Logical Non-Apriorism and the 'Law' of Non-Contradiction

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Abstract

A common response to those who question the law of non-contradiction is that it is impossible to debate such a fundamental law of logic. The reasons for this response vary, but what seems to underlie them all is the thought that there is a minimal set of logical resources without which rational debate is impossible. According to this line of response, at least, the law of non-contradiction is so fundamental that without it the logical resources available are too impoverished for debate to proceed. In this paper we argue that this response is misguided. We begin by defending non-apriorism in logic—the view that logic is in the same epistemic boat as other scientific theories. We then give an account of logical theory change in terms of this epistemology. Finally, we discuss the law of non-contradiction in terms of this account of logical theory change and we show that rational debate over this law can, and does, proceed. We also discuss some of the arguments for and against the law of non-contradiction and we illustrate how and where non-apriori considerations arise in these arguments.

1 Introduction

One of the reasons that philosophy of logic is such a difficult enterprise is that, in order to advance debates in this area, we require the very thing

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we are studying: logic. This difficulty is especially acute when engaging in the business of theory choice in logic. After all, in order to decide between two logical theories, we need to put forward evidence and arguments for each. This evidence and these arguments will need to be assessed, and the assessment will need to be conducted in the context of some logic or other. But how do we choose this latter logic? We appear to be headed for an infinite regress.

Things get even worse when we start thinking about how we might debate fundamental logical principles such as the law of non-contradiction. Here, many believe, the task is hopeless. We simply have no ground to stand on once we tentatively reject something so fundamental as the law of non-contradiction. In this paper we suggest that this pessimism is unwarranted; we argue that we *can* reasonably debate fundamental principles like the law of non-contradiction, and we show one way in which this debate can proceed. We do not propose to settle the debate over the law of non-contradiction or even attempt to settle it. Showing that it is possible to have a debate is enough.

We begin, in section 2, by arguing that logic is not a priori, so, in particular, the debate over the law of non-contradiction is not to be conducted entirely in a priori terms.[1] Then, in section 3, we provide a model of how theory change in logic can proceed. In section 4, we argue that even fundamental 'laws' like the law of non-contradiction can be debated in the framework we outline in this paper.

There is another issue that, although closely related to the topic of this paper, for the most part we shall be setting aside. This is the issue of logical monism versus logical pluralism. (The former is the view that there is one true logic, the latter that there is no unique such logic.) This debate has attracted a great deal of attention recently.[2] This monism–pluralism debate is of considerable interest and bears on the present topic in the following way. It has been argued that the variety of alternatives to classical logic—all of which seem adequate for at least some domains—presents problems

[1]The position we defend here, thus, has resonances with certain empiricist philosophies of mathematics. These empiricist philosophies of mathematics include Kitcher (1983) Lakatos (1976), Field (1980, 1993), Quine (1981) and Resnik (1997). See also Colyvan (2001), chapter 6 for a defence of the view that mathematics is empirical.

[2]See, for example, Resnik (1996), Beall and Restall (2000; 2001), Bueno (2002; forthcoming b) and Priest (2001).

for logical monism,[3] so issues about theory choice in logic are not so much about which is the one true logic, but, rather, they are about which logic is adequate to a given domain. We will not take sides on this issue (at least not in this paper). Instead, we try to present our case in terms that will be acceptable to monists and pluralists alike. Both need to embark on theory choice in logic (admittedly the interpretation of what their choices mean is different in each case), and that is all we are concerned with here. We will show that theory choice should proceed in non-apriori terms and that, given such a view of theory choice in logic, there is no reason to believe that we cannot have rational debate over apparently fundamental principles such as the law of non-contradiction.

2 Non-Apriorism in Logic

Despite the influence of Quine (1953) and Putnam (1979), the view that theory choice in logic proceeds via non-apriori terms is far from universally accepted. In this section we provide a defence of non-apriorism in logic.[4] To motivate logical non-apriorism, we will critically assess some arguments put forward *against* this view, and we will indicate why they fail. Some such arguments have been presented by Hartry Field, in his recent defence of logical apriorism (see Field 1996, 1998, and 2001). Other arguments have been around longer, such as one put forward by Alfred Tarski in his celebrated paper on the concept of logical consequence (Tarski 1936). To part company with Tarski is often not a terribly good idea, but at least we won't be in bad company if we note that some of the ideas about logical non-apriorism articulated here have also been around for quite a while, in the works of John von Neumann (1937a and b), Hilary Putnam (1979, 1983a, b, and c), and Newton da Costa (1997).

Before addressing these issues, however, let us point out a crucial feature of the logical non-apriorism articulated here. The idea is that it is possible to *revise* logical principles (or logical rules) on the basis of extra-logical considerations—which include empirical considerations. In other words, extra-logical considerations play a role in the selection and evaluation of logical

[3]For example, one of the present authors (Bueno 2002) has argued along these lines for the conclusion that paraconsistentists should be logical pluralists.

[4]Some of the points made in this section have been developed in more detail in Bueno (forthcoming a); see also Bueno (2002).

principles (or rules).[5] Some people may add a further requirement for logical non-apriorism, namely that the *justification* of logical principles should be obtained on empirical grounds as well.[6] This requirement amounts to a really strong form of logical non-apriorism—a form of apriorism far stronger than the one we will be working with here. So for the purpose of the present work, only the revisability condition above will be required.[7]

But is it plausible to suppose that logic is non-apriori in the above sense? To argue that it is, we will examine three arguments for logical non-apriorism: an argument from quantum mechanics, a critical assessment of an argument that Tarski devised for logical apriorism (implicit in Tarski's substitution requirement), and an argument that Putnam provided on the centrality of the law of non-contradiction. We will examine each of these arguments in turn.

2.1 The argument from quantum mechanics

One of the most interesting arguments for non-apriorism in logic—let's call it the argument from quantum mechanics—was explored by von Neumann (see Birkhoff and von Neumann 1936, and von Neumann 1937a and b). The main idea is that, in von Neumann's view, classical logic simply provides the wrong results when applied to the quantum domain. The overall structure of the latter domain is not adequately represented by classical logic; but it is by quantum logic.

To illustrate this, let us consider a simple example. According to standard quantum mechanics, any electron E has an angular momentum (or spin) in

[5]This is the view that we reject or revise logical principles based on empirical evidence (but we do not necessarily invoke empirical considerations for the acceptance of logical principles).

[6]This is the view that we accept logical principles based on empirical grounds.

[7]We note that our usage of 'a priori' and 'non-apriori' are somewhat nonstandard; by 'logical non-apriorism' we simply mean that extra-logical considerations come to play in theory choice in logic. As it turns out (see section 2.1), we also think that logic is non-apriori in a stronger sense (in that empirical considerations come to play). Our main purpose in this paper, however, is to defend an account of theory change in logic that allows, and makes sense of, debates about the law of non-contradiction. It's important for our case that the role of extra-logical considerations in these debates is appreciated. Some of these considerations are empirical, while others are merely extra-logical. We find it convenient to use the term 'non-apriori' to include both, but nothing hangs on this admittedly nonstandard usage.

a given direction X. Moreover, every electron has only one of two possible spin values: $+1/2$ or $-1/2$. So if we denote the spin of E in the X direction by E_X, the following disjunction is true:

$$E_X = +1/2 \vee E_X = -1/2$$

Furthermore, given Heinsenberg's indeterminacy principle, it is not possible to measure the angular momentum of E in two (distinct) directions at the same time.

Let X and Y be two distinct directions. And let us suppose that we have measured the momentum of E in the direction X, and obtained the result that $E_X = +1/2$. In other words, '$E_X = +1/2$' is true. Now, given that '$E_Y = +1/2 \vee E_Y = -1/2$' is always true (in any instant), it follows that the conjunction

$$E_X = +1/2 \wedge (E_Y = +1/2 \vee E_Y = -1/2) \tag{1}$$

is similarly true. If we assume the distributivity of conjunction over disjunction found in classical logic, it follows from (1) that

$$(E_X = +1/2 \wedge E_Y = +1/2) \quad \vee \quad (E_X = +1/2 \wedge E_Y = -1/2) \tag{2}$$

But something unexpected happens at this point. As noted above, (1) is true, but it turns out (arguably) that (2) is false (or even meaningless)! After all, given Heisenberg's indeterminacy principle, it is impossible to measure the moment of E in distinct directions X and Y at the same time. So, if we assume (as we did) that the underlying logic is classical, we are led straightaway into a conceptual difficulty (see Birkhoff and von Neumann 1936, Putnam 1979, and da Costa 1997).

There are, of course, different ways of trying to keep classical logic given the problem presented by (2). It turns out, though, that none of the current options has received unanimous acceptance. For example, one could simply change standard quantum mechanics in such a way that the problem is not generated. This amounts to the introduction of a *new* theory, for which we will need, as is the case of any new theory, independent evidence, the assurance that the theory is not *ad hoc*, and so on. But even if the theory satisfied all these requirements, the introduction of a new theory doesn't solve the problem. For it is still a problem for *standard quantum mechanics* that it seems to be in conflict with classical logic.

Alternatively, it might be claimed that it is the measurement that 'creates' the value of the spin, and so the proposition

$$E_Y = +1/2 \vee E_Y = -1/2 \tag{3}$$

is neither true nor false. But this provides no solution to the problem either—at least not for the classical logician. After all, the claim that proposition (3) is neither true nor false is in conflict with classical logic as well.

Or perhaps one could adopt a Bohmian interpretation of quantum mechanics and reject Heisenberg's indeterminacy principle (for a thoughtful discussion, see Cushing 1994). Once again, this amounts to changing the theory, given that, for example, the Bohmian interpretation and standard quantum mechanics have incompatible accounts of the nature of quantum particles. And in any case, this doesn't solve the problem either, since the original difficulty remains—there is still an apparent incompatibility between classical logic and standard quantum mechanics.

The point of these remarks is not to suggest that just by moving to quantum logic one can overcome the problems of interpretation faced by quantum mechanics—this would be unrealistically simplistic.[8] The only point we are making here is that logic may be revised on the basis of empirical considerations, which is enough to suggest that logic is not a priori.

2.2 Tarski's argument for logical a priorism and its limits

We certainly acknowledge that logical non-apriorism is not the mainstream interpretation of logic. In fact, as part of the development of what is now the orthodox account of logical consequence, Tarski also provided an argument for logical a priorism. In this section, we will discuss and assess this argument.

As a necessary (but not sufficient) condition for logical consequence, Tarski put forward what can be called the *substitution requirement*. Suppose that K is a class of sentences and that X is a given sentence. According to the substitution requirement:

> If, in the sentences of the class K and in the sentence X, the constants—apart from purely logical constants—are replaced by

[8] See Bell and Hallett (1982) for a critical discussion of some exaggerated claims made by some quantum logicians.

> any other constants (like signs being everywhere replaced by like signs), and if we denote the class of sentences thus obtained from K by K', and the sentence obtained from X by X', then the sentence X' must be true provided only that all sentences of the class K' are true. (Tarski 1936, p. 415; emphasis omitted.)

As formulated here, the substitution requirement provides one way of expressing the condition that logic should be formal. After all, it doesn't matter what extra-logical constants we replace in the original set of sentences K and in X, if X follows from K, then X' follows from K'. The requirement that logical consequence be *formal* is expressed by guaranteeing that extra-logical (in particular, empirical) considerations are in a clear sense *irrelevant* for logical consequence. If X already follows from K, a complete reinterpretation of X and the sentences in K won't change this feature. What matters for the relation of logical consequence is that the *form* of the argument is preserved. As Tarski points out:

> Since we are concerned here with the concept of logical, i.e. *formal*, consequence, and thus with a relation which is to be uniquely determined by the form of the sentences between which it holds, *this relation cannot be influenced in any way by empirical knowledge*, and in particular by *knowledge of the objects to which the sentence X or the sentences of the class K refer*. The consequence relation cannot be affected by replacing the designations of the objects referred to in these sentences by the designations of any other objects. (Tarski 1936, pp. 414–415; emphasis added, except for the italics in 'formal'.)

We don't dispute for a second that the substitution requirement is important, and that it has been part of logic since the first systematic formulation of the discipline by Aristotle (although, of course, the requirement wouldn't then be expressed in the way Tarski formulated it). After all, it is abundantly clear in Artistotle's project that logic should be formal and universal, and the only way that these two conditions can be satisfied is by guaranteeing that empirical factors do not contribute to, and are not presupposed in, determining what follows from what.

But why should we apply the substitution requirement across the board? Well, the Tarskian answers, because if we are concerned with a *formal* ac-

count of logical consequence, we simply can't tolerate the intrusion of empirical factors—what matters for logical consequence is the *form* of the arguments. But what if, by disregarding the role of empirical factors, we simply obtain the wrong results about a given domain? This is exactly the question that the quantum logician insists on asking. And the above example of what goes on in the quantum mechanics case seems to illustrate nicely the need for some revision of logic on *empirical grounds*. In other words, the unconditional application of the substitution requirement may yield a notion of logical consequence that is simply inadequate to accommodate the special features of certain domains.

Our proposal is then to *restrict* the application of the substitution requirement: one selects a domain, and the substitution requirement is then applied only to sentences/objects of that particular domain. By restricting the application of the requirement in this way, we can capture an important aspect of logic: in the context of the *particular* domain, the systematic substitution of non-logical terms does preserve the consequence relation. Moreover, the suggested restriction to the substitution requirement generates a relation of consequence that is better suited to capture the relevant features of the domain in question. After all, the *specific* features of the domain can be taken into account to yield the relation of consequence—as the quantum mechanics example illustrates with the non-distributivity of conjunction over disjunction.

By restricting the substitution requirement in this way, we allow the introduction of non-logical factors in logic selection: the parameters used in the determination of the domain bring in extra-logical factors to logic. In this sense, the present proposal provides a *non-apriorist* approach to logic, since to be a non-apriorist about logic is to acknowledge the need for extra-logical factors in the determination of the adequacy of a logic. Given that, in Tarski's hands, the substitution requirement is an emphatic expression of logical a priorism, the restriction of this requirement suggested here amounts to a defence of logical non-apriorism. And, of course, one of the main arguments for the version of logical non-apriorism advanced here is that by failing to acknowledge the role of extra-logical considerations in logic selection, we may simply end up with the wrong results—as is suggested by the quantum mechanics case discussed above.[9]

[9]In fact, all we really require for present purposes is that the substitution requirement *may* need to be restricted to particular domains. That is, we don't think that it is necessary for the requirement to hold across all domains. Thus, what counts as a logical truth *may*

But the logical a priorist will certainly complain. Logic, the a priorist will claim, is usually taken to be the most basic component of our conceptual framework; in this sense, if anything is a priori, logic certainly is. After all, the a priorist continues, logic is presupposed by any evidential system; that is, it's presupposed by any system that is taken to provide support for our beliefs. If logic is not a priori, there is no way in which it can play this role. For if logic is not a priori, any counterevidence to the conceptual system will also be a counterevidence to logic, and we would need an additional logic to adjudicate between the original logic, the counterevidence and the conceptual system. For us to be able to use logic in this way, and to avoid an infinite regress of logics here, logic should no longer be taken to be open to empirical revision—that is, logic needs to be a priori.[10]

Moreover, one can very naturally read Tarski's substitution requirement as an expression of the idea that logic is a priori (in the sense articulated in the last paragraph). After all, as Tarski points out, any constraints on logical consequence should be purely formal; such constraints don't presuppose any empirical knowledge, and so they are a priori. If extra-logical (in particular, empirical) considerations entered in the characterisation of logical consequence, instead of capturing the notion of *logical* consequence, we would be capturing the notion of *physical*, *mathematical* or *empirical* consequence.

In response to this argument, note that there is no need to require that logic be a priori for it to play the justificatory role that one expects it to play in evidential systems. For example, the *methodological decision* of not allowing logic to be open to empirical refutation will do just as well. And to consider logical a priorism only as a *methodological decision* is far weaker than to claim that logic is, in fact, a priori, since a methodological decision to treat logic as though it were a priori obviously doesn't entail that logic actually is a priori. Moreover, there are two advantages of taking logical a priorism as simply a methodological decision: (i) the decision of not holding a given logic responsible for empirical refutation is a decision that can be overthrown at any point, should we decide to do so, and (ii) the possibility of revising logic (even on empirical grounds) allows us to explore, and eventually determine, the adequacy of a logic to a particular domain. In this way, we can take into account the particular features of the domain under consideration, and use

depend on the domain under consideration.

[10] A very interesting argument for logical a priorism has been advanced along these lines by Hartry Field. See Field (1986, 1998, and 2001) for details and for a critical discussion of Field's arguments, see Bueno (forthcoming a).

the particular features of logic to explore the domain in question in a better way. Of course, to be able to do that, we need to restrict the application of Tarski's substitution in the way suggested above.

In other words, if we accept the domain dependence of the substitutional requirement, we are in a better position to select a logic that is adequate to a particular domain. But to argue—and decide—that a given logic is adequate to a particular domain, don't we need to use logic? And, once again, doesn't this highlight the crucial and fundamental role that logic plays in *any* conceptual framework?

Certainly logic plays a crucial role, but (a) logic doesn't need to be a priori for it to play this role, since, as mentioned above, the *methodological decision* of holding a logic fixed for the sake of the argument would do just as well. Moreover, (b) to use logic in the selection of a given logic—that is, to determine what consequence relation is adequate to a particular domain—is not the same as to explore the particular consequences that hold in that domain. The former presupposes the latter of course, but not the other way around. So the use of logic in the determination of the adequacy of a given logic is *only one* of many roles that logic plays, and it is by no means the most fundamental one.

In this way, it should be clear that the need for one to use logic in establishing the adequacy of a given logic doesn't require logic to be a priori. Furthermore, logic selection is only one role of logic; a role that can be played by different logics, some of which may be better suited than others to fulfil this role. And by not being logical apriorists, we can explore which logics are more adequate to certain domains than others, and so make better decisions about logic selection. (We will return to this point below in the context of our model of theory change in logic.)

But perhaps the Tarskian could try to keep logical a priorism by making one of three possible moves:[11] (a) The Tarskian could say that we can know a priori that distribution fails. (b) Alternatively, the Tarskian could claim that we can know a priori that distribution succeeds, and so the quantum logician's analysis of the quantum mechanics case is mistaken. (c) Finally, the Tarskian could perhaps say that conjunction or disjunction aren't logical constants, so the fact that truth-preservation of this form is a posteriori isn't a problem for the a priori view of logic.

These are all interesting moves, but we don't think that they ultimately

[11] We are grateful to an anonymous referee for pressing this point.

work. Option (a) is neither well motivated nor very plausible. On what grounds could the Tarskian say that we can know a priori that distribution fails? The quantum mechanics argument discussed above depends crucially on an empirical premise (namely, the measurement of the angular momentum of an electron E in the X direction). Of course, one can always say that we could have *imagined* that empirical possibility (the particular result of the measurement), and we could then run the quantum mechanics argument in exactly the same way as before. But at this point, we need to be very careful. After all, if the Tarskian were to take that route, he or she would have to be a priorist not only about logic, but also about science. For the Tarkian could just as easily say that a Newtonian physicist could have known a priori that gravity depends on the curvature of space-time, just by running the style of argument provided by Einstein in the formulation of General Relativity. Worries about incommensurability apart, the Newtonian could have *imagined* that possibility. But simply imagining this possibility is surely not enough to warrant the claim that we know a priori the results of physics.

Option (b) fares better on the plausibility score, but it still faces a serious difficulty. The option is plausible in the sense that the analysis provided by the quantum logician of the quantum mechanics case is not taken to be *necessary*. As we noted above, it is possible to keep classical logic in the quantum domain. The trouble is that to keep classical logic, we need to make changes in the interpretation of the quantum mechanics formalism (several interpretations of quantum mechanics do just that). But, as is acknowledged in the literature, every proposed interpretation of quantum mechanics faces substantial difficulties, and none of them is uniquely selected by the data. Thus, to close the possibility of the quantum logician's interpretation on a priori grounds amounts to sheer dogmatism.

Option (c) is not an option for the Tarskian. As we noted above, Tarski is explicit about the fact that logical constants are *not* open to reinterpretation (see the first quotation from Tarski 1936 above). So, he is in no position to deny that conjunction or disjunction are logical constants. But perhaps a more liberal Tarskian could entertain that possibility (of rejecting that conjunction or disjunction are logical constants). The trouble here is that this would actually *support* the kind of revision of logical principles on empirical (or, at least, extra-logical) grounds that we propose in this paper. After all, if the distributivity law fails in the quantum domain, we can't say that the substitution requirement applies across the board: some interpretations of extra-logical constants (such as that of the distributivity law in the context

of quantum mechanics) do *not* preserve logical validity. So, in the end, this would amount to a defence of non-apriorism about logic, along similar lines to the view we're advocating here.

2.3 Putnam's centrality argument

Despite the widespread acceptance of Tarski's account of logical consequence, not everyone endorsed Tarski's characterisation of logic in aprioristic terms. Putnam, for example, provides a very interesting case. Not only does he consider the possibility that logical principles may be revised on the basis of empirical considerations (arising, for example, from quantum mechanics), but Putnam even entertains the claim that *the law of non-contradiction might be revised on empirical grounds* (see Putnam 1983a). The claim is made in the context of Putnam's discussion of the centrality argument, that is, the argument to the effect that the laws of logic are presupposed by 'so much of the activity of argument itself that it is no wonder that we cannot envisage their being overthrown [...] by rational argument' (Putnam 1983a, p. 110). In response to this argument, Putnam immediately adds:

> But we should be clear about what the centrality argument does not show. It does not show that a putative law of logic, for instance the principle of contradiction, could not be overthrown by *direct observation*. Presumably I would give up the principle of contradiction if I ever had a sense datum which was both red and not red, for example.[12] And the centrality argument sheds no light on how we know that this could not happen. (Putnam 1983a, p. 110)

In a note added to the paper, Putnam elaborates on this point:

> I think it is right to say that, within our present conceptual scheme, the minimal principle of contradiction [i.e. the claim that not every statement is both true and false] is *so* basic that it cannot significantly be 'explained' at all. But that does not make it an 'absolutely a priori truth', in the sense of an absolutely unrevisable truth. (Putnam 1983a, p. 111)

[12] One of the present authors has argued that this may indeed be possible. See Beall and Colyvan (2001) for details of how contradictions such as this might be observable, given a paraconsistent account of vagueness.

We completely agree with the spirit of this passage, even though we may not agree with the letter. Putnam is correctly indicating here that, although the 'minimal principle of contradiction' is basic, it shouldn't be taken to be a priori. We couldn't agree more.

However, one point should be noted in relation to this. In this passage Putnam has a peculiar understanding of the notion of a priori: a priori truth is understood in terms of *unrevisable* truth. Clearly, this view of the a priori is not unproblematic, given that it rejects by fiat the possibility of *defeasible* a priori claims (claims that Field, for instance, insists that are part of logic; see Field 1998). And it might be complained that there is no need to tie the notion of a priori with that of unrevisability. The axiom of choice in set theory, is an example of a claim that is (arguably) known a priori but which is clearly revisable.

For our purposes here it is enough to note that, using the notion of a priori adopted in the present work, we can make perfect sense of the claim that the law of non-contradiction is not a priori in Putnam's sense. Recall that in our view, logical non-apriorism is the thesis that acknowledges the importance of *extra-logical* considerations in the selection and evaluation of logical principles. In particular, logical principles may be *revised* on empirical grounds. So in either sense of a priori (ours or Putnam's), logic turns out not to be a priori.

The point of this section is to indicate, following Putnam's lead, that even if one takes the 'law' of non-contradiction to be a fundamental logical principle, this shouldn't be taken as a reason to conceive of it as being a priori. As we will discuss below, in the debate about this 'law', both those who defend and those who criticize the 'law' typically use empirical considerations to support their claims. It's non-apriorism all the way down.

3 Theory Change in Logic

If logical non-apriorism is a live option, the question arises as to how, according to the logical non-apriorist, theory change in logic proceeds. This section will outline, in general terms, one answer to this question. Roughly, one can hold fixed some logical principles—or, as we will see, a common core of (methodological or axiological) assumptions—that are agreed upon by all parties. These (limited) resources are then used to conduct the argument

about the contentious principles.[13]

The above idea can be cashed out by using a very interesting model of scientific theory change originally developed in the philosophy of science, namely Laudan's reticulated model (see Laudan 1984). The crucial feature of the model is that scientific debates range over three main components: debates over the aims of scientific research, over methods, and over scientific theories. And typically, at any given time, the scientific community agrees upon at least one of the three components, and this shared agreement can then be used to settle the debate over the remaining two components. For example, shared aims of scientific research can be used to settle disagreements about methods (e.g. select the methods that best realise the aims), and shared methods can then be used to adjudicate between rival theories (e.g. select the theories that are best justified by the methods). But the relation between aims, methods and theories is not hierarchical: theories also constrain methods and realise aims, just as methods also realise aims (for further details, see Laudan 1984). The interrelation between the three levels can be represented by the following diagram:

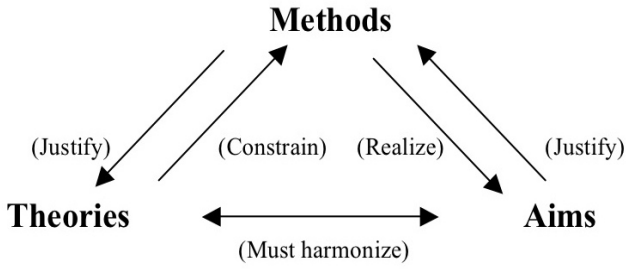

Figure1: The Reticulated Model of Justification

We believe that, suitably reinterpreted, the reticulated model provides a framework to represent how debates about logical principles can be con-

[13]As we will see, the idea is very much like Neurath's famous image of rebuilding a boat at sea.

ducted. The crucial idea is that—similarly to what goes on in science—debates about logic typically involve a common core of assumptions that are shared by the various parties in the debate. This common core includes: (i) shared logical theories (that is, logical principles and rules), or (ii) shared views about the aims of logic, or (iii) shared methodological principles (broadly understood to include metalogical properties). Although usually items (i)–(iii) are not all shared at the same time, at least *one of them* typically is. And as we will see, this provides enough common ground for debates about logic to be conducted and, in some cases, settled.

But to be able to apply Laudan's reticulated model to theory change in logic, we need first to reinterpret slightly the original components of the model. We leave the *axiological component* (the one dealing with aims) as it is, except for the fact that instead of considering aims and values of science, we will be examining the axiology of *research traditions* in logic.[14]

Moreover, we understand the *methodological component* (or the methodological principles) of the reticulated model in broad terms: it includes both methodological and metamethodological principles. That is, the methodological component incorporates not only specific methodological principles of theory construction, but also metamethodological principles to choose between alternative methods of theory construction. In the case of logic, the methodological component (broadly understood) includes metalogical principles (such as the completeness or incompleteness of a formal system, its decidability or undecidability etc.), as well as proof-theoretic and model-theoretic techniques, the use (or not) of the T-schema, and further patterns of choice of logical notions, logical rules and logical principles.

We also redefine the *theoretical component* of the reticulated model to make room for logical principles and rules of inferences. On this account, logic can be read as a theory, namely, a theory of the relation of logical consequence. One of the main issues faced by a logic is whether it provides an adequate account of the relation of logical consequence. And typically to address this issue requires the examination of further issues about the domain

[14]Laudan developed the notion of research tradition for science (see Laudan 1977). As he conceives it, a research tradition provides a whole framework for the development and implementation of scientific research in a give domain, including standards, methods, theories, principles, metaphysical assumptions etc. We think that the notion of research tradition is not restricted to science (a point with which, we take it, Laudan would agree), and the notion applies just as well to the foundations of logic—even though, thus far, it has not been so applied.

to which the logic in question is applied. The reinterpreted reticulated model can then be represented by the following diagram:

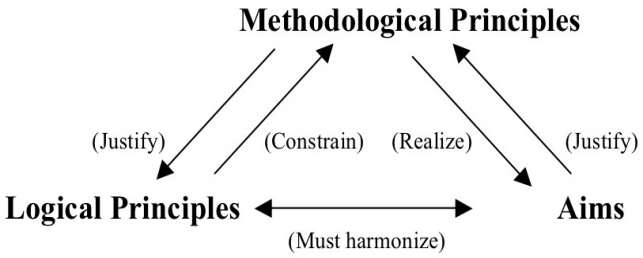

Figure 2: The Reticulated Model as Applied to Logical Theory Choice

Given the above reinterpretation of the reticulated model, we can then indicate how debates about theory change in logic can be represented and accommodated. As mentioned above, our aim here is not to resolve the debate about the law of non-contradiction, but to put forward a possible framework in terms of which the debate can be understood. And we insist that the reticulated model is *only one* among many possible ways of modelling the phenomenon in question: we advance it here only to illustrate how debates about logical principles can be carried out.[15]

The reticulated model works in the same way in logic and in science. The three components of the reticulated model (aims of logical research, methodological/metalogical principles, and logical principles) typically are not all under critical scrutiny at the same time. One could use, for example, shared aims in logical research to choose between different methodological/metalogical principles, or to adopt shared logical principles to reassess the adequacy of some aims of logic. Alternatively, one could use shared methodological/metalogical principles to choose between rival logical principles, or to employ shared aims of logic to choose between rival metalogical

[15]Other approaches along broadly naturalistic and non-apriori lines (not all of which are direct competitors with the reticulated model) may be found in Maddy (forthcoming), Mares (forthcoming), and Resnik (forthcoming). Mike Resnik's 'reflective equilibrium' approach, in particular, shares a great deal with the reticulated model.

principles. As noted above, the point here is that—just as in science (see Laudan 1984)—typically debates over rival research traditions don't involve changes in *all* of the three components of the reticulated model at the same time. At least one component of the model is fixed, and one can use the fixed component to choose among the components that are under debate.

Let us give a simple example to illustrate how the reticulated model works. Suppose that we start with the classical research tradition in logic. The *aim* of logic is taken to be to provide an account of logical consequence that captures the intuitive notion of consequence found in natural language. *Logical principles* are then formulated to realise this aim. These principles are, basically, those of classical predicate logic, and they yield a particular body of *metalogical* results (e.g. the connectives are extensional, the system is complete, the monadic fragment of the logic is decidable etc.). With the three components in place, research in logic can then be conducted.

It soon becomes clear, however, that some intuitively valid inferences in natural language turn out *not* to be valid according to the above logical principles. For example:

John plays football on weekends.

Therefore, John can play football on weekends.

This inference, although intuitively valid, is not *formally* valid. To make the above inference formally valid, one needs to change one component in the reticulated model of classical logic's research tradition. One needs to introduce *new logical principles and rules* (in particular, those concerning the modal operators of necessity and possibility—box and diamond, respectively). Note, however, that the introduction of these principles is justified by the *aim* of the research tradition: to provide an account of logical consequence that *accommodates natural language*. So the new logical principles are introduced. The trouble, though, is that the new principles require a revision in the accepted *metalogical principles*. After all, with boxes and diamonds, we no longer have extensionality. In this way, we moved from classical logic's research tradition to a new research tradition, that of modal logic. Both traditions share the same aim, but they have different logical and metalogical principles.[16]

[16] Although this example is something of a toy example, it does help illustrate the way the reticulated model of theory change in logic works. Obviously real historical examples such as the shift from Aristotelian logic to Fregean logic will be more complicated and

What this simple example illustrates is how a particular aim (to provide an account of logical consequence in natural language) may require changes in the two other components of the reticulated model—the logical and the metalogical principles. In this way, we can see how, by sharing the same aims of logical research, a community can decide about the way to change the logic and the metalogic of a given domain. It then becomes clear how a community can settle debates about logical principles on the basis of other common assumptions about the domain in question.

4 The 'Law' of Non-Contradiction

So far we've argued that logic is not a priori and, in particular, that questions about theory choice in logic are not settled on a priori grounds. We've also emphasised the fallible nature of logical theory and provided a particular model of theory change in logic—the reticulated model. Is this enough to settle the question of whether there can be meaningful debate on the law of non-contradiction? Not quite. There are a couple of concerns that arise in the context of the debate over the law of non-contradiction that go beyond the aprioricity of logic and beyond the usual issues of theory choice.[17] These concerns involve the seemingly privileged status that certain fundamental laws like the law of non-contradiction hold in any logical theory. David Lewis presents the problem like this:[18]

> No truth does have, and no truth could have, a true negation. Nothing is, and nothing could be, literally both true and false. This we know for certain, and *a priori*, and without any exception for especially perplexing subject matters. The radical case for relevance should be dismissed just because the hypothesis it requires us to entertain is inconsistent.

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we do not discuss them here for that reason. Whether such examples do fit well with the reticulated model is an interseting question that we intend taking up elsewhere.

[17] Also there's the issue that accepting or rejecting the law of non-contradiction is not, strictly speaking, an issue of theory choice. Rather, it concerns accepting or rejecting *classes* of logical theories. This detail need not concern us, though, since the same questions arise over choices between classes of theories as over choices between single theories. We are thus justified in focusing our attention on theory choice in logic.

[18] See also Lewis's 'contribution' to this volume (Lewis, forthcoming) for another typically elegant statement of the problem.

> That may seem dogmatic. And it is; I am affirming the very thesis that Routley and Priest have called into question—contrary to the rules of debate—I decline to defend it. Furthermore, I concede that it is indefensible against their challenge. They have called so much into question that I have no foothold on undisputed ground. So much the worse for the demand that philosophers always must be ready to defend their theses under the rules of debate. (Lewis 1982, p. 101)

Lewis's point is a serious one: Even falliblists, he suggests, must hold onto some principles dogmatically. To give up certain fundamental logical principles (even for the sake of debate) leaves one with too impoverished a set of logical resources to proceed. Or perhaps the suggestion is that we can proceed, but the result of our investigations are irrelevant, because what we conclude is so much less certain than what we've given up to get there.

Now clearly debate about the law of non-contradiction can and does proceed. (One need look no further than the present volume to see that.) Lewis's concern, though, is not about whether such debate does in fact occur; it's about whether such debate is fruitful or even rational. We believe that this debate is both fruitful and rational; demonstrating this, however, is a subtle matter. We begin by applying the model of logical theory choice developed in the last section to the question of the status of the law of non-contradiction. It is important to see how such a model allows debate to proceed on even the most fundamental logical laws and that we can have confidence in the results of the relevant debate—even apparently fundamental logical principles can be trumped by enough considerations elsewhere, both within logic and from further afield. The debate can proceed by employing those principles and data not under dispute. We will now run through a couple of the arguments presented for and against the law of non-contradiction and show (i) how these arguments depend on non-apriori considerations and (ii) how the various participants in these debates agree on a great deal more than Lewis acknowledges and (iii) it is this agreement that allows debates about even apparently fundamental principles to proceed.

Consider the argument that there can't be true contradictions because if there were, everything would be true; but clearly everything is not true so there can't be any true contradictions. (Call the view that everything is true *trivialism*.) Here we have an example of an argument for the law of non-contradiction based on the empirical fact that not everything is true—the

world is not flat, for example.[19] As it turns out, this argument against true contradictions is not a particularly good one, for there is a very good reply to it. The argument just presented assumes *ex falso quodlibet* (or *explosion*, as it has become known). This rather undesirable feature of classical logic is denied by dialethists like Graham Priest (see Priest 1987, 1998). Dialethists believe that there are true contradictions, yet they agree with the supporters of non-contradiction that not everything is true. Dialethists reject classical logic—in particular they object to its explosive character and opt, instead, for non-explosive paraconsistent logics.

Now you might think that this just confirms Lewis's point since these debates over the law of non-contradiction instantly reach deadlock. But this is not right. The above paragraph outlines a fairly typical philosophical debate in which both parties agree about a great deal—they both accept that the conjunction of true contradictions and classical logic is untenable. The dialethist and the classical logician, however, provide different diagnoses of the problem—the dialethist blames explosion while the classical logician blames the initial acceptance of true contradictions—but they *are* able to conduct a debate. They also agree on the reason for the untenability of the above conjunction of true contradictions and classical logic: they agree that trivialism is false. Now if this debate reaches a deadlock, that's neither here nor there; many philosophical debates reach deadlocks (at least for a time). Our point here is simply that the above 'explosion' argument and reply is a perfectly good example of a philosophical debate that makes some progress on the problem at hand.

Next consider an argument *against* the law of non-contradiction. It starts from the assumption that (i) satisfaction of the T-schema is a necessary condition on a theory of truth, and (ii) natural languages are semantically closed (that is, they contain their own truth predicates). At least the second of these is an empirical matter—it's simply an empirical fact about English, for example, that sentences such as 'it's true that South Carolina is East of

[19] You might think that we can come to the view that trivialism is false by a priori means. There are a couple of things to say to this. First, the falsity of trivialism might be know*able* a priori, but it does not follow that it is discovered by a priori means. There are straight-forward empirical means to come to this view—observing that the earth is not completely covered with water, for instance. Second, even if you think that 'trivialism is false' is not an empirical truth, our main point here still stands: there are some issues on which all parties in this debate agree. The law of non-contradiction is not philosophical bedrock, as it were.

California' are well formed (and, in this case, true). Now we consider the strengthened liar sentence:

($\star$) *This sentence is not true.*

By a well-worn argument employing the above two assumptions, we arrive at the conclusion that ($\star$) is both true and false. Most philosophers are thus led to question (i) and (ii) in various ways, or to give up on semantically closed natural languages. For instance, Tarski accepted that English was semanticall closed but chose to replace English with a heirachy of formal languages, each of which was not semantically closed. Others haved questioned the T-schema, and others still claim that, despite appearances, ($\star$) is not a sentence of English. Dialethists, on the other hand, accept the conclusion that ($\star$) is both true and false. They do so because they accept the premises of this argument and, somewhat reluctantly, give up the law of non-contradiction.

Again we draw attention to the significant agreement here. All parties accept (at least initially) the premises of the liar argument. All parties accept the reasoning involved in the argument to the contradiction (essentially *modus ponens* and that every sentence is either true or not-true). The responses to the argument are different, but we think it's fair to say that all parties acknowledge the cost of giving up what they do. Tarski did not take the denial of semantic closure lightly, Priest does not take giving up the law of non-contradiction lightly, and those who deny that ($\star$) is a sentence of English typically go to considerable length to defend this counterintuitive position.

What we're drawing attention to here is that the law of non-contradiction is not such a fundamental principle of logic that debate is impossible without it. In particular, we see that extra-logical assumptions, such as the T-schema, and extra-logical (empirical) facts also have a great deal of weight in these debates. We also draw attention to the fact that often these extra-logical considerations are a posteriori, as in the case of the universally-agreed-upon fact that not everything is true. Finally, we wish to emphasise that the reticulated model of logical theory choice, outlined in the previous section, provides an effective way of making sense of both the debates as they actually proceed and how it is that the debates can *rationally* proceed in this way.

Indeed, on this model, what counts as fundamental logical principles can change when one reassesses the aims or methodological principles. For example, in the above argument from the strengthened liar, we might start out with classical logic (including the law of non-contradiction) as our logical

principles; our aim might be to give an account of logical consequence as it arises in natural language; our methodological principles that help us achieve this goal would include the T-schema and the requirement of semantic closure. Then we find that from considerations of the strengthened liar (which is a piece of natural language), we derive a contradiction. The dialethic response is to keep the methodological principles and the aims and revise the logical principles—namely, reject the classical law of non-contradiction. Contrast this with Tarski's response to the same initial situation; he chose to revise the methodological principles in light of the paradox. In particular, he abandoned the methodological constraint that the language be semantically closed (because semantically closed languages are inconsistent). This, in turn, meant he had to revise his aims, because clearly natural languages *are* semantically closed. Tarski's new aim was thus to provide an account of logical consequence for suitable formal languages.

These examples show us how the reticulated model is able to make sense of debates about fundamental logical principles as well as debates about aims and methodological principles. It doesn't necessarily tell us the correct moves to make in such debates. For instance, the model is silent on whether Tarski's response, the dialethic response, or some other response is the most appropriate in the case above. The model merely provides a nice framework for conducting such debates (and perhaps it helps to keep track of the costs and benefits of the various strategies). But this, of course, is not trivial. The reticulated model does not rule against the revision of logical principles—no matter how fundamental they are. You might choose not to abandon certain logical principles such as the law of non-contradiction, but the *possibility* of revisions here are not ruled out.

Finally let us return to a point that's perhaps implicit in the passage from Lewis earlier in this section. The objection we have in mind is that any investigation of a fundamental logical principle, such as the law of non-contradiction, is doomed because other principles invoked in the investigation are so much less certain than the law of non-contradiction itself. Thus the results of such an investigation—whatever they happen to be—are less secure than the law of non-contradiction.[20] We have, in effect, already addressed this objection but we think it might be helpful to address it explicitly. We have two things to say. First, the objection seems to presuppose some sort of

[20]Lewis has explicitly articulated an objection along these lines in private communication.

foundationalist epistemology, according to which if principle P is less certain than Q, then P cannot be invoked to overthrow or revise Q. It would take us too far afield to rehearse the various objections to such an epistemology; suffice to say that it seems plausible, at least, that if there are many such Ps (P_1 to P_n, say), and even if each P_i is less certain than Q, there is a sense in which the weight of evidence of the P_is piles up to overthrow the (initially) more certain Q.[21] Moreover, we've shown how this can be done using the reticulated model. Second, and more importantly, it's simply not clear that the law of non-contradiction is more certain than other principles appealed to in these debates. After all, all parties agree that trivialism is false and this at least suggests that the falsity of trivialism is more certain than the law of non-contradiction.[22]

5 Conclusion

We believe that there *is* debate about the law of non-contradiction; for the most part this is *rational* debate; and there *ought* to be such debate. We hope to have shown how this debate can proceed. We've argued that non-apriori considerations are important in this and other debates about theory choice in logic, so logic can't be apriori. We thus defended non-apriorism in logic. We provided a particular model of how debate about logic—even seemingly fundamental principles of logic—can be conducted. The model we applied—the reticulated model—is a plausible way forward. Finally, we addressed the objection that the law of non-contradiction is just *too* fundamental to debate. According to this objection, without the law of non-contradiction there's no common ground left for the participants in the debate to stand on. This, we

[21] The natural way to spell out this 'weight of evidence' claim is in terms of prior probabilities and conditionalising to obtain posterior probabilities via Bayes' Theorem. We resist this temptation, though; any (classical) probability talk here is misguided. In classical probability theory (i.e. probability theories that obey the Kolmogorov axioms), *all* classical logical truths get probability one, and so we cannot talk of some logical truths being more likely than others. We thus keep probabilistic talk out of the debate (as does the objector, for s/he too wants to say that the law of non-contradiction is more certain than other logical principles (such as excluded middle), without cashing out 'certainty' in terms of (classical) probability).

[22] Of course consensus and certainty don't always coincide, but we think they do here. Surely it's the fact that everyone is certain that trivialism is false that accounts for the agreement on this issue. And surely it's because the law of non-contradiction is somewhat less certain that all parties offer arguments for or against it.

argued, is not so. There might not be much common ground, but there is enough for debate to proceed.[23]

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