KRIPKE ON THEORETICAL IDENTIFICATIONS: A REJOINDER TO PERRICK

James BUXTON

Abstract

This paper examines an argument of Saul Kripke for the necessity of theoretical identification statements and defends it against a criticism of M. Perrick ("Are Kripke's Theoretical Identifications Necessary Truths?", *Logique et Analyse*, Volume 115, September 1986, pages 381-384). It is argued that Perrick's criticism rests on a fallacy of ambiguity. Formal modal logic is used to examine a number of plausible interpretations of Kripke's argument, and Perrick's error is shown to arise from confusion concerning the scope of the modal necessity operator.

In this journal (1), M. Perrick has argued that Kripke's arguments fail to establish the necessity of theoretical statements, such as "Gold is an element with atomic number 79". While I do not agree with Kripke that theoretical identity statements are necessary, I believe that Perrick's article embodies some confusions that it is important to dispel and that, consequently, it does not fairly represent Kripke's views.

Consider the following passage from Perrick's article:

> Generally, for any object *a* and any property *P*, given that *Pa* is true, it is impossible that something which lacks *P* is identical with *a*; the assumption of the contrary involves a straightforward contradiction, whether *Pa* is a necessary or a contingent statement. ...
> Although in itself correct, Kripke's argument is still inadequate... (2)

Perrick judges this argument to be inadequate because, on his interpretation, it applies to clearly contingent statements, such as

(1) PERRICK, M., "Are Kripke's Theoretical Identifications Necessary Truths?", *Logique et Analyse*, Volume 115, September 1986, pages 381-384.

(2) PERRICK, page 382.

(A) Reagan is the 40th President of the U.S.

as well as to supposedly necessary statements, such as

(B) Gold is an element with atomic number 79.

My first criticism, a minor one, is simply this: If the argument is "inadequate", how can it be "correct"? Since the simple truth of (A) or (B) as a premise is not in question, the argument must be inadequate because it either fails to establish the necessity of (A) or, conversely, leads to the false conclusion that (B) is necessary. Therefore, either the argument is invalid, or it involves some unstated, false premise. In either case, it is not "correct".

A more serious criticism is that Perrick's argument suffers from a fallacy of ambiguity. There are at least five initially plausible interpretations of the statement that is central to the argument:

(C) ... given that *Pa* is true, it is impossible that something which lacks *P* is identical with *a*...

The differences in these interpretations are best brought out using a notation of formal modal logic. (3)

(D1) $Pa \rightarrow -M(Ex)(-Px \;\&\; x=a)$
(D2) $Pa \rightarrow (x)-M(-Px \;\&\; x=a)$
(D3) $-(Pa \;\&\; (Ex)(-Px \;\&\; x=a))$
(D4) $-M(Pa \;\&\; (Ex)(-Px \;\&\; x=a))$
(D5) $Pa \rightarrow (x)(-Px \rightarrow -M(x=a))$

(D3) is a logical truth of the predicate calculus; (D4) is a logical truth of modal systems T, S4, and S5 with or without strong identity, that is, with or without identity axioms from which the necessity of identity, "$(x)(y)(x=y \rightarrow L(x=y))$", follows; and (D5) is a logical truth of S5 with strong identity. Therefore, if we interpret the "assumption of the contrary" to be the negation of (D3), (D4), or (D5), then, as Perrick says, "...the assumption of the contrary involves a straightforward contradiction...;" – with some reservation about how "straightforward" the contradiction

(3) I write "→" for material implication, "&" for conjunction, "–Pa" for the negation of "Pa", "(Ex)" for existential quantification, "(x)" for universal quantification, "M" for the possibility operator, and "L" for the necessity operator.

is in the case of (D5). But surely Kripke does not take himself to be calling our attention to truths of formal logic when he argues for the truth of theoretical identifications. More importantly, the intended conclusion, the necessity of "Pa" ("LPa"), does not follow from "Pa" and either (D3), (D4), or (D5).

It is only on the interpretation of (C) as (D1) or (D2) that "LPa" follows with "Pa" as an additional premise. And surely it is (D1) or (D2) that best reflects the conceptual position for which Kripke argues. (4) (D1) and (D2) are logically equivalent in S5 and also in T and S4 when these latter two systems are supplemented with the Barcan Formula, "(x)L(wff) $\rightarrow$ L(x)(wff)", where "wff" represents any well-formed formula. But (D1), and therefore (D2), is not a logical truth of any of these systems either with or without strong identity.

In short, Perrick's statement that "...the assumption of the contrary involves a straightforward contradiction..." is true only when (C) is interpreted as (D3), (D4), or (D5); but his claim that "Kripke's argument for the necessity of [(B)] equally applies to the supposedly contingent statement [(A)]", that is, that "...Kripke's argument applies to any truth whatever..." (5) is true only when (C) is interpreted as (D1) or (D2), for it is only from one of these statements – in conjunction with "Pa" – that "LPa" can be deduced. Thus Perrick is guilty of a fallacy of ambiguity.

Since (D1), and therefore (D2), is not a logical truth, the truth of either of these statements must be argued on other grounds, (6) and this is precisely the point. Kripke would argue that (D1) or (D2) is true for statements "Pa" such as "Gold is an element with atomic number 79," but not true for statements such as "Reagan is the 40th President of the U.S."

Perrick presents a second argument against Kripke that rests on the same mistaken interpretation of (C). He asks us to consider the following three statements – the numbering is Perrick's:

(3) Gold is an element with atomic number 79, and a counterfac-

(4) See KRIPKE, Saul, "Identity and Necessity", in *Naming, Necessity, and Natural Kinds*, ed. Stephen P. Schwartz, Cornell University Press, 1977, pages 86-88.

(5) PERRICK, footnote 3, page 382.

(6) "...if P is the statement that the lectern is not made of ice, one knows by a priori philosophical analysis, some conditional of the form 'if P, then necessarily P'." KRIPKE, op. cit., page 88.

tual situation, in which gold were not this element, is (metaphysically) impossible.

(4) Gold is this element, but in a counterfactual situation gold might have been a compound.

(5) Given that gold is this element, nothing, however much resembling gold, but not being this element, could be gold. ([7])

Perrick's argument is as follows:

> ...(5) gives, in a concise form, Kripke's argument for the necessity of (1). [(1) is our (B) above: "Gold is an element with atomic number 79."]
>
> Does (5) lend any support to (3), that is, to the necessity of (1)? One easily sees that it does not. For although (3) and (4) contradict each other and (5) is intended as an argument for (3) i.e. for the necessity of (1), it is evident that (5) is compatible with (4) as well. That is to say, Kripke's purported argument for the *necessity* of (1) is compatible with the statement claiming the *contingency* of (1). The fact that (5) is compatible both with (3) and (4) makes it quite clear that Kripke's argument is *irrelevant* in respect of the metaphysical status of (1). ([8])

Let us ignore such niceties as the fact that (3) and (4) are contraries rather than contradictories – for example, they would both be false if gold were an element with atomic number 78 – and the fact that compatibility is usually understood to be a relation between statements, not between arguments and statements. Perrick's argument depends on the claim that (5) and (4) are compatible, that is, that (5) does not entail not–(4). In summary his argument is this: Since (3) and (4) are contraries, (3) entails not–(4). If (5) entailed (3) then, since (3) entails not–(4), (5) would have to entail not–(4). But (5) and (4) are compatible. Therefore, (5) does not entail (3).

Letting "Pa" symbolize the statement "Gold is an element with atomic number 79," the features essential to statements (3), (4), and (5) for the purpose of Perrick's argument can be symbolized as follows:

([7]) PERRICK, page 383.

([8]) PERRICK, page 383.

(3') $Pa \ \& \ -M(-Pa)$
(4') $Pa \ \& \ M(-Pa)$
(5') $Pa \rightarrow L(x)(-Px \rightarrow x \neq a)$

(5) has been symbolized as shown in (5') in accordance with our earlier discussion of statements (D1) through (D5). In fact, (5') is logically equivalent to (D1); this is to be expected since (5) is simply a rephrasing of the Kripke argument given above.

But, contrary to Perrick's assertion, (5') is not compatible with (4'). "$-M(-Pa)$", which is clearly incompatible with (4'), can be derived from (5') and "Pa". Thus Perrick's argument against Kripke relies on a false premise. Furthermore, far from being "...irrelevant in respect of the metaphysical status of (1)", Kripke's argument is conclusive for (3'), the necessity of (1). This is so because (3') can be derived from "Pa" and (5'). The derivation is valid in T, S4, and S5, and, with respect to identity, requires only the fact that *a* is necessarily identical with itself.

If there is a lesson is all this, it is that great care must be taken in analyzing modal statements. (D1) – (D5) are logically equivalent to:

(D1') $Pa \rightarrow L(x)(-Px) \rightarrow x \neq a)$
(D2') $Pa \rightarrow (x)L(-Px \rightarrow x \neq a)$
(D3') $Pa \rightarrow (x)(-Px \rightarrow x \neq a)$
(D4') $L(Pa \rightarrow (x)(-Px \rightarrow x \neq a))$
(D5') $Pa \rightarrow (x)(-Px \rightarrow L(x \neq a))$

It is interesting to note that (D1'), (D2'), (D4'), and (D5') differ only with respect to the scope of the necessity modal operator "L". (D3'), which lacks a modal operator, is logically equivalent to (D4') in T, S4, and S5, since it is a valid formula of the predicate calculus. It has already been noted that (D1') and (D2') are logically equivalent in these systems given the Barcan formula.

If we are to criticize Kripke, we must criticize the analysis which leads to his conclusion that statements such as (5) are a priori necessary truths. If such a consequential modal statement is clearly understood and then accepted as a premise, there is no fault to be found with the relatively simple argument in which it is employed. ([9])

University of Virginia James BUXTON

([9]) I should like to thank Professor James Cargile, University of Virginia, for suggestions from which this paper has significantly benefited.