Identification of antinomies by complementary analysis

Antinomy of the Cantor's Diagonal Argument

July 20, 2023

Abstract

It has been noticed that self-referential, ambiguous definitional formulas are accompanied by complementary self-referential antinomy formulas, which gives rise to contradictions. This made it possible to re-examine ancient antinomies and Cantor's Diagonal Argument (CDA), as well as the method of nested intervals, which is the basis for evaluating the existence of uncountable sets. Using Georg Cantor's remark that every real number can be represented as an infinite digital expansion (usually decimal or binary), a simplified system for verifying the definitions of real numbers, subsets, and strings was created - the Cantor Criterion, which allows faulty definitions to be pointed out. For the CDA, the connection of formulas defining objects (real numbers, subsets, strings) from outside the list with supplementary formulas was demonstrated - their indirect and indispensable nature testifies to the lack of unambiguity and gives rise to contradictions for Cantor's antinomic formulas. Thus, Cantor's theorem about the higher power of the set of all subsets, using the reductio ad absurdum proof, lost its power and it was indicated that it is necessary to correct the scheme of the Axiom of Specification, which was introduced precisely to exclude antinomies from set theory by excluding from the use of self-referential antinomies and ambiguous supplementary formulas coupled with them identified by the H hypothesis. The method of nested intervals was investigated and it was shown that every real number can be defined by the limit of nested intervals and a countable list of real numbers obtained from a countable pool of all Jules Richard's texts.

Key Words: Cantor, diagonal, argument, method, uncountable sets, self-reference, paradox, antinomies, ambiguities, complementary, complements, a new hypothesis.

Subject Classification: 03E65

Contents

1 Introduction[1][2][3][4]

Uncountable sets, different powers of infinity in set theory were introduced by Georg Cantor in two ways:
1st: Nested intervals method
2nd: A more familiar method is the diagonal method of defining objects outside the list of objects selected from a set of objects with the same properties. Cantor describes how to create new objects using the diagonal[1] method, defining a string outside the list of infinite two-character strings, using characters from the diagonal list. Then this method was carried over to reals and sets, which is especially simple when the list is binary $0-1$ and the reals are also represented in binary with the addition of a separator separating the integer parts, and the subsets with

[1] http://www.logicmuseum.com/cantor/diagarg.htm

an indicator function[2] with identical string structure.

Objections to the conclusions resulting from Cantor's Diagonal Argument CDA[3] and the method itself were raised by mathematicians and philosophers contemporary to Cantor, such as his promoter Kronecker, Poincare, Brouwer, and Wittgenstein. The intuitionist Brouwer commented, "Given that the Cantor number itself is also an element in M, it is suspicious whether the Cantor number is well-defined." Ludwig Wittgenstein considered a modified version of the CDA (in fact, it was not a modified version - just an intermediate and necessary step in determining the Cantor number) showing its ambiguity and connections with self-referential paradoxes. These connections will be explored in detail using some of the most famous antinomies.

The diagonal method was also used to demonstrate Russell's antinomy and resulted in the development of axioms protecting set theory from contradiction, but it only eliminated the universal set generated by unconstrained predicate understanding, although there were reasons to look at self-referential complements of predicates as we did for this antinomy in the article at the beginning.
The axiom of separation [4] was considered the most important of the axioms to avoid Russell's antinomy, and an important factor to prevent contradictions was not to use the set symbol (B) in the φ formula, but this protection may be insufficient for formulas with hidden self-reference.

All texts that correctly define mathematical objects (here: strings, real numbers, or sets) are included in the list of all T texts arranged according to Jules Richard's idea among many other texts. This list also includes nonsensical texts and texts that are irrelevant to our choice, even though they are relevant in other areas of life.
In order to extract interesting textual definitions of given objects from Richard's list and transfer them to the list (S, R or $P(\mathbb{N})$), we will use the Cantor Criterion (CC) which allows us to transfer only those texts that defined any number of characters of interesting mathematical objects.
The f' supplementary list will contain texts that do not.
We examine all texts - none can be excluded, but we can move the texts we are interested into the top of the list, including the Cantor object definition formulas s and s_d. Comparing the string s with the strings from the list spontaneously and mechanically shows their difference resulting from the construction of the string s in relation to the list S. At the same time, it

[2] https://en.wikipedia.org/wiki/Indicator_function
[3] https://en.wikipedia.org/wiki/Cantor%27s_diagonal_argument
[4] https://en.wikipedia.org/wiki/Axiom_schema_of_specification

seems natural that its definition is also correct, since all characters appearing in the diagonal list of valid strings, i.e. strings consisting of only corresponding characters, are replaced with other characters. This article examines the correctness of the definition of s using the Cantor Criterion and the necessary correctness of the definition of s_d - that is, the intermediate diagonal, which is also a complementary definition to s.

The specifics of real numbers and subsets differ slightly in the use of CDA, so they will be discussed separately.

2 The meaning of Antinomy in Mathematics [4][5][6]

To describe reality, including mathematical reality, we use not only digits but also many other signs, often derived from different cultures and alphabets. The language created on this basis is necessary for interpersonal communication, creating definitions, theorems, and proofs, but thanks to combinatorial redundancy it can create many meaningless or non-mathematical connections that can be relatively quickly excluded from mathematics, such as in the sentences: "$cat\ er5t?$", "$naive\ n!ail$".

The problem arises when, despite the appearance of reasonableness and truth, a sentence has nothing to do with truth and reality, as in the following sentences: "*a regular pentahedron*", "*the smallest natural number that needs at least* 15 *words to define*", or "*triangle with four equal sides*".

The real tragedy is the antinomies[5]. Ascribing truth to a sentence and recognizing it as correct, results in the implication of a true negation of the same sentence. Recognizing two contradictory statements as true results in the fact that everything becomes true - even a false statement. The correct recognition of antinomies, which are quite closely related to self-reference, and their elimination from mathematics is an important task.

3 Known antinomies and their complementary formulas

3.1 The Liar's Paradox[6]

Let's divide people into two complementary sets: $A{-}liars$ and additionally: $A'{-}truthful\ ones$

[5] `https://en.wikipedia.org/wiki/Antinomy`
[6] https://en.wikipedia.org/wiki/Liar paradox

Which set should include person X who says: X : "$I'm\ lying$" —-?

$X \notin A, X \notin A'$——— classical antinomy = contradiction

And to which set can we include person Y saying: Y : "$I'm\ telling\ the\ truth$" -?

$Y \in A'$- it is obvious, but if $Y \in A$, he would also have to say the same sentence: "*I am telling the truth*", that is: $Y \in A'$ *or* $Y \in A$, which in turn gives ambiguity in the choice = we have an excess of implementation possibilities here.

3.2 Aunt's paradox – equivalent to the barber's paradox[7]

The paradox arises from the definition of: "$Aunt\ C\ only\ likes\ people\ who\ don't\ like\ themselves$"

Let's divide people into two complementary sets: A – *people who like each other* and complementary: A' – *people who do not like themselves*

$C \notin A,\ C \notin A'$– antinomy = contradiction - no implementation possibilities.

Y – *any other person* (also *you*) can classify yourself according to your internal, current belief depending on e.g. hairstyle, carcass, achievements, etc. to any set,

$Y \in A'$ *or* $Y \in A$ - excess of implementation possibilities = ambiguity of the definition

3.3 Russell's paradox[8]

In Russell's paradox, considering the property:

"*Being your own element*",

we can create two classes of sets: $V = X : X \notin X$, let's call it a *normal class*, and a complementary class $V' = X : X \in X$, which we'll call *strange class* because it contains sets containing themselves as elements. If class V were a set, is it strange or normal? If normal, then it should contain itself as a class of all normal sets, which in turn would contradict the fact that normal is one that does not contain itself. The opposite assumption also leads to a contradiction, that is, we cannot qualify V to any of the classes. On the other hand, class V' may be normal if, of course, it was a set and then it would not contain itself, because although it would collect only strange sets, it does not have to contain itself as an element. The second possibility is that V' might be a strange class because it would contain itself as an element.

The contradiction of class V implementation is also demonstrated by the diagonal method, where numbered sets are placed in the row and column headers in the same order, and in the table itself, at the intersection of a row and a column, a 1 is placed if the set from the row is included in the set from the column - otherwise, a 0 is placed. From the diagonal of the table, we

[7] `https://en.wikipedia.org/wiki/Barberparadox`

[8] https://en.wikipedia.org/wiki/Russell

select all those sets for which there is a 0, i.e. sets that do not contain themselves as an element, and we want to create a new set V from them. Since we can put all sets in the table, and so there could also be a set V, and then we can't put any sign on the diagonal for that set. This is to indicate that such a set V does not exist. Note that the same can be done with the set of all other sets V' marked with the symbol 1 on the diagonal, collecting sets containing themselves as an element. Here, however, we can implement both signs at the same time: 0 and 1. This contradiction and ambivalence results from the self-referential feature of "including oneself as an element" and the (scheme) Axiom of Specification works well here because it excludes the existence of a universal set from which this predicate was supposed to create sets in Russell's antinomy.

4 Hypothesis H

The above observations about the self-reference of formulas allow me to formulate a hypothesis:

H: Self-referential formulas that give rise to antinomian contradictions are accompanied by ambiguous self-referential supplementary formulas.

The paradox disclosed by Russell in 1901 initiated the emergence of a system of axioms to prevent contradictions, putting an end to the naive set theory created by Cantor and his belief that simple predicates and formulas would be enough for defined objects to exist. Consider Cantor's other ideas in light of the above hypothesis and the fact that the language in which they were articulated can describe more than just Platonic reality. The modern set theory includes not only the set theories based on the ZF or ZFC axioms[9] but also the earlier Cantor diagonal method in its various versions and generating various infinity powers along with the scale of alephs and betas.

It is true that Jules Richard in 1905, i.e. a few years after the publication of the diagonal method[10] by Cantor and a few years before the introduction of set-theoretic axioms, raised objections to this method, but an explanation for this paradox[11] was found[12].

[9] `https://en.wikipedia.org/wiki/Zermelo%E2%80%93Fraenkel_set_theory`
[10] `wikipedia-diagonalargument`
[11] `wikipedia.org/wiki/Richardparadox`
[12] Error becomes an error when it is born as truth – Stanisław Jerzy Lec (Polish poet writer)

5 Important accents allowing to decide about the existence of uncountable sets

5.1 Writing difficulties. Defining objects.

A finite number of digits is sufficient to represent any rational number, with the optional addition of one minus sign and a slash, with the additional proviso that there is no zero below the slash.
Irrational numbers, which extend rational numbers to real numbers, cannot be written in the same way. They are often written as an infinite sum with a definite limit, but not all series have such a limit.

Definitions are written in a higher-level language with many extra characters. They are more compressed and even infinite strings can be written in a finite form - e.g. a string of ones as 1(1)...
The same object can be defined by several different definitions - for example, the number π can be defined[13] by various finite formulas that allow you to calculate the successive digits of its infinite digital representation.

It doesn't matter what definition you choose to represent an object - all equivalent and valid definitions define the same object. Each object is defined by an infinite number of different notations, as seen in the case of numerical definitions by multiplying them by $\frac{n}{n}$, in the case of sets by summing their multiple intersections, and in the case of words by adding a space at the end of the definition.

Definitional redundancy gives rise to the defect of apparent correctness, where, contrary to appearances, there are no objects defined by them, such as: "a regular decahedron", "a triangle with sides of 2 cm, 3 cm, 4 cm, 5 cm" or "a magnetic monopole consisting of a broken part iron ball after permanent, separate magnetization of each fragment from the convex side with the north pole".

The problem of defining (Tarski[14]) and deciding whether a given definition correctly defines the subject under study has been known for a long time.
Jules Richard noticed that the set of all texts can be put into one countable list, and although all definitions are there (including definitions of real numbers and, in a special case, the definition

[13] `https://en.wikipedia.org/wiki/Pi`
[14] `https://en.wikipedia.org/wiki/Tarski%27s_undefinability_theorem`

of Cantor numbers) the problem is to extract from the list of all texts only such an unambiguous list that contains definitions of real numbers (because only such a list is applicable for Cantor's Diagonal Argument):
(quote[15])

> Richard (1905) presented a solution to the paradox from the viewpoint of predicativisim. Richard claimed that the flaw of the paradoxical construction was that the expression for the construction of the real number r does not actually define a real number unambiguously, because the statement refers to the construction of an infinite set of real numbers, of which r itself is a part. Thus, Richard says, the real number r will not be included as any r_n, because the definition of r does not meet the criteria for being included in the sequence of definitions used to construct the sequence r_n. Contemporary mathematicians agree that the definition of r is invalid but for a different reason. They believe the definition of r is invalid because there is no well-defined notion of when an English phrase defines a real number, and so there is no unambiguous way to construct the sequence r_n.

It is worth noting that the CDA is to apply to any list of relevant objects - not necessarily containing all objects, because the conclusion is that we will not create such a list. A criterion can be used that will omit some correct definitions and examine only those that we are currently able to classify in a finite time - e.g. 1 hour of work at the computer.
Therefore, we can omit such definitions that we are currently unable to compute or solve for scientific or prior art reasons, such as "x is equal to 1 when Goldbach's hypothesis is true and 0 when it is false" because x can be defined in a different, simpler way.
Cantor noted that the various ways of defining and constructing irrational numbers (as well as all real numbers) would only be valid if they had an unambiguous (albeit infinite) digital (e.g. decimal) representation.

5.2 CC - Cantor's Criterion

This allows the generalization of Cantor's observation to similar areas, limiting the scope of its applicability to definitions decidable with the current state of knowledge and technical possibilities, and using Cantor Criterion CC to verify the definitions as follows:

for real number $\mathbb{R}$: CC:= every real number, also defined by the definition of L_i, has a unique digital representation,

[15] https://en.wikipedia.org/wiki/Richard%27s_paradox

for subsets $\mathbb{N}$: CC:= every subset $\mathbb{N}$, also defined by the definition of t_i, has a unique its indicator function,

for binary strings: CC:= every binary string, also defined by the s_i has all its signs uniquely defined.

Validation of a definition by CC or otherwise will not disgrace a good definition, but rejection by such a simple criterion as CC will result in exclusion from the exclusive club of good definitions, where missed definitions will be guaranteed entry through more complex criteria.

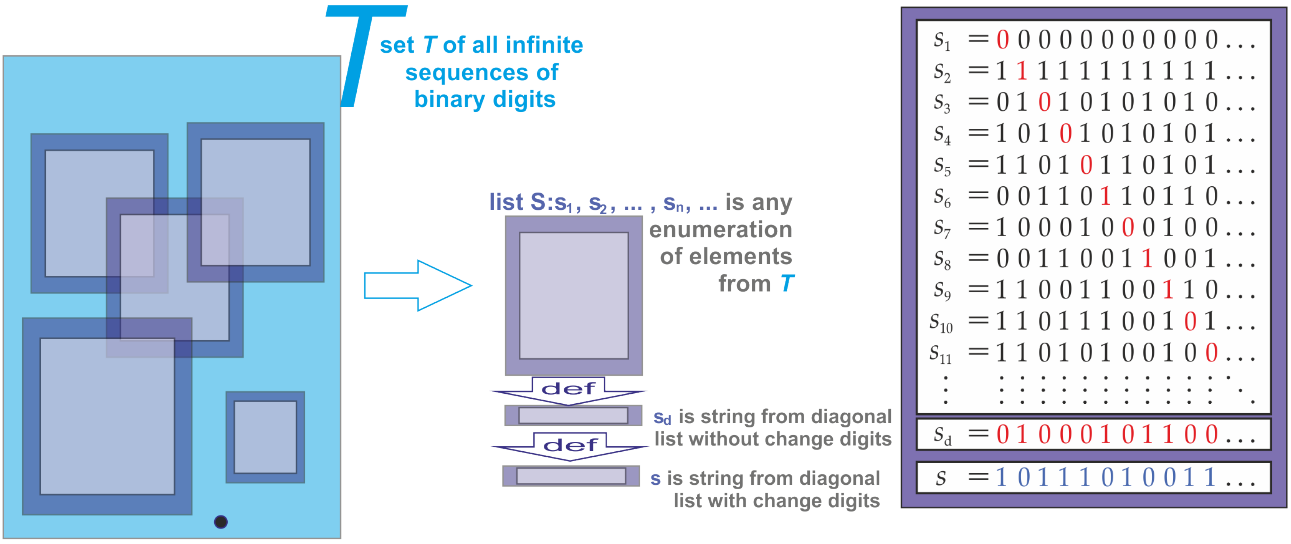
6 CDA Cantor's Diagonal Argument

In the lemma, Cantor suggested that for any list taken from a set T containing all binary strings, he could construct a binary string s from T not contained in that list. [16]
CDA seems to be an extremely simple way to define the string s since there are only 0s and 1s in the entire binary list, of the characters from the diagonal of the list are selected and converted from 0 to 1 and from 1 to 0.

Each S list can be represented as successive strings of symbolic characters:
$s_1 = a_{11}a_{12}a_{13}a_{14}a_{15}\ldots$
$s_2 = a_{21}a_{22}a_{23}a_{24}a_{25}\ldots$
$s_3 = a_{31}a_{32}a_{33}a_{34}a_{35}\ldots$
$\ldots$
where $a_{nm} \in \{0, 1\}$
and then define s as
$s = a_1a_2a_3a_4a_5\ldots$ where each $a_i = 1 - a_{ii}$,
which is
$a_{ii} = 0 \Rightarrow a_i = 1$, and $a_{ii} = 1 \Rightarrow a_i = 0$

Definition of the s string:
The sign at the nth position of string s is different from the sign at the nth position of the nth string of the examined list.

[16] https://en.wikipedia.org/wiki/Cantor%27s_diagonal_argument#cite_ref-10



CDA consists of two successive steps:

1. Extracting characters on the diagonal of a list - creating an s_d string.
2. Replacing characters in the string s_d - creating the string s.

The first step is often unexposed and omitted, and one can even imagine that the characters on the diagonal are first swapped and then placed into the string s, but the result must be the same regardless of the order of these operations that make up the Cantor string s. The existence of the string s is conditioned by the existence of the string s_d.

Important:

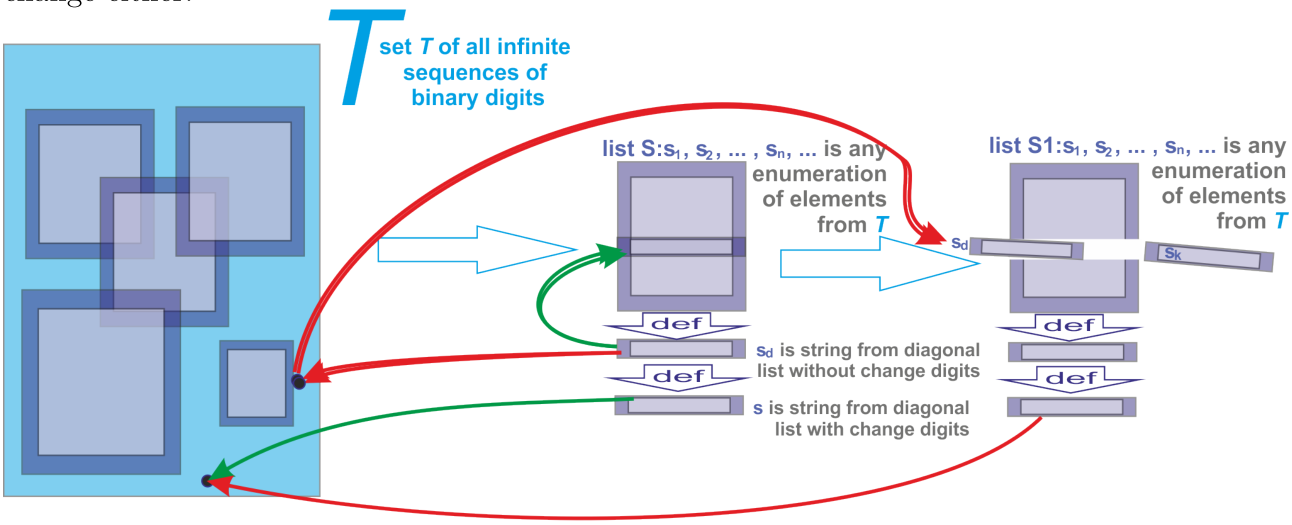
The string s is well defined when the string s_d is well defined

Ludwig Wittgenstein[17] modified the definition of generating the string s and received the definition of the string s_d, but neither he nor Chaohui Zhuang[18], analyzing his reasoning[8], noticed that the definition of s_d is necessary, intermediate, and complementary to the definition from s. The modification was ignored because such a process may contain errors. Creating new objects s_d, s described by CDA (strings, real numbers, or subsets) outside of a list requires building a proper list. In Cantor's lemma, the source of such a list of infinite two-character strings is simply the set T of all such strings.

[17] Ludwig Wittgenstein, Edited by G. H. von Wright, Translated by G. E. M. Anscombe. Zettel. University of California Press, 1970.

[18] `https://philarchive.org/rec/ZHUWAO`

We have two cases of the presence of the string s_d in list S:
1. The string s_d defined by the characters in the diagonal list is in the parsed list at some position k (green line).
2. The string s_d is not in the list S (red line). Such a string is in the set T, from where we can take it and put it in place of any word in the list S, creating a new list $S1$. Such substitution will not change the value of the signs on the diagonal, which means that the string s will not change either.



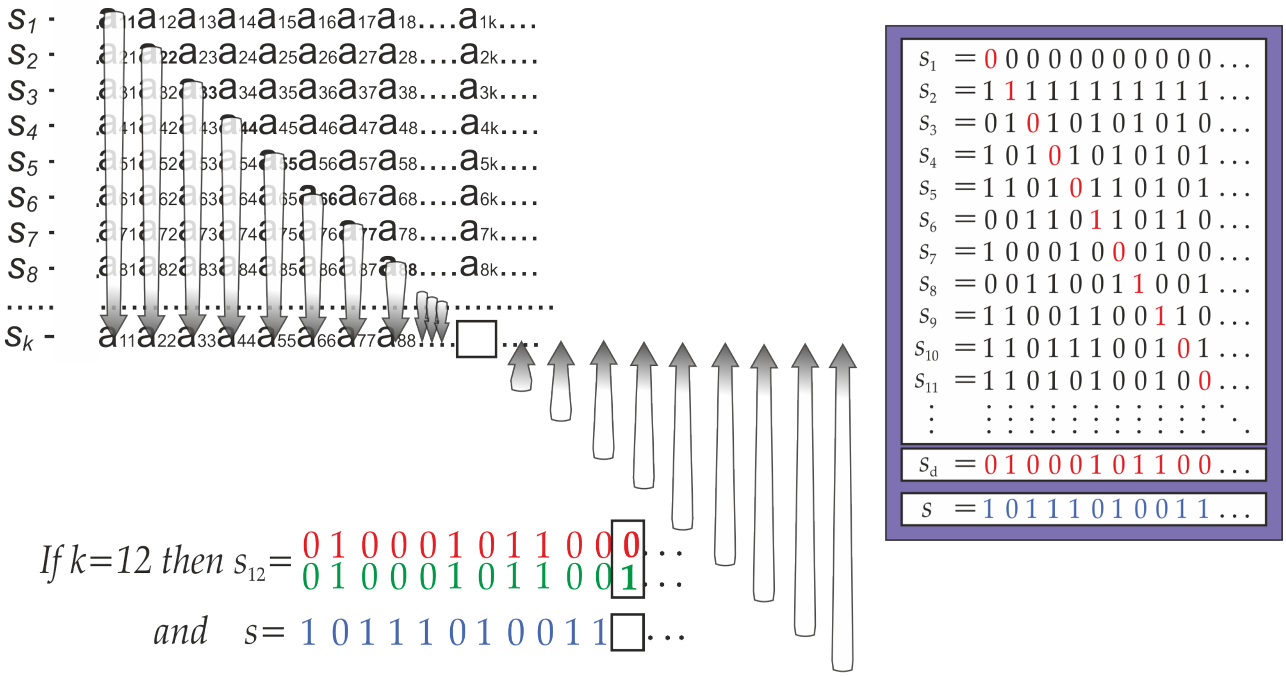
So for each string s there is a list of strings (S or $S1$) taken from T, for which the definition of s_d should uniquely define a two-character string in that list.

The definition of this string s_d is:
The sign at the nth position of the string s_d is identical to the sign at the nth position of the nth string of the examined list.

And symbolically for a list of binary strings:
$s_d = d_1 d_2 d_3 d_4 \ldots$, where $d_i = a_{ii}$ (or: $a_{ii} = 0 \Rightarrow d_i = 0$, and $a_{ii} = 1 \Rightarrow d_i = 1$)

The definition of the string s differs from the definition of s_d by changing the characters in the defined string.
The definition of s_d places the defined string in line k or list S or list $S1$, the sign at position k of this string, i.e. d_k is the same as the sign on the diagonal of the list in the k column, i.e. a_{kk} which means equality:
$d_k = a_{kk}$ and is a repetition of the definition of the sign of the string s_d in the kth position, and since each character of this string is to be identical with the characters from the diagonal of the list from the appropriate column, then in this case, the equation is undetermined and

means that the string at position k can contain any character - and in the case of a binary string, both 0 and 1. This ambiguous definition can be illustrated with a diagram:



As you can see, all the diagonal elements of the list are moved to the s_d string embedding line - except for position k where we can put all available characters as defined. For the given example of binary values, these two different strings:

010001011000...

010001011001...

meet the definition of s_d, which means that it is not a correct definition, because it is not unambiguous, and because it is an intermediate definition for the definition of the string s - this definition is also wrong, which can also be explained by the fact that if there were both different strings in one line k - then at position k in the string s_d all available states would be occupied, and for the string s at this position k no character other than the character in s_d can be found - implementation of a different character in s is not possible.

This is in line with the previously articulated hypothesis H in this paper, identifying the definition of s as an antinomy and the complementary definition of s_d as ambiguous.

The specifics of real numbers and subsets differ slightly in the use of CDA, so they will be discussed separately.

6.1 CDA for real number $\mathbb{R}$

For the most common form of representing CDA for real numbers in the range (0,1), it is enough to precede each string with a prefix consisting of the characters "0" and a separator "." - then we will receive a list of real numbers expressed in binary, a binary Cantor number s, the definition of which can be examined - taking into account, of course, the indirect and supplementary definition of the real Cantor number s_d resulting from the highlighted digits on the diagonal of the list without replacing them.

For real numbers using decimal expansion, we can say that in the kth place of string s_d, without changing the digits, as many as ten digits can be inserted, which gives ten different numbers that meet the definition of s_d, and consequently no implementation possibilities for the definition of s.

6.2 CDA for subset $\mathbb{N}$

Sequences consisting only of the sign 0 and sign 1, from the set T, are ready-made indicator functions[19] for the subsets of $\mathbb{N}$, whose unambiguous representation determines the correctness of the definition of the set in accordance with the adopted Cantor Criterion.

Subset definitions of $\mathbb{N}$ used in Cantor's Theorem[20] can be expressed in a symbolic language using the Axiom of Specification in the form: $\{x \in \mathbb{N} : \varphi(x)\}$. This axiom is supposed to guarantee the existence of the $\mathbb{N}$ subset for all elements of $\mathbb{N}$ satisfying the φ predicate, where the symbol of the defined set B does not occur freely in the φ formula.
It was introduced to liberate set theory from Russell's paradox[21] and is considered by many to be the most important axiom.

The Cantor definition $\{x \in \mathbb{N} : x \notin f(x)\}$ should define the set B for every function $f : \mathbb{N} \to P(\mathbb{N})$ outside the list f, and the supplementary definition $\{x \in \mathbb{N} : x \in f(x)\}$ should define supplementary set $B' = \mathbb{N} \setminus B$.

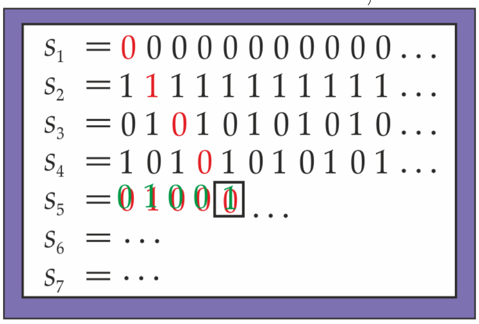
Let's note:
definition $\{x \in \mathbb{N} : x \notin f(x)\}$ **well defines the set** $B \Leftrightarrow$
definition $\{x \in \mathbb{N} : x \in f(x)\}$ **well defines the set** B' .

[19] https://en.wikipedia.org/wiki/Indicator_function
[20] https://en.wikipedia.org/wiki/Cantor's_theorem
[21] https://en.wikipedia.org/wiki/Russell%27s_paradox

We will examine the beginning of the list of objects defined using the Axiom of Specification scheme, where the fifth of the predicates will be $\varphi(x) = x \in f(x)$ which is supposed to uniquely generate a set. The analysis of the indicator function will decide whether the formula can be included in the f-list.

I present the first five formulas as candidates for the status of the correct definition of a subset of natural numbers and after analysis they will be placed on the list f (of subsets $\mathbb{N}$):
1. $\{x \in \mathbb{N} : x \in \mathbb{N} \backslash f(2)\}$
2. $\{x \in \mathbb{N} : x \in \mathbb{N}\}$
3. $\{x \in \mathbb{N} : x = 2m, m \in \mathbb{N}\}$
4. $\{x \in \mathbb{N} : x \in \mathbb{N} \backslash f(3)\}$
5. $\{x \in \mathbb{N} : x \in f(x)\}$

For the above formulas, we have the following indicator functions:



Analyzing using the indicator functions created for the formulas cited above, we find that the formula $\{x \in \mathbb{N} : x \in f(x)\}$ is ambiguous - two different strings (differing in sign in 5 positions) meet its conditions, so the formula does not generate an unambiguous subset of $B' \in P(\mathbb{N})$, and the further implication is that the supplementary formula $\{x \in \mathbb{N} : x \notin f(x)\}$ also does not define set B.

The function f exists without faulty definitions.

6.3 Summary: CDA - the source of antinomy s and ambiguity s_d.

The assumption cited in the proof of reductio ad absurdum cannot be an antinomy - because the law of excluded middle to antinomy does not apply - and we are dealing here with an antinomy, which results in the invalidity of Cantor's proof and the resulting larger size of the set composed of all subsets of the set of natural numbers. It is false to believe that f : N $\rightarrow$ P(N) cannot be surjective.

The way of identifying incorrect definitions and self-referential constructions by analyzing their complements in hypothesis H indicates the need for changes in the axiomatics.
The Axiom of Specification should be supplemented with the exclusion of antinomic and ambiguous predicates, and the commonly accepted name of the Axiom of Power should be changed to the Axiom of All Subsets - the existence of a set consisting of all subsets of a given set.

7 Cantor Method for Nested Intervals. MNI

Jules Richard showed that the set of all texts over an arbitrarily rich and finite alphabet is countable. The list of all texts includes any subsets containing objects with selected features and their definitions (e.g. real numbers, sequences, subsets, strings), but their extraction may encounter qualification problems.
By applying CC for real numbers to a list of all texts, we can extract a countable list R - the definition of real numbers, which by definition need not contain all real numbers from $\mathbb{R}$.
In the (considered constructive) proof of the uncountability of real numbers by nested intervals [22], Cantor, for any list of real numbers and any interval (a,b), constructs a number defined as $a_\infty = lim_{i\to\infty} a_i$ or $b_\infty = lim_{i\to\infty} b_i$.

7.1 PL -Separating permutation according to L

Let's take any $L \in \mathbb{R}$

The number L is a real number subject to the Cantor Criterion, and its notation can be represented by numerical expansion in exponential notation with base (positional system) p:
$L = p^k * l_1.l_2l_3l_4\dots$
$L_n = p^k * l_1.l_2l_3\dots l_n$- a number consisting of the first n significant digits of the number L.
where p - bases of the positional system (most often used: binary or decimal), k - exponent for a given number, and l_1 - first significant digit, l_2 - second significant digit, etc.

Let's put $\varepsilon = p^{k+1}$

Construction of the list X_L by induction
Step 1.

[22] https://en.wikipedia.org/wiki/Cantor%27s_first_set_theory_article#Second_theorem

From the pool of definitions of real numbers from list R, we transfer to the first position of the created list X_L a number in the form: $x_{L_1} = a_1 = p^k * l_1 - \varepsilon$, and for the second position text $x_{L_2} = b_1 = p^k * l_1 + \varepsilon$. Now, single-character definitions from the list R will be considered, i.e. those that are defined by one character (e.g. 7, 9, etc.), and if they define numbers from the range (a_1, b_1), they will remain in the list R, but if they are outside this range, will be successively placed on the X_L list as the next elements of this list, i.e. x_{L_3}, then x_{L_4}, etc. and simultaneously removed from R.
This will create a new list R_{L1} - a list of text definitions of real numbers contained in the interval (a_1, b_1) containing also all texts longer than 1 character.

Step n.
From the $R_{L(n-1)}$ list, we transfer the next two numbers denoting the next p-fold narrowing of the range, but with the center determined by the first n digits of the number L, i.e. $L_n = p^k * l_1.l_2l_3l_4 \dots l_n$, forming the interval $(a_n, b_n) = (L_n - \frac{\varepsilon}{p^{n-1}}, L_n + \frac{\varepsilon}{p^{n-1}})$. Next, we will analyze the definitions from the $R_{L(n-1)}$ list, which consist of a maximum of n characters (e.g.:8; 19; 3.1; etc.), if they fall within the new range, they will remain in the $R_{L(n-1)}$ list, and if not - they will be placed in the x_L list and removed from the $R_{L(n-1)}$ list.
The list X_L will grow and the list $R_{L(n-1)}$ will shrink.
A new R_{Ln} list will be created - a list of text definitions of real numbers in the range $(L_n - \frac{\varepsilon}{p^{n-1}}, L_n + \frac{\varepsilon}{p^{n-1}})$, containing also all texts longer than n characters.

end construction list X_L

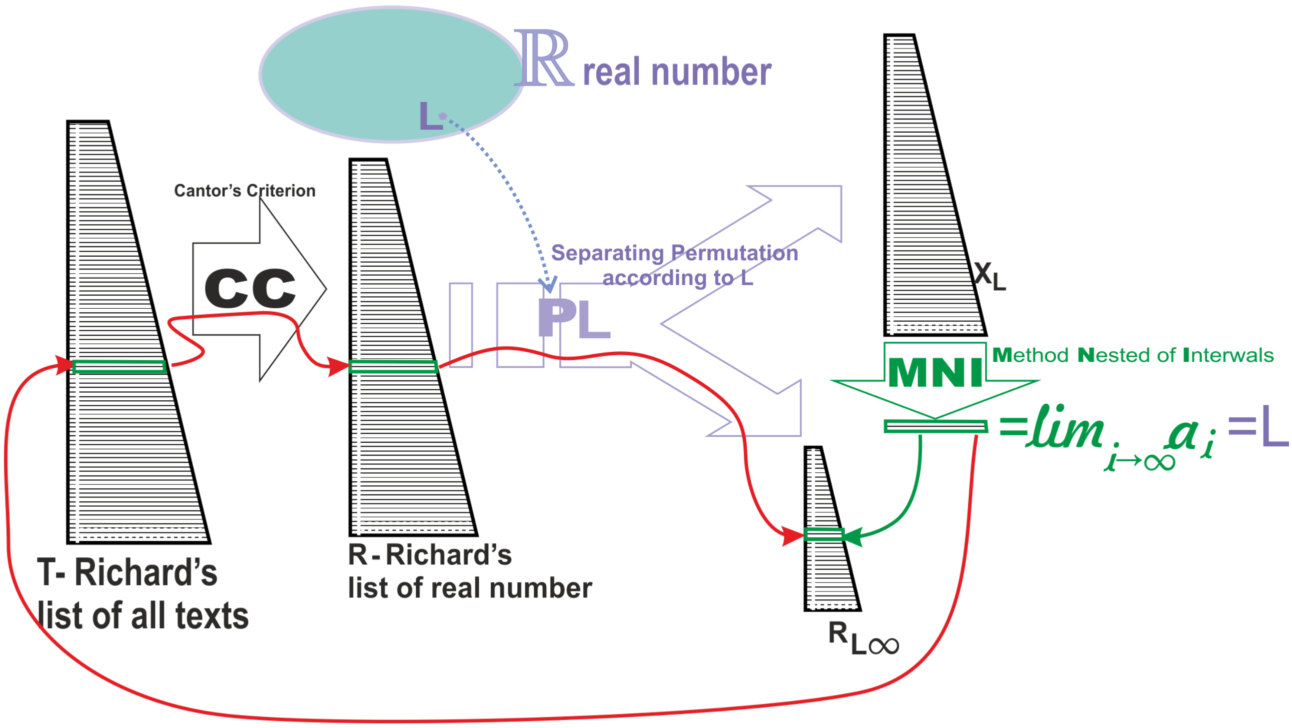
In the (considered constructive) proof of the uncountability of real numbers by nested intervals [23], Cantor, for any list of real numbers and any interval (a,b), constructs a number defined as $a_\infty = lim_{i\to\infty} a_i$ or $b_\infty = lim_{i\to\infty} b_i$.

For n tends to infinity, the limit of the sequence of nested $lim_{i\to\infty}(a_i, b_i)$ intervals formed from the list X_L will, of course, be the number $L \in \mathbb{R}$, which we assumed at the beginning and ensured by the construction of this sequence.

7.2 MNI - Method Nested of Intervals (for X_L)

Interestingly, what will remain in the list R_{Ln} as n goes to infinity? We can label this boundary list as $R_{L\infty}$.

[23] https://en.wikipedia.org/wiki/Cantor%27s_first_set_theory_article#Second_theorem



Can any definition of number $y \in R$ other than L be left on this list? No - because we can find n such that $\frac{\varepsilon}{p^n} < |y - L|$ and y will be outside all subsequent nested intervals converging to L for steps greater than n.

Can this list be empty? It can contain text that defines L as $a_\infty = lim_{i\to\infty} a_i$, (and this is shown by the green line), but it doesn't have to. The MNI for x_L is constructive and defines a number as the text $lim_{i\to\infty} a_i$. This text is obviously included in the T-list of all Richard's texts, and since Cantor's Criterion is intended to extract from the T-list only those texts that we are sure to define real numbers and place them in the R-list, we can join this definition (as well as all similarly derived ones) as correct, which will result in $R_{L\infty}$ being found on the newly created list. (Red line).

For each number L in the set of real numbers, we have shown how to separate all the elements of the list R and create a list x_L convergent by the Method Nested Intervals to the number L and a complementary list $R_{L\infty}$. Thanks to the definition of convergence obtained by MNI, we show that also this arbitrarily chosen number from $\mathbb{R}$ has an unambiguous and finite textual definition from R.

$\neg(\exists L \in (\mathbb{R} \backslash R))$ so $\mathbb{R} = R$ countable set.

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