On the Asymmetry in Relativistic Doppler Shifts Caused by Time Dilation: Proposed Two-way Experiment

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There are two factors responsible for the relativistic Doppler effect. One of these contributions is of first-order in the velocity of the light source relative to the observer and is non-relativistic in nature, but there is also a second-order (transverse Doppler) effect caused by the difference in rates of clocks at the source and in the laboratory (time dilation). It is pointed out that the traditional derivation for the relativistic Doppler effect based on the assumed Lorentz invariance of the phase of light waves is too restrictive. According to the latter argument, when two observers in relative motion send out signals to one another employing an identical light source, their respective measured Doppler shifts should *always* be the same (totally symmetric). However, an alternative derivation can also be given which indicates that the *second-order*

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contribution should in fact be *anti-symmetric,* although this analysis agrees with the conclusion that the corresponding first-order Doppler effect is symmetric. This is because there is a *reciprocal* relation between the ratios of the rates of the moving and rest clocks for the two observers. Quantitative calculations of the amount of the predicted anti-symmetry in transverse Doppler shifts are made for several examples on the basis of experimental data reported for the rates of atomic clocks carried onboard airplanes and rockets. Because of the Earth's rotation about its polar axis, the Doppler shifts can be measured in principle by using exclusively land-based atomic clocks located at widely different latitudes. Verification that the transverse Doppler frequency shifts are indeed anti-symmetric would be a definitive confirmation that the oft-cited symmetry principle of Einstein's special theory of relativity (STR) is *incorrect*, namely the claim that two observers in relative motion should each find that his/her clocks run faster than the other's. Finally, changes in the formulation of relativity theory that bring it into agreement with modern-day observations of variations in the rates of clocks with state of motion are discussed with reference to the methodology of the Global Positioning System (GPS).

Keywords: transverse Doppler effect, time dilation, Global Positioning System, objectivity of measurement, remote simultaneity, Alternative Lorentz Transformation (ALT).

I. Introduction

The relativistic Doppler effect differs from its classical counterpart in that there is not only a first-order effect depending on the speed of the source, but also one of second order. There is thus a *transverse* Doppler effect for light waves, as first verified experimentally by Ives and Stilwell [1], although no analogous phenomenon has ever been observed for sound because of the lower speeds involved in this case.

There is general agreement that this second-order Doppler effect is directly related to the time-dilation prediction of Einstein's special theory of relativity (STR [2]).

In standard texts one finds two different derivations of the relativistic Doppler effect. In the first case [2] it is assumed that the phase of light waves should be invariant to a Lorentz transformation (LT) for two different observers in relative motion, whereas in the other the time-dilation effect is taken into account directly [3]. In the following discussion attention is drawn to the relevance of experiments carried out to determine the rates of atomic clocks carried onboard airplanes [4] and rockets [5]. One knows, for example, that there is an East-West effect for these clocks caused by the Earth's rotation about its polar axis. This result raises a critical question: are the transverse Doppler frequency shifts the same (symmetric) for observers on airplanes/rockets that are in relative motion when they exchange light signals, as the invariant-phase derivation indicates; or are they equal but of opposite sign (anti-symmetric) as one should expect based on the other derivation [3]? The only definitive way to answer this question is by carrying out a two-way Doppler experiment in which signals are exchanged between observers whose proper clocks are known to run at different rates, something that has not yet been accomplished in previous work. How the proposed experiment can be carried out in practice with the highly accurate atomic clocks that have been developed since Einstein's original paper [2] is the subject of the following discussion.

II. Two Factors in the Relativistic Doppler Equations

The phase Φ of a simple harmonic plane wave in free space is defined as:

$$\Phi = \boldsymbol{k} \bullet \boldsymbol{r} - \omega t \,, \tag{1}$$

where $\boldsymbol{k}$ is the wave vector and ω is the circular frequency. The phase velocity is thus $\frac{\omega}{k} = c$ (2.99792458x10^8 m/s), the speed of light in free space. The relativistic Doppler effect has been derived [2,6] on the basis of the *assumption* that Φ must be invariant to a Lorentz transformation (LT). For the case when the light source is moving in the *x* direction along the line of observation, this means that ω and $\boldsymbol{k}$ transform according to the following equations:

$$\omega' = \gamma\left(\omega - u k_x\right) \tag{2a}$$

$$k_x' = \gamma\left(k_x - \frac{u}{c^2}\omega\right) \,, \tag{2b}$$

where u is the speed of approach of the light source toward the observer and $\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-0.5}$. In these equations the *unprimed* variables are the values measured by the observer in the laboratory when the source is moving ($u > 0$) relative to him, whereas the primed quantities are the corresponding *in situ* values obtained by the same observer when the light source is stationary ($u = 0$). Since in this example, $\omega = k_x c = kc$, one can rewrite these equations in a simpler form:

$$\omega' = \gamma\omega\left(1 - \beta\right) \tag{3a}$$

$$k' = \gamma k\left(1 - \beta\right) \,, \tag{3b}$$

with β = u/c.

The above equations contain two factors that are dependent on the speed of the light source. For $u \ll c$, γ is second-order in β and thus in actual experiments is generally far outweighed by the $(1-\beta)$ factor. If the velocity of the light source makes an angle φ with the line of observation, it is found that only the first-order term is affected ($\varphi = 0$ corresponds to head-on motion of the source toward the observer):

$$\omega' = \gamma\omega\left(1-\beta\cos\varphi\right) \tag{4a}$$

$$k' = \gamma k\left(1-\beta\cos\varphi\right) \ . \tag{4b}$$

The corresponding transformation equations for the frequency ν and wavelength λ of the light waves ($\omega = 2\pi\nu$ and $k = \frac{2\pi}{\lambda}$ are therefore:

$$\nu = \frac{\nu'}{\gamma}\left(1-\beta\cos\varphi\right) \tag{5a}$$

$$\lambda = \gamma\lambda'\left(1-\beta\cos\varphi\right) \ . \tag{5b}$$

By measuring the wavelength of light for a source moving toward ($\varphi = 0$) and away ($\varphi = \pi$) from the observer in the laboratory and averaging these two values, it is possible to eliminate the first-order effect. This procedure corresponds to an indirect measurement of the transverse Doppler effect ($\varphi = \frac{\pi}{2}$), i. e. $\lambda = \gamma\lambda'$, and the corresponding results obtained by Ives and Stilwell [1] with this method were in good agreement with Einstein's predictions [2]. The Mössbauer technique was used subsequently [7] to obtain a more precise verification of the transverse Doppler effect (to within 1%).

It is possible to derive eq. (5) in a more intuitive way [3] that does not directly involve the LT, however. First, one assumes that the period of the radiation is longer at the source than in the laboratory because of the time-dilation effect. By assuming that clocks run *slower* by a factor of γ *on the moving source* relative to those in the laboratory, it follows that both the observed period $T = \nu^{-1}$ and the associated wavelength λ are greater by this fraction than their respective *in situ* values. This accounts for the transverse Doppler effect directly ($\varphi = \frac{\pi}{2}$). It is purely a matter of time dilation on moving objects. Since the source is moving relative to the waves that it emits, there is a second effect whenever the motion is not transverse, however. If we denote the component of the source velocity *toward* the observer as u_r, this means that the observed wavelength and period are smaller by a fraction of $(1 - \frac{u_r}{c})$ than otherwise would be the case. This is the explanation for the $(1 - \beta \cos\varphi)$ factors in eqs. (5a,b), since $u_r = u \cos\varphi$. The latter result is simply that part of the Doppler effect that could be confirmed for sound waves in classical experiments and thus can be considered to be a purely non-relativistic phenomenon. In summary, the first-order effect in β is solely due to the relative motion of the source to the observer, while that of second order arises because the rate of the observer's clock is different than for those co-moving with the source.

III. The Generalized Version of the Relativistic Doppler Transformation

Both of the above derivations of the relativistic Doppler effect arrive at the same conclusion, so it might be thought that there is no way to distinguish them experimentally. This is certainly the case based on consideration of eqs. (5a,b) as they stand. There is nonetheless a potential difference that needs to be explored. The second derivation [3] assumes that clocks moving with the light source must run exactly γ times more slowly than those in the laboratory. However, the experimental data obtained for clocks on circumnavigating airplanes [4] demonstrate that *this assumption is not generally valid*. They show instead that it is not possible to successfully predict the *ratio* of the rates of the respective onboard clocks simply by knowing the speed of the two aircraft relative to one another. Instead, it is necessary in both cases *to know the speed u of a given airplane relative to the Earth's polar axis*. By assuming that the rate of the clock on the airplane is always $\gamma(u)$ times slower than an identical clock located on this (non-rotating) axis, Hafele and Keating [4] were able to predict the relative rates of clocks located on each of two airplanes to within about 10% accuracy. Vessot and Levine [5] used the same assumption successfully in analyzing their experimental results for transverse Doppler shifts of light signals emitted from a rocket.

Eqs. (5a,b) are derived under the assumption that the *same* speed u is to be used in evaluating both β and γ therein. For head-on motion of the source toward the observer ($\varphi = 0$), this allows one to make a simple cancellation so as to arrive at the following two equations:

$$\nu = \nu'\left(\frac{1+\beta}{1-\beta}\right)^{0.5} \tag{6a}$$

$$\lambda = \lambda'\left(\frac{1-\beta}{1+\beta}\right)^{0.5}. \tag{6b}$$

On this basis, one is forced to conclude that there is a definite symmetry for the Doppler effect. If each of the airplanes in the HK experiment [4] were to send out light signals of identical frequency to the other, the Doppler shift should be exactly the same for both according to this analysis.

It is sometimes argued that the above result does not follow directly from eqs. (2-6) because of the fact that the airplanes are not perfectly inertial systems (IS). The HK analysis [4] claims, for example, that the non-rotating polar axis qualifies as an IS, whereas a point on the Earth's surface away from the Poles does not. The centrifugal acceleration ($R\Omega^2$) at the Equator due to the Earth's rotation is 0.04 m/s^2 and it decreases with latitude χ in direct proportion to $\mathbf{cos}^2\,\chi$. Accordingly, the corresponding value at each of the Poles is null to be sure, but this argument nonetheless overlooks a critical point. There is also acceleration caused by the Earth's orbit around the Sun. It has a magnitude of 0.006 m/s^2 and is the same at the Poles as it is at the Equator. It is therefore incorrect to claim that the non-rotating polar axes correspond to an IS and this fact raises a number of questions about the HK [4] interpretation. In other words, where do we draw the line as to what level of acceleration is allowed for the purposes of choosing a reference frame from which to apply Einstein's formula?

It also needs to be pointed out that a counter-balancing force can always be applied in any given rest frame that makes it an IS according to the accepted definition of perfectly null acceleration (validity of Newton's First Law). The HK hypothesis would have us believe that an observer's measurement of relative clock rates depends critically on whether such a potentially miniscule force has been applied to his detector or not. The IS argument that is used to explain why the slowing of clock rates is only predicted correctly when speeds are taken relative to the non-rotating polar axis is therefore specious. Consequently, one needs to find another explanation for the uniqueness of the polar axis for this determination, and this has been done in a companion publication [8].

The key point in the present context is that there is no justification whatsoever for using the HK argument about the uniqueness of the polar axis as an IS to invalidate the use of eqs. (2-6) for the proposed two-way Doppler experiment. The relative speed (β) of the *other* airplane in the HK experiment [4] is obviously the same in each case, so since the *in situ* value for the frequency (ν') of their respective light signals is also identical, it follows from eq. (6a) that the observers on both airplanes must measure exactly the same value ν for the frequency of the other's signals. In other words, STR and the LT predict unequivocally that the relativistic Doppler effect is totally symmetric.

Consideration of the alternative derivation based on the time-dilation effect indicates that the above relationship is *not* totally symmetric, however. Even though the $(1-\beta\cos\chi)$ factor in eqs. (5a,b) is clearly the same for both airplanes, the situation is different for the second-order (time-dilation) effect. Let us assume, for example, that the observer's clocks on one of the airplanes run α_{O} times *slower* than the reference clock mentioned above for the HK experiment [4]. If identical clocks on the second airplane run α_{M} times slower than the same reference clock, it is necessary to modify eqs. (5a,b) as follows in order to be consistent with the time-dilation effect:

$$\nu = \frac{\alpha_{\mathrm{O}}}{\alpha_{\mathrm{M}}} \frac{\nu'}{1-\beta\,\mathbf{cos}\,\varphi} \tag{7a}$$

$$\lambda = \frac{\alpha_{\mathrm{M}}}{\alpha_{\mathrm{O}}} \lambda'\left(1-\beta\,\mathbf{cos}\,\varphi\right). \tag{7b}$$

This is the generalization of the relativistic Doppler transformation. It assumes that measurement is objective and rational [9], *and therefore that the rates of the two clocks are always in the same proportion* independent of who the observer is in a given case. This is the main distinction between eqs. (7a,b) and the invariant-phase result of eqs. (5a,b). Because of the assumption of Lorentz invariance in the latter's derivation, it is necessary to conclude that measurement is subjective, in particular that two observers may disagree as to which of two clocks is running slower than the other.

More details about the definition of the "clock-rate parameters" α_{O} and α_{M} in eqs. (7a,b) may be found elsewhere [8]. The latter relations reduce to eqs. (5a,b) in the special case when the light source has been accelerated to speed u relative to the observer in the laboratory ($\alpha_{\mathrm{M}} = \gamma$ when $\alpha_{\mathrm{O}} = 1$, for example). More importantly in the present context, however, this analysis indicates that if the roles are reversed for the two airplanes, that is, if the subscripts M and O are interchanged in eqs. (7a,b), *the results for* ν *and* λ *do not remain the same*. The first-order effect in β is symmetric with respect to such an exchange, *but the ratio of clock rates is not*. There is clearly a *reciprocal* relation between the latter factors for the two airplane observers *if we assume that measurement is objective* [9].

The reasoning behind the conclusion that the phase Φ in eq. (1) must be Lorentz invariant overlooks a basic fact. Since $\omega = kc$ and $\omega' = k'c$ because of the constancy of the speed of light, it follows that there is a free parameter in the Lorentz invariance condition that is invariably omitted from consideration in discussions of this point, namely:

$$\omega^2 - k^2c^2 = \varepsilon^2\left(\omega'^2 - k'^2 c^2\right). \tag{8}$$

The value of ε is completely undetermined in this equation because of the constancy of the speed of light in both rest frames, whereas the phase argument for the Doppler effect assumes the only allowed value for it is $\varepsilon = 1$. An analogous degree of freedom exists for the relativistic space-time transformation, as was first pointed out by Lorentz [10] several years before Einstein's seminal work [2]. In this case, the Lorentz invariance condition proves to be incompatible with the *principle of remote simultaneity of events* that is essential to the workings of the GPS methodology. *This observation eliminates the LT as a physically viable space-time transformation* [11,12]. Eq. (8) is compatible with eqs. (7a-b) for *any values* of the clock-rate parameters α_{O} and α_{M} (and also for any value of ε) since $\nu\lambda = \frac{\omega}{k} = \frac{\omega'}{k'} = c$ in all cases.

IV. Numerical Examples

The points discussed above can best be illustrated with some numerical examples. One airplane E heads in an easterly direction with a ground speed of 1000 kmh, while another (W) heads *westerly* with the same ground speed. Their relative speed is thus 555.6 m/s. They each send out light signals to the other with an *in situ* frequency

ν'. The Doppler shift $\Delta\nu = \nu - \nu'$ caused by the $\left(1-\beta\cos\varphi\right)^{-1}$ factor in eq. (7a) is the same for observers on both airplanes. There is a positive (first-order) shift of $\frac{\Delta\nu}{\nu'} = 1.853\text{x}10^{-6}$ for each of them. In order to compute the additional factor due to time dilation, it is necessary to know the rotational speed of the Earth around its polar axis. Let us assume that this value is 1500 kmh in the easterly direction in the present example. The speed of airplane E relative to the polar axis is thus 2500 kmh, whereas that of W is only 500 kmh. According to the Hafele-Keating analysis [4], the onboard clocks on E are running slower than the reference clock on the polar axis by the factor of $\alpha_{\text{E}} = 1 + 2.6829\text{x}10^{-12}$ on this basis, i.e. $\gamma\,(u = 2500 \text{ kmh})$. The corresponding factor for the W clocks is $\alpha_{\text{W}} = 1 + 1.0730\text{x}10^{-13}$, so that the ratio $\left(\frac{\alpha_{\text{O}}}{\alpha_{\text{M}}} = \frac{\alpha_{\text{E}}}{\alpha_W}\right)$ in eq. (7a) for the observer on airplane E is $1 + 2.5758\text{x}10^{-12}$. The corresponding ratio for the observer on airplane W is the *reciprocal* of this value. When they exchange signals, E observes a blue shift $\Delta\nu\left(\text{E}\right) > 0$, whereas W observes a red shift of the same magnitude. The fractional transverse Doppler shift $\frac{\Delta\nu_2}{\nu'}$ for their respective measured frequencies is therefore:

$$\frac{\Delta\nu_2\left(\text{E}\right)}{\nu'} = \frac{-\Delta\nu_2\left(\text{W}\right)}{\nu'} = 2.5758\text{x}10^{-12}. \tag{9}$$

The key point is that according to eq. (7a), this second-order (transverse Doppler) shift is *anti-symmetric* for the two observers rather than being in the same direction for both. In addition, it is important to note that the relative speed of E and W does not enter directly into the computation.

In the above example the clocks on the ground (Earth's surface) are moving with a speed of 416.67 m/s relative to the polar axis. The $\frac{\alpha_M}{\alpha_O}$ ratio for signals emanating from airplane E that are received by an observer G on the ground is thus $1+1.7171\text{x}10^{-12}$. If identical signals are sent from the ground to airplane E, the respective transverse Doppler frequency shifts are thus:

$$\frac{\Delta\nu_2(\text{E})}{\nu'} = \frac{-\Delta\nu_2(\text{G})}{\nu'} = 1.7171\text{x}10^{-12}. \tag{10}$$

In order to obtain this result in practice, a gravitational correction must be applied to the measured frequencies, consistent with the procedure employed in the HK study [4]. The first-order (non-relativistic) Doppler effect must also be eliminated in the actual measurements to be consistent with the above formulas. This can be done experimentally with transponder devices by employing a method used by Vessot and Levine [5] in their experiments with atomic clocks carried onboard a rocket in flight. After both the gravitational and the first-order Doppler corrections are made, the (adjusted) frequency shift $\Delta\nu_2(\text{E})$ should be positive by the fractional amount in eq. (10), whereas the corresponding value $\Delta\nu_2(\text{G})$ that the observer on the ground measures for the signals with the same *in situ* value that arrive from airplane E should be negative. The underlying reason for this anti-symmetry is the fact that the onboard clocks aboard E are known [4, 5] to run slower than those located on the Earth's surface after the difference in gravitational potential is taken into account.

V. The Earth as Rocket Ship

The Doppler experiments discussed in the previous section can be carried out without the use of airplanes or rockets. One simply needs two laboratories located on the Earth's surface at greatly different latitudes to accomplish the same purpose. For example, a clock located at the Equator is moving at a speed of 463 m/s relative to one at either Pole. Thus a laboratory (M) near the Equator can play the role of the rocket ship in the Twin Paradox in the proposed experiment, whereas that at the Pole (O) corresponds to the station from which the flight originated. The key $\frac{\alpha_{\mathrm{M}}}{\alpha_{\mathrm{O}}}$ ratio in eqs. (7a,b) thus has a value of $1+1.1926\text{x}10^{-12}$ in the proposed experiment. A transverse Doppler *blue* shift ($\Delta\nu_2$) of this amount should thus be observed at the Equator, whereas a *red shift of the same magnitude* should be measured at the polar laboratory if the assumption that the time dilation effect is anti-symmetric (reciprocal) is correct. If one assumes instead that the effect is governed solely by the Lorentz invariance condition, as discussed in Sect. II, there should be a *red (negative) transverse Doppler shift of the above magnitude* for each observer, i.e. using $u = 463$ m/s in eq. (4a). There is thus a clear difference in the predictions of the two theories.

Carrying out the proposed experiment in Earth-based laboratories has distinct advantages over using airplanes or rockets. Clearly, it is much easier to obtain optimal stabilities for the various atomic clocks if they are land-based. In addition, it is possible to minimize the influence of the gravitational red shift on the respective frequencies by having the two laboratories at close to the same gravitational potential. The usual gravitational correction ($\Delta\nu_{\mathrm{G}}$) would have to be made in any case. This contribution to the frequency shift is also assumed to be anti-symmetric in character, whereby a blue shift would be observed at the laboratory located at lower altitude, as demonstrated by the experiments of Pound and coworkers [13]. If the difference in altitude is equal to h, $\frac{\Delta \nu_{\mathrm{G}}}{\nu'} = \frac{gh}{c^2}$, where g is the acceleration due to gravity at the Earth's surface.

One problem that is not present when airplanes are used is that the light signals cannot generally be exchanged in straight lines between the two laboratories. At least in principle, this is akin to making a telephone call between them. The signals have to be transmitted with the aid of a transponder device on an appropriately located satellite that is already available for telecommunication purposes. The key point is that both laboratories must be equipped with their own transmission and detection devices so that a "two-way" Doppler experiment is achieved. This includes a device in each laboratory that can be used to determine the magnitude of the first-order Doppler shift from the incoming frequency. Vessot and Levine [5] dealt with this problem by transmitting a light signal to a transponder on the rocket and then measuring the frequency shift as it returned to the laboratory. In this way both the transverse Doppler and gravitational shifts are eliminated since the signal is emitted and detected at virtually the same point in space with a negligible time delay. The value of $\Delta \nu_1$ is just one-half of the latter frequency shift. Since two laboratories (O and M) are involved in the proposed experiment, a first-order shift needs to be measured for each of them ($\Delta \nu_{1\mathrm{O}}$ and $\Delta \nu_{1\mathrm{M}}$) relative to the communicating satellite. The total first-order Doppler shift ($\Delta \nu_1$) is the sum of these two quantities and is the same for both laboratories.

The observed fractional shift $\frac{\Delta\nu}{\nu'}$ for light signals of standard frequency ν' exchanged between the two laboratories is equal to:

$$\frac{\Delta\nu}{\nu'} = \frac{\Delta\nu_{1\mathrm{O}} + \Delta\nu_{1\mathrm{M}} + \Delta\nu_2 + \Delta\nu_{\mathrm{G}}}{\nu'} . \qquad (11)$$

According to the Lorentz-invariant theory of eq. (4a), $\Delta\nu_2$ is the same for both laboratories. In the other case [see eq. (7a)], a blue shift is expected for the laboratory closer to the Equator and a red shift of equal magnitude for the other. The rotational speed of each laboratory depends on its latitude χ_{I}. If we use the polar axis as reference, each clock-rate parameter α_{I} in eq. (7a) is given by (*R* is the radius of the Earth and Ω is its rotational frequency):

$$\alpha_{\mathrm{I}} = 1 + \frac{\left(\frac{R\Omega\cos\chi_{\mathrm{I}}}{c}\right)^2}{2} . \qquad (12)$$

As already stated, the maximum value possible for $\frac{\Delta\nu_2}{\nu'}$ on this basis is 1.1926x10^{-12}, i.e. for laboratories located at the Equator and one of the Poles. It is proportional to $\left(\cos\chi_{\mathrm{M}} - \cos\chi_{\mathrm{O}}\right)^2$ in the Lorentz-invariant theory of eq. (4a), in which case the result is invariant to an exchange of detector and source location (O and M). According to eq. (7a), however, it is proportional to $\cos^2\chi_{\mathrm{M}} - \cos^2\chi_{\mathrm{O}}$, which is clearly anti-symmetric with respect to such an exchange. The value of $\frac{\Delta\nu_{\mathrm{G}}}{\nu'}$ in eq. (11) can be accurately computed and is potentially negligible in any event. The magnitudes of the first-order shifts

$(\Delta\nu_{1O} + \Delta\nu_{1M})$ should be minimized by the nearly transverse motion of the satellite [$\varphi = \frac{\pi}{2}$ in eq. (7a)] and are to be determined by experiment. The time delay for signals to be transmitted between each laboratory and the communicating satellite needs to be taken into account in the determination of the first-order corrections, but this can presumably be accomplished to sufficient accuracy by carrying out the measurements of these quantities on a continuous basis during the course of the experiment.

Measurement of a frequency shift $\frac{\Delta\nu}{\nu'}$ of the order of one part in 10^{12} is certainly feasible with present-day atomic clocks. This is not a "null" experiment in which one looks for deviations from a specific result such as the speed of light in free space or the existence of decay positrons from protons. A definite value for the frequency shift at each laboratory can be observed that is outside the error limits of the measurement. The proposed experiment is thus a potentially definitive one. The $\Delta\nu_2$ shifts should not be time-dependent, unlike the case when airplanes or rockets are used.

If the two transverse Doppler shifts $\Delta\nu_2(\mathrm{M})$ and $\Delta\nu_2(\mathrm{O})$ both have the same value (in magnitude and direction), it would be a stunning verification of the theory (STR) that two clocks in relative motion can each be running slower than the other at the same time. This result is predicted by the invariant-phase derivation of the transverse Doppler effect that is based on the assumption of Lorentz invariance for every inertial system. As safe as this assumption might seem to some, it is in fact *not* supported by the results of the HK experiments [4]. The latter indicate that clocks run more slowly at the Equator (M) than they do at the Earth's Poles (O), from which one must conclude that a second-order red shift will be observed at the latter position and a blue one at the other. The HK formula [eq. (4) of ref. [4]] is derived on the assumption that the ratio of clock rates can be determined solely from knowledge of the respective altitudes and speeds of the laboratories relative to the Earth's polar axis. That is a quantitative indication in itself that measurement is totally objective and rational [9], contrary to what is assumed in STR on the basis of the Lorentz-invariant phase derivation of the Doppler effect.

VI. GPS Simulation of the Two-way Experiment

The GPS navigation technology makes use of a procedure for atomic clocks that can also be applied for the proposed Doppler measurements. The frequency of a clock to be carried onboard a GPS satellite is "pre-corrected" prior to launch so that upon reaching orbit it will be exactly equal to that of an identical clock left behind on the ground. In this way one can directly compare the timing results obtained at both positions in order to accurately measure elapsed times for light signals passing between them.

In an analogous way, the frequencies of the source and detector located at one latitude can be adjusted so as to return to their original values upon being transported to the other latitude in the experiment. For example, let us assume that clocks run Q times slower at the Equator (E) than at the latitude of the other laboratory (A). One would adjust the frequencies downward by this amount while the source and detector are still at the Equator. Upon moving to A their rates will speed up by the same factor, with the end result that they are brought back to their original values at E. A two-way Doppler experiment can then be carried out using the "equatorial" source and detector along with their counterparts running at the normal rate for location A. This procedure obviously eliminates both the first-order

Doppler and gravitational shifts because all detectors and light sources are at rest at the same location.

Experience with the GPS technology indicates that the transport of the atomic clocks by itself has no significant effect on their rates. The pre-correction is done on the ground simply because it is inconvenient to adjust frequencies on an orbiting satellite. In the present case it is clearly easier to alter the frequencies of the clocks in laboratory A and avoid the transport phase entirely. When signals are exchanged between the two sets of devices under these circumstances, it is obvious what will happen. The detector running at the lower frequency will record a blue shift for a signal coming from the uncompensated light source, i.e. $\frac{\Delta\nu_2}{\nu'} = Q-1 \quad (Q>1)$. The corresponding shift for the other detector will be equal in magnitude but opposite in sign ($\frac{\Delta\nu_2}{\nu'} = 1-Q$) because its frequency is Q times greater than for the source with the compensated value. One can choose a larger value for Q than is appropriate when the source and detector are actually located at different latitudes on the Earth's surface, thereby making it easier to measure the Doppler shifts under the simulated conditions. By construction, Q is equal to the ratio $\frac{\alpha_{\mathrm{O}}}{\alpha_{\mathrm{M}}}$ in eq. (7a), so the expected result is obviously consistent with this formula as well as with the predicted anti-symmetry/reciprocity of the theory from which it is derived.

VII. Conclusion

The relativistic Doppler effect is governed by two separate factors that have quite distinct characteristics. When two observers send out light signals to one another, there is a first-order Doppler shift that is the same for both since it only depends on their relative speed. There is also a second-order contribution, however, which potentially destroys this symmetry. It is proportional to the *ratio* of the rates of clocks co-moving with them. As such, there is a reciprocal rather than a symmetric relationship for the two observers for this portion of the Doppler effect. The value of the clock-rate ratio in a given case depends on a number of factors, as demonstrated by the experiments carried out with circumnavigating airplanes by Hafele and Keating [4]. Specifically, it is not always equal to the γ factor that appears in the conventional derivation of the Doppler effect based on the assumption that the phase of an electromagnetic plane wave must be invariant with respect to a Lorentz transformation.

The predicted anti-symmetry in the second-order Doppler effect is perfectly consistent with the relativity principle. The latter only requires that the *in situ* frequency of a given light source be independent of the state of motion, as guaranteed by the general expressions given in eqs. (7a,b). In this case the clock-rate parameters α_{M} and α_{O} are necessarily equal and of course $\beta = 0$, so that $\nu = \nu'$ and $\lambda = \lambda'$ in any such application. Experimental verification of the predicted asymmetry is critical since it would prove that that the LT is not a valid space-time transformation [11,12] and that the precept of STR that "everything is relative" is actually of much more limited validity than is widely assumed. If the clocks on an airplane are known to run more slowly than those on the ground, for example, there shouldn't be any question that those on the ground must therefore be running faster than the ones on the airplane. Indeed, the GPS methodology could not function properly if it was not perfectly clear that this simple rule holds [11]. The time-dilation effect proves that the *unit of time* varies from one inertial system to another, as do those for other physical quantities such as energy and inertial mass. The relativistic Doppler effect can be very useful in obtaining accurate determinations of the ratios of these units for different rest frames.

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