# Reconsidering Relativistic Causality

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**Abstract**

I discuss the idea of relativistic causality, i.e. the requirement that causal processes or signals can propagate only within the light-cone. After briefly locating this requirement in the philosophy of causation, my main aim is to draw philosophers' attention to the fact that it is subtle, indeed problematic, in relativistic quantum physics: there are scenarios in which it seems to fail.

I consign to an Appendix two such scenarios, which are familiar to philosophers of physics: the pilot-wave approach, and the Newton-Wigner representation. I instead stress two unfamiliar scenarios: the Drummond-Hathrell and Scharnhorst effects. These effects also illustrate a general moral in the philosophy of geometry: that the mathematical structures, especially the metric tensor, that represent geometry get their geometric significance by dint of detailed physical arguments.

# Contents

# 1 Introduction

This paper concerns some cases where a relativistic quantum system apparently violates relativistic causality, i.e. the requirement that causal processes or signals can travel at most as fast as light. This is a large topic, both because there are several apparent cases of such spacelike causal processes, and because there are open questions for both physics and philosophy.

To save space, I will consign to an Appendix two cases which, though "heterodox" traditions within physics, are already familiar to philosophers of physics. Namely: (i) the pilot-wave approach, in whose relativistic versions the guidance equation and the quantum potential yield non-local effects (analogous to their effects in non-relativistic versions); and (ii) the Newton-Wigner representation, in which localized states propagate superluminally (and which has other non-local aspects, such as the fact that spectral projectors of the position operator associated with spacelike related spatial regions do not in general commute).

I aim instead to draw philosophers' attention to two unfamiliar cases, in which a superluminal effect is predicted by an orthodox relativistic quantum theory, viz. quantum electrodynamics (QED):
(i): the Drummond-Hathrell effect, in which a photon travels at a superluminal speed in a curved (general relativistic) spacetime; and
(ii): the Scharnhorst effect, in which a photon travels at a superluminal speed between two parallel plates in Minkowski spacetime.

Apart from these effects' intrinsic interest, they are worth discussing for three broader reasons.
(i): Their violation of relativistic causality is very different from that associated with the familiar EPR-Bell correlations (since it occurs in a single quantum system, not a pair of them).
(ii): They illustrate an important moral in the philosophy of geometry: a moral which, incidentally, explains away the apparent contradiction in the above claim that a photon travels at a superluminal speed.
(iii): Although these effects are uncontroversial in the physics community, our present understanding of them is undoubtedly incomplete: there are open questions, for both physics and philosophy, waiting to be addressed. In particular, there are open

philosophical, or at least conceptual, questions about: (i) relations between the various formulations of relativistic causality, and thereby about which formulations these effects violate; and (ii) how these effects avoid the causal loops, and hence (by the "bilking argument") the contradictions, that are traditionally meant to follow from superluminal processes. So these open questions are an invitation to the reader for future work![1]

The plan is as follows. First, I place my topic within the general philosophy of causation (Section 2; this general discussion is not needed later). In Section 3, I discuss different formulations of relativistic causality, emphasising open questions about QED in curved spacetime and about avoiding causal loops. Section 4 prepares us for the two effects by discussing a general moral about the philosophy of (chrono)-geometry which they illustrate: that the mathematical structures, especially the metric tensor, that represent geometry get their geometric significance by dint of detailed physical arguments. I learnt this moral from Brown (2005), and we need some details of his argument for it, in order to understand the two effects. So in Section 4, I give those details. Once we have them, we will be ready for Section 5's punchline: the two effects that violate relativistic causality.[2] Finally, in the Appendix I discuss relativistic causality, or rather is violation, in the pilot-wave approach and the Newton-Wigner representation.

## 2 In the shadow of causal anti-fundamentalism

In the history both of physics, and of its relation to philosophy, causation is a perennial player, though the role it plays of course changes—dramatically. Similarly in contemporary physics, and philosophy of physics: the former seems to be up to its neck in causal talk, and accordingly much of the latter focusses on causation—indeed, an *embarras de richesse*.

But I must also admit to embarrassment, in the usual English sense! This arises from my being inclined to endorse *causal anti-fundamentalism* in the sense espoused by Norton (2003, 2006). Norton denies 'that the world has a universal, causal character

[1]I confess at the outset that I set aside a recent line of argument which "goes in the opposite direction" to what follows: i.e. which uses relativistic causality at high energies (mostly in the form of an analytic S-matrix) to select low-energy effective theories—thereby concluding from effects like those I shall discuss that QED is not embeddable in any causal high energy theory. Thanks to Hugh Osborn for this, and for referring me to Adams, Arkani-Hamed et al. (2006). That paper also discusses other superluminal effects, including one from a brane model—and how it might be detected by tiny deviations in the moon's orbit! Shore (2007) includes a response to this line of argument, drawing on his work on the Drummond-Hathrell effect: details of which are in Section 5.2 below. So far as I know, this line of argument awaits philosophers' attention.

[2]Though these effects are unfamiliar, Brown is not alone in pointing to their philosophical importance. Weinstein mentions the Drummond-Hathrell effect as one of several examples, in his recent judicious defence of superluminal signalling (2006, pp. 390, 393); and his (1996) urges a moral similar to Brown's. What follows is much indebted to them both.

such as would be expressed by a principle of causality that must be implemented in the individual sciences' (2006, Section 2). He supports this by surveying the efforts over the centuries to articulate such a principle, concluding that there is 'such a history of persistent failure that only the rashest could possibly expect a viable, factual principle still to emerge' (2003, Section 2). Indeed, Norton's summary of his survey provides a helpful thumbnail sketch of some roles that causation has played in physics, and these roles' impact on natural philosophy, over the last four hundred years. I can best set the scene for my discussion—and explain my embarrassment—by an extended quotation:

> Highlights of this survey include ... Newton's (1692/93, third letter) insistence that unmediated action at a distance is "so great an absurdity, that I believe no man, who has in philosophical matters a competent faculty of thinking, can ever fall into it." Yet the continued success of Newton's own theory of gravitation, with its lack of any evident mediation or transmission time for gravitational action, eventually brought the grudging acceptance that this absurdity was not just possible but actual. In the nineteenth century, what was required of a process to be causal was stripped of all properties but one, the antecedent cause must determine its effect: "For every event there exists some combination of objects or events, some given concurrence of circumstances, positive and negative, the occurrence of which is always followed by that phenomenon" (Mill, 1872, Bk. III, Ch.V, 2). The advent of quantum mechanics in the early twentieth century established that the world was not factually causal in that sense and that, in generic circumstances, the present can at best determine probabilities for different futures. So we retracted to a probabilistic notion of causation. Yet the principles that we thought governed this probabilistic notion were soon proved empirically to be false. For Reichenbach suggested that we could still identify the common cause of two events, in this probabilistic setting, by its ability to screen off correlations between the events. That too was contradicted by the EPR pairs of quantum theory. (Norton, 2006, Section 2)

Thus Norton rejects in particular the idea that physics provides a reduction of causation, for example identifying it with an appropriate transmission of a conserved quantity, especially energy or momentum, as in the process theory of causation of Salmon (1984) or Dowe (2000). But he does admit (2003, pp. 5-6) that this is currently the most promising candidate for a principle of causality.

I endorse much of this broad picture; (my disagreements with it will emerge below). But it implies that the principal topic in the analysis of causation, on which the evolution of physics can shed light, is determinism. And this is "embarrassing", for two reasons:

(i): In so far as determinism involves a notion of causation, it is a logically weak ("thin") notion. For example: the determining cause is the entire previous state of the system, not some logically weaker, because localized, fact or event; (so the determination claim is weaker). Also, determinism does not need, or even support, a necessitarian view of laws (not least because determinism is a feature of a theory, and so only needs a theory-relative notion of law); so that the notion of causation involved in determinism will be non-necessitarian. So various philosophers of causation, keen on facts or events as causal relata or on causal necessity, will find this notion of causation unsatisfactory. Besides, as Norton notes: according to orthodox quantum theory, even this weak notion fails.

(ii): Furthermore, some proposed weakenings of determinism, such as Reichenbach's Principle of the Common Cause, apparently fail in quantum theory.

But fortunately, there remains plenty of useful work to do. In the context of Norton's anti-fundamentalism, we can usefully distinguish four kinds of work: this paper will be an example of the fourth.

(1): *Other sciences*:— First, even if physics has thus lowered its sights as regards causation, the other sciences might nevertheless use a concept of causation that is both reasonably strong ("thick") and in common among them, or many of them. (I think Norton could and would agree with this.) Such a situation would again be embarrassing for a philosopher of physics, in so far as physics, and its philosophy, apparently has little to contribute to pinning down that concept, and fixing its scope and limits. But in any case, doing so is of course the aim of many modern philosophers of causation: more power to their elbow.

But there is also plenty to do *within* the philosophy of physics. Here I see three kinds of work.

(2): *Causation in classical physics*:— First of all, one can deny that causation within classical physics, or even some small fragment of it such as Newtonian gravitational theory, boils down to determinism (as the quotation from Norton suggests). This denial is plausible, even if one lacks a fully-fledged theory of causation such as the process theory mentioned above. One can simply consider how Newton's law of universal gravitation, $F = G\frac{m_1 m_2}{R^2}$, says that the gravitational force $F$ between two masses $m_1$ and $m_2$ depends on the distance $R$ between them. Though the law is strictly speaking a mathematical statement of instantaneous functional dependence, not a causal statement, it has usually been taken to state action-at-a-distance—and this has since Newton's time been interpreted by many, I daresay most, physicists and philosophers to be a matter of causation.

My own view is that this usual construal is right: but I agree that to defend it would ultimately require a theory of causation—a large project which I will duck out of! For this paper, I shall simply assume that instantaneous functional dependence can amount to instantaneous causation. That is, turning to the relativistic context which is our concern: I will allow that functional dependence between values associated with spacelike-related points or regions can amount to a violation of relativistic causality—in senses made more precise in Section 3.

I say 'can amount to' since we must of course allow that such functional dependence in some cases merely reflects the joint effects of a common cause, e.g. the values of an electromagnetic field at an instant on a sphere around a radiating source. But again, I shall duck out of trying to give a general criterion for when spacelike functional dependence amounts to spacelike causation, or violation of relativistic causality, rather than merely reflecting the joint effects of a common cause. For it will be clear that my two examples involve the former, not merely the latter.[3]

(3): *Denying the embarrassment*:— Also, one can reasonably deny Norton's assertions, in (i) and (ii) above, that quantum theory gives up determinism and the Principle of the Common Cause. As to (i), the pilot-wave approach to quantum theory is deterministic, and unrefuted. And as to (ii), there are versions of the Principle of the Common Cause which are not only *un*refuted by quantum theory's EPR pairs (i.e. violations of the Bell inequalities), but provably *satisfied* by some rigorous formulations of quantum theory (Redei 2002, Redei and Summers 2002, Butterfield 2007). So for both (i) and (ii), there are issues that are yet to be settled.

(4): *Accepting the embarrassment*:— Finally, there is plenty to do, even if we accept (i) and (ii). For first, the topic of determinism is very rich, for philosophical as well as technical issues: witness many of the writings of John Earman (e.g. Earman 1986, 2004, 2006) . (Also, Section 3 of Norton (2003) describes how determinism can fail even in a simple Newtonian example of a ball rolling on a dome; cf. also Norton (2006a).)

Second, there is plenty to say about *relativistic causality*, i.e. the requirement that causal processes or signals can propagate only within the light-cone. Most physicists and philosophers take modern (i.e. post-Einsteinian) physics to endorse this requirement wholeheartedly; and I agree that *most* modern physics does so. But I submit that the requirement is subtler, indeed more problematic, than usually recognized. Not only does it have various formulations; also, some examples violate some of the formulations.

Finally, I should clarify the relation of these examples to Norton's causal anti-fundamentalism. The main point is that my endeavour—scrutinizing relativistic causality, and reporting violations of it—is of course compatible with causal anti-fundamentalism. Although Norton sees relativistic causality as small beer in comparison with the ambitions of causal fundamentalism, he also thereby sees no conflict between modern physics' endorsement of relativistic causality and his own causal anti-fundamentalism. Indeed, he links the two by claiming that within modern physics, the venerable "principle of causality" (roughly: that every event has a cause) boils down to exactly the requirement of relativistic causality (2006, Section 3). And Norton aside, there is obviously no conflict between scrutiny of, or even violations of, relativistic causality, and

[3]The remarks in (2) apply equally to the Appendix's two examples, the pilot-wave theory and the Newton-Wigner representation. Thus for the pilot-wave theory: in the non-relativistic version, the guidance equation is strictly speaking a mathematical statement of how the velocity of a particle depends on the position of distant particles. But it is usually, and I think rightly, taken to indicate action-at-a-distance—and not joint effects of a common cause. A similar remark applies to the quantum potential; and there are similar equations, again indicative of action-at-a-distance, in relativistic versions. For more discussion, cf. the Appendix.

causal anti-fundamentalism.[4]

On the other hand, what follows is opposed to the drift of Norton's discussions, in two ways. First: my endeavour does not positively *support* causal anti-fundamentalism. Optimists who hope that physics might provide a "thick" notion of causation can look to extract such a notion from how a precise formulation of relativistic causality treats the notion of process or signal. (Below, we shall see some possible places to look.) Second and more important: Norton's discussions give the impression that relativistic causality is both straightforward to understand, and endorsed by all post-Einsteinian physics. And of course my main message is that this impression is false.

# 3 Formulating relativistic causality

There are three general reasons why formulating relativistic causality is difficult. And even with precise formulations in hand, it is difficult to relate the theory of Section 5's effects, viz. QED, to such formulations. I will first briefly list these difficulties (Section 3.1). Then I will discuss some precise formulations (Section 3.2), and raise the question of how to avoid causal loops (Section 3.3). But I must admit at the outset that throughout we will see questions which I will not address.

## 3.1 Difficulties

The first reason for difficulty is philosophical. It is not just that there is always a gap, and so room for contention, between a formalism and its physical interpretation. There is also the more specific trouble (discussed in Section 2, especially (2)) that causality, or more commonly in philosophy 'causation', is a much-contested concept. In particular: when does functional dependence between values associated with spacelike-related points or regions amount to spacelike causation, i.e. a violation of relativistic causality, rather than, for example, joint effects of a common cause?

Such questions naturally prompt one to turn to wave physics: to fix on the idea of the propagation of a wave (or more generally: a disturbance of a field), and to take relativistic causality to prohibit the propagation having a superluminal velocity. But again, there are good questions: for example, about the relation to the idea of a signal (e.g. a signal might be required to be in some sense controllable, while a disturbance in general need not be). Weinstein (2006) is a fine recent review of such questions. But most relevant to us (cf. Section 4.3.3.2 et seq.) will be the fact that there are various definitions of the velocity of a wave.

The third reason for difficulty is specific to quantum theory. For it introduces

[4]Given just his suspicion of causality, Norton could even rejoice at my examples violating relativistic causality: he could see them as providing another nail in the coffin of (the modern representative of) the principle of causality. But as noted in the next paragraph, Norton joins most other philosophers of physics in being gung-ho about relativistic causality.

notions and considerations independent of classical physics (in particular: independent of the physics of waves), in terms of which one can consider formulating relativistic causality. The obvious example is the commutativity of spacelike related operators.

These difficulties no doubt mean that we should expect there to be several different precise formulations of relativistic causality; and that we should *not* expect their mutual logical relations to be clear, or easily ascertained. Indeed, we shall see this in Section 3.2: which will give three formulations drawn from algebraic quantum field theory.

But for us, there is a further problem: even with Section 3.2's precise formulations in hand, there are difficulties about relating the theory used in Section 5, QED, to them. Indeed, there are two sorts of difficulty here. First, there are important open questions about formulating QED on Minkowski spacetime in a rigorous framework such as algebraic quantum field theory ('AQFT'): which is what we need for the Scharnhorst effect. Second, there are further difficulties about formulating (i) quantum field theory, in particular QED, and (ii) relativistic causality itself, in a curved spacetime: which is what we need for the Drummond-Hathrell effect. Here I shall say a bit more about the first sort of difficulty, postponing the second to Section 3.2.

So far as I know, a formulation of QED in the language of AQFT has not yet been achieved.[5] Of course, there has been impressive work. One major example is Steinmann (2000): this book rigorously derives the perturbative formulation of QED, and its scattering formalism, from axioms cast in the language of AQFT (2000, Parts II and III). But as Steinmann stresses, there are plenty of open questions. And it is not just a matter of important topics which are so far unexplored, or threaten to be problematic, within his rigorous framework; (among the examples he mentions are gauge invariance and path integrals). There is the deeper problem that no one is sure that a rigorous QED exists. Here it is worth quoting a passage from his Introduction's summary of the book:

> ... Next, a description of what we understand under the name of QED is arrived at in a partly heuristic way. The resulting definition is not as precise as one might wish. This reflects the present state of knowledge. Not only is it not known whether a rigorous theory deserving to be called QED exists, it is not even exactly known what "deserving to be called QED" means. For all we know, there may exist no rigorous QED, or a uniquely defined one, or several distinct versions having equal claim to authenticity... Since it is not known how to to fashion a coherent, exact, theory ... we set ourselves the humbler task of carrying out this program in perturbation theory, which is the most intensively studied and best understood approximation scheme in QED. ... (Steinmann, 2000, p. 3)

But so much by way of listing difficulties. I turn to

[5]In this paragraph, I am indebted to Klaas Landsman and Fred Muller for correspondence and references.

## 3.2 Some precise formulations

I shall begin with relativistic causality in classical physics, and then consider quantum theories in Minkowski spacetime (Section 3.2.1). Then I consider curved spacetimes (Section 3.2.2).

### 3.2.1 Minkowski spacetime

Most presentations of classical relativity theory assert that no "signal" or "information" can propagate outside the light cone: there are to be no tachyons. There are various more precise formulations of this prohibition, and justifications for it, in the physics and philosophy literature; and comparisons of it with other "locality" principles: for philosophical surveys cf. Earman (1987) and Weinstein (2006, Sections 1-4). But for our purposes, we need only recall one way of making it precise: viz. by invoking the hyperbolic character of the partial differential equations of a classical field theory. The pre-eminent example is Maxwell's equations for electromagnetism on Minkowski spacetime.

For such equations, the state on a spacelike patch $\Sigma$ determines the state on the *future domain of dependence* $D^+(\Sigma)$ consisting of the spacetime points $p$ such that every past-inextendible causal (i.e. timelike or null) curve through $p$ intersects $\Sigma$. This determination indicates that the state of any field at $p \in D^+(\Sigma)$ cannot be influenced by events so far away that an influence from them to $p$ would have to be superluminal. We do not need a precise statement of this determination claim, let alone its proof. Here it suffices to say that:

(i) for an introduction to the initial value problem for classical fields obeying hyperbolic equations, cf. e.g. Wald (1984, Section 10.1); (Wald also defines 'spacelike patch': p. 200);

(ii) the proof of this determination claim uses the theory of characteristics for the equations concerned: which we will touch on at the end of Section 4.3.3.2;

(iii) note for future reference that one similarly defines (a) the *past* domain of dependence $D^-(\Sigma)$; and (b) the *domain of dependence* as union of the two $D(\Sigma) := D^+(\Sigma) \cup D^-(\Sigma)$; and (c) the future etc. domains of dependence of an open region $O$ of Minkowski spacetime (rather than a spacelike patch).

Turning to relativistic *quantum* theories, the situation is more complicated. For the interpretative problems of *any* quantum theory (relativistic or not, curved spacetime or not) about non-locality and the measurement problem are widely regarded as severe, and as threatening relativistic causality. In short: non-locality looks like "spooky action-at-a-distance"; and if measurement involves a "collapse of the wave-packet", perhaps the collapse is superluminal. Besides, relativistic quantum theories raise further issues, reflecting the embarrassment, well-known in foundations of physics, that we do not have a developed theory of measurement for these theories. For example, a recent line of work argues that in such a theory only a restricted class of quantities can be ideally measured, on pain of superluminal signaling (Sorkin (1993), Beckman et al. (2001); cf. Weinstein (2006, Section 6)). But suppose we set aside these interpretative problems, say by endorsing a minimal instrumentalist interpretation, to the effect that a relativistic quantum theory prescribes probabilities for appropriate measurement results.

Given this supposition, most presentations of relativistic quantum theories in Minkowski spacetime agree that the theories incorporate relativistic causality. But still, the situation is subtle. The idea has several precise formulations; and in various rigorous formalisms, e.g. AQFT, these formulations are precise enough that one can prove e.g. that one is logically independent of another.

As I see matters (and anyway: for the purposes of this paper), the three main formulations are as follows: (the first is the analogue of the classical formulation above).

(i): *Primitive causality; hyperbolicity*: In a heuristic quantum field theory, using the Heisenberg picture, operators indexed by spacetime points are subject to Heisenberg equations of motion, while the state is fixed once for all. But these equations are hyperbolic, on analogy with classical field theories using hyperbolic dynamical equations; this means one can show, at least unrigorously, that for any state, all expectation values are determined subluminally, in that the state's restriction to the field operators in a region $O$ determines all its expectation values for operators in the future domain of dependence $D^+(\Sigma)$. In AQFT, this idea is made precise as the *Diamond Axiom*. AQFT associates to each open bounded region $O$ of Minkowski spacetime, an algebra of operators $\mathcal{A}(O)$. We are to think of the Hermitian elements of $\mathcal{A}(O)$ as the observables for that part of the field system lying in $O$, and so as measurable by a procedure within $O$. We take states as linear expectation functionals $\omega$ on the algebras: $\omega : A \in \mathcal{A}(O) \mapsto \omega(A) \in \mathbb{C}$. (We can recover Hilbert space representations from this abstract setting, primarily by the GNS construction.) Then the Diamond Axiom says that $\mathcal{A}(D(O)) = \mathcal{A}(O)$. So the idea is: *if* $O_1 \subset D^+(O), O_1 \cap O = \emptyset$, i.e. $O_1$ lies in the top half of the "diamond" $D^+(O)$, and $A \in \mathcal{A}(O_1)$, so that we could measure $A$ by a procedure confined to $O_1$, *then* we could also instead measure $A$ by a procedure confined to $O$. For thanks to the hyperbolic time-evolution, "the facts in $O_1$" are already determined by "the facts in $O$".

(ii): *Spacelike commutativity* (also called *micro-causality*): Operators associated with spacelike-related regions commute. In heuristic quantum field theory, treating fermions requires one to also allow anti-commutation; but in AQFT, one distinguishes field algebras and observable algebras, and for the latter imposes only spacelike commutativity. Thus one requires: if $O_1, O_2$ are spacelike, then for all $A_1 \in \mathcal{A}(O_1), A_2 \in \mathcal{A}(O_2) : [A_1, A_2] = 0$. The physical idea is of course that such spacelike observables should be co-measurable, and so should commute. (Think of how in elementary quantum theory, one proves the no-signalling theorem, viz. that a non-selective Lüders rule measurement of $A$ cannot affect the measurement probabilities of $B$, provided $[A, B] = 0$.)

(iii): *Spectrum*: The field system's energy-momentum operator has a spectrum (roughly: set of eigenvalues) confined to the future light-cone. Of the three conditions (i)-(iii), this is perhaps the most direct expression of the prohibition of spacelike processes.

So we have three different senses of relativistic causality. And indeed, in most relativistic quantum theories, and models constructed within them, these three senses (and even others) hold good. Besides, there are various arguments connecting these sorts of formulation. For example, one can argue for (ii), spacelike commutativity, from an analogue of (i) which one might call a "signal principle", viz. that "turning on" any unitary evolution $U$ of the field system within region $O_1$ should leave unaffected the measurement probabilities of any observable $A$ associated with a spacelike region $O_2$; along the following lines. If $O_1, O_2$ are spacelike, then this signal principle requires that for all unitary $U \in \mathcal{A}(O_1)$, all $A \in \mathcal{A}(O_2)$, and all states $\omega$, $\omega(UAU^*) = \omega(A)$. So $UAU^* = A$, i.e. $[U, A] = 0$. Since the unitary operators span the algebras concerned, this means that the algebras commute, i.e. for all $A_1 \in \mathcal{A}(O_1), A_2 \in \mathcal{A}(O_2)$, we have $[A_1, A_2] = 0$.[6]

On the other hand, there are no corresponding rigorous implications between a pair drawn from (i) to (iii). That is: truly precise statements of (i) to (iii), in the language of AQFT, are logically independent of each other. Indeed, they are independent even in the presence of AQFT's other axioms. And more is true: Horuzhy (1990, pp. 19-21) reports that all six of his basic axioms of AQFT (three of which are his versions of (i)-(iii)) are independent: for each subset comprising five axioms, there is a model satisfying all five—but not the remaining sixth axiom. And although some of these models are very unphysical, others are not: again we see that relativistic causality—formulate it as you like!—is a subtle matter in relativistic quantum theories.

#### 3.2.2 Curved spacetime

I turn to curved spacetimes. The first thing to say is that in the light of our not having a rigorous formulation of QED on Minkowski spacetime (cf. the end of Section 3.1), *a fortiori* we lack such a formulation on curved spacetimes. But on the other hand, a great deal is now understood about formulating heuristic quantum field theory on such spacetimes. I shall sketch the general situation, emphasising the case of a non-interacting field (nowadays well understood), and then reporting some recent progress on the (much harder) interacting case. In both cases, I shall emphasise ideas we need for our main question, to which I will turn at the end of the Section: how to adapt our three precise conditions, (i) to (iii), to curved spacetimes.

Broadly speaking, by the mid 1990s quantum field theory on curved spacetime could be formulated in as satisfactory a manner as heuristic quantum field theory on Minkowski spacetime, subject to three conditions. (This summary is based on Wald (1994).) These conditions are:

(a): the curved spacetime is fixed, i.e. there is no back-reaction of the field on the

[6] I learnt this argument, apparently common in the folklore of AQFT, from K. Fredenhagen and S. Summers, in conversation: to whom my thanks.

spacetime geometry; (though the curvature can be non-constant);

(b): the field is linear (i.e. not self-interacting);

(c): the spacetime is such that the corresponding classical field theory has a well-posed initial value problem.

For our purposes, condition (c) prompts two comments. First, 'well-posed initial value problem' refers to the determination of solutions by initial data, as in the classical discussion at the start of Section 3.2.1. And the most common way to satisfy condition (c) is to restrict attention to globally hyperbolic spacetimes. These are spacetimes with a *Cauchy surface*, i.e. a spacelike slice $\Sigma$ whose domain of dependence $D(\Sigma)$ is the whole spacetime. In fact, global hyperbolicity is a strong condition of causal "good behaviour"; it implies that spacetime is foliated by Cauchy surfaces, and implies several other causality conditions, including stable causality—which we will return to in Section 5. The reason for this restriction is that the main theorems securing a well-posed initial value problem for hyperbolic classical equations assume global hyperbolicity (Wald 1984, Theorems 10.1.2-3, p. 250.)[7]

Second, we need to explain 'the corresponding classical theory'. This phrase indicates a standard construction of a heuristic quantum field theory (with e.g. a Fock space of states built up from a vacuum) from the solution space of a classical theory. (The construction for Minkowski spacetime is in Wald (1994, Sections 3.1-3.2); the adaptation to curved spacetimes is in his Section 4.2.) When this construction is given for Minkowski spacetime, but in a form that is suitable for generalization to curved spacetimes, one sees that:

(i): it involves the choice of a Hilbert space (essentially, of a set of complex solutions of the classical theory), with different Hilbert spaces giving unitarily inequivalent theories; (the Stone-von Neumann theorem asserting unitary equivalence of representations of commutation relations depends on the system being finite-dimensional i.e. not a field); and

(ii): this freedom of choice is characterized by a choice of a bilinear map (inducing an inner product, and so a complex structure); but that

(iii): the Poincaré symmetry of Minkowski spacetime gives a preferred bilinear map and so Hilbert space: equivalently, a preferred vacuum with the Hilbert space being the Fock space built up from that vacuum; (Wald 1994, pp. 27-29, 39-42).

In a curved spacetime, property (iii) fails, and so one must either: (a): seek some other criterion for choosing the bilinear map (or at least a set of them corresponding to a unitary equivalence class of representations); or (b): treat the different choices (and so unitarily inequivalent representations) on a par.

In some cases, tactic (a) is sensible. For example, for stationary spacetimes, there is a natural unique choice of map; and for a spacetime with a compact Cauchy surface, a condition of physical reasonableness for the state (the Hadamard condition, discussed below) constrains the bilinear maps so as to fix a unitary equivalence class. But in

[7]For an approach to quantum field theory on non-globally hyperbolic spacetimes, cf. Kay (1992, especially Section 6).

general, there is no such choice and one must opt for (b) (Wald 1994, pp. 58-60).

This suggests that we should adopt the framework of algebraic quantum theory, which (as mentioned in (i) of Section 3.2.1) takes abstract algebras of observables as the primary notion, with states being linear functionals on them. And indeed, adapting the standard construction mentioned above to the algebraic approach, we find that the (Weyl) algebra of observables that naturally arises is independent of the choice of bilinear map (Wald 1994, p. 74f.). Furthermore, this approach is at first sight very promising for our own topic of precise formulations of relativistic causality, since this approach again associates abstract algebras of observables with open bounded regions of any globally hyperbolic spacetime (Wald, 1994 p. 84). Thus we naturally hope to carry over directly to such spacetimes the three Minkowski formulations of Section 3.2.1.

Indeed, there is no problem about the first two conditions, primitive causality and spacelike commutativity. The global hyperbolicity assumption prevents any "funny business" in the causal structure, such as closed causal curves, so that these conditions can be carried over word for word: 'domain of dependence', 'spacelike' etc. now just refer to the curved spacetime's structure.

Besides, though I have so far confined my summary to the easier, and better understood, case of non-interacting fields, the same considerations apply to the interacting case. As we shall see shortly, recent formulations of interacting quantum field theory on curved spacetimes use the algebraic approach, and again there is nothing to prevent carrying over these two conditions intact.

But there is a problem about the third condition, the spectrum condition: though it is a problem that has recently been largely solved. In effect, the problem was that no one knew how to define the spectrum condition's topic, i.e. the energy-momentum operator, in a curved spacetime: all one knew was how to define a class of physically reasonable states that gave a well-defined expectation value. But in recent years, the problem has been solved by exploiting a mathematical theory, microlocal analysis. Though the details are not needed for later discussion, I should summarize them since the problem is much more general, and thus its solution much more impressive, than my mentioning just one observable can suggest. Indeed, as I understand matters, the solution secures a perturbative formulation of interacting heuristic quantum field theory on a globally hyperbolic spacetime that is about as precise as those we have for Minkowski spacetime. So, even apart from Section 5's two effects, this achievement is worth reporting.[8]

The problem of definability arises from the fact that the energy and momentum of the field are encoded in the stress-energy tensor $\hat{T}$, which involves the square of the quantum field $\hat{\phi}$. But $\hat{\phi}$ is a distribution, and the product of distributions at a single

[8]I am very grateful to Bob Wald for teaching me what follows. One caveat: to date, the framework is secured for scalar fields only. Wald tells me he is very sure it can be extended to Dirac fields, and pretty sure for QED: a happy prospect, also for this paper's topic, since it is what we need for fully understanding Section 5's effects.

spacetime point is mathematically undefined; so that some prescription is needed in order that $\hat{T}$ make sense. Until about 2000, it was not known how to do this "directly", i.e. by enlarging the algebra of observables to include some suitably smeared version of $\hat{T}$; so one aimed only to characterize a class of physically reasonable states $\omega$ for which the expectation value $< \hat{T} >_\omega$ was well-defined. This was work enough since, in particular, the standard prescription for Minkowski spacetime (normal ordering, which corresponds to subtracting off the infinite sum of the zero point energies of the oscillators comprising the field) depends on a preferred vacuum—which is generally unavailable in a globally hyperbolic spacetime. In fact, there is a compelling characterization of such states. Since it builds on Hadamard's work on distributional solutions to hyperbolic equations, they are called 'Hadamard states'. A bit more precisely: one requires the short-distance singularity structure of the two-point function $< \hat{\phi}(x)\hat{\phi}(x') >$ to be "as close as possible" to the corresponding structure of the two-point function of the Minkowski vacuum (Wald 1994, p. 94). This definition turns out to be very successful, as shown by both existence and uniqueness results. That is: (i) any globally hyperbolic spacetime has a large class of Hadamard states, and (ii) Hadamard states satisfy some natural axioms, and do so uniquely up to a local curvature term; (Wald, 1994: pp. 89-95 for (ii), and pp. 95-97 for (i)).

But in recent years, various authors have exploited microlocal analysis so as to achieve the original goal ("direct" in the preceding paragraph). Indeed, they have defined, not just the energy-momentum, and stress-energy operators; but also the other products of field operators and their derivatives, and polynomials of such products, and time-ordered products, that are crucial in order to formulate the perturbation theory of an interacting quantum field theory. I will just gesture at what is involved, with an eye on our interest in the spectrum condition. For more detail, cf. Hollands and Wald (2001, 2002), who build on previous work, especially by Brunetti, Fredenhagen and collaborators.

The fundamental idea is to use microlocal analysis' definition of the set of directions (at each point of the spacetime) in which a distribution is singular: it is called the 'wave front set' of the distribution. Using this and related ideas, one can define the above operators so as to satisfy appropriate properties, in particular locality and covariance. To give the flavour, we need only note the following characterization of Hadamard states in any globally hyperbolic spacetime (and thereby of the spectrum condition in Minkowski spacetime): the two point function of a Hadamard state has a wave front set consisting of pairs comprising: (i) at any given spacetime point $x$, a future-pointing null vector $k$; and (ii) at any other point $x'$ on the null geodesic through $x$ generated by parallel-transporting $k$, the corresponding tangent vector, i.e. the parallel-transport of $k$ from $x$ to $x'$. (This is, *modulo* some technicalities, Radzikowski 1996, Theorem 5.1.) Hollands and Wald build on this sort of characterization so as to require of their local and covariant operators, a generalized microlocal spectrum condition, which is a microlocal analogue of the translation invariance of Minkowski spacetime (2001, Definitions 3.3, 4.1). Thus the spectrum condition, (iii) of Section 3.2.1, is carried over to curved spacetimes.

To sum up Section 3.2:— In Section 3.2.1, we saw three precise formulations of relativistic causality for quantum theories on Minkowski spacetime, which prescind from interpretative controversies about such notions as causation, or signal, or quantum measurement, and which are logically independent. In Section 3.2.2, these three formulations were adapted to globally hyperbolic spacetimes. But for both kinds of spacetime, we do not yet have a rigorous formulation of perturbative interacting QED, in which to study the fate of these conditions (and so better understand Section 5's effects). But there are good prospects for soon getting such formulations.

## 3.3 Causal loops?

Given an apparent violation of relativistic causality, there are various questions to ask. First: which of the various precise formulations of relativistic causality is false? (Of course, if there are logical connections between them, some may fall as a consequence of others, by *modus tollens.*) Section 3.2 showed that even when one sets aside interpretative problems (as in Section 3.1), this is a hard technical question for either of Section 5's effects: hardly a question for philosophers, and so a question I must leave to the reader!

But other questions are more philosophical. An obvious one is: how does the superluminal propagation avoid leading to outright contradictions via causal loops? Although Section 5 will not pursue this question, the general strategy which the investigators of the effects have used to address it, is noteworthy—and I can already explain it. Thus the investigators of each example are of course aware of the threat that:

(i): superluminal propagation would imply that causal processes could go in a loop (technically: closed timelike or null curves); and that:

(ii): such loops would yield, by a "bilking argument", contradictions.

Given this awareness, it is no surprise that they argue that the threat can be avoided: if it could not be, the example would be hopeless. But their arguments are worth noting.

For they do *not* adopt the standard philosophical tactic for avoiding the threat. That tactic addresses only (ii). Namely, by saying that a "causal loop" merely implies a severe consistency condition on the state of the system at an initial time, namely that it must evolve around the loop back to itself. Thus, taking the standard philosophical example: a contradiction seems to threaten if one can travel back in time and kill one's grandparents before one's parents were conceived. The standard philosophical reply is that indeed one could not kill them; but that means, not that time-travel is impossible, but that it is severely constrained: any initial state must evolve back to itself; (e.g. Putnam 1962, pp. 243-247; and Lewis 1976, pp. 75-79: Earman (1995, pp. 170-188), Berkovitz (2002) and Arntzenius and Maudlin (2005, Section 8) are sophisticated discussions of this reply).

Instead, the defences of these examples address (i). That is: they argue that the superluminal propagation which the example countenances could *not* be exploited to make a spacelike zig-zag, "there and back", into the causal past of an initial event: a refreshing change from the philosophical literature's repeatedly addressing only the second aspect, (ii), by invoking the idea of a consistency constraint.

Another obviously philosophical question is: given superluminal propagation, how should we think of the light-cone—what *is* its significance, now that it is not the locus of light rays, or in a quantum context, of photon propagation? This leads to the more general question, which I address in the next Section: how do the mathematical representatives of metric structure earn their physical significance?

# 4 Why is this tensor "read" by rods and clocks? Brown's moral

This general question deserves a Section of its own, for two reasons. First, what I take to be the right answer—which I learnt from Brown (2005)—is controversial. So I shall spell out what I shall call 'Brown's moral', where 'moral' connotes both its being controversial, and my endorsing it. Second, we need some details of this moral as preparation for Section 5's effects; indeed, Brown takes his moral to be illustrated by them (2005, Chapter 9.4.1, 9.5.2).[9]

Brown's moral is a general doctrine in the philosophy of (chrono)-geometry, though he develops it mostly for special relativity, and briefly for general relativity. I will first state the moral in general terms (Section 4.1), then report his treatment of it in special relativity (Section 4.2), and then turn to the case we need: general relativity (Section 4.3).

## 4.1 The moral in general

We can think of the moral as having two aspects, "negative" and "positive". It will be clearer to start with the negative aspect, since the positive aspect explains it. Negatively, the rough idea is that we should not simply postulate that a quantity in a physical theory has (chrono)-geometric significance. The point here is not just that it would be wrong to infer from a quantity's being *called* a metric that it mathematically represents (what the theory predicts about) the readings of rods, and-or clocks and-or other instruments for measuring lengths and time-intervals. That is obvious

[9]Though the words are mine, this Section is due to Brown. He urges this moral, and related ones, in various passages of his (2005); cf. also his associated papers, especially Brown and Pooley (2001, 2006). Since drafting this paper, I have read Weinstein's brief but rich (1996). This paper: (i) floats a moral like Brown's, for general relativity and kindred theories, and lists some adjacent issues; (ii) illustrates the moral with several examples of non-minimal coupling, not just two as in our Section 5; and (iii) discusses coupling in terms of action principles. By the way: though the moral is controverted by philosophers, my impression in discussion with physicists is that they endorse it—at least the way I say it!

enough: after all, a quantity might be given an undeserved, even tendentious, name. But also: we should not infer from the fact that in the theoretical context, the quantity is mathematically appropriate for representing such behaviour, that it does so. For example, on the Gauss-Riemann conception of length as given by line-integrals of $ds = \surd(g_{ij} dx^i dx^j)$, the symmetric tensor $g_{ij}$ is appropriate. More specifically, for relativity's spatiotemporal lengths, $g_{ij}$ is to have Lorentzian signature. But such a tensor might well *not* represent measured lengths or times. After all, a theory might contain two such tensors, just one of which represents such matters. (We shall see such an example in Section 5.1.1.3.)

The reason these inferences are invalid is of course that any physical (chrono)-geometry should take rods, clocks and other instruments as composite bodies whose behaviour is determined by the laws governing matter. In practice, these bodies are usually very complex and so the determination of their behaviour by laws governing their micro-constituents will be very complex. But this is not to suggest that a term like 'metric tensor' cannot be justified. We can indeed write down a idealized ("toy") model of a rod or clock etc. in terms of our theory of matter (nowadays a relativistic and quantum theory), and thereby deduce that a certain quantity in our theory—in a relativistic theory, a (0,2) symmetric tensor $g_{ij}$ with Lorentzian signature—represents their readings; and thereby earns the name 'metric'.

We need three further general points about this moral.

(1): This is not to say that the quantity must itself be derived, perhaps as an effective or phenomenological aspect of a complex instrument. Our theory can postulate the quantity "on the ground floor" in its model of the instrument; indeed, most theories do so. The point is just that the quantity only earns the name 'metric' when we "close the circle" by exhibiting how instruments' readings display it. (This point is developed in Butterfield (2001, Section 2.1.2).)

(2): Nor is it to say that *only* by representing the readings of measuring instruments for lengths and times (such as rods and clocks), can a quantity have chrono-geometric significance. For in both Newtonian and relativistic theories, part of the metric tensor's significance is that massive test-particles and light-rays travel along appropriate geodesics. (Here, I have put the point in relativistic jargon; and 'appropriate' covers relativity's timelike/null distinction.)

(3): Brown argues that this moral needs to be stressed, because in recent decades the philosophical literature about relativity theory has tended to ignore it. He is especially concerned with the moral in special relativity: it is a theme of his Chapters 2-8. For the sake of completeness, I will now report his discussion. But this report is not needed for the rest of this paper's argument: for that, one can proceed directly to the moral in general relativity (Section 4.3).

## 4.2 The moral in special relativity

In special relativity, the tensor $g_{ij}$ get its chrono-geometric significance principally through the behaviour of rods and clocks, especially the length contraction and time dilation effects.[10] So as evidence for his moral being ignored, Brown describes the current tendency to call them 'kinematical effects', where 'kinematical' is taken to connote 'prior to dynamics, and so not needing a dynamical explanation'. To put the point in more philosophical terms: many philosophical commentators on special relativity apparently conceive the Minkowski metric as encoding a property (or better: structured family of properties) of spacetime that (a) is intrinsic to it, in the sense that spacetime would have the property even in the absence of matter, and (b) suffices to explain the effects.

Brown admits that this tendency has roots, both historical and conceptual. Historically, Einstein himself called the opening Sections of his 1905 paper 'Kinematical Part', and he called special relativity a 'principle' theory as against a 'constructive' one, i.e. as not concerned with any detailed mechanisms bringing about length contraction and time dilation. And conceptually, the account of length contraction and time dilation based on a spacetime diagram with hyperbolae of constant Minkowski interval from a given point,[11] is undeniably striking. Every student feels that the diagram greatly clarifies algebraic derivations based on the Lorentz transformations; and that it also makes unmysterious the reciprocity of the effects, i.e. the fact that *each* of two inertial observers judges the other's rods to be contracted and their clocks to be slowed.

But, says Brown, these roots do not justify the tendency. Einstein himself later admitted that the kinematical part of the theory did not pre-empt the need for a dynamical explanation of the effects, and accordingly downplayed the idea of special relativity as a principle theory. And in order for the account based on a spacetime diagram and hyperbolae to explain the effects, one needs to accept (or to have previously explained) that the primed variables do indeed represent the readings of the moving rods and clocks: for only via this fact can the diagram's hyperbolae be connected to those readings. And, according to Brown's moral, it is of course just this fact that needs a dynamical explanation.

So much by way of summarising Brown's critique of a current philosophical tendency. Some highlights of this critique are at: pp. 22-25, 89-92, 99-102, 129-131, 132-139, and 143-148. For example: the critique of the Minkowskian diagrammatic 'explanation', i.e. Brown's demand for a dynamical explanation of the physical interpretation of the primed variables, is at pp. 129-131. And on pp. 132-139, Brown argues by analogy with the geometric structure attributed to other state spaces in physics, viz. curved configuration manifolds in analytical mechanics, the curvature of projective Hilbert space in quantum theory, and Carathéodory's postulates for classical

[10] One might add: and through massive test-particles and light rays following $g_{ij}$'s timelike and null geodesics, respectively. But I shall set aside this aspect, for brevity: compare (2) in Section 4.1.

[11] Originally by Minkowski (1908, pp. 77-78, 81-82), and oft repeated since, e.g. Born (1962, pp. 247-249) Torretti (1983, p. 97).

thermodynamical state-space. In these and similar cases, we naturally interpret the geometry not as causing or explaining the system's behaviour, but as codifying it: so why not also in spacetime theory?

More positively, Brown gives historical and technical details about the dynamical explanations of the effects. He describes how over the decades several authors, including Einstein: (i) have seen the need for such explanations; and (ii) have even spelt out accurately just what such an explanation requires—viz. the Lorentz-covariance of the laws responsible for the cohesion of matter, laws which after the 1920s were of course recognized to be quantum-theoretic. Some highlights of this positive story are: for (i), Einstein (pp. 113-114) and Pauli (p. 118); and for (ii), Swann (pp. 119-122) and Bell (pp. 124-126). Brown describes how both Swann and Bell realize that:

(a): The dynamical explanation does not require one to know what the laws are, but only that they are Lorentz-covariant. And:

(b): The dynamical explanation does not require a transformation to moving coordinates. For example, a Lorentz boost is interpreted as active, mapping a given solution describing a "stationary" rod in internal equilibrium, to another solution describing a longitudinally contracted rod.

## 4.3 The moral in general relativity

### 4.3.1 How the gravitational field gets its metric significance

Broadly speaking, Brown's discussion of general relativity (Chapter 9, and pp. 141-143) confirms the moral he gathered from special relativity: that the metric tensor $g_{ij}$ gets its chrono-geometrical significance, not by *fiat*, but by detailed physical arguments. For Section 5, we need the following details.

All agree that in general relativity, the metric tensor $g_{ij}$ is (or better: represents a field that is) dynamical: it acts and is acted on. They also agree that it is a special field since it couples to every other one, and also cannot vanish anywhere in spacetime. Many authors go on to say that the metric tensor represents geometry, or spacetime structure, so that geometry or spacetime structure acts and is acted on. But Brown resists this. He says that the metric tensor represents primarily the gravitational field, 'which interacts with every other [field] and thus determines the relative motion of the individual components we want to use as rod or clock. Because of that, it admits a metrical interpretation'. (This quotation, on p. 160, is from Rovelli: who is one of three distinguished interpreters of general relativity whom Brown quotes as kindred spirits.)

Brown supports this position by reviewing some of the physics that underpins this metrical interpretation: i.e. the physics that explains why $g_{ij}$ is surveyed by rods and clocks, and its null and timelike geodesics are the worldlines of light-rays and massive non-rotating test-particles respectively. This review brings out that, with the exception of this last case—the worldlines of test-particles—the metrical interpretation depends, not only on general relativity's field equations, but also on the *strong equivalence principle* (SEP).

This dependence is important for this paper. For it is precisely by violating SEP that Section 5's effects will have light propagate outside the light-cones defined by $g_{ij}$. So it will be worth first spelling out SEP, and seeing how the metrical interpretation of $g_{ij}$ uses it. The main point will be that SEP is a conjunction of two propositions; and though both are part of "textbook general relativity", only one of them (which I will call *Universality*) is essential to general relativity—and it will be by violating the *other* proposition (called *Minimal Coupling*) that Section 5's effects violate SEP, and thereby relativistic causality.

Agreed, what is essential to general relativity is partly a matter of interpretative judgment, and partly a purely verbal matter. And unfortunately, there is no sharp consensus about how to formulate the equivalence principle; nor about how to make the distinction between the weak and strong principles. But I am sure that all general relativists:

(i): would accept as reasonable the decomposition of SEP into Universality and Minimal Coupling, which Brown articulates (pp. 169-172, citing Ehlers and Anderson); and

(ii): would accept that only Universality is essential to general relativity; after all, there are many articles discussing non-minimal coupling in what the article calls 'general relativity'.

#### 4.3.2 The equivalence principles, weak and strong

So let us start with the *weak* equivalence principle. Though I will not need to formulate it exactly,[12] the basic idea is that local mechanical experiments cannot distinguish gravity and inertia. A bit more precisely: they cannot distinguish a homogeneous gravitational field from the inertial effects of uniform acceleration. Nor can they distinguish free fall, i.e. motion under gravity but under no other forces, from motion subject to no force at all. So this is the idea of "Einstein's elevator": (which Einstein called "the happiest thought of my life"). This means in particular that test-particles of different masses move in the same way under gravity alone—i.e. move identically, given identical initial conditions, and if subject to no other force. ("Galileo's law": two different masses dropped simultaneously from the Tower of Pisa fall in identical ways.) Hence the idea of treating gravity as spacetime curvature, in the sense of taking freely-falling test-particles to travel along geodesics of a curved connection.

On the other hand, the strong equivalence principle, SEP—our main concern—is about how the various non-gravitational forces relate to gravity thus treated; and in particular, how they relate to the connection induced by the metric tensor. Again, the formulation varies from one author to another. But I will simply follow Brown (and so

[12] For Brown's discussion, cf. pp 25-26 and 161-163. Cf. also Norton (1985) and Ghins and Budden (2001).

Ehlers and Anderson), with slight modifications. The main point will be that SEP is the conjunction of two propositions.

The first proposition, Universality, is that the physics of each of the non-gravitational forces picks out the same affine connection. More precisely, we envisage that the theory of any such force adopts the following framework:

(i): The theory is generally covariant. This means, roughly speaking, that it is presented in coordinate-independent differential equations for appropriate scalars, vectors and tensors representing fields.

(ii): The theory invokes an affine connection on spacetime so as to have an appropriate coordinate-independent notion of differentiation on fields.

Given this framework, Universality then says that all the theories of the non-gravitational forces are to invoke the same connection, $\nabla$ say.

This assertion is similar to the basic idea above of the weak equivalence principle, for the following reason. Suppose that each such theory asserts that a test-particle that is free—i.e. a particle that is "small" enough not to affect what is influencing it, and is subject to zero force of the kind in question—travels along a geodesic of the common connection $\nabla$. Indeed, in the light of the four-dimensional formulation of Newtonian mechanics and special relativity, that is a natural assertion.[13] Then Universality makes it very natural to assert that in a theory of some or all of these non-gravitational forces, a test-particle subject to *none* of these forces—a test-particle that falls freely, i.e. subject only to gravity—should also travel along a geodesic of the common connection. After all: if the particle did not do so, this would mean that the absence of several forces yielded, in a combined theory of the forces, a motion different from the *common* prescription of the ingredient theories. So "agreed votes" from the ingredient theories would "cancel one another out" within the combined theory: which would be distinctly odd.

The second proposition, Minimal Coupling, is in effect a bold generalization of this last assertion, that in a theory of some or all of the non-gravitational forces, a freely-falling test-particle travels along a geodesic of the common connection. This generalization occurs in three ways:

(i): Minimal Coupling concerns all matter and fields, not just test-particles;

(ii): For Minimal Coupling, the matter and fields can be interacting, i.e. subject to some or all the non-gravitational forces

(iii): Minimal Coupling makes a specific prescription about what laws govern the matter and fields in the setting of a curved connection representing gravity. Namely: the laws of the corresponding *special* relativistic theory are to be valid locally.

It is the third point, (iii), that is the heart of Minimal Coupling. To state it

[13] In both these theories, there is a distinction between timelike and spacelike curves, and so the theory asserts the particle to travel along a timelike geodesic. The main difference is that in special relativity, the connection is uniquely determined by (the requirement of compatibility with) the metric. We need not consider now the issue whether this assertion needs a dynamical or 'constructive' explanation of the kind Brown favours. But I will discuss this issue in Section 4.3.3.1.

more precisely: differential geometry teaches us that the partial derivatives in the differential equations of a special relativistic theory implicitly represent the standard flat connection of $\mathbb{R}^4$; and Minimal Coupling says that the general relativistic laws are given simply by replacing these partial derivatives by the curved connection's covariant derivatives.[14]

Broadly speaking, this prescription means that the transition to the general relativistic laws is as simple as it could be, while reducing to the special relativistic laws in the case of a flat connection. For a curved connection means that covariant derivatives add "correction terms" to special relativity's familiar partial derivatives. But Minimal Coupling says that the general relativistic equations do not add *anything else*. In particular, they do not include terms proportional to any kind of curvature (whether the scalar curvature, or one of the various curvature tensors): which *ceteris paribus* they might well do, since any such terms would be zero in the setting of special relativity's flat connection, and so would not be refuted by the empirical success of the special relativistic theory.

So Minimal Coupling represents a proposal for simplicity. And evidently, it is fallible: nothing in the framework of general relativity forbids a matter field from coupling to spacetime curvature, and so requiring a curvature term in the differential equations that govern it. And as announced, we will see examples that violate it, in Section 5.[15]

### 4.3.3 Motion along geodesics

So much by way of clarifying that the violation of SEP will involve violating Minimal Coupling. Returning to Brown's overall moral, we need to extract just two points from his review of the physics that underpins this metrical significance of the tensor $g_{ij}$. These concern: (a) motion along timelike geodesics and (b) light propagation along null surfaces.

**4.3.3.1 Timelike geodesics** There is one aspect of the metrical significance of $g_{ij}$ that is well known to be independent of SEP: viz. the motion of massive test-particles. (But again, this aspect will illustrate Brown's moral.) Thus Einstein and other general relativists initially took it as a postulate that freely-falling massive non-rotating test-particles followed timelike geodesics of (the curved connection determined by) the metric tensor. Brown remarks that this is the analogue for a theory with "geometrized gravity", of the interpretation of Newton's first Law, for Newtonian mechanics and special relativity, that he rejects: viz. the interpretation that these theories' timelike

[14] Partial derivatives are usually represented by a comma, and covariant derivatives by a semi-colon. So the semi-colon abbreviates the correction terms, and Minimal Coupling is sometimes called the 'comma-to-semi-colon' rule.

[15] *Aficionados* know several such: thanks to Steve Adler for mentioning the conformal massless Klein-Gordon field.

geodesics 'form ruts or grooves in spacetime which somehow guide the free particles along their way'; (2005, p. 24; here, 'free' of course also excludes gravity; cf. also pp. 139-143 for references to advocates of this interpretation). But from about 1918 onwards, a succession of theorems by Eddington, Einstein and others made it clear that this postulate is unnecessary: a massive test-particle must follow such a geodesic, because of the conservation of energy-momentum (more precisely: the vanishing of the covariant divergence of the body's stress-energy tensor).[16]

So for Brown, these theorems are like the dynamical explanations of length contraction and time dilation in special relativity that he favours. All are genuine physical "constructive" explanations of the chrono-geometric significance of the metric tensor—as against the pseudo-explanations that just make a postulate, for example that timelike geodesics "form ruts" for test-particles. Brown also points out that these theorems are limited, but in a way that supports his interpretation. Namely, extended free-falling bodies will in general experience tidal gravitational forces, and so will *not* follow geodesics: underlining the point that it is not "in the nature" of freely-falling bodies to follow the alleged ruts.

**4.3.3.2 Null surfaces and geodesics** The propagation of light, or more generally electromagnetic radiation, along null surfaces provides an interesting comparison with massive test-particles, Section 4.3.3.1 above. The situation is similar in that it again illustrates Brown's moral. Thus one can prove from Maxwell's equations for electromagnetism on a general relativistic spacetime that electromagnetic radiation will propagate along null surfaces. So one does not need to postulate that these surfaces "form ruts" for light to follow: there are theorems that it must do so. But the situation is also dissimilar from Section 4.3.3.1, in that the theorems invoke SEP: for it dictates the form that Maxwell's equations take in general relativity.

We can see both the similarity and the dissimilarity in the simplest of this kind of theorem: viz. where we simplify the description of the light wave, by taking the limit of short wavelengths. In this limit, light is described as consisting of rays, with each ray being characterized by a curve in spacetime; (and at each point along the curve, an intensity of the light, corresponding to the amplitude of the wave—but we need not consider intensities). This is called the 'geometric optics limit', since geometric optics (as vs. wave optics) describes light as composed of such rays.

When we take this limit, the direction of the ray is given by the wave-vector (i.e. covector, 1-form) $k$ which is the gradient of the phase: the tangent vector to the ray is the corresponding contravariant vector $k^i \equiv g^{ij} k_j$. It is straghtforward to show from Maxwell's equations, as dictated by the SEP, that:

(i): $k$ is a null vector, i.e. $k^2 = 0$; and

[16]For details, cf. e.g. Misner Thorne and Wheeler (1973, pp. 471-480), Wald (1984, p. 73), and Geroch and Jang (1975). Brown notes various subtleties about these theorems. In particular, although the form of the field equations determines $g_{ij}$ to be a (0,2) symmetric tensor, nothing in the equations dictates that $g_{ij}$ should have Lorentzian signature.

(ii): the ray is a geodesic, i.e. the ray parallel-transports its own tangent vector: $k^i \nabla_i k^j = 0$; (Misner, Thorne and Wheeler 1973, pp. 568-583).

For the purposes of Section 5, I need to take note of how these theorems can also be generalized; i.e. they hold good away from the geometric optics limit. The first point is that the propagation of waves is a complex subject, and textbooks of optics, or more generally wave physics, sport *several* inequivalent notions of the velocity of a wave. For example, one (to which we will return) is the *phase velocity*: writing the wave-vector $k = (\omega, \mathbf{k})$ as usual, $v_{\rm ph} = \frac{\omega}{|\mathbf{k}|}$. But for our concerns with causality, it seems to be agreed that the relevant notion is *wavefront* velocity (also known as: 'signal velocity'); which is essentially the velocity of the boundary between the regions of zero and non-zero excitation of the field concerned. Mathematically, the wavefront is given by the characteristics of the wave equation describing the field. So the gist of the general theorems is that the characteristics of Maxwell's equations on a general relativistic spacetime are the null surfaces defined by the tensor $g_{ij}$; (cf. Friedlander 1975: Theorem 3.2.1).

So to sum up: the moral is as before. It is by a theorem, not by an interpretive postulate, that $g_{ij}$ earns the name of 'metric'. In particular, SEP implies that Maxwell's equations take a form that makes the "physical light-cones" that are defined by the (wavefront velocity of/characteristics for) the propagation of light coincide exactly with the "geometric cones" defined by $g_{ij}(X^i, X^j) = 0$.

# 5 Two effects

## 5.1 The overall shape of the examples

The overall shape of the promised examples violating relativistic causality is now clear. At the end of Section 4, we have seen the conceptual distinction between:

(i): the *geometric light-cones* defined by the tensor $g_{ij}$, and

(ii): the *physical light-cones* that are traversed by electromagnetic radiation; (or mathematically, and generalizing from electromagnetism: the characteristics for the wave equation governing the field concerned).

And we have seen how for electromagnetism in classical general relativity, SEP makes (i) and (ii) coincide.

So we naturally look for a theory that has a regime in which SEP fails, in such a way that the physical light-cones turn out to be wider, rather than narrower, than the geometric light-cones. That is: the vectors tangent to the physical light-cones are to be spacelike, rather than timelike—understanding 'spacelike' and 'timelike' with respect to $g_{ij}$. (Here 'regime' is physicists' jargon for a certain set of ranges of values of a theory's parameters. For example, in fluid mechanics one could speak of the regime of high density and low viscosity. And sometimes, the regime is specified, wholly or in part, by specifying a state of the system concerned.)

More specifically, in terms of the decomposition of SEP (in Section 4.3.2): we look for regimes where Minimal Coupling fails, and curvature-dependent terms enter the action and equations of motion for the electromagnetic field in such a way that electromagnetic propagation is described in terms of an *effective* metric whose light-cones are wider than those of $g_{ij}$. Hence, one talks of 'superluminal light': which is not a contradiction in terms, since 'superluminal' is to be understood as greater than the speed $c$ defined by $g_{ij}$.

We consider two such regimes, both arising in quantum electrodynamics (QED): so they concern superluminal photons. In fact, they are but two of a family of effects in which vacuum polarization affects the propagation of photons, owing to the vacuum being modified by one or another external environment.[17] But I shall confine myself to these two. The first is the Drummond-Hathrell effect, which concerns QED in a general relativistic spacetime, i.e. photon propagation in an external gravitational field (Section 5.2). The second is the Scharnhorst effect (Section 5.3), which concerns photon propagation in the flat spacetime between two perfectly conducting planar plates.[18]

### 5.1.1 Limitations and alternatives

Before going into details, I should make three other general points about the overall shape of these examples. The first two are, I admit, limitations of the examples: about observability, and approximations; subsequent Sections will give more details. The third is a pointer towards other similar examples, which I will not discuss further.

**5.1.1.1 Observability?** The word 'effect' carries the connotation that one could observe it. No such luck, I'm afraid! Both effects are so tiny as to be well beyond present observation—and perhaps all future observation. Incidentally: here, 'tiny' does not mean that all photons travel at a speed greater than $c$ by a tiny amount. Rather (as one would expect for a quantum theory), it means that:

(i): there is a probability distribution for finding the photon to have travelled at various speeds; and

(ii): the probability for a photon to be found to have travelled at a speed that is greater than $c$, by a large enough margin to be observationally distinguished from $c$, is tiny. (In other words: the probability for a photon to be found a measurably large

[17]A detailed study of photon dispersion and birefringence (polarization-dependent phenomena) is Adler (1971). And in recent years, a framework for unifying these results has begun to emerge: e.g. Dittrich and Gies (1998).

[18]Brown's own discussion is on p. 165-172. Among other references, he cites Shore (2003) and Liberati et al (2002). These and Shore (2003a, 2007), and some of their references, have been my sources for what follows. I again stress, as in Sections 1 and 3.3, that our present understanding of these effects, is undoubtedly incomplete—there are plenty of open questions hereabouts, for both physicists and philosophers. A vivid illustration of this is the recent results about the Drummond-Hathrell effect (Hollowood and Shore 2007, 2007a), mentioned in Section 5.2.1.2.

distance outside the geometric light-cone is tiny.)

**5.1.1.2 An artefact of approximations?** QED is an extraordinarily accurate theory, and a long-established one. But it is very complicated, so that calculations within it often have to adopt various approximation schemes: and these effects are no exception. We shall see that the currently feasible calculations involve various approximations; of which one is perhaps especially dubious, as regards the prediction of superluminal photons. Of course, our authors stress this; (e.g. Brown 2005, p. 168; Shore 2003, pp. 508, 513, 2003a, Sections 3.2, 4.3).

**5.1.1.3 The cat out of the bag: bi-metric theories** In both these effects, the idea of two metrics is "modest", in that the second metric is "merely" effective. That is: it is a structure that helps describe a certain regime of the theory, rather than occurring in its fundamental equations describing all regimes. Nor does the structure "occur implicitly" in the fundamental equations in the sense of being mathematically determined by them, say by being a function of quantities that occur explicitly—and the same function regardless of the regime, or a choice of state.[19]

But once the idea of a theory having two metrics is out of the bag, one naturally speculates! Thus a theory might postulate *ab initio* two metrics, in the sense of two (0,2) symmetric tensors, neither of which mathematically determines the other (so neither is a function of the other); and then the theory might divide between these tensors the various roles—or something like the roles—played in general relativity by just the one tensor $g_{ij}$.

A second possibility for such a bi-metric theory is a theory with the following three features.

(1): It postulates a (0,2) symmetric tensor, call it again $g_{ij}$, that is fundamental in that it occurs in the theory's basic equations.

(2): But another tensor $\tilde{g}_{ij}$ is a function of the first, $\tilde{g}_{ij} = \tilde{g}_{ij}(g_{i'j'})$, where the function concerned is: (i) universal; (ii) exact not approximate; (iii) not invertible, or at least not usefully invertible, so that we cannot equivalently rewrite the basic equations using $\tilde{g}_{ij}$.

(3): And yet our "critters"—rods, clocks, test-particles and light-rays—"display" $\tilde{g}$ rather than the fundamental tensor $g$.

Indeed, there is a long tradition of such bi-metric theories. Weinstein (1996) is a fine philosopher's introduction to this tradition: launched, like Brown's discussion, from consideration of non-minimal coupling (cf. footnote 9). A recent example of the first possibility above is Drummond's re-formulation of variable speed of light theories (2001). But I shall follow Brown (2005, pp. 172-175) in discussing a recent example

[19]Incidentally, the emergence in relativistic pilot-wave theories of Lorentz-invariance at the observable quantum level from the non-Lorentz-invariant sub-quantum level is an example of such "implicit occurrence".

of the second possibility: the TeVeS theory of Bekenstein. This is a relativistic theory of gravity which postulates, in addition to a fundamental (0,2) symmetric tensor $g$, a vector field $U$ (which is dynamically constrained to be timelike) and a scalar field $\phi$; and these together define a tensor $\tilde{g}$. So 'TeVeS' stands for 'tensor-vector-scalar'.

The motivation for the theory lies in the tradition of modified Newtonian dynamics (called 'MOND') begun by Milgrom in the 1980s, to account for the anomalously fast rotation of galaxies and clusters without having to invoke dark matter, viz. by making gravity decrease more slowly than inverse-square for very large distances. Thus in the TeVeS theory, the scalar field $\phi$ makes gravity stronger at large distances; and the vector field $U$ enhances gravitational lensing, making the theory empirically adequate to the observations that gravitational lensing is stronger than would be expected from the lensing galaxy's visible matter: observations which are usually taken to require dark matter.

But I will not need more details of the theory's motivation. What matters for us is that the theory defines another tensor $\tilde{g}$ as a function of all three of $g, U$ and $\phi$. Roughly speaking, one obtains $\tilde{g}$ by multiplying $g$ in the spacelike directions orthogonal to $U$ by a function of $\phi$, and by dividing $g$ parallel to $U$ by the same function. From the theory's postulated action, and the resulting equations of motion, one shows that the "critters" listed above, rods etc., survey $\tilde{g}$, not $g$. So again we see Brown's moral: that it is by a detailed physical argument that a tensor—here $\tilde{g}$, not $g$—earns its chrono-geometric significance.

So much by way of discussing a heterodox, though so far unrefuted, relativistic theory of gravity. Now I turn to proposals for superluminal light propagation in QED in an otherwise orthodox relativistic setting: general relativistic for the Drummond-Hathrell effect, and special relativistic, though with a preferred rest, for the Scharnhorst effect.

## 5.2 The Drummond-Hathrell effect

Drummond and Hathrell (1980) studied the effect on light propagation of vacuum polarization in a curved spacetime. Vacuum polarization gives a photon an effective size characterized by the electron's Compton wavelength $\lambda_C = \hbar/mc$ (with $m$ the electron mass). This suggests that if the photon propagates in an anisotropic spacetime with typical curvature length-scale $L \sim \lambda_C$, its motion might be affected. Here we can already see the point made in Section 5.1.1.1, that such an effect on the motion would be tiny; since for ordinary astrophysical objects—even the event-horizon of a black hole—the gravitational field is weak enough that the curvature length-scale $L$ is vastly larger than $\lambda_C$.[20]

Drummond and Hathrell's analysis confirms (subject to three main approximations) the suggestion that the photon's motion, in particular its speed, is affected by gravity.

[20]Section 5.2.1 will give a numerical estimate for a solar mass black hole.

That is: they deduce an effective action, and corresponding (non-linear) generalizations of Maxwell equations (i.e. equations of motion for light), which display an interaction between quantized electromagnetism and spacetime curvature. So both the action and the equations contain curvature-dependent terms, and violate SEP. Besides, the equations of motion imply—again, subject to approximations—that in some scenarios (in particular the Friedmann-Robertson-Walker and Schwarzschild spacetimes) light propagates superluminally. I will first summarize these results of Drummond and Hathrell, and mention some work by later authors (Section 5.2.1). Then I will discuss how despite this superluminal propagation, causal contradictions can apparently be avoided (Section 5.2.2).[21]

### 5.2.1 Faster than light?

**5.2.1.1 Three approximations** Drummond and Hathrell showed that vacuum polarization implies an effective action for the electromagnetic field in a curved spacetime that contains curvature terms and so violates SEP. But their derivation is subject to three approximations. They are:

(i): Only one-loop Feynman diagrams are considered.

(ii): Gravity is assumed to be weak, in the sense that the effective action keeps only terms of first order in the curvature tensors $R$ (scalar), $R_{ij}$ (Ricci) and $R_{ijkl}$ (Riemann). In a bit more detail: this restriction implies that results are valid only to the lowest order in the parameter $\lambda_C^2/L^2$, where $L$ is the typical curvature length scale; so the results are more accurate, the larger $L$ i.e. the weaker is the gravitational field.

(iii): The photons are assumed to be low frequency, in the sense that the effective action neglects terms involving higher orders in derivatives of the fields.

These approximations are in ascending order of 'importance', in the sense of 'recalcitrance'! That is:

As to (i): One-loop diagrams contribute to the action terms proportional to the fine structure constant $\alpha \sim 1/137$, and higher-loop diagrams would contribute terms proportional to powers of $\alpha$—which are smaller. So we can expect the one-loop approximation to give the dominant effects.

As to (ii): There is a trade-off here. As mentioned, the effect depends on $L$ being comparable with $\lambda_C$; but our results are more accurate the larger $L$ is.

As to (iii): There are two reasons why we need to consider high-frequency photons; one theoretical and one experimental. The theoretical reason develops the remarks at the end of Section 4.3.3.2. There I reported that for questions about causality, the relevant notion of the velocity of a wave (of the *several* available!) is the *wavefront velocity* $v_{\text{wf}}$: it represents the velocity of the boundary of the region of excitation of the field, and is given mathematically by the characteristics of the field's wave equation. I also reported that with SEP, $v_{\text{wf}}$ is $c$: i.e. the characteristics for the orthodox Maxwell's equations in general relativity are the null hypersurfaces defined

[21] My main sources are Shore (2003, 2003a); cf. footnote 16.

by $g_{ij}$. But now SEP, and these orthodox equations, are gone, and so we have to ask: what is $v_{\rm wf}$ for Drummond and Hathrell's non-linear generalization of Maxwell's equations? Fortunately, a theorem of Leontovich (from 1972) states that for a large class of partial differential equations (including Drummond and Hathrell's equations), the wavefront velocity is the infinite frequency limit of the phase velocity. That is:

$$v_{\rm wf} \;=\; \lim_{\omega\to\infty} v_{\rm ph}(\omega) \;=\; \lim_{\omega\to\infty} \frac{\omega}{\mid \mathbf{k} \mid} \;. \tag{5.1}$$

(The proof is sketched in Shore (2003a, Section 3.2) and (2007, p. 10-11).) So $v_{\rm wf}$ is independent of frequency; but we need to know the high-frequency limit of $v_{\rm ph}(\omega)$.

The experimental reason relates to the fact that on Drummond and Hathrell's analysis, the correction to the photon's speed is $O(\alpha\lambda_C^2/L^2)$, with $L$ the typical curvature length scale, as before. Experimentally, this is a ratio of a square of a quantum scale $\lambda_C$ to an astrophysical scale, and is therefore minuscule. (In a black hole example below, it will be $O(10^{-34})$.) In order to assess whether we could observe this correction, Drummond and Hathrell suggest (1980, p. 354) that we make two assumptions:

(a): The typical time over which propagation can be followed is characterized by $L$, so that the length difference to be observed is given by $\alpha\lambda_C^2/L$.

(b): Observability requires this to be $O(\lambda)$ where $\lambda$ is the wavelength of the light. These assumptions suggest that observability requires that $\alpha\lambda_C^2/\lambda L \;>\; 1$, while our approximations (ii) and (iii) above required respectively that $L \gg \lambda_C$ and $\lambda \gg \lambda_C$. Agreed, assumption (b) is unduly pessimistic, since modern spectroscopy enables it to be weakened by some six, or even eight, orders of magnitude. That is: observability might require only that, say, $\alpha\lambda_C^2/\lambda L \;>\; 10^{-8}$. Nevertheless, to observe the effect we would obviously like $\lambda$ as small as possible.

So much by way of caveats about the approximations. Turning now to reporting the results, there is the proverbial "good news and bad news": results giving superluminal propagation, and results suggesting that it does not occur. Following Shore, I will report these in order.

**5.2.1.2 Good news and bad** So far as I know, the main way in which superluminal propagation is derived is by applying to Drummond and Hathrell's "Maxwell's equations" (derived from their effective action) the geometric optics (short wavelength) limit; (cf. Section 4.3.3.2). When we do this, the previous result, that $k$ is null, $k^2 = 0$, is replaced by a more complicated equation. We will not need it; but for completeness it is

$$k^2 - \frac{2a}{m^2} R_{ij}k^i k^j + \frac{8b}{m^2} R_{ijmn}k^i k^j a^m a^n \;=\; 0 \;, \tag{5.2}$$

where $a$ and $b$ are real constants (of magnitude about 1% and 0.1% of $\alpha$), $R_{ij}$ is the Ricci tensor, $R_{ijmn}$ is the Riemann tensor and $a$ is the polarization vector (a spacelike 4-vector normalized to unity). For us the important point about eq. 5.2 is that it is homogeneous and quadratic in $k$, and so we can write it in the form

$$\tilde{g}^{ij} k_i k_j = 0 \;;\; \text{with } \tilde{g}^{ij} \equiv \tilde{g}^{ij}(R_{klmn}, a^p) \;. \tag{5.3}$$

This (frequency-independent) function $\tilde{g}$ of curvature and polarization represents an effective metric, as follows. The tangent vector to a light ray is now given by $p^i := \tilde{g}^{ij}k_j$; i.e. light rays are curves $x^i(s)$ with $dx^i/ds = p^i$. This definition of $p^i$ implies

$$(\tilde{g}^{-1})_{ij}p^i p^j = \tilde{g}^{ij}k_i k_j \ = \ 0 \ . \tag{5.4}$$

So $\tilde{g}^{-1}$ defines an effective metric: (though we still raise and lower indices with $g_{ij}$). We will from now on write $G$ for $\tilde{g}^{-1}$. So our question is whether in some solutions, $G$'s light cones—in Section 5.1's terms: the physical light cones—are wider than the geometric light cones defined by $g_{ij}$. In other words: a solution exhibits superluminal light if $p$ is spacelike (with respect to $g_{ij}$)—are there such solutions?

Indeed, Drummond and Hathrell (1980, p. 354; and later authors including Shore) show that the Friedmann-Robertson-Walker spacetime, and the Schwarzschild spacetime, are such solutions. But as we would expect from the discussion above, the effect is numerically tiny. In particular, for classical electromagnetism in the Schwarzschild spacetime, there is a geometric optics solution (i.e. a solution of $k^2 = 0, k^i\nabla_i k^j = 0$) describing a light ray in a circular orbit with radius $r = 3GM$ (Misner, Thorne and Wheeler (1973) pp. 672-677); and for one solar mass at the singularity, the quantum correction to the classical speed $c$ is $O(10^{-34})$.

There are also results suggestive of superluminal propagation, for generalizations of the Drummond and Hathrell action. In particular, Shore has derived an action containing all orders in derivatives (cf. approximation (iii) above): though like Drummond and Hathrell, he retains only terms of $O(RFF)$ in curvature $R$ and field strength $F$. And he has shown that this action, applied to Bondi-Sachs spacetime (describing gravitational radiation far from its source), implies that $v_{\rm wf} \equiv v_{\rm ph}(\infty)$ is superluminal.

On the other hand, the "bad news": there are indications that these results (even Shore's about Bondi-Sachs) are artefacts of the approximations used in deriving them. For a complete analysis of high frequency propagation requires one to consider higher-order terms in curvature and field strength, not just in derivatives. And there is some mathematical evidence—just recently, strong evidence—that these terms will, for high frequencies $\omega$, drive $v_{\rm ph}(\omega)$, and so $v_{\rm wf}$, to c.

I will not need details about this evidence. But for completeness: the evidence concerns the correction to the classical light cone condition $k^2 = 0$ being an integral whose integrand contains as a factor a phase roughly of the form

$$\exp[-is^2\Omega^2(R,\omega)P(R,s)] \ ; \tag{5.5}$$

where $s$ is the integration variable, $R$ is a generic curvature component and $\Omega(R,\omega)$ and $P(R,s)$ are functions (not exactly known) of the gravitational field, and also of $\omega$ and $s$ respectively; and where $\Omega^2 \sim \frac{R\omega^2}{m^4}$. So it seems likely that whatever the behaviour of $P$ and the other factors in the integrand, high frequencies $\omega \to \infty, \Omega \to \infty$ and therefore rapid variation in the exponent, will drive the integral, i.e. the correction to the classical condition $k^2 = 0$, to zero. (For an introduction to these details, cf. Shore (2003: pp. 516, 518; 2003a, Section 4.3; 2007, pp. 28-29).)

Just recently, this evidence has been much strengthened. Hollowood and Shore (2007, 2007a) have shown by means of new techniques that $v_{\rm wf} \equiv v_{\rm ph}(\infty)$ is always $c$. But they also show that micro-causality fails, i.e. commutators of fields at space-like separations do not vanish, but fall off exponentially; so that there is violation of relativistic causality in sense (ii) of Section 3.2.1.

So much by way of reviewing the prospects for superluminal propagation in the Drummond-Hathrell effect. Of course, whether (and in what sense) it occurs, our main philosophical point—viz. Brown's moral that a tensor earns its chrono-geometric significance by dint of detailed physical arguments—is vividly illustrated. For in this effect, the light-cones, and more generally geometric structure, defined by $g_{ij}$ are at one remove (though a numerically minuscule one!) from the physical behaviour of light: which is instead described by the effective metric $G_{ij}$.

### 5.2.2 Avoiding contradictions

Assuming now that there *is* superluminal propagation, I turn to how causal contradictions can be avoided. For as announced in (B) of Section 3.3, the argument against causal contradictions is not the usual philosophical one, that a closed "causal loop" implies a severe consistency condition which one just presumes the solutions in question satisfy. The argument is rather that the kind of superluminal propagation envisaged could not be exploited to produce a causal loop, i.e. a zig-zag, there and back, into the causal past of an initial event.

More exactly, Shore makes two points. The first is general (and also made by Drummond and Hathrell 1980, p. 353); the second is specific. First, he suggests a zig-zag process from an event $A$ to a spacelike event $B$, and then from $B$ to an event $C$ that is spacelike to $B$, but in the causal past of $A$, may be expected to require 'that the laws of physics should be identical in the local frames at different points of spacetime, and that they should reduce to their special relativistic forms at the origin of each local frame' (2003, p. 511; cf. 2003a, Section 2.1).[22] But this requirement is just SEP (cf. Section 4.3.2): which of course fails in the framework of the Drummond-Hathrell effect.

Second, Shore points out that one can investigate the causal structure given by the effective metric $G_{ij}$ by using general notions and results that have been developed for classical general relativity, i.e. for the causal structure fixed by the usual metric $g_{ij}$. Thus there is a well-known spectrum of properties of causal "good behaviour"—one of which, though strong, is especially relevant to assessing the causal structure fixed by $G_{ij}$. This is the concept of *stable causality*: I shall not give the exact definition of this; (cf. e.g. Hawking and Ellis 1973, p. 198; Geroch and Horowitz 1979, p. 241; Wald

[22] I presume his idea is that only with this can we be sure that there can be a process from $B$ to $C$ which is like that from $A$ to $B$. But the threatened zig-zag might exploit different processes in its two legs. But nevermind: we will see, now and in Section 5.3.2, stronger reasons to doubt that there can be such zig-zags.

1984, p. 198). We only need the idea: that a spacetime $(M, g_{ij})$ ($M$ the manifold) is stably causal if not only does it lack closed timelike curves, but also the spacetime resulting from it by a slight opening out of $g_{ij}$'s light-cones at every point does not have any such curves. The idea is of course motivated by wanting causal good behaviour to be robust to perturbations. Stable causality also follows from global hyperbolicity, which we saw in Section 3.2.2 to be usually assumed in quantum field theory.

Clearly, stable causality can be applied to our effective metric $G_{ij}$ in two ways:

(i): If one knows $(M, g_{ij})$ is stably causal, one can expect that $(M, G_{ij})$ has no closed timelike curves, since $G_{ij}$ differs from $g_{ij}$ only by terms of $O(\alpha)$;

(ii): One can ask, more ambitiously, whether $(M, G_{ij})$ is itself stably causal; so that its causal good behaviour is itself robust to perturbations (in particular to overcoming Section 5.2.1's approximations!). In some cases, this question can be answered positively by invoking the following characterization of stable causality. As usually stated for $(M, g_{ij})$, it is that a spacetime is stably causal iff it has a "global time function", i.e. a smooth function $f : \mathcal{M} \to \mathbb{R}$ whose gradient is everywhere timelike (Hawking and Ellis 1973, Prop. 6.4.9, p. 198; Wald 1984, Theorem 8.2.2, p. 198). But this equivalence of course remains valid for $(M, G_{ij})$ with *its* definition of 'timelike'. So one can be assured that $(M, G_{ij})$ is stably causal by finding a global time function $f$. And indeed, Shore remarks (2003, p. 513; 2003a, Section 2.4; 2007, pp. 12, 29) that in the Friedmann-Robertson-Walker spacetime discussed in Section 5.2.1, the usual global time coordinate is such a function, for $G_{ij}$ no less than for $g_{ij}$. So here is a clear case where superluminal light, i.e. the physical light cones being wider than the geometric ones, harbours no causal anomalies.

## 5.3 The Scharnhorst Effect

I turn to superluminal light propagation in QED in a special relativistic setting: the Scharnhorst effect (Scharnhorst 1990; Barton 1990). Again I will first summarize results (Section 5.3.1). Then I will discuss how causal contradictions can apparently be avoided (Section 5.3.2).[23]

### 5.3.1 Faster than light?

In 1990, Scharnhorst and Barton showed that for the vacuum between two infinite perfectly conducting plates, QED predicted that photons would have a wavefront velocity $v_{\text{wf}}$ larger than $c$. More precisely, $v_{\text{wf}}$ is enhanced in the direction orthogonal to the plane of the plates. Though these derivations were based on approximations, in particular on assuming low-frequency photons (cf. (iii) in Section 5.2.1.1), there is evidence (Barton and Scharnhorst 1993, pp. 2040-2044; Scharnhorst 1998, pp. 706-707) that the result is not an artefact of the approximations, but a genuine effect, albeit an unobservably small one. (Cf. below for the small size).

[23]My main source is Liberati et al (2002); cf. footnote 16.

But unlike Section 5.2.1.1, I shall not go into details about the approximations. After all (as Barton and Scharnhorst stress: 1993, p. 2044), QED in flat spacetime is much better understood than QED in curved spacetime, and the system of two plates has been much studied for another striking effect, the Casimir effect: according to which there is a force between the plates, even when the quantum electromagnetic field between them is in the vacuum state.[24] I will only stress (as Barton, Scharnhorst and Liberati et al. do) that the superluminal propagation does not violate Lorentz-invariance of the theory (the action and equations of motion which imply the propagation), but reflects only the non-Lorentz-invariance of the vacuum state.

There are two points here.

(i): Suppose a theory obeys a symmetry in the sense that a certain transformation, e.g. a spatial rotation or a boost, maps any dynamical solution to another solution. This by no means implies that every solution should be invariant, i.e. mapped onto itself, under the transformation: after all, not every solution of Newtonian mechanics is spherically symmetric!

(ii): Agreed, the vacuum state for *empty* Minkowski spacetime *is* required to be Lorentz-invariant since it should "look the same" in a translated, rotated or boosted frame. But the presence of the plates breaks this symmetry, just as a pervasive inertially-moving medium would do: licensing a non-Lorentz-invariant vacuum state.

A simple Newtonian example illustrates both these points (given by Liberati et al. Section 2.2). A wavefront of sound spreading from a point source in a fluid at rest in a Newtonian spacetime is described as spherical in the rest frame of the fluid, but in a Galilean-boosted frame it is described as blunted (due to reduced relative speed) in the direction of the boost, and as elongated (increased speed) in the opposite direction.

I turn to numerical details of QED's predictions. Writing $g_{ij}$ for the Minkowski metric (with signature (-,+,+,+)), the effective metric between the plates, i.e. defining (to order $\alpha^2$) the physical light-cones of photon propagation, is

$$G_{ij} = g_{ij} - \frac{\xi}{1+\xi} n_i n_j \ ; \tag{5.6}$$

where $n^i$ is the unit spacelike vector orthogonal to the plates; (and we again raise and lower indices with $g_{ij}$). If $a$ is the distance between the plates, we have

$$\xi \ = \ 4.36 \times 10^{-32} \left( \frac{10^{-6}\mathrm{m}}{a} \right)^4 \ . \tag{5.7}$$

This is minuscule. But since it is positive, the light-cones of $G_{ij}$ are slightly, albeit unobservably, wider in the direction orthogonal to the plates than those of $g_{ij}$: superluminal propagation.

[24] For the history and philosophy of this effect, cf. Rugh, Zinkernagel and Cao (1999).

### 5.3.2 Avoiding contradictions

There is of course a large literature about superluminal propagation in a special relativistic setting, as part of the yet larger literature on the foundations of special relativistic kinematics. Fortunately, Liberati et al. (2002) connect the Scharnhorst effect with this literature, of which they give a judicious and detailed discussion; (as of course does Brown: 2005, Chapters 2-6). So I am happy to do no more than report some of their main points: certainly, I could not do better! I shall report four points: two are general and correspond roughly to the first point of Shore's in Section 5.2.2; two are specific to the Scharnhorst effect, and correspond to Shore's second point, about stable causality.

First, Liberati et al. emphasise that Lorentz-invariance does not preclude superluminal propagation: a speed $c$ can be invariant without being a maximum signal speed. One sees this clearly in the style of derivation of an invariant speed, and of the Lorentz transformation, pioneered by von Ignatowsky in 1910, and repeatedly rediscovered and developed since then. These derivations neither assume, nor deduce, that a signal cannot travel faster than $c$. (Rather, one deduces that there cannot be a reference frame, or a coordinate system, with a relative speed greater than $c$; cf. Liberati et al. 2002, Sections 2.1, 2.3.)

Liberati et al. also emphasise that superluminal propagation does not imply that there can be a causal zig-zag from a cause $A$ to a spacelike effect $B$, which is itself the cause of a second effect $C$ lying in the causal past of $A$. To sustain this implication, one would presumably have to require that 'in any given reference frame, the only criterion for saying that an event is the cause of another one [is] the time ordering in that frame'. Which is surely false. Although 'there are no precise definitions of [cause and effect] ... the criteria used to establish that $e_1$ is a cause of $e_2$ are based on considerations ... about the so-called "arrow of time" '(2002, Section 3.1).

But whatever the vagaries of the notions of cause and effect, there is obviously no threat of a contradiction provided that with respect to one particular reference frame, all superluminal propagation is forward in time.[25] And turning now to the Scharnhorst effect, it is straightforward to check that this is so, with respect to the rest frame of the plates. Indeed, here we connect with the notions of causal good behaviour, in particular stable causality, studied in classical general relativity and invoked by Shore, as we saw in Section 5.2.2. Thus we need only consider the spacetime $\mathbb{R}^4$ containing the two infinite plates, distance $a$ apart, and equipped with the usual Minkowski metric outside the plates, and the effective metric eq. 5.6 inside. It is obvious that the time coordinate $t$ of the rest frame of the plates has $\nabla t$ everywhere non-zero and timelike. So the spacetime is stably causal: not only are there no closed timelike curves; but also none could arise by a slight widening of the cones throughout the spacetime (i.e. between the plates, a further widening from $G_{ij}$). Obviously, a similar argument will

[25]Drummond and Hathrell also make this point (1980, p. 353). One might add: consider such propagation, or even instantaneous action-at-a-distance, in a Newtonian spacetime.

secure stable causality for scenarios with more than one pair of plates, at least if they do not move relative to one another.

Finally, what about pairs of plates in relative motion? Liberati et al. consider various scenarios, arguing that a causal contradiction will not arise. Then they end by suggesting that the general threat of contradiction should be analyzed using Hawking's Chronological Protection conjecture. The idea of the conjecture is that if a spacetime is causally well-behaved "early on", it cannot become badly-behaved later. More precisely: a region of closed timelike curves which does not extend indefinitely into the past must have a "first" closed null curve, at which—Hawking argues—uncontrollable singularities will occur, implying the breakdown of quantum field theory on curved spacetime, and the need for some sort of quantum theory of gravity; (cf. Earman 1995, pp. 188-193). Applying this idea to a spacetime in which early on, several pairs of plates are well separated, and individually, stably causal (cf. above), we infer that if some scheme for the plates' later motion seems to yield a causal contradiction, then, as Liberati et al. put it: 'causal paradoxes are the least of your worries since you are automatically driven into a regime where Planck-scale quantum gravity holds sway' (2002, Section 3.2.4).

# 6 Conclusion

By way of concluding this paper, let me briefly list three of my main claims:—

(i): In some QED scenarios, relativistic causality is apparently violated.

(ii): These scenarios raise open questions, not just about how to define relativistic causality, and how to avoid causal contradictions (in more interesting ways than by saying 'there is a severe consistency constraint'), but about the much wider question, of the future of relativistic quantum physics.

(iii): Philosophically, these scenarios illustrate Brown's moral that the mathematical representatives of geometry get their geometrical significance by dint of detailed physical arguments.

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# 7 Appendix: Two familiar examples

This Appendix reports the two examples of quantum-theoretic violations of relativistic causality which are most familiar to philosophers of physics: the pilot-wave approach, and the Newton-Wigner representation. I shall say much more about the former.

Both examples concern Minkowski spacetime. So one naturally asks which of Section 3.2.1's precise formulations are violated. But this is a subtle, and even controversial, question, since the relations of these examples' formalisms to the local field operators that are those formulations' topic, are indirect: and in some respects, unknown or controversial. So here again there are questions I cannot pursue: it must suffice that the references below are the place to begin finding the answers.

## 7.1 The pilot-wave

There are various pilot-wave approaches to relativistic quantum theory. But I shall sketch one well-developed approach which is strongly analogous to the best-known pilot-wave approach to non-relativistic quantum theory. As an example of violating relativistic causality, it is in a sense only an example "by courtesy": for it takes the relativistic light-cone structure as a "merely emergent" or phenomenological description. For this reason, and also because of its being analogous to Newtonian action-at-a-distance, this example provides an interesting comparison with the others. I shall: begin with the pilot-wave approach to non-relativistic quantum theory (Section 7.1.1); then discuss action-at-a-distance within it, and the Newtonian analogy (Section 7.1.2); and finally turn to the relativistic case (Section 7.1.3).

### 7.1.1 The non-relativistic case

Recall that the system comprises both a wave and one or more point-particles. Let us begin with the wave and how it evolves over time. The wave is a complex-valued function $\psi$ on configuration space $\mathcal{Q}$ (e.g. $\mathcal{Q} = \mathbb{R}^{3N}$ for $N$ spinless particles in euclidean space), which always evolves by the Schrödinger equation. The Schrödinger equation is local in the mathematical sense: roughly, the evolution depends on $\psi$ and its spatial derivatives but not on differences of $\psi$ at different points in $\mathcal{Q}$. But it is non-local in the physical senses that:

(i): it is defined on configuration space, not real space; and

(ii): wave-functions of even a single particle propagate instantaneously: if at time $t = 0$ a wave-function $\psi(0)$ has compact spatial support (i.e. is non-zero only in a compact spatial region), then at all later times $t$, no matter how small, $\psi(t)$ is non-zero throughout all space.

Despite the continuous deterministic Schrödinger evolution, an analysis of measurement processes demonstrates an effective collapse of the wave function, nowadays often called 'decoherence': which explains the instrumental success of orthodox textbooks' notorious projection postulate.[26]

[26]More precisely, it explains it, once allied to the pilot-wave theory's invoking particle positions to provide definite events. That decoherence alone is not enough to solve the measurement problem is argued by e.g. Bub (1997, pp. 221-223, 231-232, 236) and Adler (2003).

Each point-particle has a continuous trajectory which is determined by the wave-function according to the guidance equation. We need to note three features of the guidance equation:—

(i): *Classical analogues*: This equation is natural. Indeed, it follows from the orthodox probability current for the Schrödinger equation found (for the one-particle case) in most textbooks. It is also the obvious wave-mechanical analogue of a central equation of classical Hamilton-Jacobi theory, viz. $p = \frac{\partial S}{\partial q}$. More generally, much of the pilot-wave approach bears comparison with Hamilton-Jacobi theory. In particular, many quantum effects are due to the presence (in the analogue of the classical Hamilton-Jacobi equation) of an extra potential term dependent on the wave-function, viz. the quantum potential $U$. In the simplest one-particle case, $U := -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}$; where $R$ is given by the polar decomposition of the wave-function, i.e. $\psi(\mathbf{q}, t) = R(\mathbf{q}, t)\exp(iS(\mathbf{q}, t)/\hbar)$.

(ii): *Probability and equivariance*: The pilot-wave approach recovers the orthodox quantum probabilities by averaging over particle position using $\mid \psi \mid^2$ as the probability density. This is the Born rule, understood non-instrumentalistically; i.e. understood with probabilities like those of classical statistical mechanics. Besides: taken together with the Schrödinger equation, the guidance equation implies that this probabilistic interpretation of the wave-function, viz. that $\mid \psi \mid^2$ is the position probability density, is preserved over time.

(iii): *Non-locality*: The guidance equation also implies that in a multi-particle system, the motion of each particle is determined in part by the simultaneous actual positions of the other particles. For the guidance equation says that the particle's momentum is the gradient of the phase $S$ of the wave-function at the point in configuration space corresponding to all these actual positions. To be precise, for the $i$th particle, and the $N$ actual positions $\mathbf{q}_j, j = 1, \ldots, N$: $\mathbf{p}_i \equiv m\dot{\mathbf{q}}_i = \nabla_i S \mid_{\mathbf{q}_1, \ldots, \mathbf{q}_N}$.

Non-locality is also evident in the quantum potential for many particle systems. Thus for two spinless particles, so that $\psi(\mathbf{q}_1, \mathbf{q}_2, t) = R(\mathbf{q}_1, \mathbf{q}_2, t)\exp(iS(\mathbf{q}_1, \mathbf{q}_2, t)/\hbar)$, the quantum potential is $U(\mathbf{q}_1, \mathbf{q}_2, t) = -\frac{\hbar^2}{2m}\frac{1}{R}(\nabla_1^2 R + \nabla_2^2 R)$; and only for the special case of product states $\psi(\mathbf{q}_1, \mathbf{q}_2, t) = \psi_1(\mathbf{q}_1, t)\psi_2(\mathbf{q}_2, t)$, is the quantum potential a sum, $U = U_1(\mathbf{q}_1, t) + U_2(\mathbf{q}_2, t)$.

On the other hand, this non-locality cannot be exploited to send a signal in the sense of affecting the statistics of distant experiments—provided the system is in "quantum equilibrium", i.e. provided that $\mid \psi \mid^2$ is indeed the position probability density. That is, the orthodox quantum no-signalling theorem is recovered by averaging over particle positions with $\mid \psi \mid^2$.

### 7.1.2 Action-at-a-distance?

In Section 7.1.1's summary, two features look like action-at-a-distance: the instantaneous spreading of wave-functions, and the non-local guidance equation. (In this Subsection, I set aside the quantum potential, discussion of which would be similar to that of the guidance equation.)

The first is undeniable, but also unsurprising. For this is a feature of *orthodox* non-relativistic quantum theory which the pilot-wave approach simply inherits—and which one naturally expects to disappear in a relativistic theory, not least because in a relativistic setting such superluminal propagation seems to threaten causal contradictions. But cf. Section 7.1.3 and Section 7.2.

But we need to pause here over the second, the guidance equation. I agree that it is natural to take it as asserting action-at-a-distance; (though as just noted, this "action" in an individual process could only be used to send a signal if the system was in quantum dis-equilibrium). But I should register that this can be (and has been) resisted—for reasons that apply equally to the more familiar case of Newton's law of universal gravitation, $F = G\frac{m_1 m_2}{R^2}$, mentioned in (2) of Section 2. So I shall explain both the natural construal and the reasons for resistance, for both cases.

For both the pilot-wave and Newton's law, the value of a quantity is a function of the simultaneous value of another quantity. For gravity, the force is determined by the simultaneous value of distance (or position of the other mass); for the pilot-wave, the momentum (or velocity) is determined by the simultaneous value of many positions. (Agreed, there are other differences which are crucial for physical calculations and explanations: above all, that the size of the effects of Newtonian gravity drops off with distance, while that of the pilot-wave need not.) In both cases, it is very natural to take the mathematical statement of functional dependence as a causal statement, asserting action-at-a-distance. And it is natural to support this causal construal by appealing to counterfactuals. Thus it is natural to say that in a Newtonian world, a counterfactual like 'If the centre of the Sun were now, say, 10,000 km away from where it actually is, the Sun's gravitational pull on the Earth would *now* be in a slightly different direction' is true. And the second occurrence of 'now' (rather than 'eight minutes from now') suggests instantaneous causation. In particular, within contemporary philosophy of causation: an advocate of the counterfactual analysis of causation (Lewis 1973) would take this counterfactual to indicate instantaneous causation.

And similarly for the guidance equation. For example, for the two particles in an EPR-Bell experiment: it is natural to say that a counterfactual like 'If the L-particle were now in a different position, the velocity of the R-particle would now be different' is true, and indicates causation.

But I admit that one can resist this; in fact Dickson (1998, Section 9.4, pp. 196-208) does so. His overall position is reminiscent of Norton's causal anti-fundamentalism (Section 2). He is cautious about causal judgments, and suspicious of the counterfactual analysis of causation. Like Norton, he suggests that under determinism, the most we can safely say is that the *total* present state is the effect of earlier states: he is wary of logically weaker, localized, facts or events as causal relata.[27]

[27]Beware of a false move. We could not really avoid action-at-a-distance, by (i) allowing such localized facts or events as causal relata, and then (ii) saying that the various present localized facts or events (e.g. in the Newtonian case: $m_1$'s position, and the force on $m_2$) are joint effects of a common cause, viz. the total state at any earlier time. For one can consider a very recent time, and the state at that time of a very distant region—say, the positions and velocities of bodies in that region; and so get "as near as makes no difference" to instantaneous causation, viz. the causal "contribution" from

In more detail: Dickson agrees that in everyday life we tend to interpret counterfactuals as non-backtracking, i.e. to take a counterfactual supposition about a state of affairs at time $t$ to preserve most of the past of $t$. And this makes us endorse the counterfactuals above, for we think along the following lines: if the Sun were now in a different place, nevertheless its past and the Earth's past would be as it actually was until very recently, and so the Earth would be very nearly where it actually is, and so would indeed feel the Sun's pull in a slightly different direction. But, says Dickson, it is unclear what this interpretative tendency—apparently conventional and so alterable—has to do with causation. Thus he and the counterfactual analysis agree that if instead we take counterfactuals as backtracking, then for a deterministic theory like Newtonian gravity, a counterfactual supposition about the present implies differences from actuality indefinitely far into the past: differences which could in general grow as we go further into the past (Lewis 1973a: pp. 75). But Dickson sees this as a sign—not of how backtracking counterfactuals are irrelevant to causation—but of how poorly we understand causation, in a deterministic world no less than (and perhaps even more than!) in a indeterministic one.

So much by way of:

(i) reporting the traditional, natural construal of the guidance equation (and Newton's law of gravitation) as involving instantaneous causation; and

(ii) registering that one can nevertheless resist this.

My own view is that the traditional construal is right: but as I admitted in (2) of Section 2, to defend this view would be a large project in the philosophy of causation, which I duck out of. (And as I said there, I also do not have a general criterion of when spacelike functional dependence amounts to spacelike causation (a violation of relativistic causality), rather than merely reflecting the joint effects of a common cause. But I think it is clear that all my examples involve the former, not merely the latter.)

So to assess whether the pilot-wave approach violates relativistic causality, I turn to ...

### 7.1.3 The relativistic case

I turn to the adaptation of the pilot-wave approach in Section 7.1.1 to relativity. There have been various obstacles, various significant achievements—and there are important projects yet to be done. I shall confine myself to reporting (from Holland 1993, Bohm and Hiley 1993) some salient points of work on:

(i): single-particle relativistic wave equations: where I will emphasise the question whether sub-luminal trajectories can be defined from the current as usually defined; and

(ii): quantum field theory: where I will give more details, and emphasise the use of a preferred frame and the non-locality of the quantum potential (Section 7.1.3.1).

So in various respects, what follows just scratches the surface of a large subject.

the distant bodies to the nearby present fact or event.

For example, one topic which I will ignore is the pilot-wave account of many-particle wave equations; (for the many-particle Dirac equation, cf. Bohm and Hiley 1993, pp. 214-225, 272-286; and briefly, Holland 1993, p. 509). So I will not discuss guidance equations for particles that are non-local on analogy with that in (iii) of Section 7.1.1, viz. by taking a gradient of $S$ in configuration space at a point determined by all the particles' positions. But we will still see non-locality in Section 7.1.3.1 below, viz. in the behaviour of the quantum potential.

So first, I consider (i): single-particle wave equations. To give a pilot-wave interpretation of such equations, we naturally ask whether the formalism allows the definition of a 4-vector current $j^\mu$ with the properties that:

(a): $j^\mu$ is conserved, $\partial_\mu j^\mu = 0$;

(b): $j^\mu$'s time-component $j^0$ is positive, and so might be interpreted as a probability density;

(c): $j^\mu$ is always timelike, so that its integral curves can be the worldlines of the particle concerned, which therefore travels subluminally.

It turns out that the Klein-Gordon equation, describing a spin 0 particle, resists a pilot-wave interpretation. Its 4-vector current $j^\mu$ enjoys property (a), but violates (b) and (c). On the other hand, the Dirac equation has a current satisfying all of (a)-(c); (Holland 1993, pp. 498-509; Bohm and Hiley 1993, pp. 232-238 for the Klein-Gordon equation, and pp. 214-222 for the Dirac equation).

**7.1.3.1 Quantum field theory** The pilot-wave approach in Section 7.1.1 carries over successfully to the relativistic quantum field theory of bosons, such as a spin 0 particle or the photon. The general idea is threefold:

(a): We first formulate the theory in the Schrödinger picture, using the space representation of the field coordinate $\psi$. This formulation is entirely orthodox, though not widely presented by the textbooks.

(b): Then we make the polar decomposition of the wave-function, obtaining (as in Section 7.1.1) a conservation equation, and a quantum analogue of the Hamilton-Jacobi equation containing a new quantum potential term.

(c): Then we interpret (a) and (b), much as we did in Section 7.1.1. Namely: the orthodox wave-function $\Psi$ is a physically real (though of course mathematically complex) field on the configuration space of the field $\psi$: i.e. it is a functional of the field $\psi$. $\Psi$ always evolves by the Schrödinger equation. So far, again so orthodox; (at least as regards formalism—orthodoxy might cavil at calling a wave-function physically real). But there is also at all times an actual field configuration, which evolves by a guidance equation which is natural, derived from the formalism, and a generalization of that in Section 7.1.1. And we recover the orthodox probabilistic results by averaging over field configurations, using the Born rule understood non-instrumentalistically.

I shall say a bit more about (a)-(c), concentrating on the interpretative aspects in (c); and for simplicity, on a neutral, spin 0, massless particle, described classically by a *real* scalar function $\psi(\mathbf{q}, t)$. (For many more details, cf. Holland 1993, pp. 519-537;

Bohm and Hiley 1993, pp. 238-247, 286-295.)

(a): *Space representation*:— As usual in the transition from a quantum theory of a fixed number of particles to a quantum field theory:

(i): the role of the coordinate $\mathbf{q}$ in the former, viz. being the value of a degree of freedom, is taken over in the latter, by the field $\psi$; and:

(ii): the role of the former's label $i$, viz. labelling the degrees of freedom, is taken over by the continuous index $\mathbf{x} \equiv \mathbf{q}$.

So the configuration space is the infinite-dimensional space of possible configurations $\psi : \mathbb{R}^3 \to \mathbb{R}$; and the wave-function is $\Psi[\psi(\mathbf{x}), t] = \langle\, \psi(\mathbf{x}) \mid \Psi(t) \,\rangle$. That is: the wave-function $\Psi$ is a functional of the real scalar field $\psi$, and a function of the time $t$: it is *not* a point-function of $\mathbf{x}$.

$\Psi$ obeys the Schrödinger equation $i\hbar\, \partial\Psi/\partial t = \hat{H}\Psi$; where the Hamiltonian operator $\hat{H}$ is derived from the classical Hamiltonian by the usual canonical quantization heuristic that "Poisson brackets become commutators"; so that in a representation in which $\psi(\mathbf{x})$ is diagonal, the momentum is given by the functional derivative $-i\hbar\, \delta/\delta\psi$.

(b): *Quantum potential*:— We make a polar decomposition of $\Psi$ as $\Psi = R\exp(iS/\hbar)$, with both $R = R[\psi(\mathbf{x}), t]$ and $S = S[\psi(\mathbf{x}), t]$ being real functionals of the field. Then the Schrödinger equation yields: a conservation law suggesting that $R^2$ is a probability density for the field; and a quantum analogue of the classical field's Hamilton-Jacobi equation. This quantum analogue adds to the classical equation a new potential term, $U[\psi, t] := -\frac{1}{2R}\int d^3x\ \frac{\delta^2 R}{\delta\psi^2}$. This is the field-theoretic quantum potential. Recalling the transition to quantum field theory summarized in (a) above, we see that it is analogous to the elementary quantum potential $U(\mathbf{q}_i)$ for several particles labelled $i$: $U(\mathbf{q}_i) = -\frac{\hbar^2}{2m}\frac{1}{R}\,(\Sigma_i\, \nabla_i^2 R)$; cf. (iii) of Section 7.1.1. We also see that the field-theoretic quantum potential is inherently non-local.

(c): *Guidance equation, interpretation*:— The field-theoretic guidance equation, determining the motion of the postulated actual field configuration, takes the general form $\frac{\partial\psi}{\partial t} = \frac{\delta S[\psi(\mathbf{x}),t]}{\delta\psi(\mathbf{x})}$. Recalling again the summary in (a) above, this is clearly analogous to Section 7.1.1's guidance equation $\dot{\mathbf{q}}_i = \frac{1}{m}\,\nabla_i S$. For a neutral, spin 0, massless particle, which would classically be governed by the wave equation $\Box\psi(\mathbf{x}, t) = 0$, the guidance equation takes the form $\Box\psi(\mathbf{x}, t) = -\frac{\delta U[\psi(\mathbf{x}),t]}{\delta\psi(\mathbf{x})}$.

This guidance equation, and the Schrödinger equation $i\hbar\, \partial\Psi/\partial t = \hat{H}\Psi$ from (a), are the fundamental equations of motion for the total system which comprises both an actual classical field configuration, and a wave-functional $\Psi$ on the space of such configurations. For our purposes, we need only two points about these two equations and their interpretation. The first concerns the approximate status of Lorentz symmetry; the second concerns the classical limit. (For more details about these points, cf. e.g. Holland (1993, pp. 522-524).)

(i): Our use, from (a) onwards, of the parameter $t$ amounts to postulating an absolute simultaneity structure like that in Newtonian mechanics, Galilean quantum mechanics and the non-relativistic pilot-wave approach. In particular, the constructions summarized in (b) and (c) do *not* reveal $t$ to have been "free up to" the rotation of hyperplanes associated with a Lorentz boost: as happens for the time-evolution in orthodox relativistic quantum theories. Indeed, the guidance equation and the Schrödinger equation are not Lorentz-covariant, and the actual field $\psi(\mathbf{x}, t)$ is not a Lorentz scalar. But just as in the non-relativistic case, we recovered orthodox quantum mechanical results by averaging over possessed positions using $| \psi |^2$ as the probability density: so also here, one recovers the results of orthodox relativistic quantum theory by averaging. In particular, Lorentz-covariance is an emergent symmetry: it fails at the sub-quantum level of the field configurations, but holds good once we average over field configurations.

(ii): For the pilot-wave approach, it is the right hand side term in the guidance equation, i.e. the "quantum force" (generalized gradient of a potential), which is responsible for the characteristic differences of the quantum theory from the classical theory. Accordingly, we expect to obtain the classical limit when the magnitude and gradient of the quantum potential are both negligible. And indeed, when this is so, the guidance equation reduces to the classical wave equation $\Box\psi(\mathbf{x}, t) = 0$. But away from the classical limit, the evolution of the field $\psi(\mathbf{x}, t)$ is in general highly non-local (and non-linear). That is: how the field changes here and now depends on the present value of the field arbitrarily far away.

We can now (at last!) summarize how this pilot-wave approach to relativistic quantum theories violates relativistic causality. The situation is broadly like the action-at-a-distance in the non-relativistic theory, due to the guidance equation and the quantum potential. Namely, some fundamental equations of the theory are non-local with respect to the theory's absolute simultaneity structure; so that, setting aside doubts of Dickson's and Norton's sort (cf. Section 7.1.2), an individual process involves instantaneous causation as in Newtonian gravitation. Or in terms of the light-cone structure (which for the pilot-wave approach is emergent): individual processes involve spacelike causation. On the other hand, when we average over these individual processes, we recover the orthodox Lorentz-covariant formalism: in particular, micro-causality (Holland 1993, p. 523) and Section 3.2.1's two other formulations of relativistic causality—but now interpreted as "mere" ensemble statements.

Finally, Section 3.3 raised the question how violations of relativistic causality avoid contradictions, and announced that my examples would do so by forbidding a spacelike zig-zag, "there and back", into the causal past of an initial event—so that the usual "bilking argument" for a contradiction could not get started.

For the pilot-wave approach, the situation is straightforward. In view of the underlying absolute simultaneity structure, there is obviously no threat of a zig-zag, or of a contradiction: no more than with the action-at-a-distance in Newtonian gravity. This point is made (along of course with other points about non-locality) by e.g. Holland, and Bohm and Hiley; (cf. Holland 1993, pp. 483-487, 494-495, 531-537; Bohm and Hiley 1993 pp. 286-287; Kaloyerou 1993, p. 337). And in Section 5's discussion of the Drummond-Hathrell and Scharnhorst effects, the same point was made in a somewhat generalized form: namely, that for superluminal propagation to be free of contradictions, it is obviously enough that there be one frame of reference in which all the propagation is forward in time.

### 7.2 The Newton-Wigner representation

I turn to my second example, which I will treat much more briefly. In effect, it develops a point mentioned at the start of Section 7.1.1: that in orthodox non-relativistic quantum theory, wave-functions propagate instantaneously. The point now is: this sort of propagation also occurs in a part of *conventional* relativistic quantum theory—namely in the Newton-Wigner representation.

The Newton-Wigner representation applies equally well to a relativistic quantum theory of a fixed number of particles, and to a quantum field theory. It provides a basis of the (pure) state space (the wave-functions) consisting of strictly localized states. At first sight, this seems analogous to the Dirac delta-functions, or more generally wave-functions with compact spatial support, of the non-relativistic theory. But these states, and the spectral projectors of the corresponding position quantity, have some striking features.

(i): The states propagate superluminally. Indeed: although there is no absolute simultaneity structure, they propagate instantaneously in the following sense. If at time $t = 0$ in some inertial frame, a Newton-Wigner wave-function $\psi(0)$ has compact spatial support (i.e. is non-zero only in a compact spacelike patch, $\Sigma$ say, of the $t = 0$ hyperplane), then at all later times $t > 0$, no matter how small, $\psi(t)$ is non-zero throughout all space. Agreed, the great majority of the amplitude lies in the future light-cone of $\Sigma$; and as $t$ grows, the percentage of this majority rapidly tends to 100 percent. Nevertheless there is a "superluminal tail".

(ii): Two spectral projectors associated with spacelike related spatial regions on two different spacelike hyperplanes will *not* commute.

These features seem to imply, respectively, that:

(i'): One could signal superluminally by "releasing" a Newton-Wigner wave-packet initially confined to a compact region of space. (And here 'signalling' could be taken in a strong sense, viz. as a non-vanishing probability of a detection, triggering some pre-arranged event, at spacelike separation.)

(ii'): A (sharp, von-Neumann-style) non-selective measurement of one such projector can influence that statistics of the measurement of the other.

These features have been analysed, and the Newton-Wigner formalism much developed, especially by Fleming. Most recently, his (2003, 2004) include replies to recent literature; (the features are also surveyed in Fleming and Butterfield (1999, pp. 108-130, 153-162)). His work emphasises (among many other points) that:

(a): Though these features, and the Newton-Wigner formalism, are little known, they are *not* heterodoxies: they form part of the conventional framework of relativistic quantum theories.

(b): Various arguments can be given that (i') and (ii') are in fact *not* implied (and so causal loops are avoided). I will not go into these arguments: (Fleming and Butterfield (1999, p. 157) gives some references). Admittedly, they are partial, reflecting our lack of a developed theory of measurement for relativistic quantum theories. So here it must suffice to make three points.

First: the general theme, that violations of relativistic causality may well be indicative of future physics, also occurred in Section 5. Second: I stress that the ingredients of these partial arguments are very different from those in Sections 7.1.3.1 and 5. For example, one ingredient is to associate signalling with the group velocity of a wave (there being no superluminal group velocities in the Newton-Wigner representation); and another is to relativize the notion of localization to a hyperplane.

Third: in view of Section 3.2's formulation of relativistic causality as micro-causality, i.e. commutativity of spacelike operators, I should stress two general points of "reassurance", about the non-commutation in (ii) above.

(1): If the spatial regions associated with the projectors are on the *same* hyperplane, then the projectors are of course orthogonal, representing the fact that a single particle localized in one region has zero probability to be found in the other—and so commute.

(2): The non-commutation in (ii) is *consistent* with micro-causality. For the expression of the Newton-Wigner operators, in terms of the operators that are the topic of micro-causality, involves an integral over an entire spacelike hyperplane. The non-commutation is thus "explained" by the existence of timelike paths connecting portions of the integrands in the integrals occurring in the definitions of both spectral projectors. (Of course, the fact that the Newton-Wigner representation and the conventional one are related by an unbounded integral means that the senses in which the Newton-Wigner operators, and the conventional ones, are "associated" with regions are distinct. Accordingly, elucidating these different senses is a main theme of the recent literature: cf. Fleming (2004, Sections 3c, 4d and 5b) and references therein.)

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