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On Context Shifters and Compositionality in Natural Languages

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ABSTRACT: My modest aim in this paper is to prove certain relations between some type of hyper-intensional operators, namely context shifting operators, and compositionality in natural languages. Various authors (e.g. von Fintel & Matthewson 2008; Stalnaker 2014) have argued that context-shifting operators are incompatible with compositionality. In fact, some of them understand Kaplan's (1989) famous ban on context-shifting operators as a constraint on compositionality. Others, (e.g. Rabern 2013) take context-shifting operators to be compatible with compositionality but, unfortunately, do not provide a proof, or an argument in favor of their position. The aim of this paper is to do precisely that. Additionally, I provide a new proof that compositionality for propositional content (intension) is a proper generalization of compositionality for character (hyper-intensions).

KEYWORDS: Compositionality – context-shifting operators – indexicals – natural language semantics.

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1. Introduction

Formal semantics has recently witnessed a flurry of interest on whether natural languages contain a special type of hyper-intensional operators (i.e. context-shifting operators) or not (see Rabern & Ball 2017, Santorio 2012 and the many references therein). Ever since Kaplan (1989) there has been a certain resistance in accepting that some natural language expressions are best modeled as context-shifting operators. One reason for this resistance is that, apparently, such operators cannot be handled compositionally.

The hypothesis that natural languages have a compositional semantics is usually taken to play a part in explaining their productive feature; that is in explaining speakers' ability to produce and understand complex expressions that they have never encountered before. Compositionality is, thus, a fundamental tenet of formal semantics, one that formal semanticists are extremely reluctant to renounce. Though, of course, some philosophers of a Wittgensteinian and Austinian bent who doubt that formal semantics is a viable project (because it can't model the pervasive and radical forms of context sensitivity present in natural languages), are ready to deny that natural languages are compositional (see Travis 1996). The formulation of compositionality most commonly found in philosophical and linguistic literature is the following: a language is compositional if the meaning of each of its complex expressions is determined by their syntactic structure and the meanings of their respective constituents. Although there is no consensus on the precise interpretation this general formulation, most semanticists take it to mean that a language is compositional if the meaning of each of its complex expressions is *a function* of their syntactic structures and of the meaning of their respective constituents.[2] As Partee (1995, 153) points out

[2] There are dissenters, though. For example, Szabó (2000; and 2013) argues that the intuitive formulation of compositionality (the meaning of complexes is *determined* by the meaning of constituents and syntactic structure) is stronger than, and not captured well by the formulation of compositionality in terms of functions: the meaning of a complex expression is *a function* of the meaning of its constituents and its syntactic structure. This is correct, but the solution is to put constraints on what functions can be employed as composition functions in natural language semantics. The principle of compositionality defined in terms of functions claims only that the meaning of a complex expression is a function of the meanings of its constituents and its syntactic structure. The principle is silent with respect to what that function can

"the principle can be made precise only in conjunction with an explicit theory of meaning and of syntax, together with a fuller specification of what is required by the relation 'is a function of'." This is to say that different specifications of syntax, of what meaning is, and of what meaning rules (i.e. functions) are acceptable in natural language semantics can give rise to different interpretations of the principle, and these interpretations might settle different constraints on semantic theories. In other words, in order to give a formally precise formulation of compositionality two things are needed: an account of how complex expressions in natural languages are syntactically generated and an account of what meaning is. In what follows I'll give only the minimal details of how syntax of natural languages and their meanings can be formally modeled; details that suffice for a precise formulation of compositionality.

The complex expressions of a language are generated by a syntax Σ, which can be represented as an algebra $\Sigma = (E, A, F)$, where E is the set of linguistic expressions of the language, A the set of simple expressions of the language (thus $A \subset E$), and F the set of syntactic rules of the language. Members of F are functions defined over E and with values in E, and E is closed under operations in F (i.e. every member of E is either a member of the subset A or is the value of an operation of F on members of E). Given that natural language expressions belong to different grammatical categories, and that each syntactic rule concerns only expressions of certain grammatical categories and not of others, each member of F is defined over expressions of certain grammatical categories and with values in expressions of certain grammatical categories. In other words, each syntactic rule is specified in terms of the grammatical categories of its arguments as well as the category of its value. One way to formally implement this insight is to take the set E of expressions to be an indexed

be. But, obviously not any function can be employed as a composition function in natural languages, for although there are an indefinite number of ways in which meanings can combine, some of them could not possibly be ways in which *meanings in natural languages* combine. In other words, there must be a restriction on what functions are acceptable composition functions (i.e. ways of combining meanings) for natural languages. This amounts to restricting the class of acceptable semantic rules in natural languages. For various constraints put on composition functions see Keenan & Stabler (1996), Szabó (2000), and Dever (2006).

family of sets: $E = (E_g)_{g \in G}$ where G is the set of *sorts* which model grammatical categories, and for each $g \in G$ there is a set E_g which is the set of expressions of category g. Then each syntactic rule $\alpha \in F$ is defined only on certain sets E_g; that is it yields a value only for sequences of expressions of E which belong to certain grammatical categories. For example, in modeling English there will be a set E_N (for nouns) and a set E_V (for verbs) and a syntactic rule which is a function defined on ordered pairs $\langle e^N, e^V \rangle$ where $e^N \in E_N$ and $e^V \in E_V$ and with values in the set E_{VP} (for verb-phrases).[3]

A semantics is an interpretation function $\boldsymbol{\mu}$ that maps expressions on meanings: that is, a function defined over E and with values in a set M of meanings.[4] Then compositionality can be formulated in a precise form in the following way:

> A semantics $\boldsymbol{\mu}$ is compositional iff for any syntactic rule $\alpha \in F$ there is function f such that for any expressions $e_1, \ldots, e_n$ of E, if $\alpha(e_1, \ldots, e_n)$ is meaningful then: $\boldsymbol{\mu}(\alpha(e_1, \ldots e_n)) = f(\alpha, \boldsymbol{\mu}(e_1), \ldots \boldsymbol{\mu}(e_n))$.

What kind of entities are the members of M? What is meaning, in other words? It has been long noticed that when it comes to natural languages we should distinguish two types of meaning: what an expression means independently of any context of utterance, and what an expression means relative to a context of utterance. The first type of meaning roughly corresponds to what speakers know when they know the meaning of an expression but are completely oblivious to the details of the context in which the

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3 The rule given here for exemplification is, of course, very course-grained. A proper representation of English syntax must take into account other features like subject-verb agreement. Representing syntax as *many sorted algebras* is favored by Janssen (1983) and Hendriks (2001). But this is not the only way to model syntax. Hodges (2001) and Pagin & Westerståhl (2010) prefer to represent grammars as *partial algebras* were syntactic rules are represented as functions which are simply undefined on unwanted arguments. A note on notation: from now on, I'll ignore the superscripts for grammatical categories.

4 If we believe that some expressions (e.g. "Green ideas sleep furiously") are grammatical but meaningless we should take $\boldsymbol{\mu}$ to be a partial function: undefined for some members of E. If one believes that "Green ideas sleep furiously" is grammatical and meaningful, but false, and that there are no grammatical but meaningless expressions (as Magidor 2009 does), then one can let $\boldsymbol{\mu}$ be defined over all members of E.

expression was uttered. The second type of meaning roughly corresponds to what speakers know when they know what is said by an expression at a particular context of utterance.

Following Kaplan (1989), the established term for the latter type of meaning is *propositional content.* Propositional content can be thought of as the information that determines the extension of an expression at possible states of affairs, and thus can be modeled as an intension: a function from possible worlds to *extensions* (individuals for singular terms, sets for predicates, and truth-values for sentences). Given that expressions of natural languages are context-sensitive, they can be assigned propositional content only relative to a context of utterance. Notice that in the absence of a context the English sentence "I am hungry" fails to have express anything which is truth-evaluable, since in order to determine its truth-value we need to determine who uttered it when. Moreover, depending on who utters it, "I am hungry" expresses different things: if uttered by David Kaplan it expresses the content that David Kaplan is hungry, if uttered by Saul Kripke it expresses the content that Saul Kripke is hungry. Thus we say that "I am hungry" expresses different contents at different contexts of utterance. Then propositional content is a property of expressions at contexts, and a semantics that assigns propositional content will assign it not to expressions themselves but to expressions at contexts. Let M_I be the set of intensions, then for any meaningful e of E and context C, $\boldsymbol{\mu}(e, C) \in M_{I,}$ is such that $\boldsymbol{\mu}$ (e, C): W → Ext (where W is the set of possible worlds, and Ext is the set of possible extensions). Each $w \in$ W is an extension determining circumstance, in the sense that the extension of an expression is always given relative to a w.[5] Using the standard notation $[[e]]^w$ for *the extension of e at w*, we write the content of the expression as $\lambda w.[[e]]^w$. Notice that

5 There is a long lasting debate in formal semantics on whether we can model natural languages with sparse circumstances of evaluation, consisting only of possible worlds, or whether we need richer ones. For example, Kaplan (1989) argues that we should take circumstances of evaluations to consist of world-time pairs. Others argue that circumstances of evaluation are even richer than that (see Kölbel 2008 for an overview). If we take circumstances of evaluation to be world-time pairs we should define intensions in the following way. Let W be the set of possible worlds, T the set of time moments, and W×T their Cartesian product, then each $\boldsymbol{\mu}(e, C) \in M_{I,}$ is such that $\boldsymbol{\mu}(e, C)$: W×T → Ext.

the propositional content of some natural language expressions (e.g. descriptions like “the president of France”) can be best modeled as non-constant functions from worlds to extensions, while the propositional content of other expressions (e.g. proper names like “David Kaplan”) can, arguably, be treated as a constant function from worlds to extensions.

Also following Kaplan (1989) the established term for the meaning that expressions have independent of context is *character*. The character of an expression is the convention associated with that expression and something like a rule of use: it tells what an expression can say when used in any arbitrary context. Characters are a property of expressions themselves, and can be modeled as functions from possible contexts of utterance to propositional contents; that is, as *hyperintensions*. Let $\mathrm{M_H}$ be the set of hyperintensions, then, for any meaningful expression e of E, $\boldsymbol{\mu}^*(e) \in \mathrm{M_H}$ is such that $\boldsymbol{\mu}^*(e) : \mathrm{K} \rightarrow \mathrm{M_I}$, where K is the set of possible contexts. Contexts as formal objects of the theory have the job to represent the concrete situations in which language use takes place. Given that any use of an expression is performed by someone, at a place and time and in a possible world, then each $C \in \mathrm{K}$ is an ordered tuple of parameters consisting of an agent a, a world w, a time t and a location l: $C = \langle a, w, t, l\rangle$.[6] Notice that the character of some natural language expressions (e.g. indexicals: “I”, “here”, “now”, etc., descriptions: “the tallest man in town”, etc.) can be best modeled as a non-constant function from contexts to propositional content, while the characters of others (e.g. proper names, numerals) are best treated as constant functions.

If we distinguish between the meaning that an expression has independent of context of use, and the meaning that it has relative to a context of use, and we decide to model the first as a function from contexts to functions from possible worlds to extensions, then the extension of an expression is *double relative*. The extension of an expression is a function of the

6 Some theorists (e.g. Stalnaker and his followers) prefer to represent contexts in a finer-grained manner, as the common ground between the participants in a conversational setting (i.e. the body of information commonly available to conversational partners) which can be modeled as the set of possible worlds compatible with the presumed common knowledge of the participants. These two notions of context can be complementary: a context as a common ground is determined by a context as a tuple of parameters: it is a fact about an *agent* at a *world*, *time* and *place* that she is presupposing certain propositions and that certain propositions are common ground in the conversation she is taking part. See Stalnaker (2014, 24-26) for elaborations.

context of utterance and of the circumstance of evaluation, and the two play different roles in determining the extension.[7] The character of an expression *e* determines, relative to any context *C*, the propositional content of *e* at *C*, which in its turn determines its extension, relative to any world *w*. Then I'll write $[[e]]^{c,w}$ for the extension of *e* at *C* and *w*, and write the character of *e* as: $\lambda c.\lambda w[[e]]^{c,w}$.

Then, a theorist has two options when it comes to giving an account of natural languages. She can choose a semantics that assigns characters: a function defined over E and with values in the set of characters M_H. Alternatively, she can choose a semantics that assigns propositional content to expressions in context: a function defined over the Cartesian product of E and the set of contexts K with values in the set of propositional contents M_I. The first option is:

$\boldsymbol{\mu}^*$: E → M_H, where each member of M_H is a function $\boldsymbol{\mu}^*(e)$: K → M_I

$\boldsymbol{\mu}$: E*x*K → M_I, where each member of M_I is a function $\boldsymbol{\mu}(e, C)$: W → Extension

Observe that $\boldsymbol{\mu}^*$ is the curryied version of $\boldsymbol{\mu}$.[8]

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7 To better see the need for double evaluation, consider the sentence: "Once, everyone now alive hadn't been born yet." Observe that in order to determine its truth-value we need to determine who is alive at the context of utterance; that is we need to look at the time of the context of utterance and determine the reference of the restrictor "alive now". But this is not enough. In order to determine its truth-value we also need to look in the past: to see whether it is true that there was a time in the past at which those alive at the time of the context of utterance were not born yet. In other words, "once" takes the propositional content expressed by the sentence embedded under it, and evaluates it to all time moments that precede the time of the context, such that it yields true if there is a time previous to the time of the context at which the propositional content is true. Notice that the distinct evaluations at time-moments play different roles: one is to fix the reference of "alive now", (reference which cannot be shifted by "once") and the other to evaluate for truth-value the propositional content expressed at the context of utterance by the sentence embedded under "once".

8 Currying is a standard procedure by which the addicity of a function can be reduced. Given a function *f* of type f: (X × Y) → Z, currying it provides the function f^*: X → (Y → Z). See Curry, Feys & Craig (1958).

2. Compositionality for Content and for Character

Depending on whether we define compositionality for propositional content or for character, formulations of different strength are obtained. This is how semanticists traditionally formulated compositionality for character and for propositional content respectively.[9]

i. Compositionality of character: The character of a complex expression is a function of the character of its constituents and of its syntactic structure. More precisely, a semantics $\boldsymbol{\mu}^*$ is *character compositional* iff for any syntactic rule α there is a function f such that for any $e_1, \ldots, e_n$ if $\alpha(e_1, \ldots, e_n)$ is meaningful then:

$$\boldsymbol{\mu}^*(\alpha(e_1, \ldots, e_n)) = f(\alpha, \boldsymbol{\mu}^*(e_1), \ldots, \boldsymbol{\mu}^*(e_n)).$$

A semantics fails to be character compositional if for some expressions e_i, e_j, e_k, $\in$ E and syntactic rule α,

$$\boldsymbol{\mu}^*(e_j) = \boldsymbol{\mu}^*(e_k) \text{ and } \boldsymbol{\mu}^*(\alpha(e_i, e_j)) \neq \boldsymbol{\mu}^*(\alpha(e_i, e_k)).$$

A semantics fails to be character compositional if substitution of character-equivalent expressions within a larger one does not preserve the character of the larger expression. In plain words, a semantics fails to be character compositional if substitution of synonyms is not character preserving in that semantics. To give an illustration, a semantics of English (or of a fragment of English) fails to be character compositional if it assigns the same characters to “attorneys” and to “lawyers”, but assigns different characters to “Attorneys are rich” and “Lawyers are rich”.

ii. Compositionality for content: given that content is assigned to expression-context pairs, in order to formulate compositionality for content we need to take into account the role that context plays in the determination of the content of complexes. Standardly, compositionality for content is

9 See Kaplan (1989, 507) where both varieties are given informally. For their formal rendering see Pagin & Westerståhl (2010, 259-260) and Dever (2006, 634).

formulated in the following way. The content of a complex expression relative to a context is a function of the content of its constituents at that context and of its syntactic structure. More precisely, a semantics **μ** is *content compositional* iff for every syntactic rule α there is a function *f* such that for any expressions $e_1, \ldots, e_n \in E$ and for any context C, if $\alpha(e_1, \ldots, e_n)$ is meaningful at C then:

$$\boldsymbol{\mu}(\alpha(e_1, \ldots, e_n), C) = f(\alpha, \boldsymbol{\mu}(e_1, C), \ldots, \boldsymbol{\mu}(e_n, C))$$

A semantics fails to be content compositional if the substitution of co-intensional expressions within a larger expression does not guarantee to preserve the intension (content) of the larger expression. More precisely, a semantics fails to be content compositional if there is a syntactic rule α, some expressions e_i, e_j, e_k, and context of utterance C, such that

$$\boldsymbol{\mu}(e_j, C) = \boldsymbol{\mu}(e_k, C) \textit{ and } \boldsymbol{\mu}(\alpha(e_i, e_j), C) \neq \boldsymbol{\mu}(\alpha(e_i, e_k), C)$$

For example, a semantics that assigns to "I" relative to a context C, which has David as its agent parameter, the same content that it assigns to "David" at context C, but assigns different contents to "I am hungry" and "David is hungry" at C fails to be content compositional. If the failure condition obtains then the content of $\alpha(e_i, e_j)$ at C and the content of $\alpha(e_i, e_k)$ at C are not a function of the content of their constituents at C, for a function should returns the same value given the same argument.[10]

[10] In order to avoid a potential retort that would side-track the discussion, let me point out that there is another way in which compositionality for content can fail: as a result of *context-shift failure*. That is, a semantics fails to be content compositional if a complex expression varies its content across contexts of utterance although its constituents have unvarying contents across the very same contexts of utterance. Formally, if there are some expressions e_i, e_j, syntactic rule α and contexts C_1, C_2 such that

$$\boldsymbol{\mu}(e_i, C_1) = \boldsymbol{\mu}(e_i, C_2) \text{ and } \boldsymbol{\mu}(e_j, C_2) = \boldsymbol{\mu}(e_j, C_2) \text{ and } \boldsymbol{\mu}(\alpha(e_i, e_j), C_1) \neq \boldsymbol{\mu}(\alpha(e_i, e_j), C_2)$$

obtain.

Nevertheless, the failure to preserve content under the substitution of co-intensional parts is more calamitous than context-shift failure, in the sense that any semantics that fails to preserve intension under the substitution of co-intensional parts also exhibits context-shift failure, but not the other way around. For proofs, see Pagin (2005, Apendix 1) and Westerståhl (2012). Relatedly, some authors (e.g., Pagin 2005; Lasersohn

Given the relation between $\boldsymbol{\mu}^*$ and $\boldsymbol{\mu}$, if a semantics $\boldsymbol{\mu}$ satisfies or fails to satisfy content compositionality so does its curryied version $\boldsymbol{\mu}^*$, and vice-versa: if a semantics $\boldsymbol{\mu}^*$ satisfies or fails to satisfy character compositionality so does its un-curryied version $\boldsymbol{\mu}$.

3. Some context-shifting operators

The double-index framework introduced above allows us to treat some expressions as hyper-intensional operators, in the same manner in which it allows us to treat certain expressions as intensional operators. As it is well known, intensional operators take the intension of an expression and evaluate it at alternative circumstances of evaluation: in other words, they shift the circumstance of evaluation at which the intension they operate on is evaluated. Hyper-intensional operators work at the level character in the same fashion in which intensional operators work at the level of content. Whether natural languages contain expressions which are best treated as context-shifting operators, or whether such expressions can be added to a natural language is still a matter of debate.[11] Just as an intensional operator

2012) have proposed two readings for compositionality for content: a stronger and a weaker one, where (a) the strong one entails the weak one but not the other way around; (b) the strong version has as its negation condition, context-shift failure, and (c) the weak version has its negation condition the failure to preserve intension under substitution of co-intensional parts. If (a) is true so is its contrapositive: (d) failure to preserve intension under substitution of co-intensional parts entails context-shift failure.

[11] Kaplan (1989, 510-511) answers in negative to both questions, but Perry & Israel (1986) and Santorio (2012) argue that some epistemic modals of English are best treated as context-shifting operators. Since it is still a matter of debate whether there are English expressions that are best treated as context-shifting operators, I focus the discussion on the rather artificial construction "in some other context". To get a flavor of how such operators might look like in English, Israel & Perry (1996, 311) suggest that we consider sentences containing epistemic modals evaluated under the veil of ignorance, as when the speaker of a context is ignorant about the values of the contextual parameters. Santorio (2012, 291) imagines precisely such a scenario: Rudolf and Carl, two amnesiacs, are kidnapped. Each of them knows that he is one of them but not which one. They are subjected to the following experiment, about which are informed: both are anesthetized and a coin is tossed. If the coin lands head, Rudolf will be killed and Carl released

shifts the circumstance at which an expression is evaluated, a hyper-intensional operator shifts the context at which the sentence is evaluated. In this sense, they are *context-shifting operators*. For illustration, consider the sentence

(1) I am hungry

and the sentence (2) obtained from (1) by prefixing it with the (rather artificial) context-shifting operator "In some other context":

(2) In some other context I am hungry.

Where, by stipulation, (2) is true in the context of utterance just in case the agent of some other context is hungry at the time and world of that other context. In other words, (2) is true at the context of utterance if and only if there is some other context C^* such that (1) expresses a content that is true at circumstance determined by C^*. In other words, the operator "In some other context" operates on the character of (1): it takes the character of (1), evaluates it at alternative contexts, and yields true if and only if there is an alternative context C^* such that the content of (1) at C^* is true at the circumstances of C^*. We can introduce this operator (abbreviated as Op) in the following way:

Op(S) is true at C iff there is a context C^* such that S is true at C^*.

In the course of evaluation of Op(S) at context C we evaluate S at context C^*: what the operator Op does is to look across contexts and yield true if there is a context C^* such that S is true at C^*. Before I move on, let me point one more thing. Notice that given that the character of some natural language expressions (e.g. proper names) is modeled as a constant function

on Harvard's campus, if it lands tail, Carl will be killed and Rudolf released on Stanford's Campus. Later on, one of them wakes up and utters: "I might be in Stanford" and "But I might well be in Harvard". Intuitively both sentences are true. Then "I" refers to two different individuals, irrespective of the actual context of utterance. Schlenker (2002) argues that certain propositional attitude verbs in Amharic and English are best treated as context-shifting operators, but see Maier (2016) for a defense of Kaplanian ban on such operators.

from contexts to propositional content, if we embed such an expression under a context-shifting operator their content doesn't get shifted. Context-shifting operators shift only the content of expressions whose character is a non-constant function from context to propositional content. Compare with intensional operators: intensional operators cannot shift the extension of expressions whose propositional content is treated as a constant function (e.g. proper names, among others), but only of those whose propositional content is treated as a non-constant function (e.g. descriptions, among others).

4. Compositionality and context-shifters

Context-shifting operators are not compatible with compositionality for content, but they are compatible with compositionality for character. Remember, a semantics $\boldsymbol{\mu}$ fails to be content compositional if the following obtains: for some expressions e_i, e_j, e_n, syntactic rule α and context of utterance C:

$$\boldsymbol{\mu}(e_j, \mathrm{C}) = \boldsymbol{\mu}(e_n, \mathrm{C}) \textit{ and } \boldsymbol{\mu}(\alpha(e_i, e_j), \mathrm{C}) \neq \boldsymbol{\mu}(\alpha(e_i, e_n), \mathrm{C})$$

That is, if relative to a context C two expressions e_j and e_n are assigned the same content $\boldsymbol{\mu}(e_j, \mathrm{c}) = \boldsymbol{\mu}(e_n, \mathrm{c})$, but two complex expressions $\alpha(e_i, e_j)$ and $\alpha(e_i, e_n)$ are assigned different contents relative to C, then such a semantics is not content compositional.

Given that $\boldsymbol{\mu}^*$ is the curryied version of $\boldsymbol{\mu}$, we can write the failure condition in the following way: for some expressions e_i, e_j, e_n, syntactic rule α and context of utterance C:

$$\boldsymbol{\mu}^*(e_j)(\mathrm{C}) = \boldsymbol{\mu}^*(e_n)(\mathrm{C}) \textit{ and } \boldsymbol{\mu}^*(\alpha(e_i, e_j))(\mathrm{C}) \neq \boldsymbol{\mu}^*(\alpha(e_i, e_n))(\mathrm{C})$$

It can be easily shown that context-shifting operators are incompatible with content-compositionality. If Op(S) is a sentence that contains a context-shifting operator, the content of Op(S) at a context of utterance is not a function of the content of its constituents at that context of utterance. Rather, the content of Op(S) at a context C is a function of the content of its constituent S at a context C^*, where $\mathrm{C} \neq \mathrm{C}^*$. In more detail, if two co-intensional expressions are embedded under context-shifting operators their substitution does not guarantee, anymore, preservation of neither content nor truth. Consider a language L (which is a fragment of English) for which a content compositional semantics $\boldsymbol{\mu}$ can be given. Assume that that for the two sentences of L

(Φ) I am hungry

and

(Ψ) Kaplan is hungry

and context C_1 (where C_1 is such that Kaplan is its agent) Φ is true at C_1 iff the agent of C_1 is hungry at the time of C_1 and Ψ is true at the same context if Kaplan is hungry at the time of the context. Given that Kaplan is the agent of C_1, Ψ and Φ express the same content at C_1: a function from possible worlds *w* to truth-values which yields true if Kaplan is hungry at the time and world of C_1 and yields false otherwise. That is: $\boldsymbol{\mu}(\Phi, C_1) = \boldsymbol{\mu}(\Psi, C_1)$. Since (by assumption) L has a semantics $\boldsymbol{\mu}$ which is content compositional the contents of Φ and Ψ relative to C_1 are a function of the contents of their respective constituents at C_1 and of their respective syntactic structure.

Now consider an extension M of the initial language L. M contains all expressions of L plus the sentential operator AtC_2 and sentences constructed with the help of this operator. Thus, for any sentence *S* if *S* is a sentence of L then *S* is also a sentence of M. And for any sentence *S* of L there is a sentence of the form AtC_2 S in M. We give a semantics $\boldsymbol{\mu}_M$ for M that assigns propositional content and preserves, or inherits the assignments of semantics $\boldsymbol{\mu}$ for L. For any sentence *S* of L the semantic function $\boldsymbol{\mu}_M$ of M agrees with the semantic function $\boldsymbol{\mu}$ of L: $\boldsymbol{\mu}(S) = \boldsymbol{\mu}_M(S)$. In other words, the semantics $\boldsymbol{\mu}_M$ of M is the same as semantics $\boldsymbol{\mu}$ of L for all sentences that do not contain the operator AtC_2.

The operator AtC_2 is defined as follows. For any context C, the sentence (AtC_2, S) is true relative to C iff *S* is true at C_2. In other words, AtC_2 is a function that takes the character of S and evaluates it at context C_2 such that the complex sentence (AtC_2, S) is true at C iff the evaluation of the character of S at C_2 yields true. That is, $\boldsymbol{\mu}^*_M(AtC_2, S)(C) = 1$ iff $\boldsymbol{\mu}^*_M(S)(C_2)$

= 1. Notice that by un-currying $\boldsymbol{\mu}^*$ we can define the operator for a semantics that assigns propositional content to expression-context pairs: AtC_2 is a function such that for any sentence S and any context C, it takes the content expressed by *S* at C (i.e. $\boldsymbol{\mu}_M(S, C)$) and yields the content expressed by *S* at C_2 (i.e. $\boldsymbol{\mu}_M(S, C_2)$). In other words, for any context C, $\boldsymbol{\mu}_M(AtC_2, S), C) = \boldsymbol{\mu}_M(S, C_2)$.

It is easy to show that semantics $\boldsymbol{\mu}_M$ of the extended language M is not content compositional. Take the two sentences (Φ) and (Ψ) and build two sentences with the help of the sentential operator AtC_2. We get the following sentences in the extended language:

(ϒ) AtC_2 I am hungry

(Ω) AtC_2 Kaplan is hungry

Suppose that C_1 and C_2 differ at most with respect to the agent of the context: Kaplan is the agent of C_1 but not the agent of C_2. Then, (ϒ) is true at C_1 iff the speaker of C_2 is hungry at the world of C_2. But notice that (Ω) is true at C_1 iff Kaplan is hungry at the world of C_1. The content expressed by (ϒ) at C_1 is different than the content expressed by (Ω) at C_1:

$$\boldsymbol{\mu}_M(AtC_2, \Phi), C_1) \neq \boldsymbol{\mu}_M(AtC_2, \Psi), C_1)$$

Given that M is an extension of L, and its semantics $\boldsymbol{\mu}_M$ preserves the assignments of $\boldsymbol{\mu}$ for expressions of L, since $\boldsymbol{\mu}(\Phi, C_1) = \boldsymbol{\mu}(\Psi, C_1)$ then $\boldsymbol{\mu}_M(\Phi, C_1) = \boldsymbol{\mu}_M(\Psi, C_1)$. Then, $\boldsymbol{\mu}_M$ is not content compositional because for some expressions Φ and Ψ, syntactic structure and context C_1, $\boldsymbol{\mu}_M(\Phi, C_1) = \boldsymbol{\mu}_M(\Psi, C_1)$ and $\boldsymbol{\mu}_M(AtC_2, \Phi), C_1) \neq \boldsymbol{\mu}_M(AtC_2, \Psi), C_1)$. ■

The extended language satisfies, nevertheless, character compositionality. Remember, a semantics $\boldsymbol{\mu}^*$ fails to be character compositional if the following obtains: for some expressions, e_i, e_j, e_n, and syntactic rule α:

$$\boldsymbol{\mu}^*(e_j) = \boldsymbol{\mu}^*(e_n) \textit{ and } \boldsymbol{\mu}^*(\alpha(e_i, e_j)) \neq \boldsymbol{\mu}^*(\alpha(e_i, e_n))$$

By currying $\boldsymbol{\mu}_M$ we obtain a function $\boldsymbol{\mu}^*_M$ and it can be showed that $\boldsymbol{\mu}^*_M$ is character compositional. All we need to show is that the following holds for $\boldsymbol{\mu}^*_M$: there are no expressions e_i, e_j, e_n, and syntactic rule α such that $\boldsymbol{\mu}^*_M(e_j) = \boldsymbol{\mu}^*_M(e_n)$ and $\boldsymbol{\mu}^*_M(\alpha(e_i, e_j)) \neq \boldsymbol{\mu}^*_M(\alpha(e_i, e_n))$. Given the definition of AtC_2 operator the following holds true of (Υ): for any context C, $\boldsymbol{\mu}_M(AtC_2, \Phi)C) = \boldsymbol{\mu}_M(\Phi, C_2)$. For the $\boldsymbol{\mu}^*_M$ semantics we write this in the following way:

(1) For any C, $\boldsymbol{\mu}^*_M(AtC_2, \Phi)(C) = \boldsymbol{\mu}^*_M(\Phi)(C_2)$

and the following holds true of (Ω)

(2) For any C, $\boldsymbol{\mu}^*_M(AtC_2, \Psi)(C) = \boldsymbol{\mu}^*_M(\Psi)(C_2)$

We can show *by reductio* that $\boldsymbol{\mu}^*_M$ character compositional. Assume that $\boldsymbol{\mu}^*_M$ for M is not character compositional. Then for some expressions Φ and Ψ the following holds (which is the failure condition for character compositionality):

(3) $\boldsymbol{\mu}^*_M(AtC_2, \Phi) \neq \boldsymbol{\mu}^*_M(AtC_2, \Psi)$

(4) $\boldsymbol{\mu}^*_M(\Phi) = \boldsymbol{\mu}^*_M(\Psi)$

From (4) we get (by definition of $\boldsymbol{\mu}^*$):

(5) For any C, $\boldsymbol{\mu}^*_M(\Phi)(C) = \boldsymbol{\mu}^*_M(\Psi)(C)$

From (5) we get (by universal instantiation):

(6) $\boldsymbol{\mu}^*_M(\Phi)(C_2) = \boldsymbol{\mu}^*_M(\Psi)(C_2)$

From (1), (2), and (6) we get:

(7) For any C, $\boldsymbol{\mu}^*_M(AtC_2, \Phi)(C) = \boldsymbol{\mu}^*_M(AtC_2, \Psi)(C)$

From (3) and the definition of $\boldsymbol{\mu}^*_M$ we get:

(8) There is a C, such that $\boldsymbol{\mu}^*_M(AtC_2, \Phi)(C) \neq \boldsymbol{\mu}^*_M(AtC_2, \Psi)(C)$

But (8) contradicts (7), therefore $\boldsymbol{\mu}^*_M$ is character compositional – i.e. $\neg((3)$ & $(4))$. ■

In brief, I showed that there are semantics that satisfy character compositionality but fail to satisfy content compositionality, namely those of languages that contain context-shifting operators. Interestingly, if we can show that any semantics which satisfies content compositionality also satisfies character compositionality, then together these results show that content compositionality is a proper generalization of character compositionality.[12]

I'll show, now, that if a semantics is content compositional then it is also character compositional. This is a *proof by contraposition*. It is easy to prove that if a semantics fails to be character compositional then it also fails to be content compositional which *is the contrapositive* of the claim that if a semantics satisfies content compositionality it also satisfies character compositionality.

A semantics fails to be character compositional if for some expressions e_i, e_j, and e_n and syntactic rule α the following assignments hold:

(a) $\boldsymbol{\mu}^*(e_j) = \boldsymbol{\mu}^*(e_n)$ and

(b) $\boldsymbol{\mu}^*(\alpha(e_i, e_j)) \neq \boldsymbol{\mu}^*(\alpha(e_i, e_n))$.

Notice that by definition of $\boldsymbol{\mu}^*$

(a) entails (a'): $\forall C$, $\boldsymbol{\mu}^*(e_j)(C) = \boldsymbol{\mu}^*(e_n)(C)$ and

(b) entails (b'): $\exists C^*$ such that $\boldsymbol{\mu}^*(\alpha(e_i, e_j))(C^*) \neq \boldsymbol{\mu}^*(\alpha(e_i, e_n))(C^*)$.

The conjunction of (a') and (b') entails failure of content compositionality: there are some expressions e_i, e_j, e_n, syntactic rule α and a context C^* such that $\boldsymbol{\mu}^*(e_j)(C^*) = \boldsymbol{\mu}^*(e_n)(C^*)$ and $\boldsymbol{\mu}^*(\alpha(e_i, e_j))(C^*) \neq \boldsymbol{\mu}^*(\alpha(e_i, e_n))(C^*)$. ■

In conclusion, if a semantics satisfies content compositionality then it also satisfies character compositionality.

[12] This is an alternative proof to the one given in Westerståhl (2012). His proof, though, is significantly different than the one given here, and I believe that the proof given here is much simpler than the one he gives.

5. Conclusion

Whether English, or other natural languages, contain expressions which are best treated as context-shifting operators, or whether such operators can be added to natural languages is an open empirical question. Whatever the answer to the empirical question, the results of the paper show that one cannot ban context-shifting operators on the basis that they are incompatible with compositionality. Such operators are incompatible with content-compositionality but are compatible with character compositionality. If a theorist has good arguments to believe that natural languages contain context-shifting operators, but she desires to retain compositionality she must settle on character-compositionality. Only if someone has good independent arguments to believe that propositional content must be compositional and that compositionality is non-negotiable, can she give a principled argument against context-shifting operators. That is, only if one believes that content compositionality explains, but character compositionality fails to explain certain features of natural languages that we expect semantic theories to model is one entitled to ban context-shifting operators on the basis that they ruin the content-compositional machinery. In fact, Rabern & Ball (2017) convincingly argue that Kaplan's ban on context-shifting operators rests on his further theoretical commitments that tied him to the idea that propositional content must be compositional. More precisely, Kaplan is wedded to the idea that one and the same entity must play two roles: (a) be the object of natural language operators, such that these operators can be defined compositionally, and (b) the content of speech acts and the object of propositional attitudes. Only propositional contents, and not characters, can be the content of speech acts and the object of propositional attitudes.

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