Huygens' Principle and Computation of the Light Trajectory Responsible for the Gravitational Displacement of Star Images

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A finite time-slice computational method has been developed to describe the effects of gravity on light rays passing close to the Sun by employing a theoretical approach introduced by L. Schiff in 1960. One of the key assumptions in the latter study is that each *local* observer finds a given light ray to travel along the same straight line with the same constant speed *c*. It is concluded that the observation of star image displacements during solar eclipses is primarily a verification of the fact that the speed of light for a stationary non-local observer located in a gravity-free region of space varies with the light ray's lateral distance from the Sun. This circumstance alone is enough to cause wave fronts to be rotated by exactly the amount predicted using Huygens' principle in both Einstein's general relativity and Schiff's simpler theoretical treatment *without assuming any curvature in the light path itself*. A light refraction experiment is outlined to test this conclusion, and it is pointed out that the resulting procedure would provide a direct method for the measurement of group refractive indices of dispersive media.

Keywords: Schiff's method, gravitational coordinate scaling, wave front rotation, light refraction, black holes

I. Introduction

One of the most publicized events in scientific history was the discovery that the images of stars are displaced during solar eclipses [1]. Einstein had predicted this effect several years earlier [2] as a consequence of his general theory of relativity (GTR). Since that time there have been numerous discussions in the literature about how the trajectories of light waves are bent when they pass near the Sun or other massive objects. Soldner reported explicit calculations of the angle of curvature as early as 1803 [3] based on Newton's universal theory of gravitation, but the value obtained by him was too low by a factor of two relative to Einstein's result and the best inference from subsequent experimental observations. The purpose of the present work is to demonstrate via explicit trajectory calculations that the above experiments can be successfully interpreted by merely assuming that the speed of light for a stationary observer varies with lateral distance from the Sun. It will be shown that the observed angular displacement of star images is predicted quantitatively on this basis, and it is therefore concluded that the light rays themselves are actually *not* deflected as they pass near massive bodies, but rather are merely slowed down.

II. Assumptions in Schiff's Computational Procedure

In 1960 Schiff [4] reported a simple method for calculating the angle of deflection of light rays passing close to the Sun. A key assumption was that local observers at the same gravitational potential always measure the speed of light to have a constant value of c. Moreover, the light moves in the same perfectly straight-line trajectory for a succession of such observers, that is, the *local* light velocity is always constant in both direction and magnitude. The calculations then proceed on the basis of arguments given much earlier by Einstein [5, 6] that the unit of time varies in a well-defined manner [see eq. (5) of Schiff's paper] with the position of the observer in a gravitational field. Schiff also made an additional assumption that the unit of distance in the direction *radial* to the Sun varies in inverse proportion to the unit of time, whereas that in transverse directions is independent of gravitational potential [his eqs. (5) and (6), respectively].

Although the latter approach gives results for the angle of light deflection by the Sun that are in quantitative agreement with Einstein's predictions based on GTR [2], comparatively little attention was given to Schiff's procedure. Perhaps the main reason for this attitude of contemporary physicists was that a similarly uncomplicated method could not be presented for calculating the other key quantity obtained from Einstein's theory, the relativistic contribution to the advancement angle of the perihelion of Mercury's orbit around the Sun. This situation has changed recently [7] with the discovery that Schiff's ideas can be applied successfully for this phenomenon by introducing several assumptions regarding the way in which Newton's inverse-square law needs to be employed within the framework of Einstein's special theory of relativity (STR) [8]. On this basis, results are obtained which are in agreement with experimental data for this type of relativistic effect (43".0033/cy calc. vs. 43".2±0.9/cy obs. [9] and Einstein's value of 43".0076/cy [2,10], for example). *The same dependence on mean radius and eccentricity of orbit as well as the gravitational mass of the source is found as is the case for GTR.*

The success of the latter approach [7] gives quantitative support to the key assumption on which it is based, namely that all observers, independent of their location in a gravitational field, *who are not in relative motion to one another* must agree on the magnitude of the distance between any two objects in the universe. On the other hand, as already mentioned, it is assumed that the unit of velocity increases as the object moves to a higher potential, with the component parallel to the gravitational field changing faster than those in the perpendicular direction. It will be seen in what follows, however that this distinction between the scaling of radial and transverse velocities in Schiff's procedure is only *a mathematical artifice* whose sole purpose is to accurately compute the ratio of elapsed times observed locally and at infinity as the object changes its position in a gravitational field. Finally, it is also assumed that the units of time and inertial mass are inversely proportional to the unit of perpendicular velocity, whereas that for gravitational mass is the same for all observers (more details of this gravitational scaling procedure are given elsewhere [11]).

One of the main consequences of the above approach is that all observers who are not in relative motion to one another must agree on the path followed by any given object, which in turn must be the same as that measured by a series of local observers analogous to those employed in Schiff's procedure [4] for computing the amount of gravitational deflection of light by the Sun. *They will only disagree on the time required to travel a given distance along it.* This result leaves open only one possibility for the actual path followed by the light as it travels from the star to the observer on Earth: *it must have travelled in a perfectly straight line*. Why this conclusion is perfectly consistent with the experimental observations made during solar eclipses by Eddington [1] and subsequently by others [12], as well as with the theoretical calculations of both Einstein [2] and Schiff [4], is the subject of the following section.

III. Light Paths and Huygens' Principle

It is an interesting fact of history that a different criterion for computing the angle of deflection of light by gravitational forces was employed by Einstein [2] than was the case for Soldner [3] over a century earlier. In the latter study Newton's classical theory was applied for an object of small mass that passes close to the Sun, and the amount of deflection was obtained as the angle by which the direction of the object's motion changes as it travels between the star and the Earth. Both Einstein [2] and Schiff [4] employed a different measure for the amount of curvature, however, one that is based on Huygens' principle. There is a subtle distinction in these two methods. In Soldner's Newtonian approach the direction of the light velocity is used explicitly in the calculations, whereas Einstein and Schiff only employ the speed of the light, not its direction, in arriving at their result.

To see this it is only necessary to examine the formula for the change in angle Θ that they used based on Huygens' principle:

$$d\Theta = \frac{1}{c'}\left(\frac{dc'}{dy}\right)dx\,. \tag{1}$$

In this expression *dy* is the change in lateral distance of the light from the Sun in a given time interval, *dx* is the corresponding total distance travelled, and c' is the speed of the light measured by a stationary observer on Earth over the same infinitesimal period. In Soldner's approach, the path of the light must be curved for there to be a nonzero result for the angle of deflection, whereas in that employed by Einstein and Schiff, all that is required is that the speed of light change with lateral distance from the Sun for the above observer.

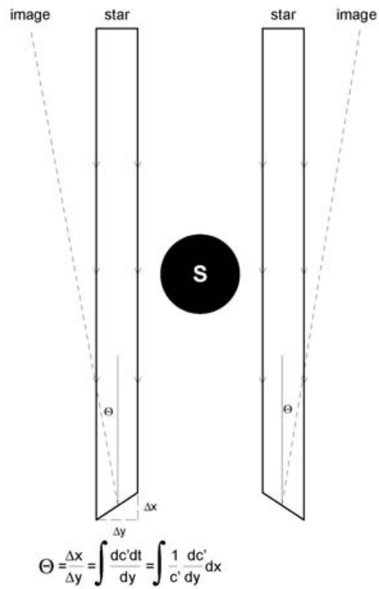

Fig. 1. Schematic diagram showing light rays emitted by stars to follow straight-line trajectories as they pass near the Sun. Because of gravitational effects the speed of the light rays c' is known to increase with gravitational potential, with the effect that the corresponding Huygens wave front gradually rotates away from the Sun. As discussed in the text, the normal to a given wave front points out the direction from which the light appears to have come, causing the star images to be displaced by an angle Θ during solar eclipses.

But can one obtain a nonzero value for $d\Theta$ in eq. (1) if the path is not curved? The answer is clearly yes, as the diagram in Fig. 1 shows. To compute the derivative $\frac{dc'}{dy}$ according to Huygens' principle, it is necessary to *compare the speeds of two different light rays* separated laterally by an amount *dy*. If we assume that the corresponding values of c' differ by dc', it is clear that the respective distance travelled in the two cases over time *dt* will also differ, namely by an amount $dc'dt$. As shown in Fig. 1, the angle which the line connecting the final positions of the two light rays makes with the corresponding one for their initial positions is thus $d\Theta = \frac{dc'dt}{dy}$. This result in turn can be used to obtain eq. (1) by simply noting that the total distance travelled is $dx = c'dt$, where c' is just the average of the two infinitesimally differing light speeds used to obtain the derivative in eq. (1). *There is nothing in this derivation that assumes that either light path is curved, only that the speeds by which the light travels along them are different.*

There is a simple interpretation of this result. The line connecting the current positions of the two light rays simulates a *wave front* in the terminology of Huygens. When the light reaches the observer, the direction from which it has come is judged by extending the normal to this wave front backward in space. Integration of $d\Theta$ in eq. (1) over the entire path therefore gives the amount by which the light *appears* to have been deflected from the straight-line path actually followed (Fig. 1). Schiff's work has shown that this angle has a value of 1".7517 for light coming from infinity which grazes the outer edge of the Sun's surface on its way to the Earth, identically the same value as obtained by Einstein [2] in 1916. Numerical calculations have also been carried out for this purpose with the (finite time-slice) procedure mentioned above for obtaining the perihelion advancement angle for planetary motion [7], and quantitative agreement is obtained with the latter value on this basis as well. A brief description of these calculations is given below.

Two light rays are assumed to start out at the same distance (10^{12} m in the present numerical example) from the Earth, as shown in Fig. 1. The lateral distance between them is Δy, which can be varied in the computer program. Both light rays travel in straight lines along the *x* axis and eventually pass by the Sun on their way to an observer of the Earth's surface. In the specific example under discussion, one of them just grazes the Sun's surface, while the other is slightly farther away from it. The origin of the coordinate system employed is placed at the solar midpoint. Since the radius of the Sun is taken to be $r_s = 0.696\text{x}10^9$ m, this means that the path of the inner light ray is the straight line, $y = r_s$, while that of the other is $y = r_s + \Delta y$, also a straight line. There is a separate local observer for each of the light rays, following Schiff's procedure [4]. Their trajectories are computed in a series of equal time intervals Δt (0.01 s in the present example) as measured on the stationary clock on Earth in each case.

At each step in the calculation the *local* velocity of light is assumed to be *c* along the *x* direction, and is resolved into its radial and tangential components relative to the solar midpoint. These are then scaled by different amounts in order to obtain the corresponding values in the system of units employed by the observer on Earth (actually assumed to be located at an infinite distance from the Sun, as in Schiff's original procedure [4]). The corresponding speed of the light (c') from the latter observer's perspective is then computed, which in the present application of the theory is always less than *c*. It is assumed that the corresponding light ray has travelled a distance $c'dt$ in the -*x* direction. The new position of the light ray is then computed on this basis and this is used as the starting point for the next time cycle (note that this position is always the same for both the local observer and his counterpart on Earth).

Since the speed of light c' is slightly different for the two light rays, it follows that they do not arrive on the Earth's surface at the same time. In the present example, with $\Delta y = 1000$ m, the outer ray travels $\Delta x = 0.008492807$ m farther than the one that just grazes the Sun's surface (all computations are done in quadruple precision). The line connecting the two end points of the rays has thus rotated by an angle $\Theta = \dfrac{\Delta x}{\Delta y}$ (see Fig. 1), which is interpreted to be the amount of angular displacement of the star's image. The value obtained for Θ with the above values for Δx and Δy is found to be 1".7517, in perfect agreement with both Einstein's [2] and Schiff's [4] result for this quantity, which in turn is well within the error bars of the measurements of this angle obtained during solar eclipses [1,12].

But if one assumes that observers at different distances from the Sun will not agree on the path followed by the light, is it not possible that the straight-line trajectory measured locally will appear bent to someone located near the Earth? In attempting to answer this question, the first point that should be noted is that the gravitational scaling of time can have no effect of the direction of the light's velocity. Simply employing a faster clock only causes all velocity components to be decreased in the same proportion, so no change in direction can be expected on this basis. The fact that the radial component of the speed of light is altered more than the transverse component near the Sun's surface as a consequence of the gravitational scaling of units does have an effect, but closer analysis shows that *this implies bending in a much different direction than what has previously been inferred from observations of star image displacements*. Since the radial component of the velocity is decreased more for the observer on Earth than the corresponding transverse component, he must therefore find a trajectory for the initial part of the light's journey from infinity that is bent away from the Sun (see Fig. 2) relative to the straight-line path observed locally (Fig. 1). After the light passes by the solar midpoint the radial component of its local velocity begins to point away from the Sun, however, so that the gravitational scaling implies that the corresponding velocity observed on Earth is now directed more toward the Sun than is its local straight-line counterpart. The result is a "Δ-shaped" trajectory (Fig. 2) quite unlike the purely concave path around the Sun that is normally assumed.

In computing the angle of "curvature" via Huygens' principle, however, the apparent direction of the light velocity from the

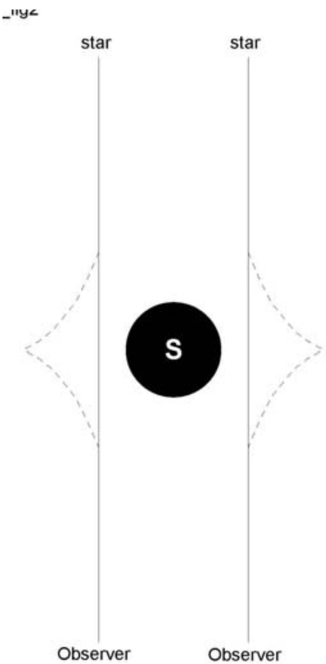

Fig. 2 Diagram illustrating the "pseudo-trajectory" inferred from the velocity vector computed for the observer on earth in Schiff's procedure (see ref. [4]). Note that on the initial approach the light appears to veer away from the Sun (convex trajectory) because the gravitational scaling reduces the magnitude of the radial component relative to the local straight-line path. This result demonstrates that the direction of the latter velocity is *ignored* in Schiff's method, which nonetheless obtains perfect agreement with Einstein's value for the angle of displacement of star images during solar eclipses because of its reliance on Huygens' principle to define this angle (see Fig. 1). Einstein employed the same definition for the displacement angle in his original work (GRT).

observer's vantage point on Earth is completely immaterial. Instead, only the change in the speed of light with lateral distance from the Sun is taken into account [2,4]. Since the light speed increases with distance from the gravitational source, as indicated in Fig. 1, the result is that the wave front of the light is rotated away from the Sun, causing the observer to have the *illusion* that the star's image has been displaced in this direction. In effect, the fact that the *direction* of the light's velocity appears to differ from one observer to another on the basis of the gravitational scaling is *totally ignored in the calculation*; only the magnitude of the velocity (speed) observed on Earth is actually considered in applying Huygens' principle. *The corresponding direction is always the same as for the local observer.* Thus the Δ-shaped trajectory obtained in Schiff's treatment [4] can simply be viewed as an artefact of the overall theoretical approach employed to compute the amount of star image displacement.

The conclusion from the above analysis is that one should carefully distinguish between a change in the direction of the trajectory that light actually follows on the one hand, and the angle of deflection indicated by photographic images of the light source on the other. Just the fact that light speed varies with lateral distance from a gravitational source, which is a clear assumption in Schiff's procedure [4], is sufficient to lead to the quantitative prediction of the observed displacement of star images during solar eclipses. The path that the light follows cannot be determined from application of Huygens' principle alone, but the assumption employed by Schiff to obtain exactly the same amount of deflection as Einstein did by using GTR, namely that the velocity of light is always the same in both magnitude and direction for a series of local observers, speaks strongly for the conclusion that its trajectory is actually a straight line.

IV. Experimental Test of the Above Interpretation

It is clear from the outset that it is a practical impossibility to measure light paths with sufficient accuracy to obtain a definitive answer to the question of whether gravitational forces really deflect light or merely create the illusion that this has occurred. One must therefore rely on indirect evidence to decide this matter as objectively as possible. One such experiment is outlined below.

The main conclusion of the preceding section is that the shifting of star images during solar eclipses is caused solely by the fact that light travels with different speeds depending on its proximity to the Sun. It is possible to make a close analogy with ordinary light refraction on this basis [13]. In conventional experiments the angles of incidence and refraction are measured by *direct observation of the path* of the light as it passes through an interface between two media. The ratio of the sines of these two angles defines the refractive index n in accordance with Snell's law. It is well known, however, that the speed of light v is not determined by n alone but rather by n_g, the group refractive index, specifically $v = \frac{c}{n_g}$ and not $\frac{c}{n}$.

The diagram given in Fig. 3 illustrates the situation in detail for light entering from free space with an angle of incidence Φ relative to the normal to the interface of a dispersive medium of refractive index n. The corresponding angle of refraction is Φ' such that

$$\sin\Phi' = \frac{\sin\Phi}{n}. \tag{2}$$

The light ray on the left of the diagram enters the dispersive medium first, at which point its speed changes from c to $\frac{c}{n_g}$, with

$$n_g = n + \omega\frac{dn}{d\omega}, \tag{3}$$

where ω is the light frequency. There is a delay before the light ray on the right enters the medium, during which time it continues to move with speed c. After both light rays have entered the dispersive medium, the line connecting them makes an angle Φ'' with the interface, but this is generally not the same as Φ' (Fig. 3). Using trigonometric identities one finds that

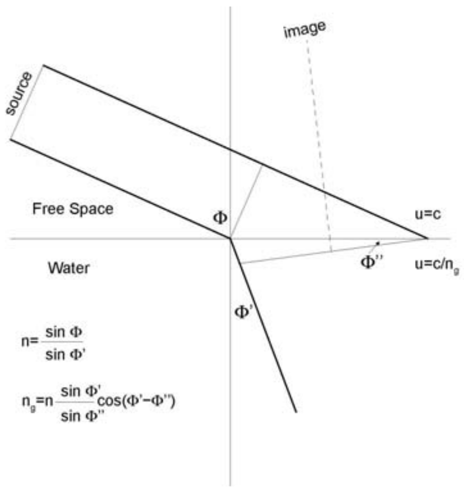

Fig. 3. Angles of incidence Φ and refraction Φ' as light enters water from free space. The corresponding angle Φ'' which the wave front makes with the interface upon entering a dispersive medium can be determined by noting the direction from which the image of the light source appears to arrive inside the medium. Note that Φ'' is generally different than Φ'. As discussed in Sect. IV, measurement of all three angles enables the determination of both the normal refractive index *n* and the corresponding group refractive index n_g of the dispersive medium.

$$\mathrm{n_g} = \mathrm{n}\left(\frac{\sin\Phi'}{\sin\Phi''}\right)\cos\left(\Phi' - \Phi''\right) \tag{4}$$

$$\cot\Phi'' = \frac{n_g}{n\sin\Phi'\cos\Phi'} - \tan\Phi' \,. \tag{5}$$

From these relations it is clear that the condition for Φ' to be equal to Φ'' is that $n_g = n$, which according to eq. (3) can only occur if the refractive index does not change with frequency.

According to the arguments of Sect. III, the image of the light source *when viewed from within the dispersive medium* will appear to lie along the normal to the wave front, i.e., at an angle of Φ'' relative to the interface. Measurement of this angle will therefore give a result that is generally different than the angle of refraction (Φ') obtained by observing the light path from outside the medium. The above equations indicate that one can determine the group refractive index on the basis of separate measurements of both these angles. The result should be the same as obtained for n_g on the basis of measurements of n at a series of light frequencies, which in turn should also agree with the value obtained by explicitly measuring the speed of light of the same ω in the dispersive medium [14-16].

In the present context, however, the most significant result of such an experiment would be to show that *the angle of deflection of the light image is generally not the same as the angle by which the light trajectory itself deviates from its initial direction.* It would therefore lend strong support to the conclusion based on Schiff's method [4] for computing the angle of gravitational deflection by the Sun that the path followed by the light on its way from the star to the observer on Earth is actually a perfectly straight line.

V. Black Holes

Even before Soldner [3] published his calculations on the gravitational bending of light, there was speculation [17] by Michell that an object might be so massive that it would become impossible for light to escape from its surface. As discussed above, the argumentation in GTR is fundamentally different than in the

Newtonian approach to gravity, but the belief still persists that such "black holes" exist and that they do not allow light to pass from them. Hawking [18] has argued that high-energy radiation can still escape from the surface of a black hole, however.

In the previous sections it has been shown that all known experiments regarding the phenomenon of gravitational light deflection can be explained quantitatively by assuming that light always travels in a perfectly straight line. It is therefore of interest to see how the theory of black holes is affected by making this assumption. First of all, it should be noted that this position is still consistent with Newton's inverse-square law provided that one takes account of the fact that the acceleration due to gravity from the *g* field on an object varies with the state of motion of the observer. Ascoli [19] has argued that when an object is moving with speed u relative to the local observer, its acceleration due to gravity is damped by a factor of $1-\frac{u^2}{c^2}$. This relation has been used successfully in the calculations mentioned above for the advancement angle of Mercury's perihelion [7]. In the case of light, for which the local value of *u* is always *c*, *this damping factor is exactly zero, so that no acceleration is to be expected*. Thus this result is consistent with both Schiff's approach [4] and the underlying theory of the present calculations.

According to this view, light can pass as closely as possible to the surface of a black hole without being deflected. The apparent shift in the position of the image of the light source will be very much larger than it is for the Sun, however. Moreover, there is no gravitational effect keeping light from escaping the interior of a black hole, so γ rays are expected to be observed, and not only those originating outside the boundary of the black hole. It should not be forgotten thereby that there is a quite high probability for photons to be absorbed because of the high density of matter, however, so on this basis the description as a blackbody is certainly applicable. It is also clear that the speed of light will be quite small in the interior of a black hole because of the gravitational time dilation, and a very large red-shift for light escaping from it is also expected for an observer located at a relatively high gravitational potential. The key point remains, however, that none of these effects need involve true gravitational deflection, as they are all consistent with a perfectly straight-line trajectory. The phenomenon of gravitational lensing is also expected on this basis, provided the light source is located directly behind the black hole. The image of the light source would be significantly distorted relative to that which would be detected in the absence of the black hole (see Fig. 1).

VI. Conclusion

The present analysis and explicit calculations indicate that one must draw a clear distinction between the angle of deflection of a star's image and the corresponding angle by which the emitted light deviates from its initial direction. Huygens' principle, which is used to define the angle of deflection in GTR, gives no information about the actual path the light follows. In order for "curvature" to be observed on this basis, it is only necessary for adjacent light rays to travel at different speeds, something that is known to occur near massive objects. This effect causes the wave front of the light emanating from the star to be rotated away from the Sun. Simply demonstrating that the images of stars are displaced during a solar eclipse therefore tells us nothing about the actual path taken by the light on the way to the observer.

The work of Schiff [4] has demonstrated that one can obtain the same value for the deflection angle as in GTR by assuming that the trajectory of the light is actually a straight line. The numerical calculations carried out in the present study show clearly that the wave front of light emitted by a star is rotated away from the Sun by exactly the latter angle, causing the star's image to appear to be displaced by this amount (Fig. 1). Since it is impossible to actually observe the path taken by the light from such a distant source, there is no direct way of either verifying or denying the hypothesis that it always travels in a straight line. Nonetheless, an experiment has been outlined for testing the conclusion that the angle of displacement of the image of a light source is not the same as the angle of refraction in dispersive media. Such a procedure should make it possible to measure the group refractive index n_g directly, instead of relying on a series of measurements of the corresponding refractive index n for different frequencies of light, as is normally done for this purpose.

The underlying basis for the present calculations of gravitational interactions is the fact that the units of time and velocity differ from one gravitational potential to another. Nonetheless, it is assumed that the distance between any two objects is the same for all observers in the Universe as long as they are not in relative motion to one another, independent of both the position of the objects and that of the observer in a gravitational field. The same relationship is assumed for gravitational masses. Following Schiff [4], however, the unit of velocity is assumed to vary with gravitational potential. It is assumed to be different for the component radial to the field than for that perpendicular to it. Analysis shows, however, that the latter distinction is only an artifice of Schiff's method whose sole purpose is to compute the ratio of elapsed times observed locally and at infinity (the Earth), respectively. It leads to a velocity of light that has the correct magnitude but the wrong direction (see Fig. 2). The true direction is the same as for the local observer, which in turn is obtained by the reverse scaling procedure in Schiff's procedure. As a consequence, all observers *not in relative motion to one another* agree completely on the path taken by the light. Once the speed of light for an observer at infinity is known, it is easy to compute its value for any other observer not moving with respect to the first simply by taking account of the difference in their respective clock rates.

In the present application, the velocity of light is assumed to be constant in both magnitude and direction, also following Schiff. A connection has been noted between this assumption and earlier work of Ascoli [19] in which he assumed that the acceleration due to gravity for an object moving with speed u relative to the observer is damped by a factor of $\gamma^{-2} = 1 - \frac{u^2}{c^2}$ relative to its local value, whereby $\gamma = \infty$ for light. When applied to optics, the latter result leads to the conclusion that photons are not accelerated by gravitational forces and therefore must always travel in a perfectly straight line according to Newton's First Law. This result is also consistent with Newton's Third Law because it indicates that the trajectories of photons are not affected by gravitational forces on the one hand, and on the other, that their null gravitational mass precludes their exerting a gravitational force on any other object.

Finally, the computational method employed to compute the angular displacement of star images during solar eclipses has also been extended to the other key application in Einstein's original work, calculation of the relativistic contribution to the advancement angle of the perihelion of planetary orbits. This objective was discussed in Schiff's original work [4], but apparently was never achieved by him. Quantitative agreement with Einstein's closed expression for the perihelion advancement angle is obtained with the present computational approach, however, thereby giving considerable support to its underlying assumptions.

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