Non-probabilistic chance?

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"Chance" crops up all over philosophy, and in many other areas. It is often assumed – without argument – that chances are probabilities. I explore the extent to which this assumption is really sanctioned by what we understand by the concept of chance.

"Chance" or "chancy mechanisms" crop up all over philosophy and elsewhere. It is often taken for granted that the proper representation of chance is probability theory. I call this claim *chance-probabilism*. There's nothing in the pretheoretic notion of chance that makes this conceptual link so tight as to block discussions of alternative theories. What arguments are offered for chance-probabilism are made in passing and none is convincing.

I try to offer an analysis of the concept of chance and I show that probabilism is not part of such a concept. I claim there is some pretheoretic concept of chance. This is, after all, what probability theory was invented to deal with. I think the current technical usage of the term is connected to this pretheoretic usage in the same way that "force" in physics or "continuity" in mathematics are connected to folk uses of those terms.[1] I think

[1] Thanks to an anonymous referee for suggesting this analogy.

there is enough of a family resemblance between a variety of chance notions that I can discuss all of them together. I try to show that it is certainly not obvious or analytic that chances must satisfy the axioms of probability theory.

There is some nontrivial groundwork that needs to be done before one can even properly *articulate* the claim that chances are probabilistic. I do this by discussing length. I claim that the two cases are analogous in certain ways. Length, like chance, is a certain kind of quantity that admits of a certain kind of structure. Discussing the less controversial length case gives us an easy warm-up for the hard case of chance. In particular, I discuss what one needs to say about the relation "is longer than" in order for there to be a function that represents length and for this function to have the requisite structure.

The idea is that if we see chance as some sort of quantity that attaches itself to events, then a similar measurement theory analysis can take place as takes place in the case of length. I argue that there could be nonprobabilistic chances. That is, I argue that the requirements one would need to place on the "is more likely than" relation in order to have a probabilistic representation are too strong to apply universally. I review some other arguments that might bolster the claim that chances are probabilistic, and I find that they also rest on assumptions that are too strong in general.

Probability theory began as the study of games of chance. So it seems that probability should be the right formalism to discuss chance. Early works on the theory of probability describe themselves as works about chance. John Venn's influential book on probability is called "The Logic of Chance". Thomas Bayes' contribution to the study of probability is entitled "An Essay towards solving a Problem in the Doctrine of Chances". So it seems that chance and probability are intimately connected. And indeed I don't deny that there are cases of chances that are appropriately modelled with probability theory. The "games of chance" that early probabilists studied – cards, dice, casino games – do seem to admit of a reasonable probabilistic interpretation. However, we must remember that Euclidean geometry began as a study of real space. It does not follow that actual space is Euclidean.

The structure is as follows: first I discuss the motivations for the project. I talk about why we might be interested in the structure of chance. Next I step back and ask the question "what *are* chances?" Once I've collected a set of "platitudes" about chances, I move on to how to properly articulate the claim that chances are probabilities. Then I move on to arguments for this claim; a direct argument first, and then some indirect arguments.

What are chances?

First, let's just ask what's at stake. Why should we care about the structure of chances? Chances come up in various places:

- discussions of determinism/indeterminism
- the related topics of randomness and unpredictability
- the debate about Humean chance
- the debate about deterministic chance
- discussions of risk
- Darwin's appeal to "chance variation" as part of natural selection
- the use of mixed acts in game theory
- von Neumann–Morgenstern and Anscombe–Aumann representation theorems
- "probabilistic causation"
- as a method of making some allocation process fair
- constraining credence via the principal principle

On top of its role in these debates, chance is an important concept in and of itself. First because we have this category of "chance" and it's worth making clear what sort of structure it has. In the same way that philosophical analyses of important concepts – causation, mind, scientific theory – are just of inherent interest, I think chance is important. Second because a proper analysis of chance makes clear some aspects of the relationship and difference between determinism and determinacy.

Is there a single chance concept? Probably not. But the chance concepts are related, I think that "chance" is a family-resemblance concept. In any case, an unexamined assumption of most uses of chance concepts is that chances are probabilities.

To take one example of where this unexamined assumption of chance-probabilism could well make a difference, let's consider Lewis' famous

Principal Principle (Lewis 1986). This says that your degrees of belief ought to conform to your knowledge of the objective chances. Without wanting to wade into the details of this tricky discussion, we can summarise the PP as saying that "If you know that the chance of X is x then you ought to believe X to degree x". Now if chances are nonprobabilistic, then your credences ought to be so too. However, there are other norms that govern credence. One important one is, awkwardly, *credence-probabilism*: your degrees of belief ought to conform to the calculus of probabilities. So it seems that in order to keep the two norms from being in conflict with each other, chances need to be probabilistic. So, if you subscribe to credence-probabilism, and to PP, then you need to argue for chance-probabilism. That is, you need chance-probabilism to be true in order to rule out your norms being in conflict. More carefully: credence-probabilism, the Principal Principle and nonprobabilistic chance are not compatible. If you were committed to both norms, then that, and the fact that your all-things-considered norms cannot be in conflict would entail chance-probabilism. One would argue as follows: "My credences are necessarily structured in a certain way, and my credences must track chances. Thus chances must be structured the same way". But this seems backwards. My beliefs should conform to how the world is, not the world to my beliefs in it. Alternatively the incompatibility might suggest that one or the other of the norms was faulty. I take this second option to be more reasonable: facts about the structure of the world are more entrenched than what we take the norms to be, so the possibility of nonprobabilistic chance suggests that one of the norms needs revision. That is: credence-probabilism, the Principal Principle and nonprobabilistic chance cannot all be held. The question is which to give up. It seems to me that conflict with putative norms shouldn't be enough to adjudicate on the truth of a claim about the world. That is, conflict with putative norms like PP shouldn't be enough to guarantee the impossibility of nonprobabilistic chances. Furthermore, in discussions of credence, credence-probabilism and the PP are both somewhat controversial, so neither norm seems as entrenched as it would have to be to block nonprobabilistic chance.

There are, I think, two main ways to understand what sort of thing chances are. You could take chances to be features of a chance set up that has certain dispositions to behave in certain ways. So a flipped coin has certain dispositional features that make it the case that the outcomes of the chance set-up – the events – have the chances that they do. Call these *dispositionalist* understandings of chance. The alternative is to understand chance as a *relational* property of events. Relative to a reference class, the chance of X is the relative frequency of Xs in the reference class. Chance is a relational property of reference classes. Call these sorts of view *relationalist* views. For the most part I am going to be speaking as if I am taking a dispositionalist understanding of chance, but I think everything I say will translate straightforwardly into the relationalist understanding.

Our next task is to work out what sort of thing a "chance" is. What are chances for? What role does something have to play to be considered as a chance? I shall first list several plausible platitudes about chance, and then discuss them. If we understand "$\mathbf{ch}(X)$" as giving some sort of numerical description of the chance of X, then something plays the "chance role" when it satisfies (most of) these criteria:

World Chance facts are claims about the world

Frequencies are evidence Observed frequencies are evidence of chances

Chances explain frequencies How the chances are should explain the frequencies we observe

Logic Chances relate to logic and truth

Theories Scientific theories tell us about the chances

Credence Chances are what constrain credences through the principal principle

Possibility $\mathbf{ch}(X) > 0$ iff X is possible

Futurity If X is an event in the past, its chance is 1 or 0

Intrinsicness If X' is an exact duplicate of X, then $\mathbf{ch}(X) = \mathbf{ch}(X')$

Lawhood Chances are determined by laws of nature

Causation "Causal chances arise within the causal interval they impact"

Schaffer (2007) offers several of the above criteria for when something plays the chance role.[2] Others I have extracted from various other discussions of chance. So this gives us some handle on what sort of things an adequate account of chance should provide. This list of criteria for the

[2] He lists the last 6 of these. He also seems to be committed to at least World. My presentation owes much to the discussion of Schaffer in Glynn (2010).

"chance role" is not uncontroversial, but it gives us some idea about what sort of properties *some* people think chances ought to have. For example, Ismael (1996) does not think that Intrinsicness should be a property of chances; Eagle (2011) takes issue with Possibility; some propensity theorists have denied the connections between frequencies and chances. In any case, I think the above list does something to triangulate the concept of interest.

I am understanding chances as being an objective feature of the world. That is, for the purposes of this paper, I am denying the possibility that chance talk is just elliptical for credence talk.[3] I want to understand chance as a quantity: a property that admits of degrees and attaches itself to things in the world. When I say "the chance of X is x" I am making a claim about the feature of the world X. A chance is a quantity that attaches itself to an event, in the same way that length is a quantity that attaches itself to a rigid body. That is, it is a property of events that admits of degrees. There can be more or less chance of some event happening, just like there can be more or less length of an object. Events for the propensity theorist are to be understood as outcomes of "chance set ups". For the relationalist, an event is a member of some reference class.[4] I am trying to be theory neutral: I want to say as little as possible about what events are. I will ascribe some properties to events that they need to have in order for some property of them to be probabilistic. But other than that, I am being agnostic about what sort of understanding one might have of events.

An important thing to know about a property or quantity is how to recognise when something has it, or to measure how much of it something has. We have various robust ways for measuring length and weight, say, and that's how we know what we're talking about when we talk about those quantities. For the case of chances, we have no such direct measurements available. We do, however, have some slightly more indirect ways of gettting a handle on how much chance certain events have. We have this in virtue of the link between chances and observed frequencies. It hardly seems worth saying that the observed relative frequency of a coin's landing heads is part of the evidence we have for the chance of heads of that coin.[5]

[3] This distinction gets fairly blurry towards the end of this paper, however.

[4] This is, of course, a thoroughly unhelpful definition without an explication of the concept of reference class.

[5] Depending on your view on chance, it might be that coins are not the sort of things that have chances attached to them. Nothing in the current paragraph hangs on this way

And likewise, it hardly seems worth pointing out that the coin's having a particular chance of heads is what explains that particular frequency. But these two ideas are an important part of the concept of chance.

If we think of logic as assigning 0 to falsehoods and 1 to the truths, then chances seem like an extension of this idea: certain kinds of propositions get intermediate values. The debate about the possibility of deterministic chances turns on whether there can be *non-trivial* deterministic chances. It is accepted that trivial chance functions – that assign only zeroes and ones – do apply to deterministic worlds. The understanding is that trivial chance functions just encode what things are true and what are false of the world. So the trivial chance functions must be bound by the same laws of logic as apply in the world. This insight will give us a way to impute a structure to chances.

Quantum mechanics contains chance-like objects, and these are what prompted to Popper (1959) develop his "Propensity theory". Propensities can be understood as a kind of chance. The debate about Humean chance involves recourse to various other chance-like objects in science. Ismael (2009) argues that chance-like entities are indispensible for physical theories. In any case, chances appear in physical theories: physical theories can give us evidence about the nature and structure of chances.

As well as relating to logic and truth, chances also connect with certain modal notions like possibility. A sufficient condition for something to be possible is for it to have non-zero chance. Schaffer (2007) goes further and argues that this is also a necessary condition. Eagle (2011) ties chances not to the standard modality of possibility, but to what he calls "CAN-claims". In Eagle's view, "CAN φ" is a kind of relative modality: relative, that is, to certain contextual features. Eagle argues that chances are also thus context sensitive. This does not mean that they are subjective, however.

> I wish to emphasise that even though "CAN" is a relative modality, this does *not* mean that it is in any way epistemic or "subjective"... [T]here is an objective fact of the matter concerning whether a certain contextual restriction is, or is not, in place with respect to a given claim "CAN φ"; and an objective fact concerning whether that restriction is, or is not, compatible with the proposition "φ". Eagle (2011, p. 284)

of thinking. Everything should work just as well here if you replace mentions of the coin with mentions of whatever it is that have chances attached.

In any case, it certainly seems that chance claims are related to claims about some sort of modality of possiblity or ability.

The Futurity condition can be seen to be related to the connection to truth and to determinism. The idea is that if we take for granted that events in the past have fixed truth values, then these must be reflected in the chance function which must then assign them trivial chances. Schaffer (2007) takes this to be part of the chance role. On some understandings of chance it makes sense, on others it needn't be part of the role chance plays.

Schaffer argues for Intrinsicness in this way: "The intuitive rationale for [Intrinsicness] is that if you repeat an experiment, the chances should stay the same." (Schaffer 2007, p. 125). We need to be careful about what we mean by the terms we use here. We need to be especially careful about what it is that we duplicate. Consider an intrinsic duplicate of a tossed coin: this duplicate has all the same intrinsic properties as the original. The duplicate, however, is in outer space, not on Earth. This duplicate wouldn't have the same chances, since it wouldn't fall in the same way. In a similar vein, Gillies (2000, p. 812) says:

> Suppose first we had a coin biased in favour of "heads". If we tossed it in a lower gravitational field (say on the Moon), the bias would very likely have less effect and [**ch**(Heads)] would assume a lower value. This shows an analogy between probability and weight. We normally consider weight loosely as a property of a body whereas in reality it is a relational property of the body with respect to a gravitational field.

The idea is that chances are going to have to be relational in this way, and thus when we are duplicating events, we need to duplicate the appropriate background conditions too. That is, when we are thinking of intrinsic duplicates, we are thinking of intrinsic duplicates of *the chance set up*, not of the coins or dice that we colloquially attribute the chances to. Ismael (1996) argues that the intrinsicness of chances, even so construed, is incompatible with a certain kind of view of chance. Roughly, the views of chance that are incompatible with intrinsicness are those that make chance relative to a reference class. This is no problem for Schaffer since he is interested specifically in a dispositional understanding of chance.

We have already discussed the idea that scientific theories give us some evidence about chances. A related principle is what Schaffer (2007) calls the "Lawful Magnitude Principle". This says that whatever the true laws of nature are, they determine what the chances are. We can see how our best guesses at the laws of nature – contained in our scientific theories – should give us evidence about chances if this principle held true. Schaffer suggests that laws determine the chance through certain true "history-to-chance conditionals". These are essentially conditionals of the form "if the world up to now has been H, the chances are **ch**" where H is some description of the history of the world and **ch** is some description of the chances.[6]

Finally, Causation. The intuition behind this aspect of chance is just that if chances are supposed to "do causal work", then they had better be at the right point in time to do that work. That is, if you want chances to explain a certain frequency, then the chance should be temporally located around the point in time where the outcomes were being determined. Put it this way: it would be very odd to explain the observed frequency of a series of coin tosses that happened yesterday in terms of some chances that are temporally located next week. This is less about chances *per se* and more just about when we expect explanations of phenomena to be temporally located.

A further two properties of chance that it is worth putting on the table now are:

Chance-probabilism Chances obey the axioms of probability theory

Determinacy Chancy events, or the outcomes of chancy events are determinate

The first of these, as we have seen, is the target of the current paper. The second of these is something that, were it true, would make chance-probabilism more defensible. But more of that later. Schaffer took Chance-probabilism to be a basic condition on chances.

Chance is among a bundle of concepts that are often used interchangibly but should perhaps be kept separate. This bundle includes the concepts "chance", "indeterminism", "unpredictibility" and "randomness".

[6] As I have done throughout, I am supressing the temporal dimension of the chance functions that Schaffer has in mind.

Earman (1986) and Eagle (2011) both contribute to trying to separate out these similar ideas.

This section has given a (partial) picture of the chance concept or concepts I am focussing on. I am going to talk as if there is a single concept for reasons of grammatical simplicity, but I don't expect anything I have to say relies on there being a unified concept. I take it that one thing these concepts have in common is a commitment to chance-probabilism: a commitment I think is generally unwarranted.

Of course, not all ordinary language uses of "chance" are going to be amenable to analysis. For example, when I say "I had a chance to go swimming with dolphins", this should be understood as referring to some sort of *opportunity*, rather than to some dispositional property with a probabilistic structure. One might want to subsume such cases into one's understanding of chances, but I don't need to do that for now.

Chance probabilism

It's interesting to note that no one has really tried to argue for chance-probabilism except in passing. Joyce (2009) says "some have held objective chances are not probabilities. This seems unlikely, but explaining why would take us too far afield." (p. 279, fn. 17).[7] Perhaps it is considered too obvious to be worth commenting on. But many people just take it for granted that chances are probabilities, so it seems like it is something people are committed to.[8] For example, Schaffer (2007) stipulates, without comment, that chances satisfy the axioms of probability theory.

There has been a lot of discussion over exactly what sort of thing chances are. Indeed, just about the only thing that seems to have been agreed on is that they are probabilities. I aim to put even that into doubt.

[7]It is perhaps unfair to single out Joyce here. Joyce is, in fact, doing better than most in even acknowledging that things could be otherwise. The reasons Joyce has for saying what he does here are effectively what I have been calling FREQUENCIES ARE EVIDENCE and CHANCES EXPLAIN FREQUENCIES (personal communication). Indeed, I owe those ideas to Joyce.

[8]Note that I am talking here about the basic idea of a chancy event. This discussion is orthogonal to that of Humphreys' paradox (Humphreys 1985; Suárez forthcoming). The issue there is to show that some conditional probabilities cannot be understood as propensities. Conditionalisation does not enter into the current discussion. What's really at stake here is the *additivity* of chances, something Humphreys does not discuss.

The plan is to try, as much as possible, to be "theory neutral". This means that I want to argue in such a way that whichever interpretation of chance you subscribe to – dispositional, relational – you can accept my arguments. The important thing is that chances are a feature of the world, and they have some relation to frequency, logic and scientific theories.

But chance-probabilism, as stated, doesn't really seem to even make sense. Chances are a feature of the world, satisfying the axioms of probability theory is a feature of real-valued functions defined over an algebra. Does it even make sense to say that this feature of the world satisfies this purely formal structure? To show that chance-probabilism makes sense, I want to focus on a simpler and less controversial example first. Before analysing the claim "chances are probabilistic" I analyse the less controversial claim "length is additive". Just like chance-probabilism, length-additivity looks like a category mistake. Length is a feature of the world that certain kinds of things have, additivity is a formal feature that certain kinds of functions can have. So there is some non-trivial groundwork to do to even articulate the claim we want to discuss. I have broken this argument down into numbered claims. The point of this is that when I come to discuss chance-probabilism I shall draw out the same structure in the argument.

1. Length is a quantity – a property that admits of degrees – that attaches itself to some kinds of things.

2. Call "sticks" things that have a length. For my purposes, pencils, arms, book spines, the imaginary line between your outstretched fingers: these are all sticks.

3. Some sticks are longer than others. Say "$X \succeq_{\mathbf{len}} Y$" means "$X$ is at least as long as Y". Define "$\succ_{\mathbf{len}}$" and "$\sim_{\mathbf{len}}$" as the irreflexive and the symmetric parts of the relation respectively.

4. There is an operation you can perform on sticks: composition. You can lay sticks end to end and parallel. Call the compound of X and Y, $X \oplus Y$. It is also a stick.

5. The set of sticks ($\mathbb{S}$) has some structure. For all X, Y, Z we have $X \oplus Z \succeq_{\mathbf{len}} X$. If $X \succeq_{\mathbf{len}} Y$ then $X \oplus Z \succeq_{\mathbf{len}} Y \oplus Z$.

6. There is a privileged stick: the null stick. No stick is shorter than the null stick, $\emptyset$. $X \oplus \emptyset \sim_{\mathbf{len}} X$.

7. Given some technical conditions, there is an additive function $\mathbf{len} : \mathbb{S} \to \mathbb{R}$ that assigns to each stick, a real numbered value: its length. **len** represents $\succeq_{\mathbf{len}}$ and is unique up to affine transformation.

8. By "**len** *represents* $\succeq_{\mathbf{len}}$" we mean that $X \succeq_{\mathbf{len}} Y$ if and only if $\mathbf{len}(X) \geq \mathbf{len}(Y)$.

9. By "**len** is additive" we mean $\mathbf{len}(X) + \mathbf{len}(Y) = \mathbf{len}(X \oplus Y)$.

Measurement theory studies this idea of representing a quantity. You get theorems that look like this: "If $\mathbb{S}$, $\oplus$ and $\succeq_{\mathbf{len}}$ have the right sort of properties, then the function **len** will have certain other properties." The properties of the function that represents the quantity tell us things about that quantity itself. For example, contrast length and temperature. Length has an additive representation. Compose two sticks in the right way – end to end and parallel – and you get a stick that has a length exactly equal to the sum of the lengths of the two component sticks. However, not so for temperature. There's no interesting physical composition procedure such that temperature is additive with respect to that procedure.[9] Put two thermal bodies in contact and the temperature of the composite body will be some sort of average of the two temperatures of the composite bodies before composition, not their sum. So this tells us that length and temperature are interestingly different *as quantities*, not just as regards their representations.[10]

There is a mathematical theorem that backs up point 7 on this list. For the technical details, the classic work is Krantz et al. (1971). See also Ellis (1966) and Kyburg (1984) for more philosophical treatment.

I want to take a moment here to emphasise that only some aspects of the representation function can be understood to be telling us things about the world. For example, there exists a particular representation function that measures the particular stick X as having a length $\mathbf{len}(X) = 42$. This is a fact about the function that we don't take seriously as a fact about the world. The X stick has no intrinsic property of "forty-two-ness". The

[9] Ellis calls length an *extensive* quantity, and temperature *intensive*.

[10] Length and temperature differ in their structural features: $X \oplus Y \succeq_{\mathbf{temp}} X$ is not true for temperature.

point is that there will be other functions that don't give X that value of length that will represent the quantity just as well. So it is only properties invariant under an appropriate class of transformations that we take to tell us something about the world (Ellis 1966; Kyburg 1984; Stevens 1946). Additivity is like this. A ten centimetre stick composed with another ten centimetre stick make a twenty centimetre stick. If we instead think of these sticks as being 3.9 inch sticks, they compose into a 7.8 inch composite stick. So additivity does not depend on the details of the representation used.

We've seen what it means to claim that length behaves additively. It is a claim about how we can represent an aspect of the world. Arguably, what representations of the world are possible depends on how the world is, so how we represent the world tells us something about how the world is.

Let's look now at what one might want to say about chance in a similar vein. The analogy isn't perfect. Chance, as a quantity, is probably more like velocity or temperature than it is like length. That is, the methods for measuring chance aren't going to be what Ellis (1966) calls "fundamental measurement". Nonetheless, a look at the measurement theory approach to chance-probabilism will be instructive.

1. Chance is a quantity – a property that admits of degrees – that attaches itself to some kinds of things.
2. Call "events" things that have chance.
3. Some events are more likely than others. Say "$X \succeq_{\mathbf{ch}} Y$" means "$X$ is at least as likely as Y".
4. There are operations you can perform on events: conjunction, disjunction, negation... If X and Y are events, then so are

   - $X \vee Y$ (X **OR** Y)
   - $X \wedge Y$ (X **AND** Y)
   - $\neg X$ (**NOT** X)

   Events have logical structure.
5. The set of events ($\mathbb{E}$) has some structure: for example, if X, Y are events then $X \vee Y \succeq_{\mathbf{ch}} X$.

6. There are two privileged events, $\top$ and $\bot$: the necessary and the impossible event, respectively. $X \wedge \top \sim_{\mathbf{ch}} X$ and $X \vee \bot \sim_{\mathbf{ch}} X$.

7. Given some technical conditions, there is a probability function $\mathbf{ch} : \mathbb{E} \to \mathbb{R}$ that assigns to each event, a value: its chance. **ch** represents $\succeq_{\mathbf{ch}}$ and is unique. See the appendix for details.

8. By "**ch** represents $\succeq_{\mathbf{ch}}$" we mean $X \succeq_{\mathbf{ch}} Y$ if and only if $\mathbf{ch}(X) \geq \mathbf{ch}(Y)$.

9. By "**ch** is a probability function" we mean:
   - $\mathbf{ch}(X) + \mathbf{ch}(Y) = \mathbf{ch}(X \vee Y) + \mathbf{ch}(X \wedge Y)$
   - $\mathbf{ch}(\bot) \leq \mathbf{ch}(X) \leq \mathbf{ch}(\top)$ for all X
   - $\mathbf{ch}(\bot) = 0$ and $\mathbf{ch}(\top) = 1$

The intricacy of the above argument, and the detail required, show that it is not trivial that all chances are probabilistic. There are two places where possibly contentious substantial assumptions are made in the above argument. The first is in the assumptions made about the event structure; second, there are the assumptions made about the "$\succeq_{\mathbf{ch}}$" relation structure.

Let's take the event structure first. One thing to note is that we need $X \wedge Y = Y \wedge X$. The event structure is such that these "and" and "or" connectives are commutative, and that the compound events are always in the event space. Typically in formal representations of quantum events, like in quantum logic, you don't have this commutativity. It is true that in all observable bases, you do have commutativity, and indeed the mod-squared amplitudes are additive, but commutativity doesn't hold more generally (Rédei 1998; Rédei and Summers 2007). Krantz et al. (1971) discuss "QM-qualitative probabilities", where these differ by not always having conjunctions. That is, it can be that X and Y are in your event structure, but $X \wedge Y$ isn't.[11]

[11] For instance, if X says something very specific about a particle's position and Y something specific about its momentum, then while $\mathbf{ch}(X)$ and $\mathbf{ch}(Y)$ might have values, $\mathbf{ch}(X \wedge Y)$ doesn't make sense, since it would violate the uncertainty principle. See p. 214 of Krantz et al. (1971). On p. 215 they state a theorem to the effect that QM-algebras can still have a probabilistic representation, but this requires a non-standard understanding of a probability space, which allows QM-algebras to be the kind of thing probabilities are defined over.

There is a second problem with the event structure. In the course of proving the uniqueness of the probability measure, some strong assumptions about the event structure are made. For instance one way to do it is to assume that: if $X \succ_{\mathbf{ch}} Y$ then there is a collection of mutually incompatible but exhaustive events Z_i each of which is so implausible that $X \succ_{\mathbf{ch}} Y \vee Z_i$ for all i (Krantz et al. 1971, p. 206). This forces the event structure to be infinite.[12] This idea that there need to collections of arbitrarily unlikely events is not something that I can find in my folk concept of an event.

So let's say we are happy with commutativity, existence of conjunctions, and collections of arbitrarily unlikely events. Let's move on to the conditions on the relation structure.

One condition requires that all chances be comparable. That is, the $\succeq_{\mathbf{ch}}$ relation must be complete. I don't see why the event "this die lands 6" needs to be comparable with the event "global mean surface temperature will rise by more than four degrees by 2080". These are both arguably events that might have chances attached to them, but I can't see why it has to be the case that they ought to be comparable. For the case of length, we have some idea what we mean when we say two lengths are always comparable. We have an intuitive idea of procedures we can follow that will establish which of two lengths is the longer.[13] In the case of length, many of the conditions can be given some intuitive weight by talk of sticks, composition and procedures of comparing their lengths. In the case of chance, it isn't so clear what the measurement procedures are supposed to be. These procedures are missing for the chance case. So there is at least some work to be done here in arguing for chance-probabilism. It seems like the procedures we would use to measure chances like "this die lands 6" will be so unlike the procedures used to estimate the chance of four degrees of warming that there is no reason to think that they will be comparable. They will both output numbers, and those numbers will be comparable. But this is putting the cart before the horse: I can "compare" the height of X with the weight of Y, but this isn't a meaningful comparison. The point is that for the two things to be comparable, there needs to be some kind of *procedure* for doing controlled comparisons. In general, I

[12] Krantz et al. (1971) offer an alternative that is satisfied by some finite structures, "although not in most" (p. 207). The axiom is not intuitive, as they admit, and in the interests of space I omit a discussion of it.

[13] Comparing lengths of, say coastlines or rivers is far from straightforward, but let's leave that aside. I claim there are extra difficulties in the chance case.

can't see that any such procedure will exist across all chances.

There is a well-known tension between the demands of probability theory and infinite spaces. Consider throwing a dart at the real unit interval. What is the chance that the dart hits a rational number? The intuitive answer is that $\mathbb{Q} \succ_{\mathbf{ch}} \bot$: that is, hitting a rational number is more likely than the impossible event. This is a long-winded way of saying that hitting a rational number is *possible*. But probability theory forces upon us the conclusion that[14] $\mathbf{ch}(\mathbb{Q}) = 0$. So it seems that probabilistic chance isn't making some distinctions we might want to make between these sorts of events. There has been some back-and-forth on hyperreal probability theory recently[15] that I don't want to get into, but it is another thing to bear in mind.

In any case, it seems like intuitions or folk conceptions of the "event concept" aren't strong enough to support the heavy duty technical conditions required here.[16] The next section will go further and argue that there are plausible cases where these conditions are indeed violated.

Chance and indeterminacy

To recap, a direct argument for chance-probabilism would involve claiming that the space of events had the requisite structure, and that the relational structure had certain properties. Neither of these claims seems reasonable in general. One might step back from this and say instead that even though the premises of the direct argument are too strong, there are independent reasons to subscribe to the conclusion. So, one would step back from the measurement theory approach, but continue to maintain that there is a function on events that represents chance; that this function is real valued; that it is bounded; and that it is additive.

In this section, I want to sketch an example that I take to involve chances in the sense we have been discussing, and I want to show that such chances do not satisfy the axioms of probability theory. That is, I want to give an example of chance that is represented by a function that

[14]There may be some way to give a probability measure on the reals such that the rationals get non-zero measure, but no measure will assign all non-empty sets non-zero measure.

[15]For example Hájek (ms.).

[16]The appendix lists these conditions.

is not additive. The example involves appeal to genuine indeterminacy or vagueness. This might make many people sceptical of the relevance of the example. However, in a later section I argue that some other views on chance have similar problems.

An urn contains ninety marbles. Thirty marbles are determinately red and thirty are determinately orange. The remaining thirty marbles are not determinately red and not determinately orange, but they are determinately red or orange. The marbles are well mixed, the drawing procedure is suitably fair, and the chance set up has all the other properties you might hope it to have. What is the chance of drawing a red marble from this urn? We can certainly say that the chance is at least one third: the determinately red marbles guarantee at least this much chance. And we can say that the chance of red is at most two thirds: even if we included all the marbles that are such that it is vague whether they are red, we would only have two thirds of the marbles in such a collection. In such a situation, one might want to say that the appropriate model of the chances makes use of, say, interval-valued functions, or sets of proability functions. So $\mathbf{ch}(\text{Red}) = [\frac{1}{3}, \frac{2}{3}] = \mathbf{ch}(\text{Orange})$ and $\mathbf{ch}(textRedorOrange) = 1$. Or one might want to take the view that the chance of red should be understood as the chance of being determinately red. Such a function would not be additive, since $\mathbf{ch}(\text{Red}) = \frac{1}{3}$ and $\mathbf{ch}(\text{Orange}) = \frac{1}{3}$, but $\mathbf{ch}(\text{Red or Orange}) = 1$ since all marbles are either red or orange. In either case, it seems like a move beyond orthodox probability theory is the appropriate response to such examples.

Let's look at what this example tells us about the measurement theory analysis of the last section. Let's imagine that we're using some sort of interval-valued or set-valued function to represent chances. Now if two such intervals overlap, then which event is likelier than the other? Arguably no relation of "is likelier than" holds between events so represented. This suggests that the requirement that the relation "$\succeq_{\mathbf{ch}}$" be complete is unwarranted. More generally, it seems possible that it can be indeterminate which of two events is more likely. This again suggests incompleteness of that relation.[17]

[17] This example also violates the "quasi-additivity" condition on the $\succ_{\mathbf{ch}}$ relation.

Indirect arguments

Frequencies are probabilistic, and frequencies are evidence of chances. Perhaps we can link chances to statistics. Hájek (1997, 2009) argues that it should at least be possible for chances to be described by frequencies, and thus that chances should at least be probabilistic. Paris (1994) offers an argument that is similar to Hájek's in that it shows how anything that is measured appropriately by statistics should be probabilistic. Note that these are *not* frequentist arguments. Whatever your attitude to chance, it seems that chances have some relation to frequencies. So even the dispositionalists can take evidence from statistics as evidence for the structure of chancy powers.

Unless we can argue that *all* chances must be amenable to statistics, then this argument can't give us what we want from it. That is, this argument can tell us nothing about those dispositional properties that aren't part of a reference class for statistics. If you are of the opinion that chances are relational properties of reference classes, then perhaps you are happy to make this move, and say that all chances must be describable by statistics: they must be determined by the statistics of the reference class. This move has the standard problem that it makes a mystery of various sorts of one-off events that we would otherwise like to assign chances to. But perhaps if you favour a dispositional account, then this move seems a little less warranted. Why ought it be the case that all such chancy dispositions be amenable to statistical descriptions? Consider the quantum case again. If you accept that the noncommutative algebras that arise there can have chances attached to them, then there's good reason to think that they can't be described by standard statistics: they aren't amenable to measurement in the right way.[18] I'm not suggesting this position is not tenable: you certainly could be a hardcore operationalist about chances, just as about other quantities. But this seems like an extreme position to adopt in order to be able to claim that chances are probabilistic.

There is, however, a stronger argument against using the fact that statistics are probabilistic to sanction chance-probabilism: statistics aren't necessarily probabilistic. Walley and Fine (1982) offer a kind of statistics that involves upper and lower probabilities: an importantly different theory

[18] Indeed, Hartmann and Suppes (2010) argue that such "impossible to measure joint events" can be modelled with a (very strange) kind of "upper probability" which is not additive.

from orthodox probability. They can do this without contradicting the arguments of Paris and Hájek by allowing that the basic events might be indeterminate, in some way. If it can be vague whether X and vague whether Y, but determinate that $X \vee Y$, then the statistics will inherit this vagueness and probabilistic representation will not be guaranteed. Consider the statistics of the vague marbles example discussed earlier. The important point to note is that to secure chance-probabilism, the frequentist needs to argue that all chances are amenable to statistics, *and* that all outcomes of chance set ups are determinate. Or one can drop a strict chance-probabilism for some sort of vague chance-probabilism: all precisifications of the vague statistics are described by probability functions. I like this last suggestion and think it deserves some thought, but it is beyond the scope of the current paper.

To clarify the statistics argument above, let's look at a slightly more general putative argument for probabilism. This relates to another of our platitudes about chance. What about the relation of chance to truth and logic? Chances are convex combinations of truth value functions, and these are always probabilistic. A theorem due to de Finetti shows that all and only the probability functions are in the convex hull of the set of classical truth valuation functions. $\mathbf{v}$ is a (classical) truth valuation function if

- $\mathbf{v}(X \vee Y) = \max\{\mathbf{v}(X), \mathbf{v}(Y)\}$
- $\mathbf{v}(X \wedge Y) = \min\{\mathbf{v}(X), \mathbf{v}(Y)\}$
- $\mathbf{v}(\neg X) = 1 - \mathbf{v}(X)$

Call $\mathbf{ch}$ a "convex combination" of valuations if:

$$\sum \lambda_i = 1$$
$$\mathbf{ch}(X) = \sum \lambda_i \mathbf{v}_i(X) \text{ for all } X$$

$\mathbf{ch}$ so defined is a probability, and all probabilities can be so characterised.

Chances have to be "somewhere between" the possible worlds that could eventuate. If these are represented by the valuation functions, then perhaps this is enough to show that chances are probabilistic. Think of this as a kind of generalisation of the intuition that relative frequencies are somehow "averages" of possible outcomes.

How intuitive is the claim that chances must be in the convex hull of (a representation of) the truth values? I don't feel my intuitions here are strong enough to use this as an argument that chances are probabilistic. In the credence-probabilism case, there are good arguments that convex combinations of possible truth values are the right structure for your beliefs (Williams 2012a). But these arguments don't really translate into the case of chances. In any case, indeterminacy undermines chance-probabilism. That is, if indeterminacy prompts you to revise you logic, then what functions are convex combinations of the nonclassical valuation functions won't necessarily be probabilities. Paris (2005 [2001]) shows that for a particular class of nonclassical valuations you get superadditive functions in the convex hull.

The relational understanding of chance

In this section, I want to suggest that the relational reading of chance has the same problems with additivity that I have discussed for the dispositional account. For the relationalist, chances are determined by the relative frequencies in the reference class. So if 47% of your sample have some property, say "brown hair", then the chance that the next person sampled has brown hair is 0.47.[19]

Perhaps the statistics you have gathered are deficient in some way. Perhaps when counting people by hair colour, the light wasn't always so good, so you weren't sure whether someone's hair was brown or black. What do you do? Perhaps you guess: you say "it might have been brown or black, let's say black". Or perhaps you'd reason as follows: "I can't tell whether Bob's hair was brown or black, so let's put him in the disjunctive 'brown or black' category, without putting him in either of the disjuncts." The statistics you build up in this way will determine a non-classical valuation function, and your chances will be superadditive, but not additive. This is what Walley and Fine (1982) do, and what Walley (1991) does in much more detail.

You might argue that there is some fact of the matter about what colour hair Bob had, and therefore some proper probability distribution that was appropriate. So this marks a flaw in our knowledge, it does not imply that

[19]This is of course a caricature of the position, but it will serve for my purposes.

there are nonprobabilistic chances. But then, there is a fact of the matter about what colour hair the next person sampled will have. So whatever argument is put against there being nonprobabilistic chances in this way also argues against there being any nontrivial chances at all. If there are "level-relative" higher-level chances, as Glynn (2010) argues, then they will be relative to the sort of information available at that level in this way. There can be, on this picture, levels with genuine chances and "epistemic vagueness". So the above discussion of vagueness on the dispositional reading is also relevant here, and it undermines chance-probabilism for the relationalist too.

In short, if your account of chance is relative to a state of information so as to allow non-trivial chances at all in a deterministic world, then it won't be able to block nonprobabilistic chance.

Kyburg and Teng (2001) also build a non-probabilistic theory of statistics. Wheeler and Williamson (2011) discuss Kyburg's "evidential probabilities" approach, as do Haenni et al. (2010). The basic intuition behind this approach is that statistical inference only happens within certain margins of error. Once we explicitly model those margins of error, we no longer have an orthodox probability measure on events. We have an "interval valued" probability. Again one might worry that the human errors shouldn't be part of our understanding of what chances are. But then, we are happy with the idea that a better knowledge of the precise initial conditions of a coin toss could allow us to predict the result. This doesn't preclude assigning non-trivial chances to it.

Hájek and Smithson (2012) suggest some other ways that chances might fail to be probabilistic. First consider some physical process that doesn't have a limiting frequency but has a frequency that varies, always staying within some interval. It might be that the best description of such a system is to just put bounds on its relative frequency. If we took a Humean perspective on what chances are, this would make it the case that its chance is nonprobabilistic. Note that such a system would be indeterminstic and chancy, but perhaps not random and almost certainly not unpredictable.

So it's not straightforward that the relationalist accounts of chance are forced to accept chance-probabilism.

As in the above example, the relative frequencies argument or the "convex hull of valuations" argument would lead to superadditive non-probabilistic chances. For various kinds of nonclassical logics, it has been explored what "chance-like" functions one builds up in the analogous way to the de Finetti result for classical logic. See Paris (2005 [2001]) and Williams (2012b) for details.[20]

As an interesting corollary to this discussion and the preceding one on frequentism, it seems to be necessary to rule out indeterminacy, in order to have well behaved indeterminism. That is, if events are allowed to be somewhat indeterminate, then it is significantly harder to justify the claims you need to have chances behave probabilistically.

The "slogan form" version of this conclusion is: "Probabilistic chances only if outcomes of chance events are determinate". That "determinate" can be read epistemically, as referring to something lacking from your state of information; or it can be read ontically, as in metaphysical vagueness; or indeed semantically as in relating to how our language matches up with the world.

What I have aimed to do in the current paper is to show that the oft assumed claim that chances conform to the probability calculus is not as obvious as it is taken to be. Second I hope to have shown how you might argue for this claim, and also what might lead you to deny it.

Appendix: Probabilistic representation

Various authors have given theorems to the effect that events with a certain compositional and relational structure are uniquely represented by a probability function (Krantz et al. 1971; Savage 1972 [1954]; Villegas 1964). My presentation follows the treatment of Savage in Joyce (1999).

The theorem states that if the following properties hold:

- Normalisation: $\top \succ_{\mathbf{ch}} \bot$
- Boundedness: $\top \succeq_{\mathbf{ch}} X \succeq_{\mathbf{ch}} \bot$ for all X
- Ranking: $\succeq_{\mathbf{ch}}$ is a partial order
- Completeness: $X \succ_{\mathbf{ch}} Y$ or $Y \succeq_{\mathbf{ch}} X$ for all X, Y
- Quasi-additivity: If $X \wedge Z \equiv \bot \equiv Y \wedge Z$ then
  - $X \succ_{\mathbf{ch}} Y$ iff $X \vee Z \succ_{\mathbf{ch}} Y \vee Z$

[20] These arguments are given in the context of credences, but they can be translated into the chance case easily enough.

– $X \geq_{\mathbf{ch}} Y$ iff $X \vee Z \geq_{\mathbf{ch}} Y \vee Z$

- Richness: If $X \succ_{\mathbf{ch}} Y$ then there exists a partition of the event space: $\{Z_i\}$ such that $X \succ_{\mathbf{ch}} Y \vee Z_i$ for all i

then there exists a unique function **ch** that satisfies the axioms of (finitely additive) probability. See the above cited papers for proof. Securing countable additivity takes a little more work, and is not something I discuss here.

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