THE NATURE AND STATUS OF LOGIC

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[Extract from an unfinished elementary textbook,
***Reasoning: the 4th "R"*]**

Logic is the science of correct reasoning in any field whatever. But what are the foundations of its laws? Are they, as some have claimed, best viewed as "the laws of thought", laws grounded in facts about human psychology? Do they have their warrant merely in the conventions for linguistic behavior? Are they, as others have claimed, grounded in facts about reality more generally? Or are they, as still others would say, grounded in facts about how this and any other possible world must be? Let's take a brief look at each of these four views.

(1) Psychologism

One version of this view was put forward by one of the pioneers of mathematical logic, George Boole (1815-64). In his introduction to *The Mathematical Analysis of Logic* (1847), he claimed that the laws of logic are "the laws of one of the most important of our mental faculties". This account was still much favoured in logic texts well into the 20th century, including the one that I studied as a young student.

Yet it leaves much to be desired. Like the Phenomenological theory of meaning, it treats logic as a branch of descriptive psychology. It is another version of what Frege called "Psychologism".

The trouble with any psychologistic account of logic is that it is plainly false.

It seems pretty clear that the laws of logic don't describe any *universal* truths about how *all* human beings actually think, reason, and use language. Would that they did! For then there would be no inferences that were fallacious, no beliefs that were self-contradictory, and all would be sweetness and light in the domain of human thought.

Nor do the laws of logic describe *non-universal* truths about how the members of *different* social and linguistic groupings actually reason.

Some think that they do. They would claim that there are "many different logics" each relative to the belief systems and modes of reasoning of different groups of people.

Thus, some claim that there is a difference between "Western logic" and "Eastern logic", between "male logic" and "female logic", between "the logic of scientific inquiry" and "the logic of religious belief", or between the so-called "logics" of different primitive tribes and the logic that is being presented

here. A whole field of inquiry, called Sociology of Knowledge, thrives on such distinctions and concludes that logic is, in some sense, "relative".

Now it is true that certain persons in fact argue in one way while others in fact argue in another. And it may even be true that certain persons regard the way they argue as "logical" while others do not. But it doesn't follow that logic is relative to one's belief-system - that there is no such thing as "objective" logic. Thinking, or saying, that a mode of reasoning is logical doesn't make it so. (As I said before, logical acceptability isn't determined by counting heads.)

Logic (i.e., deductive logic), remember, isn't just the science of reasoning; logic is the science of *correct* reasoning. So understood, logic isn't just a descriptive science, since it contains a normative element as well. It tells you how to reason correctly where the standard of "correctness" isn't determined by social acceptability but by whether or not the principles one follows lead from truth to falsity, from fact to fiction. It may be socially acceptable for primitive Azande tribesmen, for instance, to conclude that a man is causally responsible for some mishap on the basis of the fact that a pair of poisoned fowls has died.[1] But their conclusion may well be false even if their premises are true. Hence their reasoning is not correct.

I've just said that the reasoning of the Azande, when they appeal to the poison oracle to determine who did what, is incorrect. But who am I, you may object, to make such a judgement? Am I not displaying the typical chauvinism of a Western, scientifically- minded, logician (and a male one at that)? I say their reasoning is incorrect. They say it is correct. Aren't they just as entitled to their standards of correctness as I am to mine? (Here is a main source of the widespread belief in the relativity of logic.)

This sort of objection draws upon some noble anti-chauvinistic sentiments. But it is misguided nonetheless. As should be obvious from our earlier discussion of invalid inferences, the standards of correctness when it comes to reasoning aren't imposed by human beings of whatever race, gender, or intellectual sophistication. The standards of logical correctness are imposed by reality!

So far, we've seen reason for saying that both variants of account (1) are mistaken: whether one says (a) that the laws of logic are universally true descriptions of how all people in fact think; or (b) that there are different logical laws for different people, some laws describing how some people think, others describing how other people think. What's wrong with both variants is the presupposition that the laws of logic are rooted in facts about human psychology, and that they should be taken as simple descriptions of such facts.

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[1]   The case is described in the beautifully written classic by anthropologist E. E. Evans-Pritchard: *Witchcraft, Oracles and Magic among the Azande* (Oxford: Clarendon Press, 1937).

Let's now turn to the second account.

(2)   Conventionalism

Proponents of the second view like to draw attention to an aspect of logic that the first account tends to neglect.   They would point out that, in that sense of "logic" which is our concern, logic doesn't tell us how we (or others) do in fact reason. It tells us how we ought to reason if we are to reason correctly. So understood, the science of logic contains at least some prescriptive (normative) element.

The distinction between descriptive laws and prescriptive laws is pretty obvious when one thinks about it. The laws of nature - those that are investigated by sciences such as physics, chemistry, biology, and psychology - are descriptive. They are intended as descriptions of how nature works. The law of gravity, for instance, tells us how material objects in a gravitational field in fact behave. It doesn't tell such objects how such objects should behave. Unlike the kinds of laws that legislature enact - laws pertaining to criminal matters, for instance - it doesn't prescribe anything. Prescriptive laws are created; descriptive ones are discovered (when we are clever enough).

But if the laws of logic are prescriptive of how we *ought* to think, speak, and write, rather than descriptive of how we do in fact think, speak, and write, then the question arises as to *who* prescribes them and *why*. What is the warrant for these prescriptions? Why, for instance, should we *not* infer "Some of the Xs are not Ys" from "Some of the Xs are Ys"?

The answer that would be given by contemporary proponents of the prescriptive interpretation, is that the ultimate warrant for the prescriptions of logic lies in the socially instituted conventions that we have adopted for the use of words.

Thus, they would say, we shouldn't make the inference from "Some are" to "Some are not" because if we do then we'll be *violating conventions* for the use of these words. Likewise, we shouldn't commit the fallacy of denying the antecedent because if we do we violate the conventions for the use of the expression "If . . . then . . ." And so on. Belonging to a linguistic community carries with it certain obligations: one oughtn't to use words in ways that deviate from the accepted norms.

On this latter-day view, the laws of logic derive their ultimate warrant not from facts about the nature of our psychological make-up but from facts about the conventional nature of our language. Hence the term "Conventionalism".

The linguistic, conventionalist, account of logic seems to me to contain important elements of truth. It is true, for instance, that if our conventions for the use of the words "some" and "all" were reversed, then we would express our logical inferences in different ways so that it would at least *look* as though

different logical inferences were being sanctioned. We would, in this case, be able to infer what is *meant* by the sentence "All the Xs are Ys" from what is *meant* by the sentence "Some of the Xs are Ys", rather than vice versa.

But , contrary to what many claim, this doesn't show that the validity of an inference such as that from "All the Xs are Ys" to "Some of the Xs are Ys" is *just* a matter of convention and hence something that could be changed more or less at will. All it shows is that, if our linguistic conventions were different, then we would express the very same inference that we now make, but do so in different words. Changing the conventions for the use of "all" and "some" would no more make it true that the inference from "Some of the Xs are Ys" to "All of the Xs are Ys" is valid - given our current usage - than changing the conventions for the use of the words "birds" and "pigs" (so that their meanings were reversed) would make it true that pigs can fly.

What follows from what isn't up to us. It is a consequence of the way the world is, not of the way our conventions happen to be; though, if we were to change our conventions, then we would thereby also change our way of *saying* what follows from what.

The inadequacy of a conventionalist account of logic and mathematics becomes especially clear when we ask: Why is it that reality, the world as it actually is, happens to comply with human linguistic conventions?

This, I've been told, was a question that deeply puzzled the great philosopher Ludwig Wittgenstein (1889-1951) in his early years.[2] He had, you see, been working in the field of aeronautical engineering, and soon became interested in philosophical issues about applied mathematics.

The then-prevailing theories about mathematical and logical truth were conventionalist ones. The mathematical statement that $2 + 2 = 4$, for instance, was said to be "true by definition" (i.e., as a consequence of our conventions for the use of the words or symbols involved). And likewise with logical truths such as the Law of Noncontradiction. But such a theory, Wittgenstein apparently thought, made no sense of the fact that mathematics and logic are so successful in their application to the real world. Why should reality choose, as it were, to comply with our conventions? Or is this just a matter of sheer chance on a gigantic scale?

Whether or not it really was thoughts like these that drove him into philosophy, first as a pupil then as a collaborator of Bertrand Russell, I can't be sure. But the story, even if apocryphal, is a good one. Even if the young Wittgenstein wasn't puzzled by these questions, we should be. And even if the later Wittgenstein reverted to a form of conventionalism, as indeed he did, the questions remain and call for an answer.

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[2]   I seem to recall having been told this story by the distinguished historian of philosophy, John Passmore. But I've been unable to verify its authenticity or even soundness of my memory.

(3)   Actualism (the Generalized-Science interpretation)

A much more satisfactory theory is that which accords to logic and mathematics a status akin to of physics, chemistry, and biology. The laws of logic and mathematics, it is sometimes said, are like these others in so far as they deal with the nature of reality. Yet they are much more general than are the laws of physics, chemistry, and biology, having application to all that exists not just to the relatively narrow domains with which the latter deal.

This view was advanced by Bertrand Russell in his *Introduction to Mathematical Philosophy* (1919) when he wrote: "logic is concerned with the real world just as truly as zoology, though with its more abstract and general features." (p.167). Likewise, the American philosopher-logician, Willard Van Orman Quine (1908-), in his *Word and Object* (1960) described logic as a "limning [delineating] of the most general features of reality."[3] I'll call this view of logic "Actualism" since it treats logic as having to do solely with the actual world.

On this third account, the difficulties spawned by the first and second are avoided. Logical and mathematical laws aren't just artefacts of human psychology. Nor are they mere artefacts of linguistic convention. They are true because the actual world is as they say it is. To be sure, the conventions of language play a role in the *expression* of these laws: if our conventions were otherwise, we'd express these laws otherwise. But the laws of logic and language aren't true because of conventions. They are true because of facts about the world. They describe the most general facts about the world. And it is these highly general facts about the world that provide the warrant - on this view - for the prescriptive element in logical laws. We *ought* to conduct our thinking in accordance with them for the simple reason that if we don't, then we'll often pay the consequences by being led into factual error.

Note the consequences of this view, however. If the presumed laws of logic and mathematics are akin to those of the so-called empirical sciences - physics, chemistry, biology, and the like - only more general, then it would seem that like the latter, they too should be subject to revision in the light of further experience. In the past, Darwinian theory replaced Aristotelian, and Einsteinian theory replaced Newtonian. Why should it not be the case in the future, that some new theory in logic or mathematics will replace, and displace, what we now accept as true in these domains?

Quine himself has explicitly endorsed this sort of relativism regarding the status of logical and mathematical truths. To be sure, he thinks of the truths of logic as occupying the central place in what he calls the "fabric" of human knowledge and belief; and this makes them less vulnerable to revision than statements closer to the experiential periphery of the fabric. Nevertheless, he has at times claimed, they could in principle be abandoned

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[3]   A somewhat similar view had been taken a century or so earlier by the English philosopher and social reformer John Stuart Mill (1806-73).

in the light of what he calls "recalcitrant experience."

But isn't there something uniquely inviolable about the cases of logic and mathematics? Our fourth view holds that there is.

(4)   Necessitarianism (the Possible-worlds interpretation)

According to the fourth view, we can well imagine the presumed laws of physics and the rest changing, either because we discover that the world doesn't really work as they currently say it does, or because - contrary to all assumptions about the so-called Uniformity of Nature - the world itself goes topsy-turvy and starts operating in radically new ways. But the laws of logic and mathematics, it would be claimed, are quite different.

Can we imagine that 2 + 2 = 4 was really false all along or that, though it is now true, the world might change in such a way as to make it false? Can we conceive of this fate befalling the Law of Noncontradiction? These laws and their kin, it seems, are not just true of the actual world as it happens to be constituted. Rather the laws of logic are statements that are true in any possible world, any world that we can coherently conceive of.

The strongest argument for according the truths of logic and mathematics a unique status in our conceptual system is that many of them are standardly proved to be true by a method that simply doesn't work for these others. Many logical and mathematical truths can be proved true by so-called reductio ad absurdum arguments. A reductio ad absurdum argument is one that shows a statement to be true (indeed necessarily true) by demonstrating that its denial is absurd in the sense of being self-contradictory and hence not possibly true. But if the denial of a statement is not possibly true, then the statement itself is not possibly false; that is to say, the statement itself is necessarily true.

But no such proofs can be given of the truths of the empirical sciences. The truths of physics, chemistry, biology, and the like, are true merely as a matter of fact. And as the great Scottish philosopher and historian David Hume (1711-76) pointed out long ago, such statements can always be denied without contradiction. As he put it:

> The contrary of every matter of fact is still possible; because it can never imply a contradiction, and is conceived by the mind with the same facility and distinctness, as if ever so conformable to reality. *That the sun will not rise tomorrow* is no less intelligible a proposition, and implies no more contradiction than the affirmation, *that it will rise*.[4]

There is, then, a "necessity" attaching to the laws of logic and mathematics that does not belong to the laws of the natural sciences. Hence

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[4]   David Hume: *An Enquiry Concerning Human Understanding* (1748), IV, pt.i, p.26.

I'll appropriate the term "Necessitarianism" for this fourth view.

This fourth view is that which the young Wittgenstein came to advance in his *Tractatus Logico-Philosophicus* (1921, English translation 1922).[5] And it has been worked out in some detail and technical virtuosity by a number of his successors, including Rudolph Carnap (1891-1970) and the contemporary logicians Jaakko Hintikka and Saul Kripke.

Such philosophers hold that there is a fairly sharp distinction to be made between what happens to be true as a mere matter of fact and what must be true as a matter of logic. Truths of the former kind they call contingent; those of the latter kind they call necessary (or noncontingent).

Contingent truths (like those of the natural sciences) are true in the actual world, but are false in other possible worlds. Necessary truths (like those of logic and mathematics) are true in the actual world, and in all other possible worlds as well.

It is Necessitarianism, you will note, that best accommodates the account of validity which holds that a valid argument is one in which it is not *possible* for the conclusion to be false when the premises are true - one in which the conclusion *follows of necessity* from the premises. It is Necessitarianism, too, that fits best with our intuitions that logic must consider all possibilities, not just the states of affairs that happen to be the case. As Wittgenstein put it in his *Tractatus*: "Logic deals with every possibility and all possibilities are its facts." (2.0121)

When we are dealing with logical matters, it won't do to refer just to actual circumstances of truth and falsity. We need to invoke talk about how the world might have been, not just about how it happens to be. We need to speak of possible worlds, not just the actual one.

In discussing the nature and status of logic and mathematics, of course, we have been stepping outside these domains themselves and saying something about them from a broader point of view. We have been discussing issues in the *philosophy* of logic and the *philosophy* of mathematics. Disagreement about the nature and status of logic and mathematics is not likely to affect what one regards as the truths of either discipline. So, even if, after due study and reflection, you come to favour one of the rival views, that shouldn't deter you from studying Logic in the hopes of learning a little more of what - on any of these accounts - will help to make your thinking more rational and your inferences more secure, i.e., more logically literate.

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[5]   This interpretation of Wittgenstein's *Tractatus* is advanced in my book *The Nature of All Being: a Study of Wittgenstein's Modal Atomism* (Oxford and New York: Oxford University Press, 1992).