Carnap's Writings on Semantics

In "Logical Syntax of Language" Rudolf Carnap aimed to settle a firm ground for philosophy, i.e. for what he took to be the logic of science, by developing an exact syntactic method for dealing with philosophical problems. This syntactic method, which is governed by the "Principle of Tolerance", allows us to arbitrarily choose whatever axioms or rules of inference we want and this choice will determine by itself the meanings of the logico-mathematical symbols introduced by them. This method came in opposition with the customary one, which assigned meaning to the primitive logico-mathematical symbols and evaluated the correctness of logical sentences and inferences in accordance with this meaning. One of the main reasons for adopting this syntactic standpoint was that the meaning assigned by the customary procedure to the primitive symbols was expressed in natural language and, thus, taken to be inexact and ambiguous and, likewise, this customary procedure made any attempt to go beyond classical logic and mathematics deviant and in need of theoretical justification (see Carnap 1937, xiii-xv). Carnap's new standpoint widely opened the path to a pluralist approach in which the justification of a logico-mathematical system is made on practical grounds.

Later on, Carnap did no longer see the customary method of assigning meaning as inexact and ambiguous due to the developments made in semantics by Alfred Tarski and the Warsaw School. In addition, he went further to develop and apply the semantic method in "Foundations of Logic and Mathematics" (1939) and in the three volumes of his series "Studies in Semantics". The second volume of the series, "Formalization of Logic" (1943), deals with the relation between the semantic and the syntactic method in the problem of fully formalizing classical logic and, although published later, it was written before the first one, "Introduction to Semantics" (1942), which introduces the concepts necessary for the other volumes. The third volume, "Meaning and Necessity. A study in semantics and modal logic" (1947), introduces the method of extension and intension and applies it in the intensional area, to modal logic. My aim below is to discuss some central features of Carnap's semantic method, his proposal for a full formalization of classical logic, his method of extension and intension, and his semantic treatment of modalities.

Carnap did not abandon the syntactic method, but he simply supplemented it with the semantic one, one of his main aims being to investigate the possibility of the symmetry between these two fruitful methods of approaching philosophical problems, both in logic and in philosophy of science. This ideal of methodological symmetry could be described as an attempt to obtain categorical logical systems, i.e., systems that allow only the intended semantical interpretation (see Brîncuș 2022).

Carnap's Semantic Method

Tarski's success in defining the notion of adequacy for the definition of semantical concepts was one of the main reasons that made Carnap to abandon his reluctance towards these concepts. The adequacy conditions for the definition of semantical concepts –such as "designation", "truth" or "consequence"– can be precisely formulated in the metalanguage and, thus, their scientific status becomes unproblematic. For example, a predicate T in a metalanguage M is an adequate one for the concept of truth with respect to an object language S, and its definition is an adequate one, if and only if every instance of the T-schema ('p' is true if and only if p) follows from the definition of T. (see Carnap 1942, 26-29). This concept of truth, unlike the concepts of verified, confirmed, believed, etc., which belong to pragmatics, makes no reference to the persons that use it and, thus, belongs to semantics.

Semantics is defined by (Carnap 1942, v) as the theory of meaning and interpretation, but his concern is only with pure semantics, i.e. the construction and analysis of semantical systems, and not with descriptive semantics, which, like descriptive syntax, is an empirical discipline based on pragmatics. By setting up a system of semantical rules for a historically given language or for an invented one, a semantical system is thus introduced. These rules define in the metalanguage certain semantical concepts for the object language and pure semantics is nothing else than the study of these definitions and their analytic consequences. A semantical system defines truth-conditions for the sentences of the object language and, thus, determines their meanings. A semantical system usually has rules of formation, rules of designation, and rules of truth. If the system contains in addition variables, then it will also have rules of values (which specify the entities that are values of the variables) and rules of determination and fulfillment (that specify which entities will fulfill or satisfy the sentential functions). (Carnap 1942) provides nine toy-examples of elementary semantical systems, but we may think of the applications of the usual normal truth-tables for propositional logic and the substitutional and objectual interpretations of quantificational logic as paradigmatic examples of semantical systems in Carnap's sense. As (Carnap 1963, 932) will admit, unlike Tarski who worked mainly with languages without descriptive constants, he was more interested in applying the semantic method to the languages of empirical sciences.

The fundamental distinction that Carnap makes in his semantical approach is that between logical and descriptive signs and this is the root of the distinction between factual and logical truth (truth dependent upon the contingency of facts and truth dependent only on the meaning defined by semantical rules). The idea that there is a sharp division between these concepts and likewise between syntax and semantics (uninterpreted calculi and their interpretations) constitutes the main philosophical difference between Carnap's and Tarski's approaches (see Wagner 2017). In his article on logical consequence, Tarski was skeptical about the possibility of a general criterion of logicality, i.e., for the classification of signs in logical and non-logical, but later on he proposed the criterion of invariance under permutations. (Carnap 1942, 56) likewise acknowledges the fact that no general criterion of logicality is available (i.e., a definition in general semantics), but he considered that the distinction between logical and descriptive signs could be easily drawn for any given semantical system (i.e., in special semantics) by enumeration. These distinctions, however, are taken by (Carnap 1963, 64) not as assertions but as proposals for the construction of an adequate metalanguage for the language of science.

Carnap took each semantical term to have a meaning only relative to a given semantical system, its meaning being provided by the semantical rules of that system. Depending on the nature of these rules, we may distinguish between radical semantical concepts (e.g., truth, falsehood, implication, disjunction, etc) and logical semantical concepts (L-truth, L-falsehood, L-implication, L-disjunction, etc). For instance, if the system has logical rules for implication, then the semantical concept will be L-implication (the analogue of the model-theoretic concept of logical consequence, $\vDash$), otherwise just the material implication ($\rightarrow$). In this sense, logic as a theory of logical deduction and logical truth becomes a special part of semantics, the L(ogical)-semantics. These two types of concepts will also have corresponding concepts in pure syntax, i.e. in calculi, (C-truth, C-falsehood, C-implication, C-disjunction, etc) where, for instance, C-truth is theoremhood, while C-implication is logical derivability (the analogue of the proof-theoretic concept of logical consequence, $\vdash$). (Carnap 1943) reveals that there is a lack of symmetry between some L-concepts and their corresponding C-concepts and he will introduce new syntactical concepts for achieving their symmetry. A final class of concepts that Carnap introduces are the F-concepts. For instance, a sentence is factually true, i.e. F-true, if the radical concept "true" holds for it but the corresponding L-true concept does not. Thus, each sentence of a semantical system is either true (i.e., either L-true/analytic or F-true) or false (i.e., either L-false/contradictory or F-false). The F-sentences are the synthetic ones in the traditional terminology.

Hence, once we accept the division between logical and descriptive signs, then we can obtain a complete classification of the sentences from a semantical system: those whose truth values are determined only by the semantical rules alone are L-determinate and the rest are indeterminate (or factual). In (Carnap 1937), the determinate character of the logico-mathematical sentences was established by the syntactical rules, i.e., determinate are those sentences which are either theorems or contradictions, and Carnap's aim was to construct logical systems which are complete with respect to negation. For this reason he introduced the infinite rule of inference called the ω-rule, which makes the Peano Arithmetic negation-complete and, thus, Gödel's incompleteness results are partly over passed (see Warren 2020, 325-30, 274-78).

(Carnap 1939, 24; 1942, 160, 224; 1943, 143, 145) emphasized the necessity of using infinite rules of inference for obtaining an L-exhaustive calculus K for a given semantical system S, i.e., a calculus in which the C-concepts in K formalize their analogues L-concepts in S. Although (Carnap 1937) considered the infinite rules as rules of consequence (c-rules), and not ordinary rules of deduction (d-rules), in (Carnap 1942, 160-61, 247) he unified his approach by accepting the infinite rules as rules of deduction. The analytic sentences, i.e., the L-true sentences, will now correspond to those sentences that are provable by using systems with infinite rules of deduction (see Awodey 2012, Rouilhan 2012 for a discussion of analyticity). Moreover, the ideal of methodological symmetry seemed an attainable one for Carnap, as he asserted in a letter to Karl Popper from 29 January 1943 that he knows no semantical system for which no L-exhaustive calculus can be constructed. Certainly, this idea is consistent with his interpretation of Gödel's results, according to which everything mathematical can be formalized, but mathematics cannot be completely formalized by a unique deductive system, requiring an infinite progression of richer systems (see Carnap 1937, 222).

The Principle of Tolerance remained at work in Carnap's thought, but by the adoption of the semantic method it encountered a first limitation: if a syntactical system, i.e., a logical calculus, is constructed for a given semantical one, the features of the latter will determine those of the calculus in some essential respects. In addition, the conventionality of logic is also limited, since if its rules of deduction are expressed in terms of L-concepts for a given semantical system, then these L-concepts have to obey the condition of adequacy (see Carnap 1942, 218-19).

Carnap's Full Formalization of Logic

A first application of the semantic method was made by Carnap in the problem of fully formalizing classical logic. He investigated the possibility of a formalization that uniquely represents all the semantic properties of the logical symbols and discovered that the standard (i.e., single-conclusion and finite) calculi do not fully formalize the classical logic as it is semantically defined by the normal truth-tables and by the substitutional semantics. He constructed interpretations for which the logical calculi preserve their soundness (true-interpretations in Carnap's terms), but provide most of the logical terms with unintended meanings.

For propositional logic (Carnap 1943, 81) identified two mutually exclusive kinds of non-normal interpretations: one in which a certain sentence and its negation are both true (and, thus, all the other sentences are true, being consequences of their conjunction) and one in which a certain sentence and its negation are both false, but their disjunction is true (being a theorem). These interpretations are possible because the standard calculi state conditions only for C-implication (logical derivability) and C-truth (theoremhood). Thus, they can formalize only those L-concepts definable on the basis of L-implication. However, L-exclusive (two sentences are L-exclusive if they cannot both be true) and L-disjunct (two sentences are L-disjunct if they cannot both be false) are not definable on the basis of L-implication and, thus, they are not formalized by these calculi. Since these two concepts are essential for the semantical principles of non-contradiction and excluded middle, these principles will hold only in the normal interpretations.

By considering sentential classes (i.e., junctives) which may be infinite, Carnap's solution was to introduce a multiple conclusion rule and a rule of refutation:

1) $A_i \vee A_j \vdash \{A_i, A_j\}^{\vee}$
2) $V^{\&} \vdash \Lambda^{\vee}$

The first rule fixes the fourth line of the normal truth table for disjunction by requiring that at least one disjunct is true when the disjunction is true, and thus eliminates the second kind of non-normal interpretations. The second rule forbids having all the sentences true in a logical system. "$V^{\&}$" is the universal conjunctive, which is semantically defined as being true when all sentences are true, and "$\Lambda^{\vee}$" is the null disjunctive, which is always false. Thus, if we consider an interpretation in which all sentences are true, then rule 2) becomes unsound and, thus, this interpretation will not count as a permissible one.

In the case of quantificational logic (Carnap 1943, 135-50) observed that, due to the finite nature of the standard calculi, we may derive each instance from a universally quantified sentence, but we have no rule which allows us to derive a universal sentence from its infinite class of instances when it does not already follow from a finite subclass of it. Likewise, we have no rule which allows us to derive the entire infinite disjunctive class of instances from an existentially quantified sentence. This allows us to interpret non-normally a universal quantified sentence '∀xPx' as 'every individual is P and b is Q' and to interpret '∃xPx' as 'at least one individual is P or b is not Q'. In order to block the possibility of these non-normal interpretations, Carnap introduced two new rules of inference:

1) $\{A_i\,({}^{i}{}_{k})\}^{\&} \vdash A_i$
2) $(\exists i_k)A_i \vdash \{A_i\,({}^{i}{}_{k})\}^{v}$, where i_k is the only free variable in A_i.

The first rule stipulates that a sentence A_i containing a free variable i_k is directly derivable from the infinite conjunctive set of all its instances. This sentence is equivalent in Carnap's formalism with its universal closure and the deductive equivalence between a universal sentence and the class of all its instances is thus obtained. The second rule stipulates that we can pass from an existentially quantified sentence to the disjunctive class of all its instances. By adding at least one of these two rules to the standard formalizations, the deductive equivalence between the universal (and existential) sentences and all their conjunctive (disjunctive, respectively) instances is obtained, and thus the possibility of non-normal interpretations for the quantifiers disappears. (Carnap 1943, 151-54) also introduced the method of involution for obtaining a full formalization of the quantifiers (see (Kneale 1956) for a generalization of this method; see ch. Inferentialism).

Carnap's Meaning for Necessity

In 'Meaning and Necessity' Carnap introduces a new semantical method for analyzing the meaning of linguistic expressions, the method of extension and intension, and offers a semantics for logical modalities, by also considering their interaction with the quantifiers. Unlike most of the methods before and the referentialist trend afterwards, the method of extension and intension does not take the linguistic expressions, i.e. the designators (sentences, predicates, individual constants), as names for some entities, but rather as possessing extension (truth-values, classes, individuals) and intension (propositions, attributes, individual concepts). The extension is the reference or denotation of an expression, while the intension is its connotation or meaning. The extension of a designator is usually determined by empirical investigation, but if it can be determined only on the basis of the semantical rules, then the designator is L-determinate. (Carnap 1947) provides a detailed description of this method and a very useful comparative analysis with the semantic methods of Frege, Russell, Lewis, Hilbert and Bernays, Quine, and Church.

(Carnap 1942, 83-94) already outlined a definition of L-truth in general semantics by considering what he called “absolute concepts”, i.e., those concepts that are applied to the designata of expression without referring to a semantical system. An absolute concept applies to the designate of certain expressions when its corresponding semantical concept applies to those expressions. The absolute L-concepts are taken by (Carnap 1942, 92) to apply to propositions, and not only to truth-values. “L-true”, “L-false”, and “L-implication” are then taken to be synonymous with the modal terms “necessary”, “impossible” and “strict implication” in Lewis systems.

The concept of L-true is introduced by (Carnap 1947) as an explicans for the traditional concepts of logical, necessary, and analytic truth. A sentence is defined as L-true when it holds in every state-description, which means that the semantical rules are sufficient for establishing its truth. A state description for a language L is a class of sentences in L which contains for each atomic sentence only it or its negation and the range of a sentence is the class of state descriptions in which the sentence holds. These two semantic concepts will also be used by Carnap in providing a theory of induction and probability. Logical necessity (N) is by definition co-extensive with L-truth and, thus, the explicans for necessity contains in the end the state descriptions and the range rules. This understanding of logical modalities leads Carnap’s understanding of modal logic to Lewis’s stronger syntactic system for modalities, S5 (see ch. Modal Logic).

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