Heterodox underdetermination: metaphysical options for discernibility and (non-)entanglement

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Abstract

Broadly speaking, there are three views on whether Leibniz's Principle of the Identity of Indiscernibles (PII) is violated in the case of similar particles. According to the earliest view, PII is always violated (call this the *no discernibility view*); according to the more recent *weak discernibility view*, PII is at least valid in a weak sense. No and weak discernibility have been referred to as *orthodoxy*. Steven French has argued that although PII is violated, similar particles can still be regarded as individuals, or, alternatively, as non-individuals: French famously concluded therefore that *metaphysics is underdetermined by physics*. Call this thesis *orthodox underdetermination*. Most recently, some authors have turned against orthodoxy by arguing that PII is valid in more than a weak sense – call this the *new discernibility view*, also referred to as *heterodoxy*. Since heterodoxy is backed up by physical considerations, metaphysics now seems to be *determined* by physics: physics indicates that PII is valid. In this paper, I argue that with respect to entangled states, there are two ways to establish PII's validity, which yield two different ontological interpretations of entanglement. Therefore, a form of underdetermination returns within the heterodox framework. I argue that heterodox underdetermination deserves some attention, because the two ontological interpretations might yield different explanations of the violation of Bell inequalities.

Keywords: Leibniz's principle; similar particles; (non-)entanglement

1. Introduction: Orthodoxy and orthodox underdetermination

With respect to the question of whether similar particles violate Leibniz's Principle of the Identity of Indiscernibles (PII), there are three broad views in the literature. Firstly, the (earliest) view that similar particles are always qualitatively identical, so that PII is always violated; call this the *no discernibility view* (see Butterfield (1993), French and Redhead (1988), French and Rickles (2003), Huggett (2003), Margenau (1944)). Secondly, the (more recent) view that similar particles are always discernible in a weak sense – the so-called *weak discernibility view* (see Saunders (2003), (2006); Muller and Saunders (2008); Muller and Seevinck (2009)). Thirdly, the (most recent) view that similar particles are (at least sometimes) discernible in more than a weak sense; call this the *new discernibility view* (see Bigaj (2015); Caulton (2014a), (2014b); Dieks and Lubberdink (2011), (2022); Friebe (2014), (2016); Leegwater and Muller (2022)). Bigaj (2022) has recently categorized no and weak discernibility as *orthodox*, and new discernibility as *heterodox.*

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The hallmark of orthodoxy is that it presupposes a certain semantical view of the meaning of the indices of the Hilbert space formalism: namely, the view that these indices (typically ‘1’ and ‘2’) have physical meaning in that they refer to particles (particle 1 and particle 2), and directly so (*direct factorism*).[1] Direct factorism comes with the commitment that the number of indices in a given state equals the number of particles in that state (e.g., two indices = two particles). Direct factorism also licenses the use of a certain mathematical procedure (“taking the partial trace”) to calculate which state-dependent properties particle 1 and particle 2 have (see Caulton (2014a)). This procedure yields that similar particles are always in the same mixed state, e.g., a mixture of spin up and spin down in z-direction.[2] Furthermore, it can be shown that all conditional probabilities are always the same. Given that similar particles have all their state-independent properties (mass, charge, and value of spin) in common, it follows that PII (both with monadic and relational properties) is always violated.[3] Saunders ((2003), (2006)), and later together with Muller (Muller and Saunders (2008)), has argued that similar fermions are always weakly discernible, e.g., by the symmetric but irreflexive relation ‘… has opposite spin to …’. This way, PII is saved in a weak sense – even weaker, that is, than relational PII.[4] Muller and Seevinck (2009) seek to generalize the weak discernibility account to similar bosons by exploiting the fully general fact that some operators belonging to the same Hilbert space (which have the same index, accordingly) do not commute, whereas the corresponding operators belonging to different Hilbert spaces (indexed by different indices) do (see Dieks and Lubberdink (2011), (2022)). This translates to (e.g.) the weakly discerning relation ‘… has complementary location and momentum to …’ holding between two similar bosons. The fact that weakly discerning relations between operator indices are taken to translate to weakly discerning relations between particles reveals that proponents of weak discernibility, just like proponents of no discernibility, presuppose direct factorism. To sum up: orthodoxy presupposes direct factorism, which yields that monadic and relational PII are violated, but weak PII is valid.

French (see French (1989a), (1989b), (2011), (2020)), sometimes together with other authors (see French and Redhead (1988) and French and Krause (2006)) has argued that, although monadic and relational PII are violated, similar particles can still be regarded as individuals. However, as it is equally possible to argue that similar particles are “non-individuals”, French famously concluded that metaphysics is underdetermined by physics: quantum mechanics does not decide whether similar particles should be regarded as individuals or as non-individuals. Call this thesis the *underdetermination of orthodox metaphysics*, or *orthodox underdetermination* for short. To recall it in more detail, the argument from orthodox underdetermination runs as follows:

The starting point of the argument is a comparison between classical statistics (aka Maxwell–Boltzmann statistics) and quantum statistics (Fermi–Dirac and Bose–Einstein statistics). Consider the distribution of two properties (‘up’ and ‘down’, say) over two particles. The following three

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[1] The term ‘factorism’ was coined by Adam Caulton. The distinction between direct and descriptive factorism (on which more later) was introduced by Leegwater and Muller (2022).

[2] See French and Redhead (1988) for a prime example of the partial trace procedure.

[3] The distinction between monadic and relational PII is standard in the literature. Monadic PII is the version of PII which results if the property-binding quantifier in

(PII) $\forall x \forall y\ (\forall F(Fx \leftrightarrow Fy) \rightarrow x = y)$

is taken to range over monadic properties only. Relational PII results if the property-binding quantifier is taken to include relational properties. Monadic PII is usually regarded as the stronger version because it has more potential counterexamples.

[4] Relational discernibility is usually understood in terms of *asymmetric* (and irreflexive) relations, whereas weak discernibility is understood in terms of *symmetric* (and irreflexive) relations. Therefore, the version of PII which results by admitting weakly discerning relations is even weaker than relational PII. Consequently, no discernibility and weak discernibility are not opposing views, but fully compatible with each other.

combinations are possible: 1. both particles have up, 2. both particles have down, and 3. one particle has up while the other has down. In classical statistics, the third combination has statistical weight ‘2’, corresponding to the two possibilities in which this combination can be realized: (a) the first particle has up, and the second particle has down, or (b) the other way around. In other words, in classical statistics, permutations are counted. In quantum statistics (both Fermi–Dirac and Bose–Einstein statistics), however, the third combination is weighted with ‘1’, i.e., the statistical weight of the third combination is reduced by half in quantum statistics as compared to classical statistics. Historically, this reduction of statistical weight has been explained as follows: possibilities (a) and (b) are in fact not numerically distinct possibilities, but must be counted as one and the same; i.e., there is only *one* possibility of realizing the third combination. The reasoning goes as follows: if quantum particles were individuals, permutations would count; it would make a difference which particle has which property. Since permutations do not count, the particles are not individuals; instead, they are *non-individuals*.

However, as French has pointed out, there is an alternative way to explain the reduction of statistical weight. Given the requirement that states of similar bosons must be symmetric, and states of similar fermions must be antisymmetric (aka the symmetrization postulate), it follows that there is only one way in which the third combination can be realized, just as the argument for non-individuality has it. This time, however, this fact does not result from permutations (a) and (b) being identified. Rather, it holds because in the Hilbert space formalism, there is only one mathematical representation of the third combination which fulfills the respective symmetry requirements: one symmetrical state representing the third combination for similar bosons, and one antisymmetrical state representing the third combination for similar fermions. Possibility (a) and its permutation (b) correspond to non-symmetrical states, and are therefore not accessible for similar bosons and fermions, respectively. Given this alternative explanation, the conclusion to non-individuality is blocked; it is therefore still possible to regard similar particles as individuals. However, since monadic and relational PII are violated, this individuality must be understood in different terms, e.g., in terms of bare particularity or weak discernibility. In sum, we are left with two options, particles as individuals versus particles as non-individuals. Hence orthodox underdetermination.

In his latest paper on the topic, French (2020) considers what might be understood as a criterion for evaluating in which cases physics is metaphysically underdetermined in a problematic way, and in which cases physics is metaphysically underdetermined in an unproblematic way: underdetermination of metaphysics by physics is unproblematic if the metaphysical picture is still sufficiently clear, and problematic if the metaphysical picture is not still sufficiently clear.[5] Consider the following examples. Quantum mechanics deals with properties like mass, charge, and spin. As quantum mechanics itself gives no further information about how these properties are to be understood metaphysically, philosophers can argue about whether they are to be construed as tropes or as universals. Tropes, in turn, can either be understood as abstract or concrete (or both). Given these different metaphysical options, underdetermination arises. However, since it concerns metaphysical detail, the choice between abstract and concrete tropes is arguably unproblematic. Whether the choice between universals and tropes is equally unproblematic is harder to decide. One could argue that the metaphysical picture is still sufficiently clear: there are properties, and whether these properties are construed as tropes or as universals does not make much of a difference for our understanding of quantum mechanics, and what it tells us about the fundamental furniture of the world. Orthodox underdetermination, by contrast, yields a kind of underdetermination which is definitely problematic: whether quantum particles are individuals or non-individuals does not only have an impact on our understanding of quantum mechanics and what it tells us about the fundamental furniture of the

[5] French takes the idea of a sufficiently clear metaphysical picture from Chakravartty (2007). Both Chakravartty and French discuss this issue in the context of scientific realism.

world, but also on what logical and mathematical systems are apt for describing that world (classical logic and set theory for particles as individuals, non-classical logic and set theory for particles as non-individuals). Hence, given the choice between these two metaphysical options, the metaphysical picture is not clear at all.[6]

As French notes, these considerations can also be invoked to block a common objection against orthodox underdetermination, namely the objection that underdetermination already arises on the macrophysical level (e.g., macrophysical objects like chairs can either be construed as bundles of chair-properties, or as bare substrata carrying chair-properties), so that underdetermination on the quantum level does not deserve special attention. However, given the above criterion, it can be argued that it does: macrophysical underdetermination is different from quantum level underdetermination in that the former does not have an impact on the clarity of our metaphysical picture (there are chairs), whereas the latter does.

However, there is an alternative way to argue that orthodox underdetermination is not a pressing problem, at least not anymore: some authors have pointed out that orthodoxy has consequences which are undesirable from a physical point of view, and that to avoid these consequences, a heterodox view should be adopted instead. The hallmark of heterodoxy is that it rejects direct factorism, so that, as a consequence, monadic PII can be considered valid for similar fermions (and sometimes, for similar bosons as well).[7] Given the physical arguments in heterodoxy's favor, it now looks as if metaphysics is *determined* by physics: apparently, physics indicates that (monadic) PII is valid. In the rest of the paper, I am concerned with the question of whether this is indeed the case. The structure of the discussion is as follows: in section 2, I briefly recall the physical considerations against orthodoxy, i.e., for heterodoxy. Next, in section 3, I introduce the heterodox new discernibility view, and show how PII's validity is established. In doing so, I discuss a recent semantical view which has been proposed by Leegwater and Muller (2022). In section 4, I consider heterodoxy's ontological stance on non-entangled and entangled states in more detail. I argue that with respect to entangled states, two strategies to establish PII's validity are available: the *discerning defense* and the *summing defense*. I show that the discerning defense has to deal with a certain problem (intra-basis ambiguity), which the summing defense can avoid. Since the two strategies yield different ontological views of entanglement, heterodox underdetermination arises. In section 5, I discuss how both ontological views connect with the much-discussed issue of entanglement and holism. Finally, in section 6, I discuss whether heterodox underdetermination is as problematic as orthodox underdetermination. I conclude in section 7.

2. Physical arguments against orthodoxy

The most prominent argument against orthodoxy is the *argument from the classical limit*, which is due to Dieks and Lubberdink (2011, 2022). They argue as follows: the symmetrization postulate is a robust feature of quantum mechanics, which still holds in the classical limit. In combination with the symmetrization postulate, direct factorism yields that similar fermions are always in the same mixed state. In particular, similar fermions are always in a mixture of left and right, never just left and right.[8] Classical particles, by contrast, are always localized (impenetrability holds). Taken together, this yields

[6] The point here is not that every metaphysical option *by itself* is not sufficiently clear, but that, given the *choice* between two sufficiently clear metaphysical options, the *overall* picture is no longer sufficiently clear.

[7] For the sake of simplicity, I focus on similar fermions for the rest of the discussion.

[8] Without the requirement for (anti-)symmetrization, direct factorism would not yield that result – similar fermions in non-symmetric product states, for example, would be localized even if direct factorism was presupposed.

that similar fermions can never become the localized particles of classical theories (like classical electrodynamics) in the classical limit.[9] Given the reasonable demand that quantum mechanics (as the successor theory) should reproduce the results of classical theories in some appropriate limit, this result is highly undesirable.[10] Note that weak discernibility is of no help here: weak discernibility does not yield that similar fermions are localized in the sense that one particle is right and one particle is left. Consequently, the argument from the classical limit concerns both no and weak discernibility, i.e., both orthodox views. Bigaj (2022, p. 181) notes that Caulton (2014a) deems the argument from the classical limit to be the most convincing argument against orthodoxy. However, further arguments can be given.

Friebe (2014) argues that direct factorism cannot respect the distinction between physically entangled states, i.e., states which yield a violation of Bell inequalities, and states which are not physically entangled in this sense.[11] Friebe considers a measurement operator which represents a spin measurement on one side of the experimental setup (the right one, say), i.e., a spin measurement which is intuitively performed on the right particle, and this particle only. Now, this operator acts on a physically non-entangled state of two similar fermions. Given direct factorism, the measurement affects both particles in such a state: before measurement, the particles are in a certain mixed state (a mixture of right spin up in z-direction and left spin down in z-direction, say), and after measurement, *both* particles are in a *different* mixed state (a mixture of right spin up in y-direction and left spin down still in z-direction, say). Therefore, direct factorism implies that mysterious correlations arise in states which are not physically entangled; in other words, direct factorism gives rise to what might be called *spurious entanglement*. Bigaj (2022) makes a similar observation: consider a physically non-entangled state which contains two electrons, which are in a certain mixed state according to direct factorism. Now, add a third electron to the system. Direct factorism implies that the three-particle-system is in a different mixed state than the two-particle-system was before the third particle was added, thereby implying (counterintuitively) that electron states are always interdependent.[12]

Taken together, these arguments strongly indicate that orthodoxy – more specifically its hallmark, direct factorism – is problematic from a physical point of view. Therefore, direct factorism and consequently, orthodoxy, should be rejected; instead, a heterodox view should be adopted.

3. Heterodoxy: the new discernibility view

As pointed out earlier, direct factorism yields that monadic and relational PII are violated, and that only weak PII is valid. Proponents of heterodoxy therefore conclude that rejecting direct factorism is necessary in order to save PII (in more than the weak sense, that is); what proponents of heterodoxy

[9] Dieks and Lubberdink (2022) note that their argument might generalize to other properties than just location properties.

[10] The demand that a successor theory should reproduce the results of its predecessor is sometimes referred to as the *correspondence principle.*

[11] The distinction between non-entangled and entangled states in terms of (lack of) violation of Bell inequalities goes back to the physicists Ghirardo, Marinatto and Weber ('GMW') (see, e.g., Ghirardo, Marinatto and Weber (2002)), and is often referred to as 'GMW-entanglement'. Since it is widely accepted among proponents of heterodoxy, I follow suit and presuppose this understanding of (non-)entanglement in the rest of the paper. It should be mentioned, however, that GMW-entanglement is not completely uncontroversial: Leegwater and Muller (2022) consider a state which counts as non-entangled according to GMW's criterion, but nevertheless yields a violation of Bell inequalities (the hallmark of *entangled* states); in other words, they consider a potential counterexample against GMW's notion of (non-)entanglement. See Bigaj (2022, ft. 18 on p. 167) for a reply to Leegwater and Muller's criticism.

[12] An overview and more physical considerations against orthodoxy can be found in Bigaj (2022, ch. 7.1).

would deem sufficient to save PII, however, is not completely clear. To show this, consider Leegwater and Muller's (2022) distinction between *∀-factorism* and *∃-factorism*, which emphasizes the importance of specifying which Hilbert space and which factorization are considered in a given context (arbitrary ones, or specific ones). According to ∀-factorism, indices always refer to particles, no matter which Hilbert space and which factorization is considered. Put differently, indices refer to particles in any arbitrary Hilbert space. According to ∃-factorism, there is at least one Hilbert space and one factorization with respect to which indices refer to particles. Put differently, indices refer to particles with respect to one (or more) specific Hilbert space(s). Now, all proponents of heterodoxy agree that ∀-factorism is false: if the indices of the specific factorization which has been considered in the discussion of orthodoxy are taken to (directly) refer to particles (call this the *orthodox factorization*), undesirable consequences follow. Since the falsity of ∀-factorism is compatible with the truth of ∃-factorism, Leegwater and Muller (2022) aim at establishing the latter. Specifically, they refer to their position as 'descriptive factorism', because the indices in the specific factorization which they consider (call this the *heterodox factorization*) can be taken as descriptively referring particle names. However, it is not completely clear whether other proponents of heterodoxy (Bigaj, Caulton, Dieks and Lubberdink, Friebe) endorse this proposal. Caulton most certainly does, since the view which he proposes in Caulton (2014a, 2014b) is already quite close to Leegwater and Muller's (2022) proposal (as they note in fn. 23). As far as Bigaj and Dieks and Lubberdink are concerned, I think the situation is less clear. This is because descriptive factorism is committed to a certain ontological view of entangled states (to be spelled out in the next section) which both parties have their concerns about. Friebe most certainly rejects descriptive factorism for this exact reason: the ontological view which he proposes for entangled states in Friebe (2014) is incompatible with Leegwater and Muller's stance, which makes his view anti-factorist (in the sense of a negation of ∃-factorism). To spell out these claims in more detail, in the next section I discuss Leegwater and Muller's semantic and ontological views, and show what concerns other heterodox parties might have about the latter, especially with respect to entangled states. These concerns might be taken to motivate an alternative ontological view of entangled states. Hence, as I argue, heterodox underdetermination arises: the heterodox project – saving PII for similar particles – does not yield a homogeneous ontology.

3.1 Heterodox semantics: descriptive factorism

To achieve the specific factorization with respect to which indices can be taken as descriptively referring particle names (the heterodox factorization), Leegwater and Muller (2022) proceed as follows. They construct a four-dimensional subspace ("snapshot Hilbert space" $\mathcal{H}$(t), for a short time t), by means of the following four basis vectors:

a) $\frac{1}{\sqrt{2}}(|\mathrm{L}, \uparrow_z, \mathrm{t}\rangle\, |\mathrm{R}, \uparrow_z, \mathrm{t}\rangle - |\mathrm{R}, \uparrow_z, \mathrm{t}\rangle\, |\mathrm{L}, \uparrow_z, \mathrm{t}\rangle)$

b) $\frac{1}{\sqrt{2}}(|\mathrm{L}, \uparrow_z, \mathrm{t}\rangle\, |\mathrm{R}, \downarrow_z, \mathrm{t}\rangle - |\mathrm{R}, \downarrow_z, \mathrm{t}\rangle\, |\mathrm{L}, \uparrow_z, \mathrm{t}\rangle)$

c) $\frac{1}{\sqrt{2}}(|\mathrm{L}, \downarrow_z, \mathrm{t}\rangle\, |\mathrm{R}, \uparrow_z, \mathrm{t}\rangle - |\mathrm{R}, \uparrow_z, \mathrm{t}\rangle\, |\mathrm{L}, \downarrow_z, \mathrm{t}\rangle)$

d) $\frac{1}{\sqrt{2}}(|\mathrm{L}, \downarrow_z, \mathrm{t}\rangle\, |\mathrm{R}, \downarrow_z, \mathrm{t}\rangle - |\mathrm{R}, \downarrow_z, \mathrm{t}\rangle\, |\mathrm{L}, \downarrow_z, \mathrm{t}\rangle)$[13]

Then they factorize $\mathcal{H}$(t) into $\mathcal{H}$(l) and $\mathcal{H}$(r), so that the basis can be written as:

[13] It is important to note that this choice of basis vectors is biased towards base vectors which have distinct location properties. Without this bias, snapshot Hilbert space would be six-dimensional instead of four-dimensional.

$|\uparrow_z \uparrow_z, t\rangle = |\uparrow_z, t\rangle_l |\uparrow_z, t\rangle_r$; $|\uparrow_z \downarrow_z, t\rangle = |\uparrow_z, t\rangle_l |\downarrow_z, t\rangle_r$ etc.

Within this new basis, a non-entangled state that in the original formalism looked as follows

(1) $\frac{1}{\sqrt{2}} (|R\rangle_1 |\uparrow_z\rangle_1 |L\rangle_2 |\downarrow_z\rangle_2 - |L\rangle_1 |\downarrow_z\rangle_1 |R\rangle_2 |\uparrow_z\rangle_2)$

now looks like this:

(1)* $|\downarrow_z\rangle_l \otimes |\uparrow_z\rangle_r$

Similarly, an entangled state that in the original formalism looked as follows

(2) $\frac{1}{\sqrt{2}} ((|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2) \otimes (|L\rangle_1 |R\rangle_2 + |R\rangle_1 |L\rangle_2))$

now looks like this:

(2)* $\frac{1}{\sqrt{2}} (|\uparrow\rangle_l |\downarrow\rangle_r - |\downarrow\rangle_l |\uparrow\rangle_r)$

The newly appearing indices ‘l’ and ‘r’ are supposed to carry the descriptive contents ‘left’ and ‘right’, respectively. With respect to (1)*, the descriptive factorist can therefore claim that the left particle, referred to by ‘l’, has spin down in z-direction, and the right particle, referred to by ‘r’, has spin up in z-direction. With respect to (2)*, she can judge that there is a right particle referred to by ‘r’, and a left particle referred to by ‘l’, both without definite spin properties. Therefore, she can conclude that monadic PII is valid for both non-entangled and entangled states: in both kinds of states, there are two qualitatively distinct particles.

Note, however, that considering the heterodox factorization is not necessary to establish the validity of PII: state (1) already licenses the judgement that there is one particle which is right and has spin up in z-direction, and one which is left and has spin down in z-direction (given that the indices are no longer taken as directly referring particle names, that is). This result can be combined with semantic descriptivism: the particles can be named after their properties, which yields that one particle can be referred to as ‘the right-up particle’ and the other as the ‘left-down particle’. In case of non-entanglement, the orthodox factorization (*without* orthodox semantics!) therefore yields the same results as the heterodox factorization. Similarly, state (2) licenses the judgement that there is one particle on the left, and one particle on the right (both without definite spin properties), which, accordingly, can be referred to as ‘the left particle’ and ‘the right particle’. An important thing to note, however, is that (2), in contrast to (2)*, *equally* licenses the claim that there is one particle with spin up, and one particle with spin down (both without definite location properties), which, accordingly, can be referred to as ‘the up particle’ and ‘the down particle’. This seemingly unimportant difference is shown to be important in the next section.

The results thus far seem to be good news for the friend of naturalistic metaphysics: physics indicates that orthodoxy is wrong, and that heterodoxy is right. Heterodoxy establishes PII’s validity for both non-entangled and entangled states. We therefore seem to have physical evidence to the effect that similar fermions are PII-individuated. In the next section, I examine this claim – that similar particles are PII-individuated – in a bit more ontological detail.

4. Heterodox ontology

4.1 The ontology of non-entangled states

The proponent of PII-individuation wants to claim that qualitative distinctness is necessary for numerical distinctness. This is exactly what the bundle theory of objects implies when combined with universalism about properties: according to bundle theory, objects are nothing but bundles of properties. Bundles are usually understood as sets or mereological sums. In both cases, they are extensionally individuated, i.e., they are numerically distinct iff they don't contain the same elements/parts. When combined with universalism about properties, the elements/parts are (immanent) universals, i.e., entities which are numerically distinct only if qualitatively distinct (but numerically identical if qualitatively identical, which yields that universals can be multi-located). Consequently, universal bundles count as numerically distinct only if they are qualitatively distinct – in other words, universal bundle theory implies PII, and is therefore the natural ally of PII-individuation.

Note that the same does not hold when bundle theory is combined with trope theory. This is because universals and tropes have different individuation conditions, which has an impact on the extensional individuation of bundles: tropes are numerically distinct even if qualitatively identical. Consequently, trope bundles count as numerically distinct even if qualitatively identical; qualitative distinctness is not necessary. Let me illustrate this difference between universal bundle theory and trope bundle theory with a simple example: consider two red spheres and say that, for the sake of simplicity, they have only two properties, namely 'being red' and 'being spherical'. Given bundle theory, the two spheres are to be construed as the bundles {red, spherical} and {red, spherical}. When properties are understood as universals, the red of the first bundle is numerically identical to the red of the second bundle (the same goes for spherical). It follows that the "two" bundles are actually not two, but one and the same numerically identical bundle (they contain the same elements/parts). To make them count as numerically distinct, it is necessary to introduce a qualitative difference (e.g., by exchanging the red of the first bundle for blue). By contrast, if properties are understood as tropes, the bundles {red^1, spherical^1} and {red^2, spherical^2} count as numerically distinct without further ado, because they don't contain the same elements (as the superscripts indicate, red^1 is not numerically identical to red^2}. So, universal bundle theory implies PII, but trope bundle theory does not.[14]

As far as the relation between PII and trope theory is concerned, many authors claim that trope-PII is trivially true (see, e.g., Crisp and Loux (2017) and Lyre (2018)). However, this claim depends heavily on the exact natural language formulation of PII as expressed in terms of second-order predicate logic (see fn. 3); more specifically, on the natural language interpretation of '$Fx \leftrightarrow Fy$'. If this part of the principle is understood in terms of "sharing properties", then, of course, trope-PII is trivially true because tropes, by definition, cannot be shared (trope theory is a version of nominalism about properties and, as such, denies that properties are entities which can be shared by multiple objects). If it is understood in terms of "qualitative distinctness", however, a non-trivial version of trope-PII is possible: while it is still true that trope bundle theory does not imply that qualitative distinctness is necessary for numerical distinctness, the proponent of trope bundle theory can still claim a substantial version of PII by demanding that for any given pair of trope bundles, there is at least one pair of determinable tropes (color tropes, shape tropes etc.) whose tropes do not exactly resemble each

[14] Note that the former implication can be blocked by rejecting one (or both) of the above standard assumptions, i.e., 1. that bundles are individuated extensionality, and 2. that universals are numerically identical if they are qualitatively identical. See Rodriguez-Pereyra (2004) for the first option, and Rodriguez-Pereyra (2016) for the second option (note that Rodriguez-Pereyra (2016) is aware of the fact that the second option is in danger of collapsing universal theory to trope theory, and is prepared to argue against such a collapse).

other.[15] Consequently, PII-individuated objects can either be understood as universal bundles, or as trope bundles.

Now, let's apply these results to non-entangled states. With respect to non-entangled states (1) or (1)*, respectively:

(1) $\frac{1}{\sqrt{2}}(|\mathrm{R}\rangle_1\,|\!\uparrow_z\rangle_1\,|\mathrm{L}\rangle_2\,|\!\downarrow_z\rangle_2 - |\mathrm{L}\rangle_1\,|\!\downarrow_z\rangle_1\,|\mathrm{R}\rangle_2\,|\!\uparrow_z\rangle_2)$ (1)* $|\!\downarrow_z\rangle_l \otimes |\!\uparrow_z\rangle_r$

all proponents of heterodoxy agree that there is one particle which is left and has spin down, and another particle is right and has spin up. This result can be taken to indicate that a strong version of PII – PII with monadic properties – is valid: there are two qualitatively distinct particles. Given that the particles are two electrons, one could say that there are two universal bundles {m_e, q_e, s = ½, L, $\downarrow_z$} and {m_e, q_e, s = ½, R, $\uparrow_z$}. Alternatively, one could say that there are two bundles of tropes, namely {m_e^1, q_e^1, s = $½^1$, L, $\downarrow_z$} and {m_e^2, q_e^2, s = $½^2$, R, $\uparrow_z$}. Since both options represent the same reading of the above states (one particle which is left and has spin down, and another particle which is right and has spin up), the choice between universal bundles and trope bundles is arguably an unproblematic case of underdetermination.[16] As can be seen, with respect to non-entangled states, it doesn't matter which factorization (the original orthodox one, or the heterodox one) is considered: they allow for the same ontological interpretations. In the next section, I argue that the situation is different with respect to entangled states.

Before doing so, let me point out (as has been done before in the literature, see Bigaj (2016, 2022); Caulton (2014a); Friebe (2016)) that PII-individuation with respect to non-entangled states is not completely unproblematic. The problem becomes apparent if basis transformations are taken into consideration. Recall that a non-entangled state like (1) can alternatively be represented, e.g., in the following basis:

$|\Gamma\rangle = \frac{1}{\sqrt{2}}(|\mathrm{R}\rangle_1\,|\!\uparrow_z\rangle_1 + |\mathrm{L}\rangle_2\,|\!\downarrow_z\rangle_2)$ $|\Lambda\rangle = \frac{1}{\sqrt{2}}(|\mathrm{R}\rangle_1\,|\!\uparrow_z\rangle_1 - |\mathrm{L}\rangle_2\,|\!\downarrow_z\rangle_2)$

Within this basis, (1) looks as follows:

(1)' $\frac{1}{\sqrt{2}}(|\Gamma\rangle_1\,|\Lambda\rangle_1 - |\Lambda\rangle_2\,|\Gamma\rangle_2)$

Now the following question arises: in the non-entangled state under consideration, are there two particles which are represented by {m_e, q_e, s = ½, L, $\downarrow_z$} and {m_e, q_e, s = ½, R, $\uparrow_z$}, or two particles which are represented by {m_e, q_e, s = ½, Γ} and {m_e, q_e, s = ½, Λ}? Put differently, it is ambiguous in terms of which properties the particles in non-entangled states are individuated. Since this ambiguity occurs if basis transformations are considered, call it *inter-basis ambiguity*, to distinguish it from *intra-basis ambiguity* (which will be discussed in the next section).[17]

Up to now, three solutions for the problem of inter-basis ambiguity have been discussed. According to the first proposal, individuation in terms of location properties should be preferred over individuation in terms of other properties – call this the *location account*. The location account solves the problem

[15] Rodriguez-Pereyra (2014) claims that the substantial reading of trope-PII should be preferred. Note, however, that his claim is first and foremost an exegetical one: as he sees it, Leibniz understands PII as a substantial principle. Therefore, if Leibniz understands properties as tropes (which is apparently controversial), he understands PII in terms of qualitative distinctness (not in terms of sharing properties).

[16] Consequently, whenever property bundles are depicted in what follows, they can either be understood as bundles of universals or as bundles of tropes.

[17] Bigaj (2022) discusses intra-basis ambiguity in sec. 7.3, and inter-basis ambiguity in sec. 7.4. In general, he considers the problem of ambiguity as the worst problem for heterodoxy.

at hand by picking {m_e, q_e, s = ½, L, $\downarrow_z$} and {m_e, q_e, s = ½, R, $\uparrow_z$}, over {m_e, q_e, s = ½, Γ} and {m_e, q_e, s = ½, Λ}. However, Bigaj (2022, sec. 7.5) has his concerns about this account for the following two reasons. First, it is not generalizable to all cases: states which involve *the same* location properties provide counterexamples. Second, within the formalism, location properties are on a par with all other sorts of properties; therefore, preference of location properties requires justification. As Bigaj points out, preference of location properties fits nicely with some interpretations of quantum mechanics (e.g., Bohmian mechanics), but not all, which renders this solution interpretation dependent. According to the second proposal, due to Friebe (2016), a given state should never be considered in isolation, but together with the interaction that has produced it – call this the *past measurement account*. For example, a non-entangled state like (1) might result from a measurement that has been performed on an entangled state. If this measurement is, e.g., spatial-spin like, it picks {m_e, q_e, s = ½, L, $\downarrow_z$} and {m_e, q_e, s = ½, R, $\uparrow_z$}, over {m_e, q_e, s = ½, Γ} and {m_e, q_e, s = ½, Λ}. According to the third proposal, due to Bigaj (2022, sec. 7.5), individuation is not absolute, but relative to a basis. Which basis is relevant is determined by which properties we are interested in; in other words, the relevant basis is picked out by future measurements. Bigaj refers to this proposal as *perspectivalism*; I prefer to call it the *future measurement account* (to contrast it with the past measurement account). If we are, e.g., interested in spin and location properties, the future measurement account (again) picks {m_e, q_e, s = ½, L, $\downarrow_z$} and {m_e, q_e, s = ½, R, $\uparrow_z$}, over {m_e, q_e, s = ½, Γ} and {m_e, q_e, s = ½, Λ}. In contrast to the location account, both the past and the future measurement account seem promising in terms of potential counterexamples: *prima facie*, there are none, because *each* state results from an interaction, and we can *always* be interested in future properties.

4.2 The ontology of entangled states

As already stated above, with respect to entangled states as expressed in the original formalism, e.g., with respect to (2)

$$(2)\ \frac{1}{\sqrt{2}}\ ((|\uparrow\rangle_1\,|\downarrow\rangle_2 - |\downarrow\rangle_1\,|\uparrow\rangle_2) \otimes (|\mathrm{L}\rangle_1\,|\mathrm{R}\rangle_2 + |\mathrm{R}\rangle_1\,|\mathrm{L}\rangle_2))$$

the proponent of heterodoxy can claim that there is a left particle and a right particle, both without definite spin properties.[18] Alternatively, she can claim that there is an up particle and a down particle, both without definite location properties. Both options are instances of the so-called *discerning defense* of PII, which aims at establishing a qualitative difference between entities which seemed to be qualitatively identical (in this case, from an orthodox point of view).[19] Now, the fact that there are two options to establish qualitative distinctness sounds harmless at first glance (both options yield that (2) contains two qualitatively distinct particles). However, it becomes a serious problem when considered more closely.

The proponent of PII-individuation wants to claim that objects are individuated by properties which render them qualitatively distinct. As argued in the previous section, this claim can either be combined with universal bundle theory or with trope bundle theory. Now, the following problem arises: are the particles in (2) individuated in terms of the bundles {m_e, q_e, s = ½, R} and {m_e, q_e, s = ½, L}, or in terms of the bundles {m_e, q_e, s = ½, $\uparrow$} and {m_e, q_e, s = ½, $\downarrow$}? In other words, it is unclear whether the bundles

[18] Dieks and Lubberdink (2022) take the lack of definite spin properties as a reason to be hesitant about the applicability of the particle concept. This is because they think that the particle concept stems from classical physics, and within classical physics, a particle is always characterized by a *complete* set of properties. An entity which is characterized by an incomplete set of properties is therefore not a particle in the classical sense.

[19] For an overview of different strategies to defend PII, see Hawley (2009).

representing the particles are numerically/qualitatively distinct due to containing different location properties, or due to containing different spin properties – individuation turns out to be ambiguous, again. In contrast to inter-basis ambiguity (which has been discussed in the previous section), this sort of ambiguity is already apparent when one and the same basis is considered; therefore, I refer to it as *intra-basis ambiguity*.

Out of the three strategies which can be employed to solve inter-basis ambiguity – the location account, the past measurement account, and the future measurement account – only the location account can also be employed to deal with intra-basis ambiguity (the past and the future measurement account both pick one basis out of many, not properties within one basis). However, as pointed out in the previous section, the location account is limited.[20]

Now, one might think that when entangled states are considered in the heterodox factorization, e.g.:

(2)* $\frac{1}{\sqrt{2}}\left((|\uparrow\rangle_l\,|\downarrow\rangle_r - |\downarrow\rangle_l\,|\uparrow\rangle_r\right)$

intra-basis ambiguity evaporates: (2)* unambiguously favors bundles which are numerically/qualitatively distinct due to different location properties over bundles which are so distinct due to different spin properties. However, this result can only be achieved because the choice of basis vectors is biased towards vectors which contain different location properties (see fn. 13). Therefore, this way to avoid intra-basis ambiguity is only as good as the justification for said choice of basis vectors. Given that vectors which contain the same location properties would interfere with this solution to intra-basis ambiguity, their exclusion in the construction of snapshot Hilbert space seems like an *ad hoc* exclusion of problem cases.[21]

This can be taken to motivate a different solution to the problem of intra-basis ambiguity, which emphasizes another difference between the orthodox and the heterodox factorization. The descriptive factorist, in considering (2)*, is committed to the claim that there are two numerically distinct particles because there are two distinct indices ('l' and 'r'). In other words, the descriptive factorist (just like the direct factorist) is committed to inferring the number of particles from the number of indices. This inference is blocked when (2)* is considered in the orthodox factorization (2) (given that the indices '1' and '2' are *not* taken to directly refer to particles). Therefore, the proponent of heterodoxy, when considering the original formalism, has the option to perform a so-called *summing defense* of PII, which aims at showing that, in a given situation, there is only one entity which is qualitatively identical to itself. Hence, according to the summing defense, there is only one bundle object in (2), namely {$2m_e$, $2q_e$, R-L, $\hat{S}^2 = 0$, $\hat{S}_z = 0$}, which contains twice the mass and twice the charge of a single particle, is extended from left to right, has total spin zero, and zero spin in z-direction (see Friebe (2014)). The summing defense yields a vindication of PII because the summed-up bundle is, of course, qualitatively identical to itself. Friebe explicitly advocates universal bundle theory; however, the summing

[20] Note that the location account cannot be employed to solve intra-basis ambiguity for entangled states if it is spelled out in terms of decoherence: decoherence processes can roughly be understood as environmentally induced location measurements. Since entangled states collapse to non-entangled states upon measurement, decoherence cannot be invoked to argue that entangled states contain particles which are individuated by different location properties (but only to argue that non-entangled states contain such particles).

[21] This *ad hoc* exclusion might not be considered as problematic, for the following reason: the result that PII is valid in some states, but not all, is a perfectly satisfying result, and is all that we can hope for. However, as I see it, the distinction between states for which PII turns out valid and states for which it does not should make sense from a physical point of view. From a physical point of view, non-entangled states which contain different location properties and non-entangled states which do not are perfectly on a par. Therefore, ontological differences *within* non-entangled states should not be expected. To give a contrasting example: ontological differences *between* non-entangled states and entangled states make sense from a physical point of view, because there's a physical difference between them (which is then mirrored on the ontological level).

maneuver can equally well be understood in terms of trope bundle theory. This way, intra-basis ambiguity is avoided simply because the summed-up bundle contains both spin and location properties; no choice between them arises.

Therefore, the situation with respect to entangled states is the following: PII's validity can be established either by performing the discerning defense (two qualitatively distinct particles) or by performing the summing defense (one entity which is qualitatively identical to itself).[22] The former yields *particularism* (two numerically distinct particles), the latter yields *holism* (only one entity). Since 'holism' is a term which has already been used to characterize entanglement, I contextualize my use of the term in the next section.

5. Entanglement and holism

The issue of entanglement and holism is discussed in the so-called mereological debate, whose main question is whether entanglement is a case of mereological composition (i.e., whether entangled states are wholes which consist of parts). Following Näger (2021) the sort of holism introduced in the previous section can be referred to more precisely as *object holism*, which contrasts with *property holism*. The difference between object holism and property holism is the following: according to the former, the *entangled property* (i.e., the property which represents the peculiarity of entanglement, and helps in explaining the violation of Bell inequalities) is a monadic property (typically 'having total spin zero') which is carried by the entangled whole. According to the latter, the entangled property is either a (non-supervenient) relation between numerically distinct particles (typically 'having opposite spin to'), or a plural property which is carried collectively by numerically distinct particles.[23] Furthermore, Näger (2021) introduces a distinction between radical and moderate versions of object and property holism. The radical versions deny that entangled states are cases of mereological composition, i.e., the view that entangled states are wholes consisting of parts; the moderate versions assume the opposite.

Now, one might think that the possibility of moderate object holism (according to which entangled states contain a whole which is composed of parts) shows that particularism and holism as defined at the end of the previous section (two particles versus one whole)[24] are not really exclusive options: the whole might consist of parts; equally, two particles might be parts which compose a whole. However, I think that this conclusion is blocked if the one entity which the holist assumes (or, respectively, the two entities which the particularist assumes) is understood in bundle-theoretic terms – as is required by PII-individuation (see sec. 4.1). The reason is the following: given bundle theory, parts which compose a whole should be proper subsets of said whole. However, neither $\{m_e, q_e, s = ½, L\}$ and $\{m_e, q_e, s = ½, R\}$ nor $\{m_e, q_e, s = ½, \uparrow\}$ and $\{m_e, q_e, s = ½, \downarrow\}$ (the two particularist options for entangled

[22] Different strategies to save PII are, of course, already available in response to classical counterexamples like Black's (1952) spheres (see Hawley (2009)).

[23] Property holism in terms of non-supervenient relations goes back to Teller (1986); property holism in terms of collective properties goes back to Bohn (2012) and Brenner (2018) (and is also advocated by Näger (2021)). According to Näger, object holism (either radical or moderate) is advocated by Schaffer (2010), Calosi and Morganti (2016) and Calosi and Tarozzi (2014). Another important source with respect to entanglement and holism, mentioned here for the sake of completeness, is Howard (1989).

[24] To avoid confusion, in what follows I refer to the view that entangled states contain two particles represented by $\{m_e, q_e, s = ½, L\}$ and $\{m_e, q_e, s = ½, R\}$ (or $\{m_e, q_e, s = ½, \uparrow\}$ and $\{m_e, q_e, s = ½, \downarrow\}$) as *bundle-theoretic particularism*, and to the view that entangled states contain one whole which is represented by $\{2m_e, 2q_e, R\text{-}L, \hat{S}^2 = 0, \hat{S}_z = 0\}$ as *bundle-theoretic holism*.

states) are proper subsets of {$2m_e$, $2q_e$, R-L, $\hat{S}^2 = 0$, $\hat{S}_z = 0$}.[25] Therefore, I think that bundle-theoretic holism is radical in the sense that it excludes that the whole has parts. Similarly, bundle-theoretic particularism is radical in the sense that it excludes that the two particles compose a whole.

Nevertheless, bundle-theoretic particularism is still compatible with radical property holism (i.e., property holism which denies the existence of the whole): for example, two particles which are represented by {m_e, q_e, s = ½, L} and {m_e, q_e, s = ½, R} can stand in the non-supervenient relation of having opposite spin to each other. In fact, I think that bundle-theoretic particularism should be combined with some form of property holism, for the following reason: in and of itself, bundle-theoretic particularism just states that the particles in entangled states *lack* either definite spin properties or definite location properties. Now, as I see it, the most interesting task for an ontological interpretation of entanglement is to help explain why Bell inequalities are violated. Arguably, an informative account of the entangled property (be it in terms of a monadic property like 'having total spin zero', or in terms of a relational property like 'having opposite spin to') is needed to accomplish this task. However, lack of certain properties is not informative in this sense – consequently, bundle-theoretic particularism should be combined with some sort of property holism.[26] In contrast to bundle-theoretic particularism, bundle-theoretic holism automatically comes equipped with an informative view of the entangled property (the entangled property in the case at hand is 'having total spin zero'). To conclude this section, bundle-theoretic particularism yields property holism, whereas bundle-theoretic holism is a (radical) form of object holism.

6. Heterodox underdetermination

All in all, we are now faced with the following choice concerning the ontology of entangled states: 1. PII's validity is established by the discerning defense, which yields bundle-theoretic particularism, which in turn yields property holism. Call the sum of these views the *particularist package*. 2. PII's validity is established by the summing defense, which yields bundle-theoretic holism, which is a form of object holism. Call the sum of these views the *holist package*. We might now wonder whether this choice constitutes a case of underdetermination which is just as problematic as orthodox underdetermination. Recall that orthodox underdetermination is problematic because the choice between individuality and non-individuality corresponds to radically different world views (one in which classical logic and set theory are true versus one in which they are false). Does the choice between the particularist package and the holist package correspond to equally different world views?

When it comes to explaining the violation of Bell inequalities, radically different world views are definitely in play – e.g., a local world in which backwards causation is possible is radically different from a non-local world in which backwards causation is impossible.[27] However, I don't think that the particularist package and the holist package have different compatibility profiles with respect to this

[25] Note that problems of this sort already arise in classical cases. Consider a red sphere, and assume, for the sake of simplicity, that it has only two properties, 'being red' and 'being spherical'. Then, its bundle-theoretic representation is {red, spherical}. Now, consider the sphere's halves, which arguably are parts of it. The bundle-theoretic representation of both parts is {red, semi-spherical}, which is not a proper subset of {red, spherical}. Of course, this conclusion rests on the assumption that part-whole relations need to be understood in terms of subset relations when bundle theory is taken into consideration (which might be controversial).

[26] Dieks and Lubberdink (2022) drop the term 'holism' in connection with entangled states, without specifying any further how this holism manifests itself. As shown above, it might manifest itself in the form of property holism.

[27] See Näger and Stöckler (2018) for an overview of which classical assumptions have been abandoned to explain the violation of Bell inequalities.

issue. For example, both packages can provide a local common cause explanation of the violation of Bell inequalities in terms of a spatially extended entity traveling through spacetime – be it the spatially extended whole, or the spatially extended property of having opposite spin (see Näger and Stöckler (2018)) (the caveat of this solution is, of course, that one conflict with relativity is traded for another: locality is bought at the cost of privileging the reference frames in which said entity is spatially extended). Therefore, heterodox underdetermination is probably not as problematic as orthodox underdetermination.

Nevertheless, heterodox underdetermination deserves some attention. This is because the two packages might have different resources to explain the violation of Bell inequalities – e.g., the holist package might be better equipped to deal with the complexity of the violation. Here is a rough and ready argument for that claim: recall that the degree to which Bell inequalities are violated varies with measurement settings. If the measurement settings on both wings of the setup are identical (they are both set to measure spin in x-direction, say), the particle spins are anti-correlated (one up, one down) in 100% of the cases. However, if the settings are not identical (e.g., one is set to measure spin in x-direction, the other is set to measure spin in y-direction), the anti-correlation gets weaker. Therefore, it is possible that both particles turn out to have spin up (or both turn out to have spin down). How can the particularist account for that? Out of the two candidates for the entangled property – 'having opposite spin' and 'having equal spin' – neither can account for all possible measurement outcomes. The holist – who claims that the entangled property is 'having total spin zero' – has no problem accounting for that, because that property is compatible with all possible measurement outcomes. Other interesting test cases to see which package fares better at the end of the day are bosonic entanglement and single particle entanglement.

7. Conclusion

In the first part of the paper, I followed French in arguing that orthodox underdetermination is problematic because the choice between particles as individuals and particles as non-individuals prevents us from achieving a clear metaphysical picture: if quantum mechanics does not tell us which of these two ontological options is the case, we are at a loss with respect to the question of what quantum mechanics tells us about the fundamental furniture of the world. Given the physical arguments in favor of heterodoxy, one might have thought that underdetermination is no longer a problem – according to heterodoxy, similar fermions are PII-individuated; the metaphysical picture is clear. However, as I argued in the second part, there are two ways to establish PII's validity with respect to entangled states, which yield two different ontological packages: the discerning defense, which yields bundle-theoretic particularism (the particularist package), and the summing defense, which yields bundle-theoretic holism, which is a form of object holism (the holist package). Therefore, a form of underdetermination returns within the heterodox framework. Arguably, the two heterodox packages do not necessarily correspond to radically different world views (like, e.g., a non-local versus a local world). Therefore, heterodox underdetermination is probably not as problematic as orthodox underdetermination. Nevertheless, the two packages might still have different resources to explain the violation of Bell inequalities. Therefore, as I have argued, heterodox underdetermination deserves some attention.

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