Effect[1]

The term *effect* was introduced by G. Ludwig [1] as a technical term in his axiomatic reconstruction of quantum mechanics. Intuitively, this term refers to the “effect" of a physical object on a measuring device. Every experiment is understood to be carried out on a particular ensemble (“Gesamtheit") of objects, all of which are subjected to the same *preparation procedure*; each object interacting with the measuring device triggers one of the different possible measurement outcomes. Technically, *preparation procedures* and *effects* are used as primitive concepts to postulate the existence of probability assignments: each measurement outcome, identified by its *effect*, and each preparation procedure are assumed to determine a unique probability which represents the probability of the occurrence of that particular outcome. Thus, an *effect* can be taken to be the probability assignment, associated with a given outcome, to an ensemble of objects, or the preparation procedure applied to this ensemble [3].

In Hilbert space quantum mechanics, an effect is defined as an affine map from the set of states to the interval [0,1], or equivalently, as a linear operator E whose expectation value $\mathrm{tr}[\rho E]$ for any state (density operator) ρ lies within [0,1]. From this it follows that E is a positive bounded, hence selfadjoint, operator.

Two selfadjoint bounded linear operators are said to be ordered as $A \leq B$ (A is less than B) if $\mathrm{tr}[\rho A] \leq \mathrm{tr}[\rho B]$ for all states ρ. Thus, an effect E is a positive bounded operator with the property that $O \leq E \leq I$, where O and I are the null and identity operators, respectively.

Among the effects are the projection operators, P, with the idempotency property $P^2 = P$. They are singled out as those effects for which the generalized Lüders operation $\rho \mapsto E^{1/2}\rho E^{1/2}$ is repeatable, that is, $\mathrm{tr}[E\rho E] = \mathrm{tr}[E^{1/2}\rho E^{1/2}]$ for all states ρ. The condition $E = E^2$ can be expressed as $EE' = O$, where $E' := I - E$ is the *complement* effect of E. It is thus seen that for an effect that is not a projection, there is in general a nonzero probability, in a repeated Lüders measurement, of obtaining complementary outcomes. By contrast, two complementary projections P and $P' = I - P$ satisfy $PP' = O$, they are mutually orthogonal. If projections are interpreted as *properties*, then effects which are not projections are sometimes called *unsharp properties*, in an operational sense made precise in [2].

Another characterization of the set of projections is given by the fact that the set of effects is convex and the extreme elements are exactly

[1]In: *Compendium of Quantum Physics*, eds. F Weinert, K. Hentschel and D. Greenberger, Springer-Verlag, to appear.

the projections. Further details on mathematical and physical aspects of effects and their application can be found in [4, 5, 6].

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