

REASONING: GOOD AND BAD

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[Extract from an unfinished elementary textbook, *Reasoning: the 4th "R"*]

1. LITERACY, ILLITERACY, AND THE THREE "R" S.

Discuss the so-called three "R"s - Reading, 'Riting, and 'Rithmetic - with educators or the general public and you'll get debate and dissension. Is enough emphasis being paid to them, or too much? Or do our educational institutions currently give them just the right amount of attention?

These are questions we won't even try to engage here. For whatever else the disputants may disagree about, they will almost certainly agree about this: that developing the skills of reading and writing (the first two "R"s) is not only a precondition of being well-educated, but also a precondition of being able to function satisfactorily in a civilized society. Someone who cannot read or write is said to be "illiterate" in a quite strict sense of the word (or perhaps "literacy deprived", if one subscribes - as we shall not - to the passing fads of political correctness). And illiteracy, as we all know, is something to be deplored. It is one of the scourges - along with ignorance, hunger, disease, crime, social and political repression - from which most of us would like to see the world, and elements of our own society, freed.

There can, of course, be degrees of illiteracy. It ranges all the way from the extreme case of blacks in South Africa some of whom have difficulty participating in the political process because they reportedly lack even the skills to mark a voting paper with an "X"¹, to the kinds of problems that college and university teachers complain about when faced with incoming students who seem unable to construct grammatically well-formed sentences let alone string them together in well-structured paragraphs and essays.

So, too, with the third "R", arithmetic - or more broadly, mathematics. Media reports over the years have repeatedly claimed that more than a quarter of adult

¹ Reported on CBC radio prior to the epoch-making elections in which blacks and other victims of apartheid first exercised the right to vote.

Canadians lack the ability to make simple arithmetical calculations of the kind needed to check their grocery bills. Many more, though able to handle their food bills, and perhaps even their income tax, are totally lost when it comes to handling statistics or understanding the practical implications of probability theory. There are degrees, in other words, of what mathematician John Allen Paulos calls "innumeracy" or "mathematical illiteracy".

FIRST INTERLUDE

THE CONSEQUENCES OF MATHEMATICAL ILLITERACY

In his fascinating and invaluable little book entitled *Innumeracy: Mathematical Illiteracy and its Consequences*,² John Allen Paulos (who says he uses his middle name so as not to be confused with the Pope) offers numerous illustrations of how a proper understanding of numbers, and associated statistical theory, can throw light on situations that the innumerate are likely to find puzzling.

Here are some of his examples. Think about them for yourself before turning to the discussion provided at the end of this interlude.

Chance Encounters.

Suppose that two strangers from opposite sides of the United States strike up a conversation while sitting next to each other on a business trip and make the "amazing" discovery that the wife of one was in a tennis camp run by an acquaintance of the other.

Assume that most of the 200 million or so adults in the United States knows about 1500 people, and that these 1500 people are reasonably spread around the country. Is it really surprising that the two strangers are linked by a chain of two acquaintances? How would you estimate the probability? (a) close to 100%; (b) about 50%; (c) less than 20%.

Predictive Dreams.

Suppose you, or your Aunt Matilda, have a vivid dream of a fiery car crash the night before Uncle Mortimer wraps his Ford around a power pole.

Isn't this too much of a coincidence to be explained by mere chance? Since such

² Published in Canada by Collins Publishers, Toronto, 1988. Republished by Penguin Books, Markham, 1990.

dreams do occur, and occur with remarkable frequency, don't we here have evidence of precognition? What do you think?

Medical Testing.

Suppose you know that cancer afflicts about one out of every two hundred people and that a particular test for the dread disease is 98% accurate. This means, of course, that if someone does have cancer, the test will be positive 98% of the time, and that if someone doesn't have it, the test will be negative 98% of the time. Now suppose, further, that you've just taken the test, as a routine measure (not because of any worrisome symptoms), and that the doctor reports that you've tested positive.

Given this information, how would you estimate your chances of having cancer?
(a) close to 100%; (b) about 50%; (c) about 20%.

A Dice Paradox.

You are challenged to a game involving four dice A, B, C, and D, and told truthfully that A will beat B (by coming up with a higher number) two-thirds of the time; that B will beat C two-thirds of the time; and that C will beat D two-thirds of the time. These probability-estimates result from the following facts about these unusual dice:

- A has 4s on four faces and 0s on the remaining two faces;
- B has 3s on all six faces;
- C has 2s on four faces 6s on the remainder;
- D has 5s on three faces and 1s on the rest.

You are given the option of playing - for high stakes if you wish - a series of games against the challenger, and allowed to use the die of your choice. Which die should you choose?

ANSWERS AND DISCUSSION

Chance encounters:

According to Paulos (p. 29), we can easily calculate that the probability is about one in a hundred that the strangers will have an acquaintance in common, and more than ninety nine in one hundred that they will be linked by a chain of two intermediates. Seemingly strange coincidences like this, he points out, are bound to occur frequently.

Indeed it would be surprising if they did not.

These sorts of facts about probability help to undermine all sorts of factitious nonsense: the widespread belief in synchronicity (as put forward by psychologist Carl Jung, for instance)³; and New Age theories about cosmic fate. More generally, they show that we need to beware of drastically underestimating the frequency with which coincidences naturally occur; otherwise we shall too often be tempted to accord to them a significance that they lack.

Predictive Dreams.

Paulos shows, to the contrary, that such experiences are more rationally accounted for by sheer coincidence. We don't need to invoke any special precognitive powers. It suffices to make a plausible assumption about the probability of any given dream matching experience in a few vivid details.

Suppose, for example, that the probability is something like one out of every 100,000 that there's a match. Then it is easy - if we are mathematically literate - to calculate that about 3.6% of the people who dream every night will have a predictive dream once every year and hence that, in a country the size of the United States, there'll be millions of such dream-occurrences each year. (See p. 55).

As Paulos goes on to argue, the purveyors of pseudo-sciences such as this (other examples that leap to mind are parapsychology, astrology, UFOlogy, and numerology) find a ready market for their wares in the credulity of innumerate.⁴

Medical Testing.

Most of us would be inclined to fear, supposing that the test has shown it to be close to 100% certain that we've got cancer. Yet a little mathematically informed reasoning should persuade us that our prospects aren't so bad at all. Remember our assumption that one in two hundred people (0.5 %) have this particular form of cancer. Then Paulos's reasoning, as he explains it on page 66, goes like this:

³ Another of the ideas for which Jung is famous is that of the "collective unconscious". Many psychologists and most philosophers think it highly suspect.

⁴ Not surprisingly, Paulos is a Fellow of the Committee for the Scientific Investigation of Claims Of the Paranormal (acronym "CSICOP") whose periodical *The Skeptical Inquirer* (Box 703, Buffalo, NY 14226-0703) provides an invaluable service to those who want to get their thinking straight about these matters.

Imagine that 10,000 tests for cancer are administered. Of these, how many are positive? On the average, 50 of these 10,000 people (.5 percent of 10,000) will have cancer, and so, since 98 percent of them will test positive, we will have 49 positive tests. Of the 9,950 cancerless people, 2 percent of them will test positive, for a total of 199 positive tests ($.02 \times 9,950 = 199$). Thus, of the total of 248 positive tests ($199 + 49 = 248$) most (199) are false positives, and so the conditional probability of having cancer given that one tests positive is only $49/248$, or about 20 percent.

These results, Paulos points out, contain lessons not only for those who go for medical checks (e.g., taking the Pap test for cervical cancer, a test which is only about 75% accurate), but also for those legislators who contemplate mandatory testing for drugs or AIDS or the like. And, we might also note, they need to be taken into account, along with rates of natural recovery, when we set about evaluating claims of miracle cures by faith-healers, psychic surgeons, and the practitioners of quack medicine.

A Dice Paradox.

The answer seems obvious: maximize your chances of winning in the long run by picking A. Yet, when you select A and your opponent picks D, you will be perplexed (even distressed, if much money is at stake) when you find that D beats A two-thirds of the time.

How is this possible? The answer emerges when we take time to calculate the probability of D beating A. Since D has 5s on half its faces, D will beat A in approximately half of all the throws, since A's highest number is only a 4. In the other half of these throws, a 1 will turn up on D and in just two-thirds of these cases will A win (since A has a 4 on two-thirds of its faces) while D will win the remaining one-third of the time. Hence, contrary to earlier expectations, the probability of D beating A is $1/2 + (1/2 \times 1/3) = 2/3$.

Most of us, I suspect, would completely overlook this possibility. It wouldn't even occur to us that A might be beaten by D more often than not. Why? Probably we'd just jump to the conclusion that A would beat D most of the time because we'd assume that *being a more probable winner than* is a relation that behaves in much the same way as *being larger than* or *being taller than*. These latter relations (sometimes called "relationships") are said to be TRANSITIVE. Each is such that if it holds between one thing and a second and also between the second and a third, then it *must* hold between the

first and the third. Thus if A is taller than B, B is taller than C, and C is taller than D, then it follows that A is taller than D. It just isn't possible for D to be taller than A. But the relation of *being a more probable winner than*, it turns out, is different, as the above calculations show. It isn't a transitive relation but a NONTRANSITIVE one.⁵

A different example might help to make this clear. Suppose boxer A beats boxer B two-thirds of the time, that B beats boxer C two-thirds of the time, and that C beats boxer D two-thirds of the time. Does it follow that A will probably beat D? Surely, not. Of course A just might beat D most of the time, or even all of the time. But it certainly doesn't follow that he will. For D might have strengths which enable him to capitalize on A's weaknesses so as to be able to beat A most or all of the time. Until they actually do meet in the ring we simply have no warrant for drawing any conclusion about the likely winner.

So what is your best strategy with the dice, once you've worked out all the probabilities? Obviously, it is to insist that your would-be opponent make the first choice of a die (so that you can choose one that gives you odds of 2:1 for a win). Or, if your opponent refuses first choice, don't play the game at all.

There's nothing really paradoxical about any of this. Careful attention to the details of a case can help us to avoid all sorts of inferential errors involving probabilistic reasoning. There are important lessons here for those who investigate various social phenomena, e.g., the so-called "voting paradoxes" discussed by economist Kenneth

⁵ Note the use of the word "transitive" here. It is different from that in which a verb is said to be "transitive" when it takes a direct object and "intransitive" when it doesn't. Here the term is used to characterize certain logical features of relations. In this logical use, a relation that isn't transitive is either intransitive or nontransitive. To say that a relation is intransitive is to say that if it holds between A and B and between B and C, then it follows that it doesn't hold between A and C. To say that a relation is nontransitive is to say that if it holds between A and B and between B and C, it neither follows that it holds between A and C nor follows that it doesn't hold between A and C. The relation of being twice as tall as is intransitive. The relation of being an admirer of is nontransitive.

Relations may also be classified according to whether they are symmetrical, asymmetrical or nonsymmetrical. Being married to is symmetrical (if A is married to B then it follows that B is married to A). Being a mother of is asymmetrical (if A is a mother of B then it follows that B is not a mother of A). And being a sister of is nonsymmetrical (if A is a sister of B then it neither follows that B is a sister of A nor that B isn't a sister of A).

Arrow (1921-)⁶.

Paulos's book, it should be said, is full of examples - of which the above are but a sample - of ways in which mathematical illiteracy can lead to mistaken inference and thereby to false belief. Read it for yourself, and learn - as I have - just how many misconceptions innumeracy fosters. Learn, too, just how damaging those misconceptions can be to sane theory and practice. As Douglas Hofstadter has said: "Our society would be unimaginably different if the average person truly understood the ideas in this marvellous and important little book."⁷

2. REASONING: THE FOURTH "R" .

Illiteracy of any of the three kinds so far discussed is surely lamentable. Hence it is that our schools, at both primary and secondary levels, offer courses in each of the three "R"s.

Yet there is a fourth "R" which few of us ever even think about, let alone learn in a systematic way. I refer, of course, to the matter of correct Reasoning.

In the temporal order of things, reasoning comes first among the "R"s. Our children learn some semblance of reasoning before they learn to read or write or do arithmetic. As for the individual, so too for the species. Archeological evidence abounds that our primitive ancestors could reason in the elementary ways required for tool-making and communal living well before they invented language, let alone the skills to read and write it. Even chimpanzees (with whom we share nearly 99% of our genetic make-up) are reported to have rudimentary powers of reasoning; and they certainly can't even read or write in any ordinary senses of those words. There is good reason to suppose certain of our own preliterate ancestral stock no less intellectually gifted than contemporary chimps. If it were not the case that early members of our species could think and reason, at least about simple matters, well before they could read, write, or perform arithmetical calculations, why else would they develop languages in which they could express their thoughts and reasonings?

⁶ In *Social Choice and Individual Values* (1951), Arrow shows that it is not in general possible to aggregate the preference-orderings of individuals into a single combined preference-ordering for society as a whole.

⁷ Hofstadter, another CSICOP Fellow, is author of the Pulitzer Prize-winning *Godel, Escher, Bach* (New York: Basic Books, 1979).

Not only is the ability to reason temporally prior to, and in this sense more basic than, the other three "R"s; it is also more important.

- We need to be able to read. But it is more important by far that, when reading, we reflect on what has been written and assess it for its consistency and its implications.
- We need to be able to write. But it is more important by far that, when writing, we present our subject-matter in a coherent and cogent fashion so that readers can follow the train of our thoughts.
- We need to be able to reason with numbers. But it is more important by far that we be able to reason, more widely, about all those other subject-matters that comprise the rich tapestry of human thought - matters in the fields of science, morality, art, religion, politics, and the rest of our experience.

More generally, we rely on our general powers of reasoning whenever we reflect on our past or present, make plans for the future, or resort to persuasion rather than violence to resolve our disputes. Civilization itself is one of the fruits of reasoning - including, but not limited to, reasoning in mathematics.

Yet unlike these others, the fourth "R", Reasoning, is seldom taught to those who need it most - the young, whose impressionable minds are so in need of the critical skills that will protect them from credulousness and irrational belief. To be sure, a few inroads are being made. At Monclair State College in New Jersey, the Institute for the Advancement of Philosophy for Children offers Masters degrees for future teacher-educators⁸; a similar program is offered by the University of New England, in Armidale, Australia⁹; and seminars in critical thinking are becoming the fashion for high school teachers in various parts of the world¹⁰. As the University of New England brochure puts it:

Philosophy in the classroom aims to develop reasoning skills in the areas of

⁸ Candidates must have at least an undergraduate degree in philosophy. Inquiries may be made to the Director, IAPC Programs, Institute for the Advancement of Philosophy for Children, Montclair State College, Upper Montclair, New Jersey 07043.

⁹ Here the only prerequisite is a university degree or relevant three-year diploma. The certificate is offered jointly by the Philosophy Department and the Department of Social, Cultural and Curriculum Studies. No previous study of philosophy is necessary. Inquiries should be addressed to the Administrative Officer, Faculty of Education, Nursing and Professional Studies, University of New England, Armidale NSW 2351.

¹⁰ All too often at the risk of producing "instant experts" who simply don't comprehend how superficial their understanding is.

reading and oral comprehension, conceptual analysis, imaginative thinking, formulating and solving problems, giving and evaluating reasons, and establishing sound judgement. Its basic assumption is that children will most readily become active and skilful thinkers by learning together through dialogue and discussion.

All this is commendable. But we have a long way to go before the study of reasoning assumes its rightful place, alongside the other "R"s, in primary and secondary school curricula around the world.

Nor is the situation much better in colleges and universities. There the study of the principles of correct reasoning - mainly conducted in Philosophy departments - is too often thought to be the special concern of those few students who will go on to advanced studies in Philosophy, Mathematics, or Computing Science. But such students aren't the only ones whose powers of reasoning need to be developed. The ability to reason correctly is surely just as important - if not more so - for students of the natural and social sciences, the humanities, technology, law, medicine, architecture, ethics, politics, religion, and other areas of human knowledge and belief. And that ability is best learned by studying the discipline of Logic, a science whose principles of correct reasoning embrace every subject-matter whatsoever.

SECOND INTERLUDE

REASONING WHILE YOU ARE READING

Some years ago, the then Dean of Arts at Simon Fraser University invited a number of literacy experts from elsewhere in Canada and the United States to give advice as to how we might deal with increasing problems of illiteracy among our students. What were the causes of their illiteracy? How could we, at the University, best address the difficulties which they, and we as their teachers, were experiencing?

Various remedies were canvassed. One had to do with the ways in which students are taught to read. It was suggested that students often didn't realize the need to adjust the pace of their reading to the nature of the subject-matter being read. There are times, it was pointed out, when so-called "speed-reading" is wholly appropriate - when reading a novel, perhaps, or scanning through a newspaper to get a quick overview of its contents. But equally, it was insisted, there are times when one needs as it were to "get into low gear", pausing frequently to reflect on the significance of what one is reading, asking

one's self such questions as:

- Do I understand exactly what the author is getting at?
- Is what the author is saying here consistent with what he/she says elsewhere?
- Is what the author is saying here consistent with what other authors say about the same topic?
- Is it consistent with what I believe to be true?
- If not, where does the truth lie?
- What evidence, if any, is given - or might be given - for the truth of the claims I am reading?

In asking, and trying to answer, questions like these, one is actively exercising one's reasoning powers in ways that enhance understanding. And maybe, it was suggested, teaching students to read in this sort of thoughtful, reflective, and critical way, would help them to become more literate.

Some advice: The subject-matter of Philosophy in general - and Logic in particular - is especially in need of this low-gear approach. You would do well to adopt it in your reading of the present text.

An anecdote: Here's a simple little story which may help you remember this advice. It was told by the then Chair of the History Department who was present at our discussions of the illiteracy problem. To the best of my recollection, this is how it went:

An historian colleague of mine, at the university where I taught some years ago, breezed into the Faculty lounge at lunch-time and reported, "This morning, I read the whole of Hegel's *The Phenomenology of Mind* [a notoriously obscure work much stressed by Marxist historians and philosophers]." This provoked a philosopher to look up from his arm chair and protest, "You call that reading? I've just spent the morning on the first paragraph!"

Why do you think these two academics approached the same subject-matter in such different ways? What rationale would you give for each?

Test yourself: Now put your own reading, and reasoning powers, to the test by trying to answer these questions:

1. What differences, if any, in meaning and truth do you perceive between the following two typographically very similar sentences?

- (a) All blackbirds are black.
- (b) All black birds are black.

[Note how easy it would be, were one speed-reading, to fail to see any distinction between the two, and hence to fail to reflect on the question whether they have the same meaning and truth-value.¹¹]

2. Suppose you are on a ship making its passage through the Red Sea, and a child asks, "How come the Red Sea isn't red?" How would you answer?

Think about these questions for yourself before turning to the suggested answers given below.

ANSWERS AND DISCUSSION

1. **Blackbirds and black birds.**

A good way of approaching the question, "How do sentences (a) and (b) differ in meaning?", is to ask yourself, for each sentence in turn, "In what possible circumstances would this sentence say something true, and in what possible circumstances would it say something false?"

Evaluated in this light, sentence (a) - "All blackbirds are black" - would obviously be true only if every member of the species of bird that we call blackbirds had the color black; otherwise it would be false. Are there, then, any *possible* circumstances in which it would be false? Surely the answer is "Yes". We can certainly envisage the possibility of a member of that species turning out to be an albino, or even some other color. [In fact, ornithologists tell us that albino blackbirds, though rare, do in fact occur. But even if they didn't occur, we can certainly imagine what it would be like to come across a non-black blackbird. Our experience of such an exception to the generality that blackbirds are black in color, would be akin to the experience of the first ornithologists to visit Australia. All the swans they'd ever come across in Europe were white. Yet there - in this strange antipodean land - were some black swans! Being white certainly wasn't part of the strict definition of "swan". So they had to acknowledge that the statement that all swans are white was not only conceivably false, but actually false as well.] Sentence (a),

¹¹ Sentences or statements are said to have one or other of two truth-values: truth, or falsity.

then, even if it had in fact been true, certainly needn't have been. Its truth or falsity is - as we sometimes say - CONTINGENT on what the facts turn out to be, i.e., contingent on whether or not all members of the species of blackbirds are indeed colored black.

By way of comparison, sentence (b) - "All black birds are black" - can't be imagined false. It isn't about the species, blackbird. Rather, it is about all those birds that have a certain color, viz., black; and it says of such birds that they are black! Such a sentence - as we sometimes say - is or expresses a TAUTOLOGY or NECESSARY TRUTH. It would be incoherent to deny the truth of such a statement, for in denying it you would be saying that there are non-black black birds - birds which both are and are not black! You would be contradicting yourself. Since (b) says something which is true in all possible circumstances. It is not only true; it is necessarily true.

Sentences (a) and (b), then, differ both in meaning and in truth-value. Although the expressions "blackbirds" and "black birds" are typographically so similar that one might - on a rapid reading - fail to discern any difference, they have quite different meanings. The term "blackbirds" functions as a NAME: it refers to a species of bird. [True, the species might well have been so named because most (or even all) its members are black in color. But, as we have seen, their having that color isn't a necessary condition of belonging to that species.] The expression "black birds", however, functions not as a name, but as a DESCRIPTION - a description of certain birds in terms of their color. It is, of course, because of its descriptive role in sentence (b) that (b) expresses a tautology.

2. The Red Sea and red seas.

In light of the distinctions just drawn between sentences (a) and (b), you may now be in a better position to answer the question, "How come the Red Sea isn't red?" [Think about it for a moment before continuing reading. Can you see how the distinction between names and descriptions might help resolve the seeming puzzle? Try to explain its relevance in your own words.]

And now that you've tried for yourself, let's see how the answer might go. Obviously, someone who is puzzled as to why the Red Sea isn't red must be taking the expression "Red Sea" to be a description, not a name. For if it were a description, then the Red Sea couldn't fail to be red - it would just *have* to be red! On the other hand, if we

recognize that the expression "Red Sea" is a name, indeed a proper name, then the puzzle should evaporate. For the name as such doesn't imply anything about the properties of the thing named. True, the Red Sea may, for all I know, have been so named because at the time of naming it was full of the reddish micro-organisms that are hereabouts known as "red tide". But even if that were the case, once the name has been appended, the thing named could well come to lack that color. For that matter the expression "Red Sea" doesn't even *have* to stand for a sea or any other body of water! Compare the case of the name "Dartmouth". As the great logician, John Stuart Mill pointed out in his *System of Logic* (1843), this name was given to a certain town because at the time of naming it stood at the mouth of the river Dart. Yet today the town of Dartmouth lies many miles from the mouth of the Dart. So too, the name "Red Sea" could eventually come to denote a dried-up valley between Saudi Arabia and the north-east coast of Africa.¹²

The short answer to the question, "How come the Red Sea isn't red?", is that which one of my two young sons (both under six years of age) gave to the other who had asked the question while we were passing through the Red Sea in 1960. His explanation was, "Because that's its name, silly."

The strict distinction between denotation and connotation

The importance of this distinction between names and descriptions has often been pointed out by philosophers of language. We need, they would say, to distinguish between two quite different senses of the expression "the meaning of a term", viz., the DENOTATION of a term and the CONNOTATION of a term.

Roughly speaking, the denotation of a word is what (if anything) the term refers to, i.e., its REFERENT. [As to whether the denotation of a term counts as part of its "meaning" at all, is itself a matter of dispute in some quarters, some theoreticians preferring to restrict talk of meaning to the logician's sense of "connotation".] In logic, the connotation of a word has to do with the ESSENTIAL PROPERTIES of the thing (or things) that the word refers to. Thus logicians and philosophers of language usually would say that the denotation of the word "mother" is all mothers that happen to exist (whether this counts as part of the meaning being another matter), while the connotation

¹² We don't have to rely on imaginary examples to make the point. In the dry interior of Australia, for instance, numerous lakes and rivers lack water for decades on end.

of "mother" involves the essential properties of being human, being female, and being a parent.

Bringing the strict denotation/connotation distinction to bear on our examples, then, many philosophers would claim that although descriptions may have both connotation and denotation, proper names (and names of species or kinds) have denotation only, their sole semantic function being to pick out or label the bearer of the name. That is why, they would say, the Red Sea needn't be red, and blackbirds needn't be black.

The looser, psychological sense of "connotation".

Outside logic, the connotations of a word are shifting, subjective, things having to do more with the "overtones" that are rung in the minds of hearers than the strict meaning of the word or what speakers intend by its use. In this subjective, psychological sense, the connotations of a word like "mother" are taken to include the properties which people usually associate in their minds with the word - properties like being loving, caring, and a home-maker. In this looser sense, the properties connoted by the word "mother" needn't invariably be found in mothers. Indeed, in some communities, they could well come to shift in such a way as to give the term a pejorative (deprecatory) ring, the term "mother" coming to be associated, perhaps, with over-indulgence, excess of piety, or whatever else is characteristic of mothers at a particular time and place. Poets, preachers, and politicians make much fine play with these sorts of connotations. So do the proponents of political correctness, those who would have us change our terminology to keep pace with every altering shade in the emotional colorings of words.

3. LOGIC: THE SCIENCE OF CORRECT DEDUCTIVE REASONING

Is logic descriptive, or prescriptive, or both?

The term "logic" is sometimes used in an extremely loose way such that it comes to be thought of as a virtual synonym for "reasoning." Thus we find writers in the popular media, and elsewhere, using such expressions as "according to this logic", where it is evident from the context that they mean nothing more than "according to this way of

reasoning" or "according to this way of thinking".¹³ Such usage ought to be abjured: it is grounded in careless thought and inept expression, and can only give rise to confused thinking about the nature of logic itself.

A better account of logic is that which describes it simply as "the science of reasoning". But this doesn't get it quite right, either. Describing it as the science of reasoning still makes it sound as though the discipline of Logic sets out to tell us how people do *in fact* reason. If that were the case, then logic would be a branch of Psychology, and there would be as many logics as there are ways of thinking or reasoning. Now to be sure, some cultural anthropologists, and others, do seem to conceive of logic in this sort of way.¹⁴ But they are not the experts. Those who are truly familiar with the discipline of Logic certainly don't think of it in this way.

By way of reaction to the foregoing errors, some have said that logic isn't a factual or descriptive science at all, but a purely normative or prescriptive one. On this account, Logic doesn't purport to tell us how we do in fact think and reason but only how we ought to think and reason. But even that doesn't quite get it right. For we are inclined to think of norms and prescriptions as things that humans lay down, and hence also as things that humans can revoke. Think of logic in this way and you are likely to ask:

- Who sets the norms for correct reasoning?
- What makes them the authorities?
- Isn't it presumptuous of logicians to prescribe norms of reasoning for everyone else?
- Why shouldn't I pick my own or change them in accordance with my own preferences?
- Couldn't there be different, and perhaps even inconsistent, rules of reasoning for

¹³ Here is a nice example of this sort of confusion. Ottawa lawyer, Eugene Oscapella, in the *Opinion* page of the *Vancouver Sun*, Thursday, April 21, 1994, laments the fact that most parliamentarians and members of the general public think that liberalizing the strict prohibition against drug use would increase drug-related violence. Here is the way he expresses his objection: "Simple, self-evident logic. However, that logic is as flawed as it is simple. It is based on the premise that the use of illegal drugs generates most of the violence associated with these drugs." His criticism is a good one, but poorly expressed. What he is attacking is the truth of a certain premise in these people's thinking, not the logic they employ in drawing certain conclusions from it. In short, what he is attacking is their reasoning, not what he calls "their" logic.

¹⁴ See our discussion, in Chapter 2, of the reasoning of primitive Azande tribesmen.

different people, different subject-matters, or different cultures?

Describing Logic as a purely normative science makes it sound as though its principles are up to us to stipulate in a more or less arbitrary way. And it fosters, once more, the idea that there are no "absolute" standards for correct reasoning.

Later we will discuss more carefully these, and related, issues about the nature and status of the science of logic. For the moment, suffice it to say that, on the view taken in this book, logic does indeed contain a normative, prescriptive element (which is why it is important to describe it as the science of *correct* reasoning), but that the norms of correctness that it prescribes, far from being conventionally determined, are grounded in facts about reality - facts that can be stated in terms of objective descriptive truths.

By way of rough illustration of how a prescription can be grounded in objective facts, consider the way in which the prescriptive principle "Don't jump off that precipice!" is grounded in physical facts about how gravity affects physical objects. In something like the same sort of way - I shall argue - the principles of correct reasoning are grounded in logical facts about what follows from what. In neither case is the prescriptive element merely a consequence of some more or less arbitrary decision on our part about what we think persons should or should not do.

A better analogy is to be found by considering the principles of mathematical reasoning. Obviously, one would be making an indisputable mathematical error if one were to reason that the sum of two odd numbers must itself be an odd number. Similarly, if one were to argue that since prime numbers (numbers divisible only by themselves and one) occur less and less frequently as one progresses through the series of integers, the series must be finite, coming to an end with a greatest prime number. It is a fact of mathematics that the sum of two odd numbers is always even. And it is a fact of mathematics (first proved by Euclid) that there can't be a greatest prime number since there are infinitely many of them. It isn't up to us - in any interesting sense - to make these mathematical facts otherwise (though, of course, it is entirely up to us how we *express* these mathematical facts).

Likewise with Logic. Here, too, there are standards of correctness which are set not by us but by the logical facts themselves. Obviously, one would be guilty of an indisputable error of logic if from the fact that all dogs are mammals one were to conclude that all mammals are dogs. The fact of the matter is that although it is true that

all dogs are mammals the converse is just plain false, and hence the stated conclusion does not follow from the stated premise. Again, one would be arguing fallaciously if one were to suppose that if HIV is a cause of AIDS, then it is the *only* cause. Here the truth of the conjectured premise is quite compatible with the falsity of the conclusion. Hence its being the only cause of AIDS doesn't follow logically from its being *a* cause of AIDS.

As in the case of mathematical reasoning, so too in the case of reasoning about other subject matters, what follows (and what doesn't follow) from given premises is determined independently of us. The science of logic, then, has as its goal the discovery of objective truths that are in no way at the mercy of our subjective whims.

Reasoning and Arguments

Reasoning is a bit like travelling: there is a starting point, called the PREMISE or PREMISES; and there is a finishing point, called the CONCLUSION. One INFERS the conclusion from the premise(s). [Hereafter, for convenience, we'll often simplify matters by speaking of "premises" even if there is only one.]

The premises and conclusion of a piece of reasoning can be set out in the form of an ARGUMENT. Thus, for instance, we would say that a person who inferred that HIV is the only cause of AIDS from the premise that HIV is a cause of AIDS, is presenting an argument for his conclusion. Here the use of the term "argument" is estranged from its use when we talk of two or more people "having an argument". The latter use usually connotes (in the loose psychological sense noted earlier)¹⁵ dissension, angry words, even shouting at one another. The former does not. In logic, an argument is nothing more than a sequence of statements one of which, the conclusion, is said to follow from the other(s), the premise or premises.

Presenting arguments

Often, when we present arguments, we state the premises first and then the conclusion. When this is the order of presentation, we usually indicate which statement has the status of conclusion by preceding it with some such word as "therefore", "hence", "so", "thus", "consequently", or "it follows that". Sometimes, however, we present our

¹⁵ See page 13 above.

arguments with the indicator-words at the start. For instance, our arguments may have the form "Because [premises], [conclusion]." Or again, "Since [premises], [conclusion] follows." And other variants of this general pattern are possible.

There is, however, nothing sacrosanct about this ordering, i.e., stating premises before the conclusion. It is adopted in most Logic texts, but purely as a matter of convenience. Other orders of presentation are permissible, and frequently occur in ordinary oral and written discourse.

There is no reason why we shouldn't state the conclusion first, and then the premises. Consider, for instance:

All whales are warm-blooded. [Conclusion]

All whales are mammals. [Premise]

All mammals are warm-blooded. [Premise]

Here, of course, it would be natural to express the argument by saying something like, "All whales are warm-blooded, since [or because] they are mammals and all mammals are warm-blooded."

Again, we might state one of the premises first, followed by the conclusion, and then finish with one or more other premises. For instance, we might choose to say, "All whales are mammals. Hence, they must be warm-blooded, since all mammals are warm-blooded."

The order of presentation - it is important to note - does nothing to affect the quality of the argument, whether it is a good one or a bad one. That depends solely on whether the conclusion does or does not follow from the premises.

Deductively Valid and Invalid Arguments

We shall say that an argument is DEDUCTIVELY VALID if and only if it is not possible for those premises to be true while that conclusion is false. Here is an example of a deductively valid argument:

All whales are mammals. [Premises]

All mammals are warm-blooded.

All whales are warm-blooded. [Conclusion]

It is deductively valid because there's no possibility of the premises being true and the

conclusion false.

By way of contrast, consider the argument:

HIV is a cause of AIDS.	[Premise]
HIV is the only cause of AIDS.	[Conclusion]

Obviously, this argument is not deductively valid. Rather, it is DEDUCTIVELY INVALID. Its conclusion does not follow from its premise since it is perfectly possible for the premise to be true and yet the conclusion false. Even if the premise were true, its truth wouldn't give us any **guarantee** that the conclusion was true. And even if the conclusion were in fact true, its truth wouldn't FOLLOW from the stated premise since we can well envisage the possibility of there being other causes as well - the possibility, that is, of the premise being true but the conclusion false.

There is a rough intuitive test for whether or not an argument is deductively valid. Ask yourself: Are there any possible circumstances in which the premises are true and the conclusion false? If the answer is "No", then the argument is deductively valid, i.e., the premises imply the conclusion, i.e., the conclusion follows from the premises. But if the answer is "Yes", then the argument is deductively invalid, i.e., the premises do not imply the conclusion, i.e., the conclusion does not follow from the premises.

Concepts related to that of Deductive Validity

The concept of deductive validity belongs to a network of closely related concepts. We have already drawn on some of these other concepts in passing. But the relationships between many of them can be set out, more formally, in the form of definitions. Each definition will take the form, "An argument is deductively valid if and only if . . .", where the blanks are filled by an equivalent clause involving some related concept.

The expression "if and only if" calls for some comment. In effect, it functions as a means of condensing two conditional statements, one the converse of the other. If we say something of the form, "P if and only if Q" (where the letters "P" and "Q" stand for statements or clauses) we are saying both:

- (i) "P if Q" (which means the same as "If Q then P"),

and

(ii) "P only if Q" (which means the same as "If P then Q").

Conditional statement (i) asserts that the truth of P is a NECESSARY CONDITION of the truth of Q (i.e., that Q wouldn't be true unless P were also true). Conditional statement (ii) asserts that the truth of P is a SUFFICIENT CONDITION of the truth of Q (i.e., that the truth of P suffices to bring it about [in some sense or other, perhaps logical, but maybe only causal]¹⁶ that Q will be true). Condensed into the form "P if and only if Q", these two conditionals state the necessary and sufficient conditions for P's truth (and for Q's).

Note, for future reference, that it follows from what we have just said that IF THE TRUTH OF P IS A SUFFICIENT CONDITION OF THE TRUTH OF Q THEN THE TRUTH OF Q IS A NECESSARY CONDITION OF THE TRUTH OF P.

Logicians frequently abbreviate the expression "if and only if" to "iff." We will follow that practice here.

Here are some of the more important definitions of conceptual relationships:

1. **Implication.**

An argument is deductively valid iff its premises **imply** its conclusion.

Note two points:

(i) In the strict logical sense of the term that will be employed here, implication is a logical relation (relationship) between sets of statements, not an activity performed by people. True, in ordinary language, we speak of people implying things when they act or speak in certain ways - as, for instance, when we say "She implied that she didn't know him." But this is a loose sense of the word, and seldom occurs in logic texts.

(ii) The strict sense of "imply" needs to be distinguished, also, from the term "infer". Inferring is indeed an activity performed by people. Thus we speak, correctly, of a person inferring a certain conclusion from certain premises. But it is grammatically as well as logically incorrect to speak of premises inferring [!] conclusions.

¹⁶ Ingesting a large quantity of potassium cyanide is causally sufficient (though not causally necessary) for bringing about death. But it isn't logically sufficient since no CONTRADICTION is involved in the supposition that someone has ingested this large quantities of this poison and yet survived.

2. **Following from.**

An argument is deductively valid iff its conclusion **follows from** its premises.

Following from is also a relation that holds, or may hold, between sets of statements. Clearly, the relation of following from, which holds between conclusion and premises of a deductively valid argument, is the CONVERSE of the relation of implication in much the same sort of way as the relation of *being shorter than* is the converse of the relation of *being taller than*.

3. **Necessitation.**

An argument is deductively valid iff its premises **necessitate** its conclusion.

In saying that the premises of a valid argument necessitate its conclusion we are once more describing a logical relation between premises and conclusion. We are not saying that the conclusion itself is necessarily true, i.e., that it couldn't possibly be false. What we are saying is rather that the conclusion couldn't possibly be false IF the premises were true.

4. **Necessity and Necessary Truth.**

An argument is deductively valid iff the corresponding conditional statement of the form "If [premises], then [conclusion]" is **necessarily true**.

In saying that the conditional corresponding to a deductively valid argument is necessarily true we are saying that it **MUST** be the case that if the premises are true then the conclusion is true, i.e., that this conditional statement is **TRUE IN EVERY POSSIBLE CIRCUMSTANCE**, every possible situation. Deductively valid arguments, then, correspond to necessarily true conditionals.

5. **Impossibility and Possibility**

An argument is deductively valid iff it is **impossible** for its premise to be true and its conclusion false.

In speaking of the impossibility of the premises of a deductively valid argument being true while its conclusion is false, we are saying that there are NO POSSIBLE CIRCUMSTANCES in which the premises are true and the conclusion false. The relevant concepts of possibility and impossibility involved here will be sharpened up in Chapter 2.

6. Self-contradictoriness and Self-contradiction

An argument is deductively valid iff it would be **self-contradictory** to assert its premises and deny its conclusion.

and

An argument is deductively valid iff the conjunction of its premises with the negation of its conclusion involves a **self-contradiction**.

Consider the deductively valid argument:

All whales are mammals. [Premises]

All mammals are warm-blooded.

All whales are warm-blooded. [Conclusion]

Obviously if you were to assert the premises of this argument, you would be committing yourself logically to asserting the conclusion (even if you didn't state that conclusion explicitly). That is to say, you would be implicitly asserting:

All whales are warm-blooded.

But now suppose you were to assert the premises and deny the conclusion. Then, implicitly, you would be both asserting that all whales are warm-blooded and denying it. In short, you would be contradicting yourself, and thereby saying something that couldn't possibly be true, viz., that whales both are and are not warm-blooded!

The second formulation differs from the first in avoiding any reference to persons asserting or denying the component statements in the argument. It introduces two notions: that of CONJUNCTION and that of NEGATION. In talking about the negation of a statement P, we are talking about the statement that one would assert if one were to deny P. For instance, the negation of the statement that all whales are warm-blooded is the statement that not all whales are warm-blooded, i.e., that some whales are not warm-blooded. In talking about the conjunction of the premises of an argument with the negation of its conclusion, we are simply talking about taking them together. Our second

formulation, then, says that since the premises of a deductively valid argument CONTRADICT the negation of the conclusion (whether or not anyone asserts or denies any of these statements), the argument taken as a whole involves a SELF-CONTRADICTION .

For the case of the example just given, our second formulation says that the argument

All whales are mammals.
All mammals are warm-blooded.
Therefore, all whales are warm-blooded.

is deductively valid because its premises

All whales are mammals.
All mammals are warm-blooded.

contradict the negation of its conclusion, i.e., contradict the statement

Not all whales are warm-blooded.

In other words, it is deductively valid because the set of statements consisting of the premises conjoined with the negation of the conclusion contains a contradiction within itself, i.e., is a self-contradictory set of statements.

7. Inconsistency and Consistency

An argument is deductively valid if and only if it would be **inconsistent** to assert its premises and deny its conclusion.

and

An argument is deductively valid if and only if the truth of its premises is **inconsistent** with the negation of its conclusion.

Contradiction - as we shall see in Chapter 5 - is just one form of inconsistency. Hence if it would be self-contradictory to assert the premises and deny the conclusion of an argument, it would also be inconsistent to do so, i.e., one would be involved in a form of inconsistency if one were to do so.

Once more, the second formulation avoids any reference to persons and their acts of asserting or denying. It says, in effect, that the truth of the premises themselves is NOT CONSISTENT with the negation of the conclusion (whether or not anyone asserts any of these statements).

Deductive validity, invalidity, and truth

Note that, according to the account given of deductive validity, a piece of reasoning (an inference, an argument) doesn't have to have true premises, or even a true conclusion, in order to be deductively valid. In saying that an argument is deductively valid, remember, all we are saying is that IF the premises are true, then so must the conclusion be true. We are not saying that the premises ARE true.

This means that the only combination of truth-values for premises and conclusion that one CANNOT have for a deductively valid argument is:

Premises = True; Conclusion = False.

Any other combination is compatible with the deductive validity of an argument. That is to say, a deductively valid argument CAN have:

(a) True premises and true conclusion:

Example:	All mammals are warm-blooded.	(True)
	All dolphins are mammals.	(True)
	So, all dolphins are warm-blooded.	(True)

(b) False premises and false conclusion:

Example:	All vodka drinkers are Russians.	(False)
	All Russians read Tolstoy on weekends.	(False)
	Hence, all vodka drinkers read Tolstoy on weekends.	(False)

(c) False premises and true conclusion:

Example:	All Russians are from Quebec.	(False)
	Jean Chretien is a Russian.	(False)
	Therefore, Jean Chretien is from Quebec.	(True)

By way of contrast, a deductively invalid argument can have any combination of truth-values for premises and conclusion.

What all this means is that simply by looking at the truth-values of the premises and conclusion of an argument you will not be able to infer anything about the validity or invalidity of that argument except in one case: the case where the premises are true and

the conclusion is false. In this case, and this case alone, you can validly draw a conclusion about the validity-status of the argument, viz., that it is invalid.

[Note that it is arguments that are said to be valid or invalid, statements that are true or false. It is incorrect to speak of arguments as true or false, and equally incorrect to speak of statements as valid or invalid.]

Validity, Soundness and Proof

Deductive validity is not the same as SOUNDNESS. The distinction between them can be expressed in the following definition:

An argument is **sound** if and only if it is both (a) deductively valid, and (b) has true premises.

It follows from this definition that sound arguments have true conclusions, not just true premises. [Try to explain simply and clearly *why* this follows.]

Logicians, as such, are usually concerned only about assessing arguments for their validity or invalidity, not for their soundness or lack of it. This is because, outside the domains of logic and mathematics, the question whether the premises of a given argument are true or false is usually one that a logician will prefer to leave to others who are expert in the subject-matter concerned.

The concept of soundness is closely related to that of PROOF. For if an argument from premises P to conclusion C is sound, then that argument is said to constitute a proof of C. More formally, we can define the general concept of proof¹⁷ thus:

An argument for a conclusion C constitutes a **proof of C** iff the argument for C is sound (is valid and has true premises).

[Note that it is conclusions of arguments that are said to be proved, arguments themselves that are said to be sound.]

¹⁷ In formal, logico-mathematical contexts a refinement of this general concept is employed. We need not concern ourselves with it here.

Needless to say, not all arguments that are put forward as proofs are successful. Throughout the history of religious reasoning, theologians and philosophers have repeatedly put forward what they consider to be proofs of the existence of God. Among the proposed proofs, two of the simpler and more persuasive are: (i) the argument from the supposed need for a First Cause of all that exists, and (ii) the argument that there must be a Great Designer who is responsible for all the order and design we see in the universe. Do these arguments really constitute proofs of God's existence? They will do so only if they are sound. And the arguments for them will be sound only if the arguments themselves are not only valid (deductively valid, that is) but also proceed from true premises. Both alleged "proofs" are therefore susceptible to attack in two ways: by questioning their validity, and by questioning the truth of their premises. It is only fair to say that theologians and philosophers of religion who have examined these "proofs" on these two scores, have almost always conceded that neither the First Cause Argument nor the Great Designer Argument is sound. This doesn't mean, of course, that their conclusions are false. It means only that their conclusions haven't been proved true, by these arguments. So the way is still open for other theologians and philosophers to try to come up with arguments for God's existence which really will prove the desired conclusion. In some circles, the enterprise goes on; in others, it is thought doomed to failure.

4. IS THERE A LOGIC OF INDUCTIVE REASONING?

It is clear that oft-times we argue in ways that don't satisfy - and aren't intended to satisfy - the criteria of deductive validity, viz., that the conclusion we reach should follow from (be implied by) the premises from which we start. We aren't always trying to show that our conclusions must be true if the premises are. Sometimes we are trying only to show that our conclusions are made likely, or more PROBABLE, by the premises, i.e., that the premises provide good evidence - though not conclusive evidence - for those conclusions. Such reasoning is standardly referred to as Inductive Reasoning, and the associated arguments are called Inductive Arguments.

We have already come across some instances of inductive reasoning - the inferences which Paulos invites us to examine. These, you will recall, had to do with the significance of chance encounters, apparently predictive dreams, medical tests, and games of dice.

It is clear that if logic were - as it is often said to be - the study of correct reasoning in general, it should encompass inductive reasoning as well as deductive reasoning. For inductive reasoning is certainly no less important in our daily lives, as well as in our abstract thinking, than deductive. One of the roles of inductive reasoning (though only one) is to yield generalizations from experience - generalizations of the sort that must surely have featured in the reasoning of our prelinguistic ancestors, and generalizations of the sort that feature today in our more sophisticated scientific theorizing.

Yet most textbooks on Logic - this one included - have little, if anything, to say about induction. There are two main reasons for this.

1. Inductive reasoning is standardly dealt with in textbooks on scientific methodology, statistics, and probability theory. Paulos's little book provides a highly accessible introduction to some of its central notions and the ways in which they can, and should, be applied.
2. Many thinkers, including many logicians and philosophers of science, would insist that there is no such thing as a logic of inductive reasoning. Logic, in their view, provides a set of principles - a set of algorithms, if you like - which *guarantee* us of the truth of our conclusions IF our premises are true. But no set of inductive principles can do that. At best they can only give us less than conclusive reasons for thinking certain statements true: their premises can **PROBABILIFY** their conclusions, but they cannot necessitate or imply them. Hence inductive arguments are evaluated on a different sort of scale from deductive ones. They are said to be "strong" or "weak", not "valid" or "invalid". There can, of course, be degrees of strength and weakness. The notions of deductive validity and invalidity, by way of contrast, don't admit of degrees, validity being an all-or-nothing matter. According to this widely accepted view, all logic - in the strict sense of the word - is Deductive Logic. Hence, on this view, the very term "deductive" and its cognates (near relatives) - as it occurs in expressions like "deductive logic" and "deductively valid" - is to be taken as understood. It is much simpler to speak just of logic and validity.

In short, there are good - if not compelling - reasons why most logicians confine their subject to the study of principles and truths definable in terms of such related

notions as those of validity, implication, and self-contradiction.

Cautionary remarks about the terms "induction" and "deduction".

As we have already noted on a couple of occasions, terms which are fairly precisely defined within the discipline of logic are sometimes confusedly misappropriated, or misused, by those who are inexpert in the field. The term "logic" itself has often suffered such a fate. So, too, have the terms "induction" and "deduction".

Even lexicographers, who are supposed to be experts about the meaning of words, sometimes make the mistake of highlighting one particular sort of use of a word to the exclusion of others. Thus, for example, my 1974 copy of *Webster's Collegiate Dictionary* tells us that **induction** is "the act, process, or result, or an instance of, reasoning from a part to the whole, from particulars to generals, or from the individual to the universal." And the 1965 Thorndike-Barnhart *High School Dictionary*, more succinctly, tells us that induction involves "reasoning from particular facts to a general principle", and that **deduction** is "inference from a general rule or principle." Not surprisingly, therefore, those who think that dictionaries can be relied upon to provide accurate definitions of the meanings of words, are likely to jump to the conclusion that all inductive reasoning goes from particular to general while deductive reasoning goes the other way, from general to particular.

Yet in both cases our lexicographers have been guilty of a certain sort of intellectual myopia: they haven't taken into their field of vision the wide range of arguments and inferences that are classified, by experts in the field, as "inductive" and "deductive".

Their error should already be evident regarding the term "deduction". The argument:

All whales are mammals.

All mammals are warm-blooded.

Therefore, all whales are warm-blooded.

was cited above as an example of a deductively valid bit of reasoning. Yet, if its premises are to count as "general" or "universal" (as surely they must), then so too must its conclusion. Here, then, we have a counter-example to the claim that deduction takes us *to* particulars.

Again, consider the deductively valid argument:

John is married to Sue.

Therefore, Sue is married to John.

Here neither premise nor conclusion is in any sense "general", since each makes a statement about a particular individual. So deduction needn't take us *from* so-called generals, i.e., general statements.

Once more, it is easy to think of deductively valid arguments which satisfy our dictionaries' definitions of "induction" in so far as they go from the particular to the general. Here is one such argument:

John is married.

Sue is married.

John and Sue are the only persons registered for counselling.

Hence, all the persons registered for counselling are married.

It is just plain false, then, that all deductive arguments proceed from general to particular.

Similar errors of lexicographical description infect the above definitions of the term "induction". Indeed, our last example (that of a deductive argument which goes from particular to general) already establishes the point conclusively. Though it is clearly deductively valid, our dictionary definition would have us say that it isn't deductive at all, but inductive.

Other counter-examples are easy to think of. Here is an example of an inductively strong argument that goes in the 'wrong' direction, from general to particular:

All the ravens anyone has ever seen are black.

So, the next raven you see will be black.

An important lesson can be learned from this sad little story of lexicographical misadventure. Those who compile dictionaries don't legislate the meanings of words; they merely report what, in their oft-times limited experience, they believe to be the meanings that people conventionally attach to words. Their empirical surveys and reports of the conventions of common usage can, as we have seen, be erroneous. True, their reports can usually be relied upon to highlight some of the more important uses of words. But the "definitions" they supply should not be regarded as definitive!

Lexicographers, and others, are especially likely to go wrong when they purport to define terms that have been appropriated for strict, sometimes technical, use. Consider, for instance, the term "mass" as it is used in ordinary parlance; and compare its stricter use in Physics.

Few disciplines are as prone to misuse of their vital vocabularies as are Philosophy in general and Logic in particular. Both subjects - precisely because of their centrality to much of our thinking - lend themselves to Reader's Digest treatment by writers in other disciplines, and especially those in so-called "interdisciplinary" studies. Sadly, such writers all too often have no first-hand, in-depth, knowledge of what they are writing about, and their second-hand, even third-hand, accounts of philosophical and logical distinctions often involve conceptual caricature and terminological misuse. Beware, then, of the temptation to "do" your Philosophy or your Logic by consulting an ordinary dictionary or a would-be authority from another discipline.¹⁸

POST-SCRIPT

REASON AND THE ROMANTIC TRADITION

At one point in *Innumeracy*, John Allen Paulos makes the following claim: "Romantic misconceptions about the nature of mathematics lead to an intellectual environment hospitable to and even encouraging of poor mathematical education and distaste for the subject." (p. 89)

Unfortunately, the Romantic tradition's antipathy to mathematics extends, perhaps even more vehemently, to logic.

What do I mean by "Romanticism"? In *A Dictionary of Philosophy*, ed. Antony Flew, (London: Pan Books, 1983), we find Romanticism described thus:

A broad movement of thought in philosophy, the arts, history, and political

¹⁸ On the other hand, you will find it well worth your while to consult professional dictionaries written by specialists in these fields. Two of the best are: *A Dictionary of Philosophy*, edited by Antony Flew (available in paperback from Pan Books), and *The Philosopher's Dictionary*, written by Robert M. Martin (also available in paperback, from Broadview Press).

For more detailed explorations of philosophical and logical topics, you might like to consult *The Encyclopedia of Philosophy*, edited by Paul Edwards (New York and London: Macmillan Publishing Co., Inc. and the Free Press, 1967).

thought, at its height in Germany, England, and France towards the end of the 18th and in the earlier part of the 19th centuries. A reaction against the rationalism and empiricism of the Enlightenment, Romanticism is best characterized by its idealist celebration of the self, by its respect for the transcendental, and by its conviction of the power of the imagination and of the supreme value of art.

Romantic sentiments are prominently expressed in the writings of poets like Blake, Coleridge, Wordsworth, Shelley, and Goethe, and even - in varying degrees - those of philosophers like Rousseau, Kant, Schelling, Schopenhauer, and Hegel.

Many aspects of the Romantic tradition are appealing. It can sensitize us to much that we might otherwise be inclined to ignore: the beauty of nature; the nobility of great art, music, poetry, and philosophy; or the possibility of things beyond our current comprehension.

Yet all too often, it is anti-intellectual to the point of adulating an idealized subjective world of private thoughts, feelings, and emotions, at the expense of any genuine attempt to understand the objective world of scientific, mathematical, and logical fact. These sentiments live on and thrive in the thinking of many so-called "literati" (men and women of letters, devotees of literature) today.

Let's examine some of the misconceptions embodied in these sentiments, by considering some typical objections and how we might reply to them.

Objection: "Logic deals with cold abstractions not with matters of flesh and blood."¹⁹

Reply: It is unclear, for a start, why the abstractions of logic should be thought of as "cold". What, we might ask, is the function of this adjective when thus juxtaposed with an abstract noun? Are we to take it that some abstractions are warm and that it is a peculiar defect of logic (and perhaps mathematics also) that the abstractions it deals with happen to be cold ones? The problem, of course, doesn't so much lie in the figure of speech itself, but in the fact that it is hard to know what to make of it.

Yet doesn't the core of the objection remain? Isn't it true that logic deals only

¹⁹ This objection is almost word-for-word the same as one which Paulos sees the need to discuss in *Innumeracy* (page 90).

with abstractions, be they "cold" or otherwise? That is to say, isn't it true that logic, by virtue of its abstractness, has nothing to say about the "flesh and blood" concerns of daily life?

By way of providing a more adequate reply to the objection, let's distinguish two importantly different aspects of logic: PURE LOGIC, and APPLIED LOGIC. This distinction is akin to that between pure mathematics and applied mathematics. Pure mathematics is indeed an abstract science, dealing with such abstractions as wave equations, matrices, and integral equations. Yet these very same abstractions have yielded useful applications - in radio transmissions, economics, and quantum theory, respectively.²⁰ In much the same sort of way, pure logic is also an abstract science dealing with highly general principles of correct reasoning. Yet these same abstract principles and truths have application to the concerns of daily life, and much else besides. Indeed, on reflection it is easy to see that the abstractness of logic's most general truths is not something to be deplored, since it is precisely because of its abstractness that its truths have so wide a range of application, application to matters of flesh and blood as well as all the rest.

Let me illustrate by taking the case of the so-called LAW OF NONCONTRADICTION. This law was first formulated by the Greek philosopher Aristotle (384-322 BC) thus:

The firmest of all first principles is that it is impossible for the same thing [property] to belong and not to belong to the same thing at the same time in the same respect.

More generally, it simply says that a statement and its negation can't both be true. To be sure, this law of logic doesn't itself say anything about such "flesh and blood" things as life, love, or liberty. But this does not mean that it has no *application* to such matters. Try - by repeatedly contradicting yourself in the things you say to your friends, your lover, or (perhaps worst of all) the police - to violate the Law of Noncontradiction, and see for yourself what the practical consequences are.

The fact is that since logic is the science of valid reasoning, and reasoning plays a guiding role in nearly all we do, say, or even think, there is little that logic does not have

²⁰ Ian Stewart, in *Concepts of Modern Mathematics*, (Penguin Books, 1975) provides a brief but fascinating account of these and other applications of pure mathematics.

a bearing upon, no matter how abstractly its most general principles may be expressed.

Objection: "The strictures of logic are a threat to freedom and spontaneity and all that is finest in the human spirit."

Reply: How easy it is to write stuff like this, throwing words together in a sort of euphoric prose-poetry! It all sounds so deep, so sublime, so spiritual!

The objection isn't just a piece of imaginary rhetoric on my part. In *Man and Superman* (1903), George Bernard Shaw expressed it (tongue-in-cheek) thus:

The man who listens to Reason is lost:
Reason enslaves all whose minds are not strong enough to master her.

Similar sentiments are expressed (this time in all seriousness) by Georg Novack in *An Introduction to the Logic of Marxism* (1971):

The fourth defect in the laws of formal logic is that they present themselves as absolute, final, unconditional laws. Exception to them is impossible. They rule the world of thought in a totalitarian fashion, exacting unquestionable obedience from all things, claiming unlimited authority for their sovereign sway. (p.49)²¹

During the thirty six odd years I've been teaching philosophy and logic, I've found many in whose souls - even minds - these sentiments echo (vibrate?) profoundly. But what does the objection really amount to? Are there any good reasons for believing it sound?

Why should anyone think that Reason²² or logic poses a threat to human freedom and spontaneity? Here is a reconstruction of the kind of reasoning that many have offered:

Logic tells me that if I accept certain statements as true then I have *got* to accept any other statements that are their logical consequences; that is to say,

²¹ Paradoxically, Novack elsewhere adopts the view - criticized earlier - that the laws of logic describe "the activities of the thought process which goes on in human heads". (p. 17). Take the two views in conjunction, and it follows that our thought processes are never illogical!

²² Note that reason, here, is both reified (treated as if it were something concrete, not abstract), and personified (treated as if it were a person). This, too, is characteristic of the Romantic way of thinking. Reification and personification have a place, of course, in poetry - but hardly in the sober mind or treatise.

it tells me that I'm not free to accept the premises of a valid bit of reasoning and deny the conclusion. It tells me that if I do, then I'll be contradicting myself. But I reject the so-called "Law of Noncontradiction" which tells me that I mustn't contradict myself. Indeed I reject all other such constraints on my right to think as I wish. The laws of logic are just subtle surrogates for the thought police envisioned in George Orwell's novel *1984*.

Such reasoning, however, involves some fairly obvious confusions. Unlike Orwell's thought police, the Spanish Inquisition, or the arbiters of political correctness, logic puts no constraints on what you can in fact think or believe. But it does indeed put constraints on what you can think or believe COHERENTLY. It leaves you perfectly free, for instance, to believe both that the earth is flat and that it is spherical; but it tells you that if you believe both these things then you'll be contradicting yourself and hence believing something FALSE. It doesn't even preclude you from *believing* that the Law of Noncontradiction (which implies that one can never truly assert a contradiction) is false; but it does preclude you from believing TRULY that it is false. Even an omnipotent God, most theologians have long agreed, is bound by the laws of logic. No-one, not even He, can lift a stone that is too heavy for Him to lift. The laws of logic, then, involve no curtailment of genuine freedom or spontaneity. We can hardly complain about lacking a "freedom" which even the Almighty (if there is one) can't have.

Objection: "Logic is antithetical to, and diminishes, our feelings and emotions, our creative and imaginative powers."

Reply: This expresses the essence of the Romantic antipathy to reason in its various guises: science in general, and logic in particular. The Romantic tradition sees things through dichotomous lenses. It likes to pit its favorite reifications (abstractions, thought of, and ascribed proper names, as if they were concrete entities, [hence the use of capitals]), Faith, Imagination, and Feeling, against Reason, Science, and Logic. Not only are members of the two groups taken to be antithetical to one another; members of the first group are thought of as somehow superior to (or alternatively, as "deeper than") members of the second.

But is the objection well founded? In what sense, if at all, are members of the two groups "opposites" of one another?

I'm not going to pursue these questions here, preferring to leave it to you to try to resolve them in our own thinking. You may find it helpful, however, to consider a few

dissenting views from those who work within the fields concerned and so have a more intimate knowledge of their workings than do those who view them from afar.

1. Regarding the alleged opposition between Faith and Religion, on the one hand, and Reason and Rationality, on the other:

In so far as religious faith is a matter of holding certain beliefs . . . it is open to rational consideration. Even the belief that faith is immune from critical examination must itself be critically examined.²³

This comes from a group of theistically-inclined authors of a thoughtful new text in Philosophy of Religion, *Reason and Religious Belief*. Later, they add:

Simply ignoring such challenges leaves one open to the suspicion that one is really not interested in what is *true*, but only in holding on to one's present beliefs regardless.

2. Regarding the alleged opposition between Imagination, on the one hand, and Scientific Reasoning, on the other:

Scientific reasoning is . . . at all levels an interaction between two episodes of thought - a dialogue between two voices, the one imaginative, the other critical; a dialogue . . . between the possible and the actual, between proposal and disposal, conjecture and criticism, between what might be true and what is in fact the case.²⁴

The author, Sir Peter Medawar, won the 1960 Nobel Prize in Medicine and Physiology for his work on tissue transplantation. He has written voluminously about scientific method and its abuses, and has long been a trenchant critic of muddled thinking wherever he finds it - among scientists and philosophers as well as the literati.

3. Regarding the alleged opposition between Feeling, on the one hand, and Logic, on

²³ Michael Peterson, William Hasker, Bruce Reichenbach, and David Basinger, *Reason and Religious Belief* (New York and Oxford: Oxford University Press, 1991), pp. 10 and 40.

²⁴ The quote is from Sir Peter Medawar's "Science and Literature", an essay in which he subjects the Romantic tradition to incisively critical examination. That essay, along with many others of similar wit and insight, is to be found in his book *Pluto's Republic* - not to be confused with Plato's *Republic* - published by Oxford University Press, 1982, and reprinted in paperback, 1988.

the other:

Of all the superficialities, the opposition of feeling and logic is the silliest.

The author, Bernard Bosanquet, was an Idealist philosopher (1848-1923) who was much troubled by the tendency of many in his time to mouth such thoughtless aphorisms as "Life is more than logic" without really knowing anything at all about the subject they were denigrating. As a logician himself, who wrote two books on the subject, Bosanquet came to a contrary view of the matter: Logic, he said, is "the spirit of totality"; and as such it is "the clue to reality, value and freedom."²⁵

I want to add my own voice briefly to Bosanquet's. Logic requires an exercise of the imagination and a lively sense of what is possible, since most mistakes in inference arise just when we fail to take into account *possible* (not just actual) circumstances in which our premises would be true but our conclusion false.

The view that reason and imagination (and the other allegedly antithetical pairs) are polar opposites of one another is a simplicistic myth - a myth perpetuated by those who are so in the thrall of the Romantic world-view that they can't bring themselves to venture outside it.

²⁵ Quoted by John Passmore, *A Hundred Years of Philosophy*, (Markham, Ontario: Penguin Books, 2nd edition, 1978), p. 86.