicates to me clearly a connection to the Goal-Year method. Therefore, I now claim that the hypothesis has been confirmed, namely that $\Phi = 100 + \hat{\Sigma}$ **was taken to be shift in time between Lunar Eclipses and other oppositions taking place 1 Saros apart**.

This understanding of Φ also explains the existence of Text S, which contains Φ values together with a primitive mean-value model for the solar movement. Φ may have been introduced before the solar movement was modelled by the system A step function B, i.e. it can have been introduced independently of and before the whole system A was developed. I think that we are a long step further in the reconstruction of the Babylonian astronomy, but we have not yet been able to completely reconstruct the R–S schemes with its interpolation advices, and we are still far away from a complete understanding of how the linear zigzag function $\Phi = 100 + \hat{\Sigma}$ was derived from observations of the Lunar Four. Which techniques were used, and which observations? Maybe the Sirius year was involved? In any case, is it obvious that lunar eclipses were involved, but I surmise that also consecutive lunations, for which the sum of the Lunar four changed with a high and approximately constant difference, were important.

12 Outlook

In case of Φ, we know that this function only gives the lunar component for the time shift between eclipses taking place 1 Saros apart; the larger solar component was neglected. The solar component was also neglected by the components G and Λ. They only give the lunar components of time shifts $\Delta^1 t$ and $\Delta^{12} t$, since they were deduced from Φ alone. Evidently, the Babylonians must have noticed that something was missing and given a correction. Such a task was not too difficult for them, taking their practice of recording into account: we know that the Babylonian listed all relevant observables as a function of the month—in other words, the month was the independent variable: all observed or calculated quantities were recorded as functions of the month. Observations undertaken from month to month would show a systematic difference between predicted and observed full or new moons, and this holds exactly for those months of the Babylonian calendar during which the sun was moving slowly. Such observations could lead to the correcting subtraction J from G, which we find in system A texts. System A calculates corrections J and Y to G and Λ for the duration $\Delta^1 t$ and $\Delta^{12} t$ of 1 or 12 months, respectively. No such correction for the Saros has ever been found. Maybe the reason is that it was no longer necessary: system A calculations delivered from month to month all relevant data for lunar eclipses

I am convinced that the data recorded on the Goal-Year tablets were used heavily for the construction of the mathematical astronomy. We must try to use these data more in our effort to reconstruct the numerical functions of the Babylonian mathematical astronomy. It should also be emphasized strongly that we shall try to work with the means and concepts known to the Babylonians: they were very skilled calculators, but they did not have anything like our algebra, where quantities can be presented by abstract notation and used for calculations or in equations. At

some time, maybe after the invention of Φ, the function for the solar movement was found, and the position of the moon (at conjunction or opposition) was calculated as function of its position in the zodiac. It is important also to keep the schematic astronomy in mind. Note that some of the old concepts and methods were kept all the way through the development of the Babylonian astronomy, from the schematic astronomy in the early texts EAE XIV and MUL.APIN to the fully developed lunar theory of system A and system B. The schematic day = 1/30 schematic months is found in the later unit "tithi" = 1/30 synodic month. The schematic astronomy seems to play a more important role in the reconstruction of the mathematical astronomy than what was supposed until now. As an example, I can mention that a linear zigzag function for the rising and setting times of ecliptic arcs can easily be found when the partial sums (ME + GE$_6$) and (ŠÚ + NA) are listed as functions of the month.[19] (Remember: in the schematic astronomy the schematic months were identified with the corresponding zodiacal signs.)[20] This may be a fruitful starting point for reconstructing the calculations behind the duration of daylight given in column C. A technical detail may also be very helpful, when we try to reconstruct the numerical zigzag or step functions, the smallest interval δ.

12.1 The smallest interval δ between the numbers of a numerical function

In the fully developed Babylonian astronomy, we find many numerical step functions and linear zigzag functions. The smallest distance between numbers in each of these functions is important: if all numbers of a linear zigzag function are sorted according to their magnitude and listed within one period, then we can find the smallest interval δ, for example, between successive values of Φ. The values of Φ are situated at the constant distance δ. Similarly, there are smallest distances δ for each linear interval of a step function, as shown by Aaboe (1964). He pointed at the importance of such smallest intervals and saw them as a sensible starting point in the construction of the step functions. I am convinced that he was right and that the smallest interval δ also was used extensively in the construction of linear zigzag functions. I do not know how—but the Atypical Text C[21] may become important in the reconstruction of column Φ (see Brack-Bernsen and Steele 2011). Here, the so-called Q-polygon is introduced, which depicts the Φ values situated equidistantly on the circumference of a circle, the distance being δ. It is shown how such a representation and the interval δ can be used for easy calculations of Φ numbers and for finding their differences over, for example, 1, 12, 14, and 223 months, expressed in units of δ. Text C lists a quite particular sequence of 10 Φ values, which reveals a very clear hint to the interval δ. In addition, all the dates which correspond to the Φ values are situated some multiple of 14 months apart, and their distances to the truncated values of Φ are also a multiple of 14 months. To this adds that the Atypical Text C contains a list of 14 numbers,

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[19] I have presented this method at the 4th Regensburg Workshop on Mesopotamian Astral Sciences, held in Berlin in May 2014.

[20] Brack-Bernsen (2003, p. 25).

[21] Neugebauer and Sachs (1967).

called igi.gub.ú.meš. The Sumerian word, igi.gub, can be translated as "constant factor" or "coefficient". They signify relevant constants or numbers which are essential for special calculations. The igi.gub.ú.meš numbers in Text C are all special multiples of δ, and nine of them are equal to the change in Φ value after n times 14 months. The interval of 14 months is a quite good approximation to the period of the lunar velocity. Maybe this 14-month period played a larger role in the construction of Φ than we have imagined until now. It must also be stressed that Text C contains early data. We have two possibilities to connect the given values of Φ with Babylonian dates, depending on our choice of letting the Φ values be situated on the ascending or on the descending branch of Φ. In both cases, three of the dates correspond to eclipse dates. Clearly, eclipse observation must also have played an important role in the construction of the column Φ. This is evident, since the fine and round value 2, 0 of Φ corresponds to the date of a lunar eclipse. Normally, Φ numbers have up to 6 sexagesimal places. Even if Text C should happen to be a text written later, for which purpose ever, then it is worthwhile to be analysed more careful. Each time we are able to find connections, see how the Babylonians analysed data or utilized them, we learn more of their concepts and methods, which brings us nearer to a deeper understanding and reconstruction of their fascinating Astronomy. To sum up, we still have many interesting challenges left for future research.

Compliance with ethical standards

Conflict of interest The author declares no conflict of interest.

Appendix 1

"Lunar Six" time intervals and the Goal-Year method for their prediction

Babylonian astronomy was especially concerned with the moon in the time around full und new moon. Lunar and solar eclipses were of great interest but also the rising and setting times of the full and old/new moon. Six special time intervals were observed regularly over hundreds of years, the "Lunar Six".

KUR and NA$_1$ were observables in the days before and after **conjunction**:

KUR = time from last visible rising of the old moon to sunrise.
NA$_1$ = time from sunset to the first visible setting of the new crescent.

Around **opposition**, four time intervals, the "Lunar Four", were observed:

ŠÚ = time from moonset to sunrise, measured at last moonset **before sunrise.**
NA = time from sunrise to moonset, measured at first moonset **after sunrise.**
ME = time from moonrise to sunset, measured at last moonrise **before sunset.**
GE$_6$ = time from sunset to moonrise, measured at first moonrise **after sunset.**

These intervals, measured in time degrees, are evident and easy to observe, but they are quite complicated quantities from an astronomical point of view. Each of them depends on the momentary latitude $\beta_\mathbb{C}$ of the moon, on its momentary velocity $v_\mathbb{C}$, on its position $\lambda_\mathbb{C}$, in the ecliptic at conjunction or opposition, respectively, and on the time difference Δt between the time of observation and conjunction or opposition, respectively. Taking the interval NA$_1$ as an example, we write it as a function of these four variables:

$$NA_1 = NA_1 (\beta_\mathbb{C}, v_\mathbb{C}, \lambda_\mathbb{C}, \Delta t) \tag{5}$$

Figure 3 illustrates the situation at the western horizon when the new crescent is about to set at that evening, when NA$_1$ could be measured. This was an important moment, since the new crescent announced the beginning of a new Babylonian month. Obviously, the distance between the sun and the new crescent is determining for the length of NA$_1$, and this length depends on the lunar velocity $v_\mathbb{C}$ and on Δt. But it also depends on the momentary angle between the ecliptic and the horizon, which is a function of $\lambda_\mathbb{C}$, and finally, it depends also on the lunar latitude $\beta_\mathbb{C}$.

At latest from 643 BC onward, the Lunar Six were observed regularly. They played an important role within the Babylonian astronomy. One of the aims of mathematical lunar table texts was to calculate the values of the lunar six. That the Babylonians were able to cope with such complex quantities illustrates how elegant, powerful and to which high level their astronomers had developed their numerical astronomy. But even in the times before the fully developed astronomy, they could cope with these time intervals. The skill of the early Babylonian astronomers became evident when the elegant method for the prediction of the Lunar Six time intervals was discovered (Brack-Bernsen 1997, 1999).

We have called these prediction rules for "the Goal-Year method", since all data necessary for such predictions for a whole year were collected on the Goal-Year tables. Such a table collected observed data of special astronomical events for the planets and the moon. The colophon at the end of each table indicates that all data were to be used for predictions of the same events in a year Y to come. On the rear of such a table, the lunar events which had taken place in year Y-18 were recorded: lunar and solar eclipses (or eclipse possibilities), and all Lunar Six time intervals were listed under the day numbers within the Babylonian month at which they had
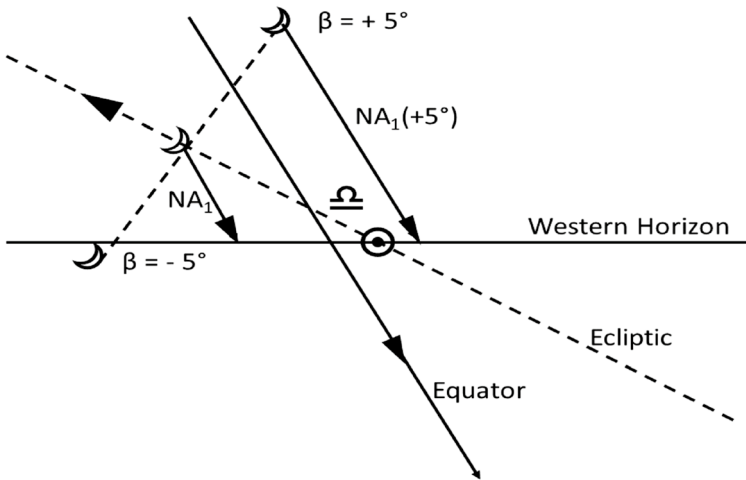
Fig. 3 The western horizon at the moment of sunset on an evening shortly before fall equinox. The sun, ☉, is setting. The solid long line shows the equator and the arrow the direction of the daily revolution of the sky. The parallel arrows indicate the direction along which the moon will set. The dashed long line is the ecliptic with an arrow indicating the direction in which the sun ☉ and the moon ☽ move. The moon has passed the sun around a day before. In case of lunar latitude of 0 or 5°, respectively, the new crescent will be visible for the first time after conjunction, and it will set $NA_1=NA_1(0°)$, respectively $NA_1(+5°)$, time degrees after sunset. In case of $\beta=-5°$, the still invisible moon has already set, so that the new crescent only can be seen on the next evening

taken place. Some partial sums, $(\text{ŠÚ}+NA)$ and $(ME+GE_6)$, were also noticed, and these measure the daily retardation of the setting and rising full moon, respectively (see Fig. 4). These time delays were used in the prediction rules in connection with the Saros period.

We note that the Saros = 223 synodic months ≅ 18 years, besides being a well-known time interval between eclipses, also is an import lunar period:

$$223 \text{ syn. m.} = 6585 \ 1/3 \text{ day} \approx 239 \text{ anom. m.} \approx 241 \text{ drac. m.} \approx 242 \text{ sid. m.} \approx 18 \text{ Years}$$

The Goal-Year method utilizes the fact that both, the lunar latitude $\beta_{\mathbb{C}}$ and the velocity $v_{\mathbb{C}}$, will repeat after one Saros = 223 months and that conjunction (or opposition) will take place at almost the same place $\lambda_{\mathbb{C}}$ in the ecliptic as in year Y-18. Only the time of opposition (or conjunction) will be shifted by 1/3. This results in a shift in time of both ŠÚ and NA by 1/3 of the daily retardation $(\text{ŠÚ}+NA)$ of the setting full moon, and similarly in a shift in time for both ME and GE_6 by 1/3 of the daily retardation $(ME+GE_6)$ of the rising full moon. Hence, the new Lunar Six values, i.e. those of the new year Y to come, were calculated from the old values from year Y-18, listed on the Goal-Year tablet for year Y. This corresponds to the following calculational procedures:
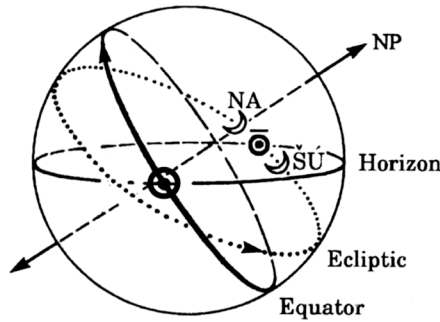
Fig. 4 The situation on the celestial sphere at sunrise on two mornings $m1$ and $m2$ around full moon: the symbol ☉ on the eastern horizon shows the rising sun. The symbol for the "anti-sun", situated at the western horizon, is about to set. On the first morning $m1$, the moon was in position ŠÚ and has already set. On the next morning $m2$, the moon has moved with respect to the sun and is at position NA at the moment of sunrise. On morning $m1$, the moon sets at ŠÚ time degrees before sunrise, and on the next morning $m2$, it sets NA time degrees after sunrise. Evidently, ŠÚ + NA measures the daily retardation of the setting moon, namely how much later with respect to sunrise the moon sets on day 2 than on the first day

$$\mathrm{NA}_1(\mathrm{new}) = \mathrm{NA}_1(\mathrm{old}) - 1/3\big(\check{\mathrm{S}}\acute{\mathrm{U}} + \mathrm{NA}\big)(\mathrm{old}\text{-}6) \tag{6}$$

$$\check{\mathrm{S}}\acute{\mathrm{U}}(\mathrm{new}) = \check{\mathrm{S}}\acute{\mathrm{U}}(\mathrm{old}) + 1/3\big(\check{\mathrm{S}}\acute{\mathrm{U}} + \mathrm{NA}\big)(\mathrm{old}) \tag{7}$$

$$\mathrm{NA}(\mathrm{new}) = \mathrm{NA}(\mathrm{old}) - 1/3\big(\check{\mathrm{S}}\acute{\mathrm{U}} + \mathrm{NA}\big)(\mathrm{old}) \tag{8}$$

$$\mathrm{ME}(\mathrm{new}) = \mathrm{ME}(\mathrm{old}) + 1/3\big(\mathrm{ME} + \mathrm{GE}_6\big)(\mathrm{old}) \tag{9}$$

$$\mathrm{GE}_6(\mathrm{new}) = \mathrm{GE}_6(\mathrm{old}) - 1/3\big(\mathrm{ME} + \mathrm{GE}_6\big)(\mathrm{old}) \tag{10}$$

$$\mathrm{KUR}(\mathrm{new}) = \mathrm{KUR}(\mathrm{old}) + 1/3\big(\mathrm{ME} + \mathrm{GE}_6\big)(\mathrm{old}\text{-}6) \tag{11}$$

Sometimes, the procedure required corrections, namely when the subtractions in Eqs. (6), (8), or (10) would lead to negative numbers. Then, the Babylonians solved this problem elegantly: they realized that when NA(old) < 1/3(ŠÚ+NA)(old), then NA(new) would occur 1 day later, and the difference 1/3(ŠÚ+NA)(old)−NA(old) was correctly identified as ŠÚ(new). [For further details, see Brack-Bernsen (1999) and Brack-Bernsen and Hunger (2008).]

Appendix 2

Development of linear zigzag functions

The atypical text E (Brack-Bernsen and Hunger 2005/2006) is concerned with lunar latitude. It started with a simple rule of thumb for finding the position of the lunar

nodes—and it utilized the schematic year of 12 months of 30 days and often identified dates with positions. We have showed how the schematic year and the corresponding schematic movement in latitude of the moon may have been utilized as a practical tool for the construction of a linear zigzag function. The important point in this connection is that the Babylonians started with a period P which was a little too long. Instead of shortening the period by the appropriate amount a, they first calculated the latitude by means of the scheme with period P. Then, they used the scheme once more to find the change over the interval a, and so they found the correction to the period by dividing by 12 for each month. In our vocabulary: they used successive approximations. I shall illustrate here how this method works if we start out with a schematic zigzag function with the period of a year $= 12$ months, and then add a correction which is due to the fact that 12 months is about 1/3 months shorter that the solar year.

I shall argue on an imaginary function with the period of 1 year $= 12$ months. Let our imaginary function have the minimum $= 28$ in month 1, maximum $= 30$ in month 7, and minimum again in month 13. The change over 6 months equals to $2 = 30$–28; therefore, the change per month would be $2/6 = 0; 20$. This numerical function has the period $P = 12$ months and the following values:

Month	1	2	3	4	5	6	7
Value	28	28; 20	28; 40	29	29; 20	29; 40	30
Month	13	12	11	10	9	8	7

We now subtract a correction for the fact that the solar year is not equal to 12 months but rather to 12 1/3 month. The change per month, $d(\text{old}) = 0; 20$ leads to the change over 1/3 month being equal to 1/3 times $0; 20 = 0; 6,40$. This is the correction for the period of 12 months. The correction for 1 month would therefore be 1/12 times $0; 6,40 = 0; 0, 33,20$. We have here constructed a linear zigzag function with a non-integer period.

Month	1	2	3	4	5	6
Value	28	28; 19,26,40	28; 38,53,20	28; 58,20,0	29; 17,46,40	…

Let me add some technical details, using the letters introduced by Neugebauer in ACT for the characterization of Babylonian numerical functions.

The starting point, a simple zigzag function, had the amplitude 2, the monthly change, $d(\text{old})$, equals $0; 20$, and the period $P = 12$ months, which was also the number period Π, so that $z = 1$. Our new zigzag function still has the amplitude 2 (still varying between 28 and 30), but the other parameters are different: the monthly change, d equals $d(\text{old}) - d(\text{old})/(3 \times 12) = d(\text{old}) \times 35/36$: $d = 0; 19, 26, 40 = 0; 20$–$0; 0, 33, 20 = 0; 20 \times 35/36$.

We have now 36 times more values; therefore, $\Pi = 12 \times 36 = 432$, $Z = 35$, and the new period $P(\text{new}) = \Pi/Z = 432/35 = 12; 20,57,34, \ldots$ which is non-integer. The smallest distance δ between values of this function comes out naturally: it is just the correction $\delta = 0; 0, 3,20$. Note that this very primitive numerical function is

quite similar to the linear zigzag function A of system B. Therefore, it gives us an idea to how some of the linear zigzag functions, which we find in the ACT material may have been developed. It may also give us a hint to how one can reconstruct $\Phi = \hat{\Sigma} + 1;40$ from observations of the sum Σ of the lunar four. The 14-month period for the lunar velocity was probably known to the Babylonians. Maybe it had been noticed that this period was a little too large, so that a correction was introduced. The sum Σ varies very irregularly, so it is not easy to construct a linear zigzag function which fits the data well. It is possible that special eclipses were chosen, situated n Saros apart, for which the sum Σ of the Lunar Four was changing rapidly. This is the case when the difference to the value of Σ for the month before and for the month after was large. But this is another story—or another research project.

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