The Utility of Knowledge

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Abstract. Recent epistemology has introduced a new criterion of adequacy for analyses of knowledge: such an analysis, to be adequate, must be compatible with the common view that knowledge is better than true belief. One account which is widely thought to fail this test is reliabilism, according to which, roughly, knowledge is true belief formed by reliable process. Reliabilism fails, so the argument goes, because of the 'swamping problem'. In brief, provided a belief is true, we do not care whether or not it was formed by a reliable process. The value of reliability is 'swamped' by the value of truth: truth combined with reliability is no better than truth alone. This paper approaches these issues from the perspective of decision theory. It argues that the 'swamping effect' involves a sort of information-sensitivity that is well modelled decision-theoretically. It then employs this modelling to investigate a strategy, proposed by Goldman and Olsson, for saving reliabilism from the swamp, the so-called 'conditional probability solution'. It concludes that the strategy is only partially successful.

1 Introduction

1.1 The Value Problem

As if the task of analysing knowledge was not already hard enough, recent work in epistemology has created a new obstacle to negotiate. Gone are the days when such an analysis could survive merely by cohering with our intuitions about cases (a challenge which, even by itself, has seemed to some insurmountable). Now it must do something else as well: it must explain the putative datum that knowledge is better than true belief. On a traditional approach, where knowledge is understood as true belief plus something else, this elusive extra ingredient must be the sort of thing which is plausibly thought of as *value-adding*; it must be to true belief as icing is to cake. This new obstacle is sometimes called the 'value problem', or the 'extra value of knowledge' problem.[1]

1.2 Reliabilism Gets Swamped

One proposed analysis of knowledge (or family of such analyses) which is widely thought to fall at this new hurdle is reliabilism. On this analysis, what mere true belief lacks, compared to knowledge, is the property of being formed by a *reliable* process, one which can be depended on to regularly produce true beliefs.

That this additional property is not value-adding is shown, it is argued, by the so-called 'swamping problem'. The argument is usually presented via an analogy like the following.[2]

[1] For a helpful overview of the topic, see Pritchard 2007; the term 'extra value of knowledge problem', often shortened to 'EVOK', is from Goldman and Olsson 2009.

[2] The example is originally due to Zagzebski 2003, p. 13.

Imagine you face a choice between two cups of coffee about which your information is limited: all you know is that one cup, call it A, was produced by a coffee machine which reliably produces good-tasting coffee, whereas the other, B, was produced by an unreliable machine. Naturally, you prefer A—assuming you're a coffee-lover, or at least a coffee-liker. Now suppose you get some more information: as it happens, A and B taste equally good. (This is one of those lucky occasions when the unreliable machine doesn't malfunction and use cold water instead of hot.) Now your preference evaporates: you're equally happy to have either cup of coffee.

This illustrates the phenomenon that epistemologists call 'swamping'. At the start, the property of being produced by a reliable machine was a value-adding property; it was what made A preferable to B. But later, after you learned that A tasted no better than B, this property ceased to be value-adding. Its value was 'swamped' by the value of tasting good.

Something similar is supposed to happen in the case of belief, assuming reliabilism. Given a choice between two beliefs, knowing only that one was formed by a reliable process and the other wasn't, you would prefer the former. But upon learning that both are true, you would be indifferent. The value of truth swamps the value of being formed by a reliable process.

1.3 The Value of Truth

Some have suggested that the true culprit here is not reliabilism, but rather a certain view of epistemic value, sometimes called 'veritism'. On this view, the only ultimate good, from an epistemic perspective, is *truth*; our sole epistemic goal is to have true beliefs (and perhaps to avoid having false beliefs). Anything else is epistemically valuable only insofar as it is appropriately connected to truth. In particular, the property of being formed by a reliable process is valuable only to the extent that beliefs so formed are more probably true.

As Alvin Goldman and Eric Olsson observe, *taste* is plausibly all that matters ultimately in choosing a cup of coffee, and that is why swamping occurs in the coffee example as described. For the analogy to hold up, therefore, *truth* must be all that matters ultimately in forming beliefs. Goldman and Olsson conclude that the swamping problem arises not from reliabilism alone, but from the combination of reliabilism with veritism (Goldman and Olsson 2009, p. 24).

Duncan Pritchard goes further still. Not only is reliabilism on its own not sufficient for the swamping problem, Pritchard argues, it is not necessary either. So long as veritism is assumed, *any* analysis of knowledge—provided at least that it retains the traditional format, true-belief-plus-X—will face the same fate: the value of X, whatever it may be, will always be swamped by the value of truth. What the swamping problem shows, according to Pritchard, is that *veritism* (which he calls 'epistemic value T-monism') must be abandoned (Pritchard 2009).

One avenue of defence for the reliabilist would thus be to abandon veritism. She might become a pluralist about epistemic value, holding that something other than truth is ultimately good. Perhaps she could say that the property of being formed by a reliable process is also valuable in itself, in addition to its value as a sign of truth. But this move, I think, would strike many as implausible. It seems just very intuitive that the value of this property is owed solely to the fact that beliefs which have it are more likely true.

Goldman and Olsson pursue a different strategy. They propose a solution to the swamping problem which, though they don't say so explicitly, seems intended to be compatible with veritism. (Actually, they propose two solutions, but I'll consider only one of these here, since it seems uniquely amenable to the sort of decision-theoretic analysis I want to pursue.) Here's the basic idea. Suppose a given belief of some subject was formed by a reliable process. Then this fact is evidence, not only for the truth of this belief, but for the truth of other beliefs of the same subject; for the processes by which different beliefs of the same subject are formed are likely to be similarly reliable. So this feature of the belief may add value in (at least) two ways: by making it more likely that this belief is true, *and* by making it more likely that some other belief is true. Since the former may be swamped while the latter is not, knowledge may be better than mere true belief, even given both reliabilism and veritism. Goldman and Olsson call this the 'conditional probability' solution (Goldman and Olsson 2009, pp. 27–31).

1.4 Decision Theory

I believe these issues may be illuminated by using the mathematical tools of decision theory. The value-judgements involved display a sort of 'information-sensitivity' that is well captured, I want to suggest, by decision-theoretic analysis.

In the coffee example, what prompted your change of attitude was not any change in the objects themselves, the cups of coffee. Rather, it was a change in what you knew about them, your *information*. Before you knew how they tasted, you preferred the one which, given what you did know, was more likely to taste good, the one which you knew had been made by the reliable machine. This property of the cup of coffee had what is sometimes called 'signatory value': though not of any value *in itself*, it was a *sign* of something else valuable, namely, tasting good.[3]

Here's an analogy. We choose bananas on the basis of their colour, preferring yellow ones to green, not because we care about the colour in itself (we may in fact like green more than yellow), but because colour is our best guide to what we do care about, the ripeness of the banana's flesh hidden inside its skin. Likewise, when we must choose between cups of coffee without first tasting them, we must be guided by other, more accessible properties, e.g. what sort of machine made them.

But the signatory value that this property had for you was lost when you found out that the cups of coffee tasted equally good. You don't need evidence of what you already know.

Discussions of the swamping problem tend not to be very precise about what triggers the 'swamping effect'. Here, for example, is a typical passage from Jonathan Kvanvig (italics added my me):

> The central feature of this argument against a reliabilist account of the value of knowledge is, to repeat, the swamping effect that the value of truth has over the value of reliable belief. *Once truth is in place*, it has a kind of value that makes the value of reliability otiose. (Kvanvig 2003, p. 48)

Echoing this same phrase, Goldman and Olsson write:

> *Once truth is in place*, its value appears to swamp the value of reliability, thus making the combination of truth and reliability no more valuable than truth itself. (Goldman and Olsson 2009, p. 22)

But what is it for truth to be 'in place'? Does it mean simply that the belief in question is true? This doesn't seem right, because the phrase suggests that there was an earlier time when truth was not in place (hence 'once'), but, presumably, the belief was true as soon as it was formed.

In later work, Kvanvig uses less metaphorical language:

> [O]nce the truth of a belief *is a given*, the importance of likelihood of truth is swamped by the presence of truth itself. (Kvanvig 2009, p. 310)

This seems consonant with the picture I sketched above. Something is 'a given', in this sense, when it is included in the information on the basis of which one's preference or value judgement is formed, when it is part of what one knows about the options.

[3] On signatory value, see Bradley 1998.

Decision theory provides a nice model of the information-sensitivity of preferences.[4] It shows how an agent's preferences can (and perhaps *should*) change over time as new information comes in. A central concept here is *expected value*. Suppose you're interested in the value of some object about which you're partially ignorant: there are different possible ways it could be, consistent with all you know about it, some perhaps being more probable than others. The *expected* value of the object is then a weighted average of the values it would have in these possibilities, where the weights are the probabilities of the possibilities. Now if you get new information, e.g. you learn something which rules out one of the previously open possibilities, then expected value might change. If the possibility ruled out was a bad one, then expected value will rise; if it was good, then expected value will fall.

The general goal of this paper is to show that this sort of decision-theoretic picture can give us a nice model of the swamping phenomena that have exercised epistemologists. A more particular goal is to use this model to investigate Goldman and Olsson's conditional probability argument. My strategy will be to assume that reliabilism is true, and then to see whether veritism is compatible with the view that knowledge is better than mere true belief, taking into account the sorts of conditional probability effects highlighted by Goldman and Olsson. As we shall see, the answer is not straightforward. The statement 'knowledge is better than mere true belief' can be read in two ways. On one reading, which I call the 'individualistic' reading, it is perfectly compatible with veritism. But on the other, the 'collectivistic' reading, it is not.

2 A Decision-theoretic Framework

2.1 Some Propositions

Suppose you wish to evaluate, epistemically, the beliefs of a certain believer, Bea. In our mathematical model, we represent your evaluation by an assignment of utilities to propositions, the latter being constructed as follows.

Let $L = \{l_1, l_2, \ldots, l_n\}$ be the set of all beliefs held by Bea. (For simplicity, I assume that the number of Bea's beliefs, n, is fixed and finite.) Let a *state* be an ordered pair $\langle V, R \rangle$, with V and R both subsets of L. V contains those beliefs of Bea's which are in this state *true*, and R those which are formed by a *reliable* process.[5] These states thus represent the various possible combinations of true and reliably formed beliefs that Bea might hold. Finally, let a *proposition* be a set of states.[6]

For each belief l_i, let us define the following propositions:

$$\begin{aligned} V_i &= \{\langle V, R \rangle : l_i \in V\} \\ R_i &= \{\langle V, R \rangle : l_i \in R\} \\ K_i &= V_i \cap R_i \end{aligned}$$

V_i is thus the set of all states in which Bea's belief l_i is true, or, as we might say more normally, the proposition that l_i is true. Similarly, R_i is the proposition that l_i is formed by a reliable process. And K_i is the proposition that l_i is both true and reliably formed, i.e. that it constitutes *knowledge* (recall, we are assuming reliabilism).

Let $u(\cdot)$ be a utility function whose domain is the set of all consistent (i.e. non-empty) propositions. This represents a ranking of propositions in respect of their epistemic value: for any propositions X and Y, X is at least as good as Y iff $u(X) \geq u(Y)$. By the epistemic value of a

[4] For simplicity, I elide the distinction between preferences and value-judgements. If a person may prefer X to Y without thinking X better than Y, the difference seems unimportant here.

[5] I use 'V' as in 'veridical', instead of 'T' as in 'true', to save confusion with '$\top$', the tautological proposition, on which more below.

[6] So the set of all propositions is $\mathcal{P}(\mathcal{P}(L)^2)$, where $\mathcal{P}(X)$ is the power set of X.

proposition I mean how pleased you would be, from an epistemic point of view, to learn that this proposition is true. X is better than Y, for example, iff you would be more pleased (or less displeased) to learn that X is true than you would be to learn that Y is true.

2.2 Two Conditions

I now define two conditions that might be satisfied by your evaluation of Bea's beliefs.

The first is that knowledge is valued more highly than true belief, which I here call the 'Meno Assumption', borrowing this name from J. Adam Carter.

Meno Assumption. For all beliefs l_i, $u(K_i) > u(V_i)$.

This says that it is better that a belief is both true and reliably formed than that it is true.

The second is a specific form of veritism according to which Bea's sole epistemic goal should be to maximise the expected number of true beliefs she holds.[7] One proposition X is better than another Y iff the expected number of true beliefs is greater conditional on X than on Y.

Let $p(\cdot)$ be a probability function whose domain is the set of all propositions. This may represent either 'objective' probability, if you believe in it, or subjective credence, degrees of belief. Then this version of veritism (expected truth maximisation) may be defined as follows.

Veritism. For all consistent propositions X,

$$u(X) = \sum_{i=1}^{n} (p(V_i|X) - p(V_i)).$$

This says that the utility of a proposition X is the expectation of true beliefs conditional on X minus the prior expectation.[8] (See the appendix below for more detailed explanation of the above formula.)

Subtracting the prior expectation, because it is a constant, has no effect on the ranking of propositions, on which proposition is better than which. All it does is shift the zero-point in the utility scale. This provides a convenient way to differentiate good, bad, and neutral propositions. It is common to classify a proposition as good, bad, or neutral according as it is, respectively, better than, worse than, or equal in value with the tautology $\top$ (i.e. the proposition that contains every state).[9] What we might call the 'informational content' of a proposition can be measured by the number of states it excludes. The fewer states are contained in a proposition, the greater its informational content. At one extreme, the tautology $\top$ excludes no states at all, and so conveys no information. In a sense, then, we may think of $\top$ as 'nothing': if all I tell you is that either a belief is true or it isn't, for example, then in effect I've told you nothing. So on this approach, 'good' means better than nothing, and so on. Because $p(X|\top) = p(X)$, by definition, veritism implies that $u(\top) = 0$. Fittingly, the utility of nothing is zero. So a positive utility indicates that a proposition is good, a negative utility that it is bad, and a zero utility that it is neither good nor bad, i.e. neutral. According to veritism, then, X is good if it raises the expectation of truth, bad if it lowers the expectation, and neutral otherwise. And X is better than Y iff X either raises the expectation more, or lowers it less, than Y.

There is another way to think about veritism. The difference $p(V_i|X) - p(V_i)$ may be thought of as a measure of the 'amount' of evidence provided by X either for or against V_i. When the difference is *positive* (i.e. when $p(V_i|X) > p(V_i)$) X is evidence *for*, or *confirms*, V_i; when *negative*, X is evidence *against*, or *disconfirms*, V_i. Thus the utility of X, according to veritism, might be

[7] Note, since I've assumed that the total number of Bea's beliefs is fixed, this goal, maximising the expected number of true beliefs, is equivalent to the goal of minimising the expected number of false beliefs.

[8] The prior expectation is the expectation conditional on $\top$.

[9] See e.g. Jeffrey 1983, pp. 81–82.

described, alternatively, as the total balance of evidence provided by X for or against the truth of beliefs, i.e. the total evidence for truth minus the total evidence against.

This version of veritism might seem to some a bit crude. One reason is that it assumes that all true beliefs are equal, whereas it could be that, say, one profound true belief is worth as much as two trivial ones. Another reason is that it assumes the marginal value of true beliefs is non-diminishing, whereas it could be that, say, a guarantee of one true belief is better than a gamble with equal chances of either two true beliefs or zero. Nonetheless, such refinements would be unnecessary complications for present purposes.

3 Is Veritism Compatible with the Meno Assumption?

3.1 One Belief

Begin with the simplest case. Suppose Bea has only one belief.

In this case veritism is not compatible with the Meno assumption. V_1 guarantees the truth of one belief. In the present case, since there are no other beliefs one could expect to be true, this is the most truth one could possibly expect. So the expectation of truth conditional on V_1 is the greatest expectation possible, and therefore, according to veritism, V_1 is at least as good as any proposition: $u(X) \leq u(V_1)$ for all X. But this plainly contradicts the Meno assumption, which implies that $u(K_1) > u(V_1)$. Veritism implies that

$$u(V_1) = u(K_1) = 1 - \sum_{i=1}^{n} p(V_i),$$

and hence that truth and knowledge are equally good.

This doesn't make reliability completely worthless, however. It is entirely possible, and perhaps quite likely, that $p(V_1|R_1) > p(V_1)$, this reflecting the plausible idea that a belief is more likely true when formed by a reliable process, in which case veritism would say that $u(R_1) > 0$. Thus we have here classic swamping behaviour. R_1 is better than nothing, but $V_1 \cap R_1$ is no better than V_1. In the absence of truth, reliability has some value, but in the presence of truth, it adds no value.

Here's an analogy. Suppose you're thirsty and I offer you a small a glass of water which will quench your thirst partially but not fully. Naturally, you would accept it. Though it's not the best you could hope for, it's better than nothing. But now suppose you've already got a large glass of water which is sufficient on its own to completely quench your thirst. In this case, you would no longer want the small glass of water. Though the small glass of water on its own provides some relief from thirst, it adds nothing to the relief provided by the larger glass, since the latter already provides the maximum relief. Similarly, though R_1 on its own provides some increase in the expectation of truth, it adds nothing to the expectation provided by V_1, since the latter already provides the maximum expectation.

3.2 Two Beliefs

It must be rare, however, for a person to have only one belief. To take a slightly more realistic case, then, suppose now that Bea has two beliefs.

The addition of a second belief is sufficient to make veritism compatible with the Meno assumption. Though, as before, V_1 guarantees the truth of one belief it doesn't guarantee the truth of the other belief, and so the expectation of truth conditional on V_1 need not be the maximum. Veritism implies that $u(V_1) \geq u(K_1)$ iff $p(V_2|V_1) \geq p(V_2|K_1)$. It is plausible (a) that Bea's second belief is more likely to be formed by a reliable process if her first belief was, and (b) that her second belief is more likely to be true if formed by a reliable process. Together,

(a) and (b) support the conclusion that $p(V_2|V_1) < p(V_2|K_1)$, and therefore, given veritism, that $u(V_1) < u(K_1)$.[10] Suppose you're marking two students' exams. You know that both students answered the first question correctly. You also know that one of the students arrived at her answer by employing some sound method, whereas the other student, for you all you know, simply made a lucky guess. Naturally you would now have a higher expectation that the former student has also answered the second question correctly, other things be equal. Similarly, knowing that Bea formed her first belief by a reliable method may raise your expectation that her second belief is true.

Veritism is also compatible with the negation of the Meno assumption. It seems plausible, however, that assumptions like those above will hold at least in *normal* cases, and hence that veritism implies a weaker, yet still not insignificant, thesis: that knowledge is *normally* better than mere true belief.[11]

3.3 Collectivising Meno

Does this save reliabilism from the swamp? Perhaps not. Another reading of the statement 'knowledge is better than true belief' may make more trouble.

The Meno assumption, as defined above, is 'individualistic' in a certain way: it compares knowledge and truth with respect to all beliefs *individually*, one at a time. But we can also define a 'collectivistic' version, which compares these with respect to all beliefs *collectively*, all at once:

Collective Meno Assumption. $u(K_1 \cap K_2 \cap \ldots \cap K_n) > u(V_1 \cap V_2 \cap \ldots \cap V_n)$.

This says that having knowledge with respect to all issues is better than having true belief with respect to all issues. (Recall, n is the total number of Bea's beliefs.)

Clearly, veritism is incompatible with this, since it implies:

$$u(K_1 \cap K_2 \cap \ldots \cap K_n) = u(V_1 \cap V_2 \cap \ldots \cap V_n) = n - \sum_{i=1}^{n} p(V_i).$$

This might seem somehow paradoxical. It may be tempting to think that the collective Meno assumption simply *follows* from the original. Take the case where $n = 2$. If K_1 is better than V_1 and K_2 is better than V_2, then, it may be tempting to think, $K_1 \cap K_2$ must be better than $V_1 \cap V_2$. If a whole is composed of two parts both of which are better than the corresponding parts of another whole, then the first whole, so this line of reasoning goes, must be better than second. Think of weight as an analogy: if x_1 is heavier than y_1, and x_2 is heavier than y_2, then $x_1 + x_2$ (i.e. the combination, or sum, of x_1 and x_2) must be heavier than $y_1 + y_2$. As with 'heavier than', it might be thought, so with 'better than'.

But in fact, as we all know, this line of reasoning is fallacious. There are familiar counterexamples: each player on the student football team is better than her counterpart on the staff team; yet collectively the staff team is better than the student team—perhaps because the staff play more co-operatively, rather than each seeking individual glory.

Normally, intuitions about the value of knowledge are elicited by scenarios in which a single issue is involved, the classic example being, of course, Plato's Meno, where the issue is which path is the way to Larissa. To test the collective Meno assumption, however, something more elaborate is required. Consider, then, two subjects, Dee and Dum, both of whom, sadly, are now dead, and so have formed all the beliefs they ever will. By some amazing conincidence, Dee and Dum formed exactly the same beliefs throughout their lives, and all their beliefs were

[10]Given parallel assumptions (i.e. by simply transposing the subscripts '1' and '2'), veritism would also implies also that $u(V_2) < u(K_2)$.

[11]Cf. Goldman and Olsson 2009, p. 31.

true. However, whereas Dee's beliefs were formed by some impeccably reliable process, Dum's were formed by the least reliable method—by consulting an astrologer, say. Dum simply had incredible epistemic luck. So Dee has knowledge with respect to every relevant issue, while Dum has mere true belief (assuming reliabilism, as before). The question is, from an epistemic point of view, who had the better life? While veritism implies that their lives were equal, many will feel intuitively, I imagine, that Dee's life was better.

3.4 Infinite Beliefs?

I've assumed that L is *finite*. What if this assumption were dropped? The short answer is that this would be too complicated to be discussed adequately here. For one thing, simply *defining* veritism would be more difficult. If there were some non-zero probability of Bea having infinitely many true beliefs, then her expected number of true beliefs would be infinite, and we would, therefore, have problems of infinite utility similar to those made famous in Pascal's Wager. So I leave the infinite-issue case for another time.

4 Conclusion

Goldman and Olsson's conditional probability solution is only partially successful. As we've seen, it deals nicely with the individualistic Meno assumption (assuming more than one belief), but not with the collectivistic version. Reliabilists will need a new solution for the latter case.

In any event, I hope to have shown some of the utility of decision theory in thinking about issues of epistemic value.

Appendix

In this appendix I briefly explain why the expected number of true beliefs conditional on X is equal to

$$\sum_{i=1}^{n} p(V_i|X).$$

First, for any set of beliefs A, let $E_X(A)$ be the expected number of true beliefs in A conditional on X. Then for sets of beliefs A and B, if $A \cap B = \emptyset$, $E_X(A \cup B) = E_X(A) + E_X(B)$, this following from the 'additive law of expectation'.[12] So

$$E_X(L) = \sum_{i=1}^{n} E_X(\{l_i\}).$$

And $E_X(\{l_i\}) = p(V_i|X)$. So

$$E_X(L) = \sum_{i=1}^{n} p(V_i|X).$$

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[12] See e.g. Hodges and Lehmann 1970, p. 151.

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