

The Speculative Generalization of the Function: A Key to Whitehead

James Bradley

Speculative as opposed to analytical philosophy is centrally concerned with the concept of activity, understood as the activity of actualization which makes things what they are. Moreover, speculative philosophy has characteristically maintained that the activity of actualization is self-explanatory in the sense that it is defined in terms of a distinct kind of entity (substance, God, the Absolute) which has necessary existence or whose existence is not derived from anything except itself. It will be argued here that the hitherto unrecognized significance of Alfred North Whitehead (1861-1947) [1] resides in the fact that he fuses together a speculative philosophy of activity and logical analysis by drastically reinterpreting the nature of the mathematical function and redefining the self-explanatory in terms of the applicability or descriptive adequacy of his functional analysis to the nature of things.

1. The Generalization of the Function

It is no accident that Whitehead describes philosophy as “imaginative generalization” (Whitehead, 1978: 5), and sees his thought as a “generalized mathematics” (1947: 109). He states that “the algebraic method” - the “examination of pattern with the use of real [free or unquantified] variables” (1947: 130-31) - is the rubric under which he elaborates his speculative philosophy, and he insists that “Logic prescribes the shapes of metaphysical thought”. [2] What he means by this is evident throughout *Process and Reality* (1929), which is the revision and culmination of his earlier mathematical and philosophical work, in particular his collaboration with Bertrand Russell in *Principia Mathematica* (1910-12). For in *Process and Reality* and subsequent writings, Whitehead builds on the brilliant success of the Frege-Russell generalization of the mathematical function and develops his philosophy on that basis. Moreover, like other Cambridge

figures such as Frank P. Ramsey in his last writings and Ludvig Wittgenstein after 1929, the position Whitehead develops from around the middle of the 1920's is markedly constructivist in character. [3] The difference resides in the way Whitehead generalizes functional structure, which he does in two distinguishable senses.

First, Whitehead generalizes the *meaning* of the function. That is, he seeks the highest or most general description of the nature of the function in order to provide a meta-functional analysis of the nature and conditions of any function at all. As strictly descriptive (1967a: 92), the analysis is not guilty of dogmatically assuming, or making an *a priori* appeal to, the principles of necessary or sufficient reason. Secondly, on the basis of his general description of the function, Whitehead is able to generalize the *range* of the function over any entity, so that his analysis of the function is intended to provide an account of the nature of all that is.

Whitehead's generalization of the function makes the claim that the generalized function has 'ultimate' status. This means in part that, with Gottlob Frege, Russell, and the early Wittgenstein, he regards the generalized function as irreducible (as not further definable or derivable from any higher principle), and as transcendental (as universal in range or application). However, Whitehead's transcendental theory of the function is a theory, not of cognition, but of the constitution of all order. Moreover, the claim to the irreducibility of the generalized function rests not on an appeal to any kind of *a priori* rational intuition, nor is it a matter of its logical 'primitivity', for in Whitehead the ultimacy of the generalized function is not such that it could be held to be inexplicably given. Rather, his generalized description is intended to establish the function as irreducible by showing it to be a particular kind of self-explanatory ultimate, which he terms 'process' or 'creative process'. The way he sets about establishing this extraordinary claim is best explained in a series of three steps. The first step considers the concept of creativity in its own right as 'the category of the ultimate'.

2. The First Step: the Function as Mapping Activity

In *Process and Reality*, Whitehead develops his position by the elaboration of a "speculative scheme" (1978: 3) or "matrix" (1978: 7-8) of categories, which, after the style of *Principia Mathematica*, he presents and defines at the start of the work. There, he defines his "category of the ultimate" (1978: 21-22) as a matter of 'creativity, many, one'. Creativity is said to be 'the ultimate principle', whereby 'the many become one and are increased by one'.

In defining the ultimate by reference to the relation of many and one, plurality and unity, Whitehead is taking up a theme which has been basic to speculative philosophy since the Greeks. In his view, however, this theme can be more adequately treated than hitherto on account of the modern mathematical discovery of the function (1947: 97-113, 127-131). That is, when Whitehead defines the ultimate by reference to the relation of many and one, *he is defining it in terms of the set-theoretical definition of the function as the class of many-to-one relations.*

A mathematical function, matrix, or schema is a rule of 'mapping' in which the elements from one set or 'domain' are matched to elements of another set or 'codomain'. Take the function 'is the square of' symbolized as $(x)^2$, where x is the variable of the argument of the function. The square function is a rule under which the numbers 2 or -2, the values of the variable in the domain, can be mapped onto or matched with the value 4 of the variables in the codomain. Hence the definition of the function as the class of many-to-one relations or mappings. [4] However, a function or mapping-rule can also be defined as an infinite set of ordered pairs $\langle x, y \rangle$, with x belonging to the domain and y to the codomain, where a correspondence is mapped between the member of the domain and the member of the codomain (so for $(x)^2$ we could have the ordered-pair values $\langle 2, 4 \rangle$, $\langle 3, 9 \rangle$, and so on). By the time Whitehead was writing *Process and Reality*, this had become, and still is, the generally preferred definition of the function. [5] Yet although mathematically respectable, it is not a definition which he could accept as philosophically fundamental.

First, the definition of the function as the class of many-to-one relations aligns the analysis of the function with the philosophical question as to the nature of plurality and unity, thereby lifting the concept of the function to the highest level of metaphysical generality. When Whitehead defines the ultimate by reference to many-to-one relations, he is at once laying out the fundamental issue which a philosophical account of the nature of the function must address, and defining the concept of the ultimate as the concept of *the function in general.*

The significance of this basic strategy cannot be underestimated. For, secondly, in distinguishing 'creativity' from 'many, one', Whitehead's point is that the function in general is the *mapping* of the relation between a domain and a codomain. The function in general is the concept of mapping in general, and mapping in general establishes or maps to structure or order.

Whitehead is here rejecting the view that the concept of set is the basic concept, and that the function is to be defined in terms of sets of ordered pairs of a certain sort. His claim is that the generalized concept of mapping as the mapping of order is distinguishable from that of any specific order, for it is the process whereby order is generated. Mapping is not any set of ordered pairs, but the concept of the ordination of order, of the ordering of pairs into sets. It is not any specific relation or rule, but the concept of the configuration of any specific relation or rule. It is not any specific difference or form, but the concept of the differentiation of difference, the formation of forms. In consequence, as the very term suggests, the concept of mapping in general is the concept of an activity. Because mapping is distinguishable from the domain out of which it proceeds, from the codomain to which it proceeds, and from any relation or rule which it establishes as such, the concept of mapping is the concept of the activity of actualization as the actualization of relations or rules. The concept of mapping is the concept of mapping activity, and it is not reducible to, explicable by, or exhaustively analyzable in terms of, any of its components. It is the ultimate, underivable condition of transformation or composition, the universal principle of construction or actualization, understood as the activity of establishing a relation between the structure of a result and its bases.

What Whitehead has done here is to begin the generalization of the meaning of the function exactly where Frege does: with the concept of mapping. [6] But he does not follow the extrinsic Neo-Kantian limitation Frege lays upon his generalization, which defines mapping as the 'thought' or concept in general and thus restricts it to a theory of cognition. Instead, Whitehead develops the dynamical aspect of the concept of the function, which in Frege makes it the principle of cognition and in Russell makes it the principle of logical construction out of sense-data, and he unrestrictedly generalizes it over any relation or order. The claim is that, just as there is a distinction between a mapping rule and its particular applications, so also there is a distinction between the activity of mapping and a particular mapping rule, and it is this all-important distinction which is lacking in the tradition of logical analysis. In other words, the Frege-Russell generalization of mapping is an insufficient or improper generalization of the meaning of the function in that it treats functional mapping as nothing other than a matter of particular mapping rules. Mapping is indeed absolutely inseparable from particular rules, as will become apparent. But the point is that the proper generalization of the meaning of the function as the activity of mapping in general discloses at the very heart of functional analysis the ineradicability and irreducibility of the concept of the activity of actualization, now defined as the mapping activity of differentiation. The concept of the ultimate or the function in general is equivalent to *the mapping of a*

domain, and Whitehead's speculative generalization is intended to ensure that no term in this commonplace formulation of the nature of the function is tacitly suppressed or ignored.

3. Activity as Mapping Activity

Defined as mapping, the concept of activity in Whitehead “means the origination of patterns of assemblage” (1947: 106). [7] The way he understands the nature of ultimate creativity so defined can be stated as follows.

First, from the nature of mapping as an activity which is the universal principle of construction and is ultimate in that it is not derivable from any higher concepts, it follows that it is a particular kind of ultimate principle: as the universal principle of the structuration of structure, “all characters are more special than itself” (1978: 31). This means that, as the highest description possible of the nature of the function, the concept of universal mapping activity is not a generic property or universal but a supra-generic property or universal. In other words, it is 'characterless' in the positive sense that it is a nondetermining property or predicate, and the claim is that it is universally applicable to any specific determination. [8] Universal mapping activity is thus properly to be described as a transcendental property in the Kantian sense, for it is that activity of actualization which is a ground or condition of any determination. More precisely, it is a condition of any other transcendental predicates there may be, as well as of all real or generically determining properties or predicates. In short: universal mapping activity is “the universal of universals” (1978: 21).

We must be careful about the concept of a 'universal', however. For, secondly, the universal mapping activity of actualization is a non-determining transcendental condition in a further, radical sense: as the principle of ordination or differentiation, it is prior to logic and is a condition of logical order. More precisely: since differentiation is the basic condition of inconsistency, exclusion, or negation, in the concept of universal mapping activity “the whole movement of logic is provided for” (1968: 52), since difference, inconsistency, or negation is the fundamental definition of determination of any kind. In consequence, while mapping activity is universal, it is not a universal form but a universal activity. That is, the concept of universal mapping activity is not to be understood as a unity of essence and existence, nor of logic and ontology, for the activity of differentiation is the ground or condition of the principle of noncontradiction. It does not follow, however, that the claim to the primordially of activity collapses into non-rationality or ineffability. Yet before considering the question

of the rationality of the concept of universal mapping activity, another, equally crucial, feature of that concept must be introduced: infinity.

By definition, thirdly, the concept of universal mapping activity cannot be exhaustively analyzed in terms of any instance of mapping, for it is a condition of there being any instance of mapping at all. Now, the generalized concept of mapping is the concept of a universal activity, and the concept of a universal activity which is inexhaustible by any of its instances is the concept of an infinite activity. What kind of infinity is this?

Considered in itself or in complete abstraction from its instances, the concept of infinite mapping activity is the concept of a merely negative infinite in that it is indeterminate, or lacking determination of any kind. As a merely limitlessly determinable *apeiron*, it would be characterless, not in the positive, but in the negative sense that it is utterly empty or vacuous, simply a nonentity. However, that would not be activity at all. For activity is a relative term: "there is no entity which is merely "any"" (1947: 110), no such thing as activity (or power, agency, or causality) in general, nothing which in its own right is *a priori*. And what that means is that there is infinite mapping activity only relative to its instances (1947: 105-6). The concept of universal mapping activity is thus the concept of an infinite activity in the sense that it is the concept of an activity which is transcendent relative to any of its instances. That is, universal mapping activity is the concept, not of a complete, but of a syncategorematic, potential, or relative infinite activity. To begin to see what such a notion of the infinite might imply, we need to consider the second step in Whitehead's speculative analysis of the function. It will emerge that Whitehead's analysis of the function does not merely conform to the definition of the function as the mapping of many-to-one relations, but constitutes a constructivist metaphysical proof of that definition carried out in the realm of ontology or the theory of actualization.

4. The Second Step: the Activity which Maps to Structure

The ultimate reason (1967b: 179) of any instance of mapping is universal and infinite mapping activity. However, universal and infinite mapping activity, as the formation of form, maps to structure itself. Mapping to structure constitutes for Whitehead the primary instance of universal and infinite many-to-one mapping, and he defines it in an unusual and controversial way.

Whitehead's interpretation of mapping to structure is conceptualist in respect of universals or what he calls 'eternal objects', and stands in the Cambridge tradition of speculative theology along with the work of his friends and contemporaries. [9] Thus

mapping from many eternal objects to one structure is defined as the operation of the 'primordial nature' of God, on the aristotelian principle that there must be something in act to realize form or potency (1978: 32). God maps from "the many eternal objects conceived in their bare, isolated multiplicity" (1978: 349; cf. 1967a, Chapter X) to their unity in an ordered infinity of possible mutual relations which is all-embracing, non-exclusive, and non-selective. This unity is not a fixed, necessary ideal order to which all things must approximate (1978: 84), but is a matrix of all possible orders or relations which, in a late essay, Whitehead describes under Luitzen Egbertus Jan Brouwer's concept of a "spread involving an infinitude of dimensions" relative to any finite determination (1947: 88). Whitehead's conceptualism holds at the same time that universals are reals and that they need to be ordered by a constructive act of divine mapping.

Divine mapping or "valuation" (1978: 31) is held to be "infinite" in that it is not limited by any determinate actuality, and in this respect is also "free" (1978: 345): that is, it is a causa sui in respect of how it maps to structure (1978: 88, 222). Here Whitehead stands close to the British voluntarist tradition: "no reason can be given for just that limitation which it stands in [God's] nature to impose" (1967a: 178). However, there is no subscription to the voluntarist notion of an ineffable divine act of will. Divine mapping involves no conscious act on the part of God: it is a necessity of His nature to impose structure upon possibilities as an instance of universal mapping (1978: 345). Whitehead's intention here is to avoid both the anti-rationalist tendencies of voluntarism and the problem of impredication which Russell's Paradox might raise for the concept of a divine mind - or any mind - which surveys all orders of possibility. [10]

In this connection, it would be a mistake to think that Whitehead's theory of "general potentiality", as he calls it (1978: 65), is a theory of the real infinite. [11] For he describes the divine mapping of the realm of eternal objects as "deficiently actual" (1978: 345), both in the sense that there are other essential features of the divine nature (as will emerge), and in the sense that the realm of eternal objects is not self-existent or independent of its finite realizations. On the contrary: not only, in contrast to platonism, is there is no such thing as general potentiality in its own right, but also, in contrast to Aristotle, eternal objects are not active but wholly passive. This has the effect of completely reversing the notion of participation: the participation of eternal objects in the actual depends wholly on their finite determinations. Like universal mapping activity, general potentiality is thus to be conceived, not as a categorematic, but as a syncategorematic or potential infinity which "presupposes" (1978: 349; cf. 225, 257), or is essentially relative to, its finite determinations. It is here that general potentiality

takes on the character of “real potentiality” (1978: 65) without which it would be “mere vacancy” (1947: 106).

5. The Third Step: Finite Occasions of Mapping Activity

Infinite mapping is essentially relative to its instances, and its primary instance is divine mapping to general potentiality. General potentiality is in its turn essentially relative to its finite instances, which Whitehead calls 'actual occasions' of mapping (where 'actual' is the adjectival form of 'act'). On account of the essentially incomplete, interdependent, and correlative nature of these three basic elements in his ontological generalization of the function, Whitehead can say that “Finitude is the condition of activity” (1947: 105-6).

Whitehead's account of occasions of mapping has close structural similarities to what the Wittgenstein of the *Tractatus* calls an 'operation', which is the condition of particular functions. Although the distinction between the set-theoretic concept of the function and the concept of an operation was still being developed in mathematics at the time they wrote, both Whitehead's occasions and Wittgenstein's operation are concepts of mapping as a 'process' (the term is also Wittgenstein's) which takes its own results as its domain or input. [12] That is, although its content is otherwise undefined, the basis of an occasion or operation is the result of an antecedent occasion or operation. A Whiteheadian occasion of mapping, like a Wittgensteinian operation, can thus be described as *iterative* in that it takes the results of an antecedent occasion of mapping as its basis.

As iterative, occasions of mapping are for Whitehead intrinsically *serial* or ordered in nature. In principle, this means no more than that any occasion of mapping stands in a single iterative series of occasions, and inherits the entire contents of that series as iterated in each successive occasion. Yet there are good reasons, based on scientific theory (1967a: 121) and on empirical observation (1967a: 124), for maintaining that as a contingent matter of fact there are multiple, contemporary or parallel series of occasions. On the multiple series hypothesis, any given occasion includes in its domain the iterative series of preceding occasions which constitute its environment as much as it includes the iterative series of predecessor occasions in the particular series of mappings in which it stands. Any occasion of mapping is thus intrinsically serial in nature in that it is necessarily a member of a single series of occasions, which in fact is one of a multiplicity of series which constitute its environment. These are the multiple and intrinsically complex routes of inheritance of any occasion of mapping and constitute its genealogical conditions. In consequence, all occasions necessarily have at

least the following characters: they are asymmetrical as many-to-one constructions; they are transitive, in that the relations between them are many-to-one relations; they are connected, in that they have predecessors; they are consecutive in that their immediate predecessors are occasions; and they stand in a “cumulative” relation to their predecessors (1978: 237), for, as iterative, they 'contain' their predecessors in their domains. [13] In the nature of the case, there is no such thing as a single, solitary occasion (1967a: 174), and because any occasion stands in a cumulative route of iterations, it necessarily has a complex domain. In other words, it is akin to what mathematicians call a 'functional', at least in the sense that its domain (and thus its codomain as well) is constituted by sets of functions or mappings. The concept of series of occasions can thus be regarded as an ontological generalization and constructivist reinterpretation of the Plato-Frege theory of numbers as serial relations, for it installs serial relationality as an intrinsic feature of the nature of things by defining both an occasion and its relata as many-to-one configurations or connectives in series. [14] In the iterative series, that is, occasions switch roles from being a successor mapping, or synthetic subject, to being the predecessor, object, or basis, and thus the subject or argument, of a consequent successor mapping. The result is that subject and object, as well as subject and predicate, are not here fixed ontological opposites (1978: 157-59, 1967b: 175-77). They are not fundamentally different in nature or kind. Rather, they are the basic states or sequential relations of occasions of iterative mapping.

Another aspect of the significance of Whitehead's theory of occasions, as of Wittgenstein's operation, is that it sidesteps Russell's Paradox, the paradox of the class of all those classes which are not members of themselves. For, defined iteratively, no occasion includes itself; it includes in its arguments only the results of antecedent occasions and its own results are included only by its successors. In other words, an occasion is intrinsically a matter of iterative activity, so there is in serial analysis no completely realized real to generate reflexive paradoxes.

There are, however, fundamental differences between Whitehead's and Wittgenstein's theories of functional process. One difference is that, whereas in Wittgenstein the base of an operation is an elementary proposition which is not generated by the application of the operation, in Whitehead's account of finite mapping there are no irreducible logical atoms: there are only complex occasions of mapping. The crucial difference, however, is between Whitehead's theory of functional process as essentially a matter of activity and what Wittgenstein graphically calls an 'automatic' (algorithmic) process, understood as describable independently of any appeal to a principle of activity in the speculative sense. [15]

On Whitehead's account, an occasion of mapping is not exhaustively traceable to infinite mapping activity, for it requires a domain or input. Moreover, infinite mapping activity is a non-determining predicate, which means that it does not determine how an occasion maps from its domain or given content. Equally, how an occasion maps is not exhaustively determined by its domain, for it maps from that domain as its given content. Nor is how an occasion maps exhaustively determined by the structure or rule which it maps, for it is the act of mapping of that structure or rule. It follows that the way any occasion maps is free or spontaneous relative to infinite mapping activity, its domain, and its rule. It is free or spontaneous both in the negative sense that how it maps is not exhaustively traceable to any or all of its conditions, and in the converse, positive sense that it is to this extent irreducibly self-actualizing or self-causing. No occasion is sole cause of itself, for it derives from infinite mapping activity and a given domain. As Whitehead puts it, "there is no such fact as absolute freedom" (1978: 133). [16] But every occasion is a *causa sui* in respect of how it maps from its given domain (1978: 222). So it is not universal and infinite mapping activity which is self-actualizing here, but its occasion as relative to a given domain. There are no ways of mapping as such, only instances of ways.

In consequence, any occasion of the infinite mapping activity of differentiation is not merely a transitive configuration but an intransitive difference or singularity. That is, any finite instance of infinite mapping activity is irreducible to any or all of its conditions and is *qualitatively different* from any other instance because it is, to the extent defined, self-actualizing; it is *novel* because it is a 'never before'; and it is *unique* in the sense that its activity is unrepeatable - it is a 'never again' as well as a 'never before'. Whitehead's ontological generalization of the function here provides a way of elaborating a theory of difference, singularity, or *haecceitas* from *within* functional structure itself, independent of the problems of reference presented by standard functional analysis (as in Charles Peirce or Russell). [17]

6. The Significance of Whitehead's Generalization

The basic structure of Whitehead's metaphysics is now evident. It includes three types of syncategorematic and correlative infinity: the characterless infinity of universal many-to-one mapping, which is primarily instantiated and characterized by the divine mapping of structure, which in its turn is instantiated and actualized by the iterative mapping of series of occasions. Moreover, each of these three interdependent elements is itself threefold in nature. The ultimate is a matter of "creativity, many, one", or

mapping, “novelty”, and synthesis or “togetherness” (1978: 21). God's primordial nature is held to be inseparable from his 'consequent nature', by which he maps from occasions to their timeless preservation and redemption, and the consequent nature is held to be given to the world as part of the domain of each occasion (1978: 87-88, 351). Finally, each occasion is triadically analyzed in terms of a given domain, its private synthetic act as a *causa sui*, and its public or communal role as an object of a successor occasion (1978: 87-88). [18] It is by way of this unusual analysis of the complex structure of functional activity that Whitehead would explain what for Frege and logical analysis is indemonstrable: the existence of value-ranges or sets.

One obvious feature of Whitehead's speculative generalization of the function as mapping activity is that it criticises logical analysis in all its forms for completely taking for granted the “invariableness” or “self-identity” both of the connectives and of the variables of the function (1947: 127-28; 1968: 106-107). In other words, he holds functional analysis to be too closely tied to the view that the connectives and variables of a function in its different instantiations are merely numerically different. This means that there has been no proper consideration of what may be involved in the topic-relativity of the concepts of connective and variable, which is arguably implicit in the fact that the function is meaningless apart from its instances. The claim is that the usual accounts of functional analysis are basically aristotelian in interpreting the function as merely numerically different in its instances. The only difference is that 'substance' has been neutralized into an instance of a variable. Thus Whitehead's functional analysis is not only a generalization of the mathematical-logical function; it is also a critique of the standard interpretation of the mathematical-logical function in that it rejects the metaphysically-loaded use of the term 'constant', understood as that which is fixed, in contrast to the term 'variable', understood as a mere gap waiting to be filled. For Whitehead, the function as employed and usually understood in mathematics and logic is a highly abstract and by no means fundamental product of conscious occasions of many-to-one mapping (1967b: 254).

A second central feature of Whitehead's generalization of the function is that, perhaps for the first time in the history of speculative metaphysics, the concept of the activity of actualization is not analyzed in terms of any kind of emanation, self-differentiation, or divine creation *ex nihilo*. Whitehead is critical of such notions as basically nonrelational and thus ineffable (1967b: 130, 169, 236). In their place, he puts the notion of many-to-one mapping activity, which is not non-rational but nonconceptual (1978: 22). That is, it is a complex, dynamic correlation of whole and part, an irreducible “togetherness” (1978: 21-22, 189) which is given as such and is neither derivable from concepts nor a

product of the synthetic operations of the mind. Yet it is an intelligible given or “fact” (1978: 211), for all its elements are thoroughly relational in that they can be defined not only as 'that from which another comes' (*a quo alius*), but also and necessarily as 'that which comes from another' (*qui ab alio*). [19] It would thus seem that the iterative economy of Whitehead's functional analysis is the proper 'concrete universal', for its unity is generated through the correlativity and topic-saturatedness of the parts that make it up, and not in virtue of 'containing' or being 'contained in' an underlying, identical substance, often thought of as having all the reasons for its existence in itself.

In this respect, thirdly, no instance of mapping activity is necessary in Whitehead. Only the absence of any mapping activity at all is inconceivable. His theory of iterative seriality breaks the traditional link between the speculative notion of the self-explanatory on the one side, and, on the other side, the notions of an independent and completely realized reality with its own fully-furnished interior of possibilities. Moreover, the self-explanatory in Whitehead does not depend on an a priori appeal to the principle that nothing is without a reason. The functional theory of mapping activity indeed contains in itself all the reasons needed to explain why there is existence. Yet each of its reasons are relational elements which are in their own nature incomplete and partial. Only together, and by way of their descriptive adequacy to experience (1978: 4), do they render the fact of existence self-explanatory. In other words, Whitehead's functional scheme constitutes an ontology of necessary reasons strictly in the sense that, granted the descriptive adequacy of the analysis, it shows no more than why, necessarily, *something* exists - and why all concrete 'somethings' are contingent. The speculative theory of the function agrees with empiricism and logical analysis on the primacy of the actual. But it stands against them in claiming that reasons can be discovered for the primacy of the actual (1967a: 92). And it holds that such reasons can never be anything more than necessary reasons, for the necessitarian view that everything has a sufficient reason is denied, and the different kinds of free activity involved in divine and finite mapping are among the reasons posited to explain the fact of actuality. In line with Whitehead's characteristic method of appropriating empiricist and analytic methods for the derivation of speculative results, metaphysical necessity becomes in his hands wholly a matter of the range and applicability of the speculative description of the function.

It follows fourthly that, for Whitehead as for Russell, what is philosophically ultimate is what is ontologically ultimate, without any submerged or noumenal depths. However much they may differ on the nature of the function, they do not disagree that there is absolutely nothing beyond or behind functional structure. Consequently, unlike the

Tractarian Wittgenstein, Whitehead does not regard the givenness of the world as an ineffable mystery. As he puts it, explicitly challenging the functional analysis of the *Tractatus*, “the purpose of philosophy is to rationalize mysticism” (1968: 174).

Finally, Whitehead's speculative theory of the function is self-referentially inclusive and can define itself as a finite, revisable or fallible, and non-exclusive construction. It is real so far as it can be consistently and comprehensively constructed or applied (1898: 5-12, 1978: 3-4) and it is ideal and hypothetical so far as, in the nature of the case, there can be no final construction, application, or exclusion of alternatives in an infinitely proceeding, aeviternal reality. Thus Whitehead is a constructivist to the extent that he denies the applicability of the principle of bivalence to speculative descriptions (1978: 8), and he replaces the traditional concept of “proof” with that of the “working hypothesis” that does not negate alternative analyses (1978: xiii, 7-9; 1967b: 222-223). Yet his fallibilist theory of construction does not lead him to deny the principle of *tertium non datur*, the principle that there can be no circumstances in which a speculative claim can be recognized as being absolutely undecidable or absolutely neither provable nor refutable (1978: 7-9, 274-75). [20] As itself an historically-situated, finite construction, the speculative theory of functional activity has no “pretensions to the achievement of final truth”, which Whitehead regards as “pathetic” (1947: 125).

Notes:

[1] For discussions on various topics in this paper, I am deeply indebted to Andre Cloots, Guy Debrock, J.-C. Dumoncel, the late Dorothy Emmet, Roland Farber, Lewis Ford, Stephen Gardner, Peter Harris, Charles Lewis, Ilie Parvu, Julian Roberts, Philip Rose, David Scott, Johan Siebers, Peter Trnka, Jan van der Veken, Luca Vanzago, and Michel Weber. I would also like to thank the “Tijdschrift” referees for their acute and helpful comments.

[2] Whitehead's “Foreword” to Quine (1934), ix-x. See also Whitehead (1968: 174). To my knowledge, the first writer to recognize the importance of algebraic method in Whitehead's metaphysics is Wolfe Mays (1959), especially Chapter 5. See also Mays (1977).

[3] See Ramsey (1990). On Wittgenstein's constructivism, see Marion (1998).

[4] On this definition of the function, both one-to-many relations and one-to-one relations are analyzed as special cases of many-to-one relations.

[5] See, for example, Quine (1960), Section 53, "The ordered pair as philosophical paradigm".

[6] See Bergmann (1968: 42-67, especially 49-52).

[7] 'Assemblage' is a coinage of Whitehead and Russell to describe the mapping of sets. See, for example, Russell (1914), Lecture IV.

[8] Because it is characterless, Whitehead likens mapping activity both to Aristotelian matter (except that it is not passive), and to the neutral stuff or neutral monism of the later William James and Russell, in that it is not a subject-object relation considered in its own right, but is the condition of subject-object relations (Whitehead, 1978: 31). Whitehead points out, however, that the many-to-one connectivity of mapping activity means that, unlike neutral stuff, it is not be conceived as standing in external relations, for it is intrinsically relational. What this means will be spelled out in the course of this paper.

[9] See Ward (1911); McTaggart, (1921, 1927); Tennant (1928).

[10] The problem of impredication will be addressed in the next section of this article.

[11] This is clearly not the case with Whitehead's account of extension as the "extensive continuum", which he defines in constructivist fashion as potential and not real (Whitehead, 1978: 35, 61, 283). In his late writing, he also defines the mathematical concept of infinite series in constructivist terms (1968: 54, 82, 91-92, 97).

[12] On Wittgenstein's concept of operation, see Marion, (1998), and Sundholm (1992: 57-76).

[13] Whitehead analyzes the notion of containment in terms of a direct realist and singular-causal ontology of 'perception' or information in what he calls his theory of 'feeling'. His account of the domain or genealogical conditions of any occasion means that every occasion iterates and inherits or 'prehends' (which is not primarily a matter of consciousness) the entire previous history of the universe. Thus there is only one universe for Whitehead (Whitehead, 1978: 4).

[14] On Plato and Frege, see Taylor (1926), Chapter XIX. Whitehead regularly cites Taylor's work on Plato, and his interest in the late Plato's theory of mathematics is explicit (Whitehead, 1947: 97-113).

[15] For Wittgenstein's view of 'process', see Wittgenstein (1983), especially 68, 69, 95, 246. I leave open the complex question as to the nature of Wittgenstein's treatment of activity, and to what extent he regards activity as lying outside the range of functional analysis.

[16] This is a fundamental difference between the theories of activity of Peirce.

[17] Because Whitehead's theory of occasions of mapping as synthetic subjects provides a universal account of actualization, his theory of the subject is not to be conflated with the cognitive or conscious subject of idealism or phenomenology, nor is it vulnerable to the critique of the philosophical subject characteristic of systems theory. Consciousness for Whitehead is a high-level instance of universal many-to-one mapping (Whitehead, 1978: 157-67, 308).

[18] For examples of one-to-many mappings considered as special cases of many-to-one mappings in Whitehead, see Whitehead (1978: 348-49, 1968: 20, 51).

[19] I am here referring to the brilliant analysis of the concept of *innascibilis* (ungenerated or unbegotten) developed in respect of the Father or First Person of the Trinity, by Thomas Aquinas, *Summa Theologiae*, Q. 32, Art. 3.

[20] In this context, it is noteworthy that while Whitehead regards *ex absurdo* arguments as justified in mathematics, he castigates their 'misuse' in philosophy. In philosophy "it is rashly assumed without further question that the peccant premise can at once be located", in disregard of the fact that "every premise in a philosophical argument is under suspicion" due to the inherent generality and referential indeterminacy of philosophical concepts (Whitehead, 1978: 8).

References:

Bergmann, Gustav. "Frege's Hidden Nominalism", in Klemke, E.D. (ed). *Essays on Frege* (Urbana, University of Illinois Press: 1968).

Marion, Mathieu. *Wittgenstein, Finitism, and the Foundations of Mathematics* (Oxford, Oxford University Press: 1998).

Mays, Wolfe. *The Philosophy of Whitehead* (London, Allen and Unwin: 1959).

--. *Whitehead's Philosophy of Science and Metaphysics* (The Hague, Nijhoff: 1977).

McTaggart, J.E. *The Nature of Existence*, 2 vols. (Cambridge, Cambridge University Press: 1921, 1927).

Quine, W.V. *System of Logistic* (Cambridge MA, Harvard University Press: 1934).

--. *Word and Object* (Boston, MIT Press: 1960).

Ramsey, F.P. *Notes on Philosophy, Probability, and Mathematics*, ed. M.C. Galvotti (Naples, Bibliopolis: 1990).

Russell, Bertrand. *Our Knowledge of the External World* (London, Allen and Unwin: 1914).

Sundholm, G. "The General Form of the Operation in Wittgenstein's Tractatus", *Grazer Philosophische Studien*, 42 (1992).

Taylor, A.E. *Plato* (London, Methuen: 1926).

Tennant, F.R. *Philosophical Theology*, 2 vols. (Cambridge, Cambridge University Press: 1928).

Ward, James. *The Realm of Ends or Pluralism and Theism* (Cambridge, Cambridge University Press: 1911).

Whitehead, Alfred North. *A Treatise on Universal Algebra* (Cambridge, Cambridge University Press: 1898).

--. *Essays in Science and Philosophy* (New York, Philosophical Library: 1947).

--. *Science and the Modern World* (New York, The Free Press: 1967a).

--. *Adventures of Ideas* (New York, The Free Press: 1967b).

--. *Modes of Thought* (New York, The Free Press: 1968).

--. *Process and Reality: An Essay in Cosmology* (Corrected Edition, New York, The Free Press: 1978).

Wittgenstein, Ludvig. *Remarks on the Foundations of Mathematics*, ed. von Wright, G.H. R., Rhees, G.E.M. and Anscombe (Cambridge, MA, The MIT Press: 1983).