# Why Can't There Be Numbers?

**David Builes** 

Forthcoming in *The Philosophical Quarterly* 

## Abstract

Platonists affirm the existence of abstract mathematical objects, and Nominalists deny the existence of abstract mathematical objects. While there are standard arguments in favor of Nominalism, these arguments fail to account for the necessity of Nominalism. Furthermore, these arguments do nothing to *explain* why Nominalism is true. They only point to certain theoretical vices that might befall the Platonist. The goal of this paper is to formulate and defend a simple, valid argument for the necessity of Nominalism that seeks to precisify the widespread intuition that mathematical objects are somehow 'spooky' or 'mysterious'.

#### **1. Introduction**

Are there any abstract mathematical objects? Platonists say 'yes', and Nominalists say 'no'. Here, I will be concerned with the case for Nominalism. One standard argument for Nominalism is an *epistemic* argument, according to which Platonism should be rejected because there is no plausible epistemology for how we come to know about a realm of causally-inert abstract objects. Another standard argument for Nominalism is a *uniqueness* argument, according to which Platonism should be rejected because it does not provide unique referents to mathematical singular terms like '2'.<sup>1</sup> According to Balaguer (2009), 'these are widely regarded as the two most important objections to Platonism' (87). Perhaps the next most common argument is an argument from parsimony, according to which Platonism should be rejected because an ontology without abstract mathematical objects is simpler than an ontology that includes such objects.

There are two serious problems with all of these arguments. First, none of these arguments account for the *metaphysical necessity* of Nominalism, which is the orthodox Nominalist position. It would certainly be nice if the Platonist could avoid these three objections. However, there is no modal requirement that we can't be in a skeptical scenario (e.g. there are possible worlds with brains in

<sup>&</sup>lt;sup>1</sup> Benacerraf (1965, 1973) is the classic source for both of these arguments. For more recent discussions, see Warren (2017) and Cowling (2017: 162–86).

vats), nor is there a modal requirement that our singular terms must have unique referents, nor is there a modal requirement that the world must be parsimonious. Second, none of these arguments provide any satisfying explanation as to *why* there aren't any abstract objects. They only point to certain theoretical vices that might befall the Platonist.

It is surprisingly hard to find a compelling argument for Nominalism that avoids these two problems.<sup>2</sup> Because these arguments are so hard to find, some philosophers have argued that Nominalists should simply abandon orthodoxy and regard their thesis as a *contingent* truth.<sup>3</sup> However, many Nominalists don't base their belief in Nominalism (or its necessity) on any particular *argument*. Rather, they simply have a basic intuition that there couldn't be abstract mathematical objects.<sup>4</sup> If pressed, they may simply say that such objects are too 'mysterious' or 'spooky' to be admitted into one's ontology.

It is easy to be dismissive of such inchoate intuitions. However, I will argue that there is a fairly straightforward, valid argument for Nominalism that can be rescued from these intuitions of mysteriousness. Of course, like almost any other philosophical argument, its premises can be resisted. However, each of the premises are at least *prima facie* plausible. Moreover, each of the premises can also be supported by philosophical views that are *independent* of the debate between Platonists and Nominalists. Lastly, the argument avoids the two serious problems that plague the (arguably) most popular arguments in favor of Nominalism. For all these reasons, I believe the argument deserves to be a major contender in the Nominalist's arsenal.

<sup>&</sup>lt;sup>2</sup> There are some arguments that purport to do this that are far less popular than the standard three arguments above. For example, Goodman thought that the existence of sets was somehow unintelligible because different entities couldn't be composed of the same constituents (e.g.  $\{O\}$  and  $\{\{O\}\}\}$ )). Even if this argument could be made to work, in order to establish the necessity of Nominalism it would have to be supplemented with an argument that there couldn't be mathematical objects that aren't sets. Some have thought that abstract objects couldn't exist because they lack clear 'identity conditions' (e.g. Quine 1960: 200–9). At least prima facie, however, sets do seem to have precise identity conditions (e.g. two sets are identical if they have the same members). Jubien (1996) and Merricks (1998) also argue against the general requirement for informative criteria of identity. One might also think that Platonism should be rejected because it leads to set-theoretic paradoxes. However, there are standard ways to avoid these paradoxes, and even if none of them succeed, this does not explain why other mathematical objects like the natural numbers can't exist.

<sup>&</sup>lt;sup>3</sup> For example, Rosen (2006) criticizes the view that Nominalism should be necessary, but he describes the 'Standard Conception' of metaphysical necessity as one which is committed to such a claim. Clarke-Doane (2019a, 2019b) argues that even if Nominalism is regarded as 'metaphysically' necessary, there is a broader sense of genuine possibility on which it should be regarded as contingent. Balaguer (1998) also argues for a similar conclusion in saying that 'it is doubtful that our mathematical theories are necessary in any interesting sense' (44).

<sup>&</sup>lt;sup>4</sup> See, for example, Goodman and Quine (1947: 105).

Baldly stated, here is the argument<sup>5</sup>:

- 1. Necessarily, there are no bare particulars.
- 2. Necessarily, if there are abstract mathematical objects, then there are bare particulars.
- 3. Therefore, necessarily, there are no abstract mathematical objects.

The argument brings together two largely separate strands of philosophy. The first premise draws on very general metaphysical considerations concerning the relationship between objects and properties, and the second premise draws on the increasingly popular 'structuralist' approach to the philosophy of mathematics. In the next two sections, I will clarify and motivate both premises.

### 2. The First Premise

The intuitive idea of a bare particular is an object that has no intrinsic properties: no color, no shape, no size, no mass, no charge, no causal powers, no conscious experiences, etc.<sup>6</sup> However, this intuitive idea needs to be precisified. Wouldn't such an object have the property of *being non-red*, or the property of *being self-identical*, or the property of *being such that all ravens are ravens*? In order to avoid unintended 'counterexamples' like these, one needs to be operating with a sparse conception of properties. How one decides to precisify the intuitive idea of a bare particular will be sensitive to one's underlying metaphysical resources. For example, if one believes in universals (sparsely construed), then one may say that a bare particular is an object that instantiates no monadic universals.<sup>7</sup> Alternatively, if one believes that there is a distinguished class of properties that are (perfectly) natural, then one may say that a bare particular is an object that has no (perfectly) natural intrinsic properties.<sup>8</sup> Other philosophers may recognize a distinction between properties that are genuine *qualities* (e.g. *being red*) and other 'properties' that merely play the same semantic role as qualities in our language (e.g. *being such that there are no unicorns*).<sup>9</sup> On this approach, we may say that a bare particular is an object that there is.<sup>10,11</sup>

<sup>&</sup>lt;sup>5</sup> A precursor to this argument can be found in the following quote by Russell (1903): 'If [numbers] are to be anything at all, then they must be intrinsically something' (86).

<sup>&</sup>lt;sup>6</sup> These examples are meant to be intuitively intrinsic, but philosophers have questioned each of them. For more on the distinction between intrinsic and extrinsic properties, see Marshall and Weatherson (2018).

<sup>&</sup>lt;sup>7</sup> For more on universals, see Armstrong (1989).

<sup>&</sup>lt;sup>8</sup> For more on natural properties, see Lewis (1983) and Dorr and Hawthorne (2013).

<sup>&</sup>lt;sup>9</sup> For more on qualities, see Heil (2012: 53–83)

<sup>&</sup>lt;sup>10</sup> Those philosophers who follow Goodman (1955) in thinking that any such distinction among (abundant) properties is a mere projection of our conceptual scheme will be unable to make sense of the notion of a bare particular.

<sup>&</sup>lt;sup>11</sup> Philosophers use the term 'bare particular' in different ways. Sometimes it is simply used to refer to any underlying 'substratum', within the context of the substratum theory. My usage of the term resembles Sider's (2006) notion of a 'truly bare particular' and Perovíc's (2017) notion of a 'genuinely bare particular'.

I take it to be at least *prima facie* plausible that there can't be bare particulars. A wide variety of philosophers, both historically and in contemporary times, have thought that bare particulars are in some way incoherent or unintelligible.<sup>12</sup> Try to positively conceive of a world that contains nothing but a single bare particular. Such a world would be completely devoid of colors, shapes, masses, particles, fields, conscious experiences, etc. Now conceive of a world that contains nothing at all. Did you manage to conceive of two clearly distinct possibilities? To many, the answer will be no. Insofar as one finds bare particulars to be inconceivable, this gives one some (defeasible) reason to think that bare particulars are not genuinely possible.<sup>13</sup> In writing about the 'obvious incoherence' of bare particulars, Strawson (2017) writes, 'Clearly there can no more be objects without properties than there can be closed plane rectilinear figures that have three angles without having three sides...to be is necessarily to be somehow or other, i.e. to have some nature or other, i.e. to have properties' (69). Armstrong (1997: 109–10) claimed that bare particulars were 'vicious abstractions' and was explicit about their impossibility when building his own metaphysics. Although Sider (2006) defends the intelligibility of bare particulars, he aptly summarizes a common attitude towards bare particulars in saying that 'bare particulars are widely regarded as the grossest of metaphysical errors' (392).

As well as being pre-theoretically attractive, the first premise can also be *derived* from metaphysical views that are independent of the debate between Platonists and Nominalists. For example, metaphysicians disagree about how individuals (or 'objects' or 'particulars') relate to the properties that they have. On one view, the substratum theory, individuals and properties belong to two separate ontological categories, and individuals 'instantiate' properties. On a rival view, the bundle theory, there are really only properties (either tropes or universals), which may be bundled together by a relation of 'compresence'. When the substratum theorist says that there is some individual *a* that instantiates properties *F* and *G*, the bundle theorist dispenses with *a* and simply says that *F* and *G* are compresent. Various different versions of the bundle theory have been defended by many different philosophers.<sup>14</sup> All of them, however, imply that bare particulars are impossible. If there are no properties to bundle together, then there is no corresponding object.

In recent times, many other anti-individualist views have been developed in addition to the bundle theory. For example, Dasgupta (2009, 2017) defends a view called 'Algebraic Generalism' that accounts for some standard objections to the bundle theory. Turner (forthcoming) develops a metaphysical theory that dispenses with individuals in favor of a theory that is described using 'predicate functors'. Strawson (2008) argues that we should collapse the object-property

<sup>&</sup>lt;sup>12</sup> See, for example, Plato's *Timaeus* (48c–53c), Aristotle's *Metaphysics* (1029a20–33), Locke (1977), Russell (1996), Mertz (2003), Lowe (2003: 86), Bailey (2012), and Giberman (2012).

<sup>&</sup>lt;sup>13</sup> See Chalmers (2002) for more on the nature of positive conceivability and its relationship to possibility.

<sup>&</sup>lt;sup>14</sup> See, for example, Russell (1940), Williams (1953), Campbell (1990), Bacon (1995), Paul (2002, 2017), and Keinänen and Tahko (2019).

distinction altogether. On his view, 'the being of an object is literally identical with the being of its propertiedness' (281).<sup>15</sup>

The reasons why metaphysicians have been attracted to anti-individualist views are many and varied. Some are attracted by the elegance and simplicity of having a one-category ontology over an ontology consisting of both individuals and properties (e.g. Paul 2002, 2017). Some wish to avoid 'haecceitistically' distinct metaphysical possibilities that only differ about *which* individuals have which properties. Not only do these possibilities involve empirically undetectable differences, but they also threaten the viability of determinism. For example, the famous *hole argument* in the context of general relativity purports to show that the theory of general relativity fails to be deterministic because of certain haecceitistically different possibilities.<sup>16</sup> Dasgupta (2009, 2017) has also argued that individuals should be eliminated for the same reasons that absolute velocity should be eliminated from our best physical theories.<sup>17</sup>

This is not the place to fully assess the costs and benefits of these different metaphysical approaches. The main point I wish to make is simply that, in addition to being pre-theoretically intuitive, the first premise can also be supported by independent metaphysical arguments.

### 3. The Second Premise

The second premise receives *prima facie* support from the standard negative characterization of abstract objects. Abstract objects are supposed to lack any physical properties, mental properties, spatiotemporal properties, or causal properties.<sup>18</sup> Bare particulars lack all such properties as well, since they lack all properties simpliciter. However, one might argue that there are distinctively *mathematical* properties that count as intrinsic qualities, or intrinsic (perfectly) natural properties, which serve to distinguish mathematical objects from bare particulars. For example, perhaps the number 4 has the intrinsic quality of *being even*, and the number 7 has the intrinsic quality of *being prime*.

In response to this suggestion, the best defense of the second premise involves a *structuralist* approach to mathematical properties, which is becoming increasingly popular in the philosophy of

<sup>&</sup>lt;sup>15</sup> See Strawson (2021) and Builes (2021) for further development of this view.

<sup>&</sup>lt;sup>16</sup> See Norton (2019) for an overview of the hole argument. Perhaps the most popular way to secure determinism while holding on to haecceitism appeals to facts involving the essences of space-time points. See Teitel (2019) for arguments against these essentialist approaches. For more on the relation between space-time and haeceeitism, see Dasgupta (2015). See Hawthorne (2006) for general arguments against determinism given haecceitism.

<sup>&</sup>lt;sup>17</sup> Although Dasgupta's version of 'Quantifier Generalism' seems to allow the existence of bare particulars, his 'Algebraic Generalism' seems to rule out bare particulars, insofar as it describes the world in terms of which qualities are stitched together with which other qualities.

<sup>&</sup>lt;sup>18</sup> For more on the abstract-concrete distinction, see Rosen (2020).

mathematics.<sup>19</sup> In slogan form, structuralism is the thesis that mathematics is the study of purely *structural* or *relational* features of things, and, as such, it does not concern itself with the intrinsic nature of the particular objects that stand in such relations. For example, to say that the number 4 is 'even' is simply to say that the number 2 stands in the *is-a-divisor-of* relation to 4. To say that the number 7 is prime is simply to say that there do not exist any numbers (other than 1 or 7) that stand in the *is-a-divisor-of* relation to 7. Here are some representative structuralist remarks:

To *be* the number 3 is no more and no less than to be preceded by 2, 1, and possibly 0, to be followed by 4, 5, and so forth. (Benacerraf 1965: 291)

Mathematics is concerned with structures involving mathematical objects and not with the 'internal' nature of the objects themselves. (Resnik 1997: 529)

[There] is no more to the individual numbers 'in themselves' than the relations they bear to each other. (Shapiro 1997: 73)

Although these remarks might seem applicable to numbers, one might initially question whether they apply to all mathematical objects. Don't the objects of Euclidean Geometry, such as lines and shapes, have an intrinsic nature? Can't we speak of functions being intrinsically continuous or differentiable? Don't sets 'contain' other sets as part of their internal structure? Don't sets have intrinsic 'sizes' corresponding to their cardinality? Although these kinds of cases might seem like counterexamples, further reflection on them only supports the claim that mathematical objects are bare particulars. By definition, abstract mathematical objects are entirely *non-spatial*. The Euclidean abstract triangle, for example, cannot *literally* have three sides or three angles (or any other geometrical properties) if it has no spatial extension whatsoever! It is likewise misleading to visualize a continuous or differentiable function as analogous to a continuous or differentiable line on a piece of paper. Lines on paper have all sorts of intrinsic geometrical properties, but abstract functions cannot have any geometrical properties, since they also completely lack spatial extension. In response, one could associate these geometric objects are bare particulars reduces to the question of whether set-theoretic constructions of numbers (such as  $\mathbb{R}^3$ ) are bare particulars.

In the case of sets, we might analogize sets 'containing' other sets by reference to ordinary physical objects containing various others. However, the 'membership' relation in set theory is exactly that: a *relation*. To say that the empty set is a member of the singleton of the empty set is only to say that one object stands in a certain asymmetric relation to another. Moreover, in claiming that a set has a certain intrinsic 'size' or cardinality, one is only claiming (by definition of cardinality) that there exists a bijective function that *relates* the original set to another set (a cardinal). In fact, once one recognizes that the only non-logical notion in the language of set theory is a binary relation (the 'membership' relation), and once one acknowledges that the whole of mathematics can be

<sup>&</sup>lt;sup>19</sup> For an overview of structuralist approaches to mathematics, see Reck and Schiemer (2020).

done in the language of set theory, it follows that mathematical claims only ever make claims about which objects stand in which relations to others.<sup>20</sup>

As John Burgess (2015) argues in detail, the advent of the modern axiomatic method of mathematics, in which mathematical structures are characterized by formal axioms, sought to replace the kinds of informal geometric, spatial, and other forms of intuition that used to be prevalent in mathematical proofs and definitions. Although these kinds of informal intuitions about mathematical objects can certainly be helpful to the practicing mathematician, the quest for a rigorous foundation of mathematics has squarely favored the axiomatic method. For our purposes, the most important consequence of the axiomatic method is that formal axioms can only ever characterize mathematical structures *up to isomorphism*. Since the intrinsic qualities of objects are not invariant across isomorphic structures, it follows that mathematical practice is not concerned with the intrinsic qualities of the objects that it studies.

In light of these sociological observations about contemporary mathematical practice, philosophers of mathematics have developed many different (philosophically controversial) structuralist approaches to mathematics. Some of these approaches are Platonist, such as Shapiro's (1997) *ante rem* structuralism, and others are Nominalist, such as Hellman's (1989) modal structuralism. Structuralism *per se* is therefore neutral with respect to the debate between Platonists and Nominalists. However, one might naturally wonder whether Platonist versions of Structuralism are committed to the existence of bare particulars. How have Platonists responded to this kind of threat? Although the literature on Platonist versions of Structuralism is vast, it is worth making three points in response to this question.

First, it should be pointed out that the notion of a 'bare particular' is a technical notion from the metaphysical literature on objects and properties that is seldom discussed in the literature on the philosophy of mathematics. For this reason, it's hard to know exactly how Platonist versions of Structuralism respond to the threat of bare particulars. Several philosophers have emphasized that Structuralist versions of Platonism *should* endorse the view that mathematical objects have intrinsic properties, such as the property of *being abstract*.<sup>21</sup> However, following our discussion in the previous section, merely claiming that an object has an intrinsic property does not establish that an object is not a bare particular. After all, bare particulars (if they could exist) would have the intrinsic properties of *being non-red* and *being non-massive* and *being non-spatial*, etc. Making sense of the notion of a bare particular requires a metaphysical distinction among intrinsic

<sup>&</sup>lt;sup>20</sup> My discussion of sets is intended to be limited to what philosophers sometimes call 'pure' sets as opposed to 'impure' sets. Pure sets ultimately contain only sets, whereas impure sets ultimately contain concrete objects (a set x 'ultimately contains' y if it x's transitive closure contains y). Maddy (1990) has argued, for example, that sets of physical objects are located in space and time and have causal powers (because they can be perceived by the senses). More generally, views on which impure sets of concrete objects are somehow 'made up of' or 'constituted' by these

concrete objects (and thereby inherent some of the intrinsic qualities of these concrete objects) are outside the scope of my argument. My aim is only to argue against the existence of *abstract* mathematical objects, and impure sets of this kind are arguably not abstract.

<sup>&</sup>lt;sup>21</sup> See Burgess (1999), Reck (2003: 406-409), MacBride (2005: 583-584), and Linnebo and Pettigrew (2014).

properties, which rules out 'merely negative' intrinsic properties such as *being non-red*. If an object has the intrinsic property of *being abstract* in virtue of (say) *being non-spatio-temporal*, *being non-mental*, and *being non-causal*, then *being abstract* will simply be akin to the property of *being non-red*.<sup>22</sup>

Second, a standard way for Platonists to accommodate structuralist intuitions is by claiming that allegedly singular terms like '2' do not successfully refer to unique objects. For example, on popular versions of set-theoretic structuralism, the only mathematical objects are sets, and '2' may be taken to refer to any set that plays the 'role' of 2 in any set-theoretic structure that satisfies the axioms of arithmetic.<sup>23</sup> In saying that '2' does not uniquely refer to any particular set, set-theoretic versions of structuralism can accommodate the structuralist intuition that it would be entirely arbitrary to identify the number 2 with any particular set. However, these versions of Platonism only move the bump under the rug as far as bare particulars are concerned. If *sets* fail to have any intrinsic qualities, then the hierarchy of sets will simply be a hierarchy of bare particulars that stand in 'membership' relations to one another.

Lastly, some Platonists of a Structuralist bent have simply claimed that it is unproblematic for mathematical objects to lack any intrinsic nature (however 'intrinsic nature' is ultimately understood).<sup>24</sup> While I don't think there is any knock-down argument against this kind of attitude, it should be stressed that the possibility of bare particulars is bound up with all sorts of very general debates that metaphysicians have been having for a long time. Even if one finds the idea of a bare particular to be pre-theoretically intelligible, one's ultimate verdict concerning the possibility of bare particulars should be sensitive to the considerations involved in these more general metaphysical debates (such as the debate between the substratum theorist and the bundle theorist). The viability of a Platonist view according to which Plato's Heaven is filled with an infinity of bare particulars that stand in various relations to one another cannot simply be decided on grounds that are internal to the philosophy of mathematics.

In any case, let us turn our attention back to the second premise of the argument:

2. Necessarily, if there are abstract mathematical objects, then there are bare particulars.

<sup>&</sup>lt;sup>22</sup> Different structuralist strategies for providing a defensible analysis of the slogan that 'mathematical entities only have structural properties' in some ways mirror the kinds of attempts to precisify the notion of a bare particular. For example, these strategies typically appeal to metaphysical distinctions involving 'constitutive' properties (e.g. see Reck 2003) or 'fundamental' properties (e.g. see Linnebo and Pettigrew 2014). It's a difficult metaphysical question whether negative properties, such as *being non-massive* should count as 'non-fundamental', given that it's unclear how to ground the fact that [x is non-massive] in any more fundamental property of x.

<sup>&</sup>lt;sup>23</sup> See Pettigrew (2018) for a recent defense of this kind of set-theoretic structuralism.

<sup>&</sup>lt;sup>24</sup> For example, in response to the quote by Russell in footnote 5, Leitgeb (2020) claims that "it is simply not true that in order for [mathematical objects] to be anything at all, they would have to be 'intrinsically something'" (13).

My main argument in favor of the second premise takes the form of a dilemma. On the first horn, one could object to the second premise by saying that mathematics itself is in the business of describing the intrinsic qualities of mathematical objects like the number 2. The problem with taking this first horn is that it seems to fly in the face of mathematical practice. Insofar as mathematicians are only concerned with properties that are preserved across isomorphic structures, mathematics is not in the business of describing the intrinsic qualities of mathematical objects. According to the second horn, one could grant that mathematics does *not* describe any intrinsic qualities of mathematical objects. However, on this view, it is exceedingly unclear how one can deny the second premise. Because abstract objects are non-physical, non-mental, non-causal, and non-spatiotemporal, they don't have any intrinsic qualities that are studied by any other nonmathematical field of inquiry. If the number 2 does have some intrinsic quality, it follows that it must have an intrinsic quality that isn't investigated by any field of inquiry *at all*, mathematical or otherwise. Such a position seems to be committed to a kind of *mysterianism* about the number 2. If no field of inquiry can investigate the intrinsic qualities of the number 2, then such intrinsic qualities seem unknowable. Not only do we not know which intrinsic qualities the number 2 has, but we don't even have a positive conception of what such an intrinsic quality could possibly be like. These intrinsic qualities are entirely *ineffable* to us. Even apart from these epistemological problems, it's not even clear that there *could* be intrinsic qualities that are non-physical, nonmental, non-mathematical, non-causal, and non-spatiotemporal for the number 2 to have.

In sum, those who deny the second premise must *either* reject all structuralist approaches to mathematics in a fairly extreme way *or* countenance the existence of ineffable and unknowable intrinsic qualities of mathematical objects that are not studied by any field of inquiry. Both options involve significant costs for the Platonist.

#### 4. Conclusion

The standard arguments for Nominalism have serious problems. They don't explain *why* mathematical objects don't exist, and they don't account for the *necessity* of Nominalism. I have argued that the widespread intuition that abstract objects are somehow 'spooky' or 'mysterious' can be precisified into an argument that satisfies these two basic desiderata. The reason why abstract mathematical objects can't exist is because bare particulars can't exist. After all, in order to *be* something, you have to be *like* something!<sup>25</sup>

<sup>&</sup>lt;sup>25</sup> Many thanks to Mark Balaguer, Michele Odisseas Impagnatiello, and two anonymous referees for their helpful feedback.

#### References

Armstrong, D. (1989) Universals: An Opinionated Introduction. Boulder, CO: Westview Press.

Armstrong, D. (1997) 'Against "Ostrich Nominalism": A Reply to Michael Devitt', in D.H. Mellor and A. Oliver (eds.) *Properties*, 101–11. New York: Oxford University Press.

Bacon, J. (1995) Universals and Property Instances: The Alphabet of Being. Oxford: Blackwell.

- Balaguer, M. (1998) *Platonism and Anti-Platonism in Mathematics*. New York: Oxford University Press.
- Balaguer, M. (2009) 'Realism and Anti-Realism in Mathematics', in A. Irvine (ed.) *Philosophy of Mathematics (Handbook of the Philosophy of Science)*, 35–101. Amsterdam: North Holland.
- Bailey, Andrew M. (2012) 'No Bare Particulars', Philosophical Studies, 158: 31-4.
- Benacerraf, Paul. (1965) 'What Numbers Could Not Be', Philosophical Review, 74: 47-73.
- Benacerraf, P. (1973) 'Mathematical Truth', Journal of Philosophy, 70: 661–79.
- Builes, D. (2021) 'The World Just Is The Way It Is', *The Monist*, 104: 1–27.
- Burgess, J. (1999) 'Review of Stewart Shairo's philosophy of mathematics: structure and ontology', *Notre Dame Journal of Formal Logic*, 40: 283–91.
- Burgess, J. (2015) Rigor and Structure. Oxford: Oxford University Press.
- Campbell, K. (1990) Abstract Particulars. Oxford: Blackwell.
- Chalmers, D. (2002) 'Does Conceivability Entail Possibility?', in T. Gendler and J. Hawthorne (eds.) *Conceivability and Possibility*, 145–200. New York. Oxford University Press.
- Clarke-Doane, J. (2019a) 'Metaphysical and absolute possibility', *Synthese*. <a href="https://doi.org/10.1007/s11229-019-02093-0">https://doi.org/10.1007/s11229-019-02093-0</a>
- Clarke-Doane, J. (2019b) 'Modal Objectivity', Noûs, 53: 266-95.
- Cowling, S. (2017) Abstract Entities. New York: Routledge.

- Dasgupta, S. (2009) 'Individuals: An Essay in Revisionary Metaphysics', *Philosophical Studies*, 145: 35–67.
- Dasgupta, S. (2015) 'Substantivalism Vs Relationalism About Space in Classical Physics', *Philosophy Compass*, 10: 601–24.
- Dasgupta, S. (2017) 'Can We Do Without Fundamental Individuals? Yes.', in E. Barnes (ed.) *Current Controversies in Metaphysics*, 7–23. New York: Routledge.
- Dorr, C. and Hawthorne, J. (2013) 'Naturalness', in K. Bennett and D. Zimmerman (eds.) *Oxford Studies in Metaphysics Vol.* 8, 3–77. New York: Oxford University Press.
- Giberman, D. (2012) 'Against Zero-Dimensional Material Objects (and Other Bare Particulars)', *Philosophical Studies*, 160: 305–21.
- Goodman, N. (1955) Fact, Fiction, and Forecast. Cambridge, MA: Harvard University Press.
- Goodman, N. and Quine, W. V. (1947) 'Steps Toward a Constructive Nominalism', *Journal of Symbolic Logic*, 12: 105–22.
- Hawthorne, J. (2006) 'Determinism De Re', *Metaphysical Essays*, 239–244. New York: Oxford University Press.
- Heil, J. (2012) The Universe as We Find It. Oxford: Oxford University Press.
- Hellman, G. (1989) *Mathematics Without Numbers: Towards a Modal-Structural Interpretation*. Oxford: Oxford University Press.
- Jubien, M. (1996) 'The Myth of Identity Conditions', *Philosophical Perspectives*, 10: 343–56.
- Keinänen, M. and Tahko, T. (2019) 'Bundle Theory with Kinds', *Philosophical Quarterly*, 69: 838–57.

Leitgeb, H. (2020) 'On Non-Eliminative Structuralism. Unlabeled Graphs as a Case Study,

Part B', Philosophia Mathematica, nkaa009, <https://doi.org/10.1093/philmat/nkaa009>

- Lewis, D. (1983) 'New Work for a Theory of Universals', *Australasian Journal of Philosophy*, 61: 343–77.
- Linnebo, Ø. and Pettigrew, R. (2014) 'Two Types of Abstraction for Structuralism', *Philosophical Quarterly*, 64: 267–83.
- Locke, J. (1977) An Essay Concerning Human Understanding, edited by R. Woolhouse. London: Penguin Books.
- Lowe, E.J. (2003) 'Individuation', in M. Loux and D. Zimmerman (eds.) *The Oxford Handbook of Metaphysics*, 75–95. New York: Oxford University Press.
- MacBride, F. (2005) 'Structuralism Reconsidered', in S. Shapiro (ed.) *The Oxford Handbook of Philosophy of Mathematics and Logic*, 563–589. New York: Oxford University Press.

Maddy, P. (1990) Realism in Mathematics. New York: Oxford University Press.

Marshall, D. and Weatherson, B. (2018) 'Intrinsic vs. Extrinsic Properties', in E. Zalta (ed.) *The Stanford Encyclopedia of Philosophy*, Spring 2018 edn.

<https://plato.stanford.edu/archives/spr2018/entries/intrinsic-extrinsic/>

Merricks, T. (1998) 'There Are No Criteria of Identity Over Time', Noûs, 32: 106–24.

- Mertz, D. W. (2003) 'Against Bare Particulars A Response to Moreland and Pickavance', Australasian Journal of Philosophy, 81: 14–20.
- Norton, J. (2019) 'The Hole Argument', in E. Zalta (ed.) *The Stanford Encyclopedia of Philosophy*, Summer 2019 edn.

<https://plato.stanford.edu/archives/sum2019/entries/spacetime-holearg/>

Paul, L. A. (2002) 'Logical Parts', Noûs, 36: 578-96.

Paul, L. A. (2017) 'A One Category Ontology', in J. Keller (ed.) Being, Freedom, and Method:

*Themes From the Philosophy of Peter van Inwagen*, 32–61. New York: Oxford University Press.

Perovíc, K. (2017) 'Bare Particulars Laid Bare', Acta Analytica, 32: 277–95.

Pettigrew, R. (2018) 'What we talk about when we talk about numbers', *Annals of Pure and Applied Logic*, 169: 1437–56.

Quine, W. V. (1960) Word and Object. Cambridge, MA: MIT Press.

Reck, E. (2003) 'Dedekind's structuralism: An interpretation and partial defence', *Synthese*, 137: 369–419.

Reck, E. and Schiemer, G. (2020) 'Structuralism in the Philosophy of Mathematics', in *The Stanford Encyclopedia of Philosophy*, Spring 2020 edn.

< https://plato.stanford.edu/archives/spr2020/entries/structuralism-mathematics/>.

Resnik, M. (1997) Mathematics as a Science of Patterns. Oxford: Oxford University Press.

- Rosen, G. (2006) 'The Limits of Contingency', in F. MacBride (ed.) *Identity and Modality*, 13–39. Oxford: Oxford University Press.
- Rosen, G. (2020) Abstract Objects, in E. Zalta (ed.) *The Stanford Encyclopedia of Philosophy*, Spring 2020 edn.

<https://plato.stanford.edu/archives/spr2020/entries/abstract-objects/>

Russell, B. (1903) Principles of Mathematics. Cambridge: Cambridge University Press.

Russell, B. (1996) An Inquiry Into Meaning and Truth, 2nd edn. London: Routledge.

- Shapiro, S. (1997) *Philosophy of Mathematics: Structure and Ontology*. New York: Oxford University Press.
- Sider, T. (2006) 'Bare Particulars', *Philosophical Perspectives*, 20: 387–397.

Strawson, G. (2008) 'The Identity of the Categorical and the Dispositional', Analysis, 68: 271–82.

Strawson, G. (2017) The Subject of Experience. New York: Oxford University Press.

Strawson, G. (2021) 'Identity Metaphysics', *The Monist*, 104: 60–90.

- Teitel, T. (2019) 'Holes in Spacetime: Some Neglected Essentials', *The Journal of Philosophy*, 7: 353–89.
- Turner, J. (Forthcoming). 'On Doing Without Ontology: Feature-Placing on a Global Scale', in J. Cumpa (ed.) *The Question of Ontology*. Oxford: Oxford University Press.

Warren, J. (2017) 'Epistemology Versus Non-Causal Realism', Synthese, 194: 1643-62.

Williams, D.C. (1953)' On the Elements of Being: I', Review of Metaphysics 7: 3-18.