On Specifying Truth-Conditions*

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Consider a *committalist*—someone who believes that assertions of a sentence like 'the number of the planets is 8' carry commitment to numbers—and a *noncommittalist*—someone who believes that all it takes for assertions of this sentence to be correct is that there be eight planets. The committalist wants to know more about the noncommittalist's view. She understands what the noncommittalist thinks is required of the world in order for assertions of simple sentences like 'the number of the planets is 8' to be correct, but she wants to know how the proposal is supposed to work in general. Could the noncommittalist respond by supplying a recipe for *translating* each arithmetical sentence into a sentence that wears its ontological innocence on its sleave? Surprisingly, there is a precise and interesting sense in which the answer is 'no'. We will see that when certain constraints are in place, it is impossible to specify an adequate translation-method.

Fortunately, there is a technique for specifying truth-conditions that is not based on translation, and can be used to explain to the committalist what the noncommittalist thinks is required of the world in order for arithmetical assertions to be correct. I call it 'the $\phi(w)$-technique'. A shortcoming of the $\phi(w)$-technique is that it is of limited dialectical effectiveness. From the perspective of a committalist, the $\phi(w)$-technique only delivers suitable noncommittalist truth-conditions on the assumption that numbers exist. So the $\phi(w)$-technique is not particularly promising as a maneuver for convincing a committalist to give up mathematical Platonism. But I hope it will earn its keep by providing the committalist with an illuminating account of the noncommittalist position, and by clarifying the role of noncommittalism in addressing certain philosophical puzzles.

The $\phi(w)$-technique turns out to have applications beyond arithmetic. It can be used to shed light on the truth-conditions of second-order and set-theoretic sentences. It can also

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be used to clarify the notions of ontological commitment and sameness of truth-conditions.

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It will be useful to begin with discussion of second-order languages as a warm-up case.

1 The Parable of Charles and George

Charles and George disagree about the commitments carried by second-order sentences.[1] Charles believes that in order for '$\exists X(X(\text{Susan}))$' to be intelligible, it must carry commitment to an object other than Susan (perhaps to an *attribute* had by Susan, or to a *set* with Susan as a member). George believes that it can be understood in such a way that it carries commitment to Susan, and nothing else. There is no disagreement, however, about matters of ontology. Charles and George both believe that Susan exists, they both believe that there are sets and neither of them believes that there are attributes.

Substantial efforts were made to settle the debate. George invited Charles to read second-order quantifiers as English plural quantifiers, and proposed the following paraphrase for '$\exists X(X(\text{Susan}))$':

> There are some things such that Susan is one of them.

"And, of course"—George suggested—"all it takes for there to be some things such that Susan is one of them is for Susan to exist. So all that is required in order for the truth-conditions of '$\exists X(X(\text{Susan}))$' to be satisfied is that Susan exist."

Charles was unmoved. He complained that his best effort to understand plural quantifiers involved treating them as first-order quantifiers ranging over sets. He claimed, in particular, that his best effort to make sense of George's paraphrase involved reading it as:

> There is a set with Susan as a member.

George attempted to block this line of response by appealing to domains of discourse too big to form a set. He asked Charles to consider sentences like 'There are some things such that all the sets are among them', where it was to be understood that the domain of discourse consisted of absolutely everything there is. But Charles had long been skeptical of the legitimacy of domains too big to form a set, and requested that such domains not be taken into consideration for the purposes of the debate.

Just when an impasse was starting to look inevitable, George came up with a new idea. He decided to give up the project of trying to find *paraphrases* for second-order sentences that Charles might find both intelligible and non-committing. He would instead

[1] Beware: there are important respects in which the views I attribute to Charles and George differ from those of certain non-fictional namesakes. The same is true of the fictional characters I introduce below. Names are meant as mnemonic devices only.

attempt to explain to Charles—in a metalanguage that was not under dispute—what the truth-conditions of sentences in the second-order object-language should be taken to be.

George began with the observation that one can specify truth-conditions for a sentence like 'there are elephants' by making any of the following three statements:

1. The truth-conditions of 'there are elephants' consist of the requirement that there be elephants.

2. What is required in order for the truth-conditions of 'there are elephants' to be satisfied is that there be elephants.

3. What is required of the world in order for the truth-conditions of 'there are elephants' to be satisfied is that it contain elephants.

The second statement is simply a long-winded version of the first. And when 'the world' is an taken to abbreviate 'the mereological sum of everything there is'—as it will be throughout this paper—the requirement that there be elephants comes to the same thing as the requirement that the world contain elephants (as parts). So the third statement yields the same result as the other two.

Having clarified that he would use statements of these three kinds interchangeably, George set out to explain his proposal. The $\phi(w)$-technique, as he liked to call it, proceeds as follows:

1. *Preliminaries*

   Let w be a variable ranging over *representations* (stories, or plays, or paintings, or, on some accounts, possible worlds). If w is the only variable occurring free in a formula $\phi(w)$, we shall say that $\phi(w)$ is a *w-formula* just in case every occurrence of w in $\phi(w)$ is the subscript in some subformula of the form $\ulcorner [\psi]_w \urcorner$, where '$[\ldots]_w$' is read 'according to w, ...'.

   Here two examples of w-formulas:

   (E1) [there are elephants]$_w$
   (*read:* according to w, there are elephants)

   (E2) $\exists x$(x is the largest elephant $\wedge$ [x trumpets]$_w$)
   (*read:* there is a largest elephant and, according to w, that individual trumpets)

   A w-formula may be thought of as imposing a requirement on how the world ought to be represented. Say that the *primary condition* corresponding to $\phi(w)$ is the requirement that the world be represented as w would have to represent it in order to satisfy $\phi(w)$, given how the world actually is. The primary condition corresponding to (E1), for instance, is the requirement that the world be represented as containing elephants. And the primary condition corresponding to (E2) is the requirement, regarding the largest elephant, that the world be represented as being such that that individual trumpets. (If Cornelius happens to be the largest elephant, this is the requirement that the world be represented as being such that Cornelius trumpets.)

The primary condition corresponding to a w-formula is a requirement on how the world ought to be *represented*. But a w-formula can also be used to specify a requirement on how the world ought to *be*. If the primary condition corresponding to $\phi(w)$ is the requirement that the world be represented as being a certain way, the *secondary condition* corresponding to $\phi(w)$ is the requirement that the world *be* that way. (Equivalently: the secondary condition corresponding to $\phi(w)$ is the requirement that the world be as $\phi(w)$'s primary condition requires it to be represented.) The secondary condition corresponding to (E1) is therefore the requirement that the world contain elephants. And—on the assumption that Cornelius is the largest elephant—the secondary condition corresponding to (E2) is the requirement that the world be such that Cornelius trumpets.

One should *not* think of the secondary condition corresponding to a w-formula as somehow capturing the w-formula's truth-conditions. w-formulas are *open* formulas, so they can only be said to have truth-conditions relative to a parameter. By taking a w-formula's free variable to designate the world, one might be able to get something along the lines of truth-conditions *simpliciter*. But there would still be something amiss, since it isn't immediately clear what '$[\ldots]_w$' should be taken to mean when w is the world, rather than a representation. What is it for something to be the case *according* to the world? Perhaps one could say that p is true according to the world if and only if p. If so, the 'truth-conditions' of (E2) would consist of the requirement that there be a largest elephant who trumpets. Note, however, that this is crucially different from the requirement that Cornelius trumpet, which—on the assumption that Cornelius is the largest elephant—is the secondary condition corresponding to (E2). Moral: the secondary condition corresponding to a w-formula is one thing; the w-formula's truth-conditions (relative to a parameter) are another.

Here are some additional examples of w-formulas. (I assume throughout that Susan actually exists.)

(a) $[\text{Susan runs}]_w$

Primary condition: that the world be represented as containing Susan, and as being such that she runs.

Secondary condition: that the world contain Susan and be such that she runs. (Equivalently: that Susan run.)

(b) $\exists x(x = \text{Susan} \wedge [x \text{ runs}]_w)$

Primary condition: same as (a).

Secondary condtion: same as (a).

(c) $\exists x(x = \text{the finest cellist} \wedge [x \text{ runs}]_w)$

Primary condition: same as (a), if Susan happens to be the finest cellist. (What if there is no finest cellist? Then, on a Russellian reading of the definite description, the primary condition is unsatisfiable.)

Secondary condition: same as (a), if Susan happens to be the finest cellist.

(d) $[\exists x(x = \text{the finest cellist} \wedge x \text{ runs})]_w$

Primary condition: that the world be represented as containing a finest cellist who runs.

Secondary condition: that the world contain a finest cellist who runs. (Equivalently: that there be a finest cellist, and that he or she run.)

(e) $\exists \alpha(\text{Susan} \in \alpha \wedge \forall x(x \in \alpha \rightarrow [x \text{ runs}]_w))$

Primary condition: same as (a), on the assumption that Susan's singleton exists. (To see this, note that what it takes for w to satisfy the formula is for there to be some set α containing Susan which is such that, for any member of α, w represents the world as containing it and as being such that it run. But if Susan's singleton exists, one can let α be Susan's singleton. So all it takes for w to satisfy the formula is for w to represent the world as containing Susan and as being such that she runs.)

Secondary condition: same as (a), on the assumption that Susan's singleton exists.

2. *The $\phi(w)$-technique*

The $\phi(w)$-technique is a way of using the secondary condition corresponding to a w-formula to specify truth-conditions for a target sentence s. One does so by laying down an instance of the following schema:

> What is required of the world in order for s's truth-conditions to be satisfied is that it meet the secondary condition corresponding to $\phi(w)$.

Substituting (E1) for '$\phi(w)$' delivers the result that what is required in order for s's truth-conditions to be satisfied is that there be elephants. And—on the assumption that Cornelius is the largest elephant—substituting (E2) for '$\phi(w)$' delivers the result that what is required in order for s's truth-conditions to be satisfied is that Cornelius trumpet.

How is this supposed to help George with the task of specifying truth-conditions for '$\exists X(X(\text{Susan}))$'? The crucial observation is that the $\phi(w)$-technique can be applied on the basis of either of the following two w-formulas:

Wide Scope

$[\text{there is a set } \alpha \text{ such that Susan} \in \alpha \text{ and for any member } x \text{ of } \alpha, \exists y(y = x)]_w$

Narrow Scope

there is a set α such that Susan $\in \alpha$ and for any member x of α, $[\exists y(y = x)]_w$

The primary conditions corresponding to Wide Scope and Narrow Scope are importantly different. The former is the requirement that the world be represented as containing a *set* with Susan as a member. In contrast, the latter is the requirement that the world be represented as containing Susan (since—on the assumption that Susan's singleton exists, as Charles and George believe—that is how w must represent the world in order to satisfy Narrow Scope). Accordingly, the $\phi(w)$-technique will result in different specifications of truth-conditions for '$\exists X(X(\text{Susan}))$' depending on whether it is applied on the basis of Wide Scope or Narrow Scope:

- Applied on the basis of Wide Scope:

  What is required in order for '$\exists X(X(\text{Susan}))$'s truth-conditions to be satisfied is that there be a *set* with Susan as a member.

- Applied on the basis of Narrow Scope:

  What is required in order for '$\exists X(X(\text{Susan}))$'s truth-conditions to be satisfied is that Susan exist.

I would like to insist that it is important not to confuse the truth-conditions that are specified for '$\exists X(X(\text{Susan}))$' by way of Narrow Scope with the truth-conditions of Narrow Scope itself (relative to some w). As we have seen, all that is required in order for the truth-conditions specified by way of Narrow Scope to be satisfied is that Susan exist. But if, like Charles and George, one favors a standard semantics for set-theoretic discourse, then one thinks that (relative to any w) satisfaction of Narrow Scope's truth-conditions requires the existence of sets. As a result, one should think that only those who believe in sets are in a position to use Narrow Scope to specify set-theoretically innocent truth-conditions for '$\exists X(X(\text{Susan}))$'. (It is a good thing that Charles and George both believe in sets!)

By making use of Narrow Scope, George was able to succeed in the task he set himself: he was able to specify set-theoretically innocent truth-conditions for '$\exists X(X(\text{Susan}))$' by using a language that was not under dispute. But Charles couldn't understand why George would use something as complicated as the $\phi(w)$-technique when the same effect could be achieved by simpler means. "Why not say the following instead?" he asked:

> The truth-conditions of '$\exists X(X(\text{Susan}))$' consist of the requirement that Susan exist.

George was happy to concede that Charles's suggestion would be as effective as his own in this particular case. But he went on to reveal a decisive advantage of the $\phi(w)$-technique: that it admits of generalization. As long as the domain of discourse is not too big to form a set, the $\phi(w)$-technique can be used to assign the desired truth-conditions to any sentence in the language.[2]

To illustrate the procedure's generality, George invited Charles to consider the Geach-Kaplan Sentence: 'some critics admire only one another', or, as George liked to put it:

Geach-Kaplan

$$\begin{aligned}&\exists X[\exists y(Xy) \wedge \\ &\quad \forall y(Xy \rightarrow \textsc{Critic}(y)) \wedge \\ &\quad \forall y \forall z(X(y) \wedge \textsc{Admires}(y, z) \rightarrow X(z) \wedge y \neq z)]\end{aligned}$$

(In plural terms: there are some things such that (a) each of them is a critic, and (b) if y is one of them and y admires z, then z is one of them and $y \neq z$.)

George then set forth two open formulas, paralleling Wide Scope and Narrow Scope above:

GK (Wide Scope)
[There is a non-empty set α such that: (a) for any y, if $y \in \alpha$ then y is a critic, and (b) for any y and any z if $y \in \alpha$ and y admires z, then $z \in \alpha$ and $y \neq z]_w$

GK (Narrow Scope)
There is a non-empty set α such that: (a) for any y, if $y \in \alpha$ then [$\dot{y}$ is a critic]$_w$, and (b) for any y and any z if $y \in \alpha$ and [$\dot{y}$ admires $\dot{z}$]$_w$, then $z \in \alpha$ and $[\dot{y} \neq \dot{z}]_w$

The primary condition corresponding to GK (Wide Scope) is the requirement that the world be represented as containing a non-empty set of critics every member of which admires only other members of the set. By contrast, the primary condition corresponding to

[2]Here's how. For s an arbitrary second-order sentence, one stipulates that what is required of the world in order for s's truth-conditions to be satisfied is that it meet the secondary condition corresponding to $(s)^*$, where $(\ldots)^*$ is characterized as follows:

$$\begin{aligned}(\ulcorner P_i^n(x_{j_1}, \ldots, x_{j_n})\urcorner)^* &= \ulcorner[\hat{P}_i^n(\dot{x}_{j_1}, \ldots, \dot{x}_{j_n})]_w\urcorner \text{ (where } \ulcorner\hat{P}_i^n\urcorner \text{ translates } \ulcorner P_i^n\urcorner) \\ (\ulcorner X_i(x_j)\urcorner)^* &= \ulcorner x_j \in \alpha_i\urcorner \\ (\ulcorner \phi \wedge \psi\urcorner)^* &= \ulcorner(\phi)^* \wedge (\psi)^*\urcorner \\ (\ulcorner\neg\phi\urcorner)^* &= \ulcorner\neg(\phi)^*\urcorner \\ (\ulcorner\exists x_i\, \phi\urcorner)^* &= \ulcorner\text{there is an object } x_i \text{ such that } ([\exists y(y = \dot{x}_i)]_w \wedge (\phi)^*)\urcorner \\ (\ulcorner\exists X_i\, \phi\urcorner)^* &= \ulcorner\text{there is set } \alpha_i \text{ such that } (\forall z(z \in \alpha_i \rightarrow [\exists y(y = \dot{z})]_w) \wedge (\phi)^*)\urcorner\end{aligned}$$

The dots on variables are introduced to deal with a complication arising from the fact that the world might have contained objects it doesn't actually contain. The issue is considered at length in the appendix, but I encourage the reader to ignore the dots until section 3.1. Doing so will result in a more comfortable read, and carries little risk of misunderstanding. (I have made the simplifying assumption that the second-order language under discussion contains no function letters or individual constants.)

GK (Narrow Scope) is—*modulo* a technicality mentioned in footnote 2—the requirement that the world be so represented that for some non-empty set α: (a) each member of α is represented as being a critic, and (b) for any z and any y in α, if y is represented as admiring z, then z is in α and y and z are represented as being distinct. Accordingly, whereas the secondary condition corresponding to GK (Wide Scope) requires the existence of sets, the secondary condition corresponding to GK (Narrow Scope) does not.

What exactly does the secondary condition corresponding to GK (Narrow Scope) require of the world? George would be happy to formulate the requirement thus: "that there be critics who admire only one another". But this is not a neutral way of putting it, since Charles believes that there need to be sets in order for 'there are critics who admire only one another' to be true. In fact, Charles takes himself to speak a language that is not expressive enough to formulate the requirement directly. For any direct statement of the requirement would constitute a paraphrase of George's understanding of 'there are critics who admire only one another', which is an example of something that cannot be captured using only first-order quantifiers (see (Boolos, 1984)). Fortunately, George and Charles don't have to agree on a formulation of the requirement in order to take advantage of the $\phi(w)$-technique. George can explain how he thinks Geach-Kaplan ought to be understood by stating the following: "what is required of the world in order for the truth-conditions of Geach-Kaplan to be satisfied is that it meet the secondary condition corresponding to GK (Narrow Scope)". For although Charles takes himself to be unable to produce a direct formulation of the secondary condition corresponding to GK (Narrow Scope), he has a strategy for coming to understand it. He can use GK (Narrow Scope) to grasp a requirement to the effect that the world be *represented* as being certain way, and conclude that this is how the world must *be* in order for the corresponding secondary condition to be satisfied.

Charles was beginning to see why the $\phi(w)$-technique might constitute genuine progress. For years it had seemed to him that, insofar as Geach-Kaplan was intelligible, it meant simply that there is a non-empty set of critics any member of which admires only other members. And for years he had listened to George complain that the sentence could be used to mean something quite different, having nothing to do with sets. Time and time again, Charles had asked George to explain this alternative meaning. But all George would do in response was assert things like "there are some critics such that *they themselves* admire only one another", with the occasional table-thumping. It had always seemed to Charles that this was no help at all, since his best effort to understand George's plural quantifiers involved thinking of them as first-order quantifiers ranging over sets. Now, however, Charles was in a position to see what George had been going on about all that time. By using the $\phi(w)$-technique, Charles was finally able to see what truth-conditions George was hoping to associate with it.

Charles was warming up to the proposal, but there was a worry he couldn't quite put his finger on. All too often, he had been in the company of philosophers who attempted to solve problems in the philosophy of mathematics by adopting unorthodox metaphysical outlooks, and he was worried that George might be doing just that. "Wait a minute!"—he finally asked—"Is this going to be one of those proposals where it turns out that there is more than one type of existence, or where it turns out that there are objects that exist but don't exist 'in' the world?"

"Absolutely not."—George explained—"What I have set forth is an account of truth-conditions for second-order sentences, not a metaphysical theory." Just to make absolutely clear that nothing metaphysically untoward was going on, George offered to tell Charles about his metaphysical views. "I make no distinction between existing and being. I take everything there is (including sets) to exist in the very sense of 'existence' in which people, volcanoes and electrons exist. But the view I have been trying to defend does not depend on this unremarkable metaphysics: all that matters is that the existence of sets not be part of what is required for the truth-conditions of typical second-order sentences to be satisfied. As for the view that there are objects that exist but don't exist 'in' the world"—George continued—"I believe that everything there is is part of the world. So from my point of view there is no difference between existing and existing in the world, or between existing and being contained in the world, or between existing and being a part of the world. I am also a modal actualist: I believe that everything that exists actually exists. But, again, I don't think the view I have been trying to defend depends on my actualism. I have set forth a view about the truth-conditions of second-order sentences, not about the non-linguistic facts."

Charles was still uneasy. There was a dictum that had served him well for many years, and now he was worried that George might be asking him to give it up: To Be is to Be a Value of a Variable. (More precisely: a sentence carries commitment to Fs if and only if Fs must be admitted among the values of the variables in order for the sentence to be true.) On George's proposal, the values of second-order variables are taken to be subsets of the domain of discourse. This means, in particular, that sets must be admitted among the values of the variables in order for a sentence like '$\exists X \exists y (Xy)$' or GEACH-KAPLAN to be true. So it would appear to be an immediate consequence of the dictum that '$\exists X \exists y (Xy)$' and GEACH-KAPLAN both carry commitment to sets.

George was ready with a reply. "First of all"—he explained—"you are misapplying the dictum. The dictum, as originally intended, is a criterion of ontological commitment for *first-order* languages. It has nothing to say about the commitments carried by the sentences of a non-first-order language, such as the one we are discussing. If one wants to say something more general about ontological commitment, what one should say is this: a sentence carries commitment to Fs if and only if the world is required to contain Fs in order for the sentence's truth-conditions to be satisfied. It is a happy feature of first-order languages that there is a straightforward connection between the objects that must be counted amongst the values of the variables in order for the sentence to be true, on the one hand, and the objects whose existence is required for the sentence's truth-conditions to be satisfied, on the other. But there is no good reason for thinking that this feature will carry over beyond first-order languages."

"In general, one must distinguish between the objects that form part of the semantic machinery that is used to specify the truth-conditions of a sentence, on the one hand, and the objects whose existence is required in order for the truth-conditions specified to be satisfied, on the other. Take, for example, the case of the logical connectives. On certain standard semantic theories,[3] the semantic value of a logical connective is a certain kind of function in the domain of the theorist's language. But nobody would thereby expect a sentence like 'Roses are red *and* violets are blue' to carry commitment to functions. The semantic value of 'and' forms part of the semantic machinery that is used to specify truth-conditions for 'Roses are red and violets are blue', but its existence is not required in order for the resulting truth-conditions to be satisfied. Similarly, standard semantic theories take the semantic value of a predicate like 'runs' to be an extension (or some variant thereof), but it would be a mistake to conclude on those grounds alone that 'Susan runs' carries commitment to extensions. On my view"—George continued—"the values of the second-order variables should be thought of in this spirit. They form part of the semantic machinery that is used to specify truth-conditions for second-order sentences, but their existence is not in general required for the resulting truth-conditions to be satisfied."

Charles was not convinced. But then again, philosophers rarely are. At least he conceded that his objections had been met.

2 Arithmetic

The objective of this paper is to do for arithmetical discourse what the Parable has George doing for second-order discourse. I aim to show that one can use the $\phi(w)$-technique to specify ontologically innocent truth-conditions for arithmetical sentences.

There are several respects in which my task will be more difficult than George's. Firstly, the debate I consider below will lack a simplifying feature of the Parable. Whereas the Parable stipulated that Charles and George were in full agreement about matters of ontology, some of the positions I will consider below disagree about what there is. Secondly, the discussion in the Parable was pretty informal; below I give a more rigorous treatment of the notion of truth-conditions and the notion of ontological commitment. Finally, my discussion of arithmetical discourse will concern regions of the philosophical terrain that are very well travelled, and it will be necessary to explain how my proposal fits in with the rest of the debate.

2.1 Committalists and Noncommittalists about Arithmetic

When it comes to the commitments of arithmetical assertions there are two broad schools of thought According to *committalists*, marketplace arithmetical assertions commit the speaker to the existence of numbers; according to *noncommittalists* they do not.[4] Committalists believe, for example, that a marketplace assertion of 'The number of the planets

[3]See, for instance, (Heim and Kratzer, 1998). This is the textbook I shall have in mind whenever I speak of a 'standard semantic theory'.

[4]A marketplace assertion is an assertion in a context in which speakers are not concerned with matters of ontology. The qualification is needed because a noncommittalist might think that even though marketplace assertions of 'the number of the planets is eight' carry no commitment to numbers, assertions of 'there are numbers' in the course of a conversation about ontology carry commitment to numbers. I shall have more to say about this dual attitude in section 4.3.

Noncommittalist Postion	**The correctness-conditions of an assertion are ...**
Eliminativism	its truth-conditions; but truth-conditions don't correspond to what surface grammatical structure would suggest.
Fictionalism	its fictional-adequacy-conditions; mathematical assertions are set forth in a spirit of make-believe.
Pragmatic Instrumentalism	its instrumental-adequacy-conditions; mathematical assertions are set forth on the understanding that one is only committed to their adequacy as tools for distinguishing between possibilities that are relevant for the purposes at hand.
Empiricism	its empirical-adequacy-conditions; mathematical assertions are set forth on the understanding that one is only committed to their empirical adequacy.
Modalism	its categorical-adequacy-conditions; mathematical assertions are set forth on the understanding that one is only committed to the claim that the assertion would be true if there were extra objects playing the role of numbers.

Figure 1: Noncommittalist Positions

is eight' commits the speaker to the existence of the number eight. Noncommittalists disagree. But it is not that they thereby believe that the assertion would be 'correct' however the world happened to be. Noncommittalists think that there have to be exactly eight planets in order for the assertion to be correct.[5] What is distinctive about the view is that it takes the existence of numbers to be *irrelevant* to the correctness or incorrectness of marketplace arithmetical assertions.

Different versions of noncommittalism spell out the notion of correctness in different ways.[6] (See figure 1.) But it is important to distinguish the question of how the notion of correctness is to be spelled out—for instance, whether fictionalism is to be preferred over instrumentalism—from the question of what one thinks is required in order for the correctness-conditions of an arithmetical assertion to be satisfied. The focus of this paper will be on the latter: I will be concerned with the question of what it takes for the correctness-conditions of arithmetical assertions to be satisfied, while remaining neutral on the question of how the relevant notion of correctness should be understood.

It is also important not to conflate the distinction between committalism and noncommittalism with the distinction between Platonism (the view that there are numbers) and nominalism (the view that there are no numbers). Whereas committalists and noncommittalists disagree about the correctness-conditions of mathematical assertions, Platonists

[5]As usual, I take 'there are exactly eight planets' to have the same truth-conditions as '$\exists!_8 x$ Planet(x)', where $\exists!_1 x(\phi(x)) \equiv_{df} \exists x \forall y(\phi(x) \leftrightarrow \phi(y))$ and $\exists!_{n+1} x(\phi(x)) \equiv_{df} \exists x(\phi(x) \wedge \exists!_n y(\phi(y) \wedge x \neq y))$.

[6]Texts germane to the varieties of noncommittalism listed in figure 1 include, respectively, (Hodes, 1984), (Hodes, 1990a); (Yablo, 2001); (Eklund, 2005), (Yablo, ript); (Van Fraassen, 1980); (Dorr, ming).

and nominalists disagree about what there is.

2.2 The Devil in the Details

When asked to explain what is required for the correctness-conditions of arithmetical assertions to be satisfied, it is easy enough for noncommittalists to rehearse a familiar set of toy examples:

- What is required for a marketplace assertion of 'the number of the planets is eight' to be correct is that there be exactly eight planets.
- What is required for a marketplace assertion of 'the number of the cats is the number of the dogs' to be correct is that there be just as many cats as there are dogs.

But what one really wants to know is how the proposal is supposed to work in general. We want a recipe for specifying the correctness-conditions of arbitrary arithmetical assertions.

In fact, there are a number of such recipes. The simplest is this: an assertion of ϕ is correct if and only if ϕ's universal Ramseyfication is true.[7] Others have been set forth in (Hodes, 1984), (Fine, 2002), (Rayo, 2002a) and (Yablo, 2002). These recipes all share a distinctive feature: they only deliver the standard truth-values for arithmetical sentences on the assumption that there are infinitely many objects within the range of the variables.[8] (For instance, if there are only finitely many objects within the range of the variables, then the universal Ramseyfication of any arithmetical sentence will be trivially true because the antecedent of its main conditional will be unsatisfiable.) Hereafter I shall only consider versions of noncommittalism that wish to avoid such infinity requirements. (The reason for this restriction will become apparent in section 5.)

Might it be possible to find a strategy for specifying the correctness-conditions of arithmetical assertions whereby infinity requirements are avoided altogether? As it turns out, there is a formal result that places significant limits on the noncommittalist's options. One can show that there is no function f satisfying the following constraints:[9]

[7] ϕ's universal Ramseyfication is the universal closure of $\ulcorner(\mathcal{A} \rightarrow \phi)^*\urcorner$, where $\mathcal{A}$ is the conjunction of suitable list of axioms, and ψ^* is the result of uniformly replacing the arithmetical vocabulary in ψ by variables of appropriate type. For the case of pure arithmetic, the (second-order) Dedekind Axioms are suitable. For the case of applied arithmetic, the axiom system that Richard Heck calls Two-Sorted Fregean Arithmetic (plus definitions) is suitable—see (Heck, 1997a).

[8] In the case of Yablo's recipe, the problem emerges because when there are only n objects within the range of the variables, the result of applying the procedure to $\ulcorner n+1=n\urcorner$ is trivially true, even though $\ulcorner n+1=n\urcorner$ is false on the standard interpretation. (Hodes, 1990b) and (Rayo, 2002a) discuss the infinity assumptions of the relevant recipes. The proposals in (Fine, 2002) II.5 and (Rayo, 2002a) yield equivalent results. For an illuminating discussion of noncommittalist strategies, see (Burgess and Rosen, 1997).

[9] *Proof Sketch:* Let L be the language in which sentences in the range of f are couched, and assume, for *reductio*, that constraints 1 and 2 are both met, and that constraints 3 and 4 are both satisfied in some model M. Since constraint 4 is satisfied in M, M must have a finite domain. Since, by constraint 1, L is an nth-order language with finite non-logical vocabulary, and since M has a finite domain, the set of sentences of L that are true in M is effectively specifiable. By constraint 1, this means that the set of

1. f takes each sentence of elementary arithmetic to a sentence in some first- or higher-order language with finite non-logical vocabulary;

2. f is effectively specifiable;

3. f preserves truth-values;

4. there are only finitely many objects within the range of the variables of sentences in the range of f.

An immediate consequence of this result is that it is *impossible* for the noncommittalist (the finite being that she is) to specify a first- or higher-order paraphrase for arbitrary arithmetical sentences in a such a way that truth-values are preserved and infinity assumptions are avoided. Mathematical truth is just too complex.

2.3 Intensional Operators

Might infinity assumptions be circumvented by wheeling in intensional operators? A particularly straightforward strategy would be to claim that an assertion of ϕ is correct if and only if the result of adding a box in front of ϕ's universal Ramseyfication is true. But this would only deliver the right truth-values for sentences of *pure* arithmetic. ('The number of the planets is 8' would turn out to be false, for example, since there are infinite worlds with no planets.) In order to accommodate applied arithmetic, something more elaborate is needed. Here are two familiar proposals:

> THE MODALIST METHOD
>
> The correctness-conditions of marketplace assertions of s are the literal truth-conditions of the following counterfactual:
>
> > If the world contained an infinity of extra objects playing the role of numbers, s.

arithmetical sentences that are mapped by f to true sentences in M is effectively specifiable. But, in light of constraint 2, it follows that the set of true arithmetical sentences is effectively specifiable, contradicting Gödel's Theorem.

A version of the result will continue to hold when the sentences in the range of f are allowed to contain any of the standard pieces of modal vocabulary. What one gets is that there can be no f satisfying constraints 1–3 when there is a finite upper bound to the number of objects a possible world can contain. [*Proof Sketch:* A sentence containing any of the standard pieces of modal vocabulary (including counterfactuals) can be paraphrased as a sentence with no modal vocabulary but quantifiers ranging over possible worlds. When there is a finite upper bound on the number of objects a possible world can contain, any model for the resulting language is equivalent to a finite model. So the previous result applies. (This assumes that the accessibility relation for '$\Box$' takes indistinguishable worlds to be equally accessible (as is trivially the case in S5), and that the 'closeness' relation for counterfactuals takes indistinguishable worlds to be 'equally close'.)] For discussion of the case in which every world has finitely many objects but there is no limit to the number of objects a world can contain, see (Hodes, 1990b).

The Fictionalist Method

> The correctness-conditions of marketplace assertions of s are the literal truth-conditions of the following sentence:
>
> > According to a fiction whereby the world is as it actually is except for the addition of an infinity of extra objects playing the role of numbers, s.

The first thing to note is that these strategies do not deliver the same correctness-conditions as the non-intensional strategies we considered above. On the (non-modal) universal Ramseyfication method, for example, what is required of the world in order for marketplace assertions of 'the number of the planets is 8' to be correct is that it contain either exactly eight planets or only finitely many objects (since that is what is required of the world in order for the truth-conditions of the sentence's universal Ramseyfication to be satisfied). On the intensional strategies, by contrast, what is required of the world in order for marketplace assertions of 'the number of the planets is 8' to be correct is that it contain exactly eight planets (since that is what is required of the world in order for 'the number of planets is 8' to be true according to the relevant fiction or possible world). And the point generalizes. It is an immediate consequence of our limitative result that the correctness-conditions delivered by the intensional methods described in this section will differ from those of *any* strategy based on an effectively specifiable first- or higher-order paraphrase of arithmetical sentences.

It seems to me that, properly construed, the intensional strategies can both be made to deliver the desired assignment of correctness-conditions. There is, however, an important respect in which the intensional strategies are both unilluminating. Consider the problem of explaining, from the intensionalist's perspective, what is required of the world in order for the correctness-conditions of an assertion to be satisfied. It can be divided into the following two subtasks:

1. explaining what is required of a fiction or possible world in order for the sentence asserted to be true at the fiction or possible world;

2. explaining what is required of the world in order for the addition of extra objects playing the role of numbers to yield a fiction or possible world at which the sentence asserted is true.

When it comes to the first subtask, intensionalists have an admirably perspicuous story to tell: they can give a standard compositional semantics for the language in question, and relativize it to fictions or possible worlds. The problem is that they have yet to supply a systematic way of addressing the second subtask. It is, of course, easy enough to rehearse the familiar set of toy examples. One can say, for example, that what is required of the world in order for the addition of extra objects playing the role of numbers to yield a possible world at which 'the number of the planets is eight' is true is that there be exactly eight planets. But what one really wants to know is how the proposal is supposed to work in general—and *that* task is no easier than the original task of supplying a noncommittalist specification of correctness-conditions for arithmetical statements.

Here is a different way of putting the point. The intensionalist's proposal involves two different requirements. The first is a requirement on how the world ought to be *represented*:[10]

- *Requirement 1*

  That the world be represented (by a fiction or possible world) as being such as to make the target sentence true.

Because the intentionalist is able to address the first of the two subtasks mentioned above, this requirement is clear enough.

The second requirement is a requirement on how the world ought to *be*:

- *Requirement 2*

  That the world be such that adding extra objects to play the role of numbers would yield a fiction or possible world satisfying Requirement 1.

But because the intensionalist lacks a systematic way of addressing the second subtask, she is unable to give an informative account of how the world would have to be in order for Requirement 2 to be satisfied. The best she can do in the general case is this:

> The world must be as Requirement 1 requires it to be represented, *ignoring those aspects of Requirement 1 that concern numbers.*

And this is no help at all. For if the noncommittalist was able to factor requirements into arithmetical and a non-arithmetical components, she could simply characterize an arithmetical assertion's correctness-conditions as the non-arithmetical component of the truth-conditions of the sentence asserted, and bypass talk of fictions and possible worlds altogether.

As a last resort, an intensionalist might claim that she is being asked to do too much. The most one can do in the general case to shed light on the correctness-conditions of an arithmetical assertion is insist that the world must be such that a certain counterfactual turns out to be true, or such that a certain fiction turns out to be apt. In the following section we will see that there is no need to settle for this sort of answer.

[10] *Notation:* I say that a possible world represents the world as being such that p just in case, according to (or at) that possible world, p.

2.4 Specifying Correctness-Conditions

Imagine a debate between Kurt and Steve. Kurt is both a Platonist and a committalist. Steve is a noncommittalist. (His stance *vis-à-vis* Platonism is irrelevant for present purposes.) Kurt kicks off the debate with the following: "I don't care how you understand the notion of assertoric correctness. Whether you're a fictionalist or an instrumentalist, an eliminativist or an empiricist, it's all the same to me. What I'd like to understand is what you think is required for the correctness-conditions of marketplace arithmetical assertions to be satisfied."

Steve knows better than to answer the challenge by wheeling in a few toy examples: he knows that what is sought is a specification of correctness-conditions for assertions of *arbitrary* sentences in the language of applied arithmetic.[11] He therefore decides to try out the $\phi(w)$-technique. As in the Parable, the first thing to note is that there are two different w-formulas on the basis of which the $\phi(w)$-technique might be used to specify correctness-conditions for assertions of a sentence like 'the number of the planets is eight':[12]

> PLANETS (WIDE SCOPE)
> [The number of the planets $= 8]_w$
>
> PLANETS (NARROW SCOPE)
> The number of the xs such that [$\dot{x}$ is a planet$]_w = 8$

But Steve notices a complication in the dialectic. When George set forth $\phi(w)$ formulas in the course of his exchange with Charles, there was no disagreement about the satisfaction-conditions of the $\phi(w)$ formulas themselves. In contrast, Steve and Kurt disagree about the correctness-conditions of PLANETS (in either of its versions) because PLANETS contains arithmetical vocabulary and Steve and Kurt disagree about the correctness-conditions of arithmetical assertions. So there is no guarantee that Steve and Kurt will agree about the primary and secondary conditions corresponding to PLANETS (in either of its versions). Fortunately, the difficulty can be contained. Steve's objective is to get Kurt to understand what the noncommittalist correctness-conditions consist in. And for this it is sufficient that Steve identify a statement which *as understood by Kurt* will allow Kurt to see what the

[11] I take the language of applied arithmetic to be a two-sorted first-order language, with the following features: (a) the arithmetical predicates '0', 'S', '+', '×', '<' and '*Finite*' take only arithmetical variables ($n_1, n_2, \ldots$) as arguments; (b) the n-place non-arithmetical predicates $\ulcorner P_i^n \urcorner$ take only non-arithmetical variables ($x_1, x_2, \ldots$) as arguments; (c) the identity predicate '=' must be flanked either by two arithmetical variables or by two non-arithmetical variables; (d) the expression $\ulcorner Num_v(n_i, \phi)\urcorner$ is well-formed just in case v is a variable, $\ulcorner n_i \urcorner$ is an arithmetical variable and ϕ is well-formed; (e) there are no additional atomic predicates, function-letters or individual constants; and (f) other than the restrictions noted above, well-formed formulas are defined in the usual way. (Some of the expressive capabilities of this language are described in section 4.3.)

[12] As in the Parable, the dot in PLANETS (NARROW SCOPE) is introduced to deal with a complication arising from the fact that the world might have contained objects it doesn't actually contain. I consider the issue at length in the appendix. But, as in footnote 2, I encourage the reader to ignore the dots until section 3.1. Doing so will result in a more comfortable read, and carries little risk of misunderstanding.

noncommittalist correctness-conditions consist in. Whether or not Steve would understand the statement differently is irrelevant to whether Kurt is able to use it to acquire the desired understanding.

Having finessed the dialectical situation, Steve proceeds to make the following observation. "As understood by you, Kurt, different secondary conditions correspond to Planets (Wide Scope) and Planets (Narrow Scope). The secondary condition corresponding to Planets (Wide Scope) requires the existence of the number eight. But the secondary condition corresponding to Planets (Narrow Scope) does not: all it requires is that there be exactly eight planets. Here then is the explanation you seek: what is required of the world in order for assertions of 'the number of the planets is eight' to be correct is that it meet the secondary condition corresponding to Planets (Narrow Scope)."

And, crucially, the point can be generalized. It is possible to assign correctness-conditions to every sentence in the language of pure and applied arithmetic in such a way that the following three constraints are satisfied: (*a*) the assignment is compositional (and therefore finitely specifiable), (*b*) every sentence in the language gets assigned the correctness-conditions that the noncommittalist thinks assertions of the sentence should have, and (c) the correctness-conditions are stated in such a way that it is clear what categorical features of reality would be responsible for the correctness or incorrectness of the assertions.[13]

Kurt is warming up to the proposal. But, like Charles, he wants to know whether different types of existence are required, or the possibility of things existing without existing 'in' the world, or an otherwise unorthodox metaphysics. By paralleling George's response to Charles, Steve is able to explain why his proposal is compatible with an unexciting metaphysics. Steve's is a proposal about the correctness-conditions of arithmetical asser-

[13]Here's the relevant transformation:

$$\begin{aligned}
(\ulcorner P^n_i(x_{j_1},\ldots,x_{j_n})\urcorner)^* &= \ulcorner[\hat{P}^n_i(\dot{x}_{j_1},\ldots,\dot{x}_{j_n})]_w\urcorner \text{ (where } \ulcorner\hat{P}^n_i\urcorner \text{ translates } \ulcorner P^n_i\urcorner)\\
(\ulcorner x_i = x_j\urcorner)^* &= \ulcorner[\dot{x}_i = \dot{x}_j]_w\urcorner\\
(\ulcorner Num_v(n_i,\phi(v))\urcorner)^* &= \ulcorner\text{the number of the } v\text{'s such that } \phi(v)^*\text{is } n_i\urcorner\\
(\ulcorner 0(n_i)\urcorner)^* &= \ulcorner n_i \text{ is the number } 0\urcorner\\
(\ulcorner S(n_i,n_j)\urcorner)^* &= \ulcorner n_i \text{ is the successor of } n_j\urcorner\\
(\ulcorner +(n_i,n_j,n_k)\urcorner)^* &= \ulcorner n_k \text{ is the sum of } n_i \text{ and } n_j\urcorner\\
(\ulcorner \times(n_i,n_j,n_k)\urcorner)^* &= \ulcorner n_k \text{ is the product of } n_i \text{ and } n_j\urcorner\\
(\ulcorner n_i < n_j\urcorner)^* &= \ulcorner n_i \text{ is smaller than } n_j\urcorner\\
(\ulcorner Finite(n_i)\urcorner)^* &= \ulcorner n_i \text{ is a finite number}\urcorner\\
(\ulcorner n_i = n_j\urcorner)^* &= \ulcorner n_i = n_j\urcorner\\
(\ulcorner \phi\wedge\psi\urcorner)^* &= \ulcorner(\phi)^*\wedge(\psi)^*\urcorner\\
(\ulcorner \neg\phi\urcorner)^* &= \ulcorner\neg(\phi)^*\urcorner\\
(\ulcorner \exists x_i\,\phi\urcorner)^* &= \ulcorner\text{there is an } x_i \text{ such that } ([\exists y(y=\dot{x}_i)]_w\wedge(\phi)^*)\urcorner\\
(\ulcorner \exists n_i\,\phi\urcorner)^* &= \ulcorner\text{there is a number } n_i \text{ such that } (\phi)^*\urcorner
\end{aligned}$$

(Regarding the use of dotted variables in the metalanguage, see footnote 12.)

tions, not about the non-linguistic facts. "Hold on!"—cries Kurt—"I agree with all that. But surely your proposal is committed to Platonism! For even if the secondary condition corresponding to Planets (Narrow Scope) does not require of the world that it contain numbers, Planets (Narrow Scope) itself is up to its neck in arithmetical vocabulary. So Planets (Narrow Scope) can only be used to specify the desired condition on the assumption there are numbers."

"It is true"—Steve responds—"that, *as understood by you*, Planets (Narrow Scope) can only be used to specify the desired correctness-condtions on the assumption that there are numbers. You are, after all, a committalist. (It is a good thing you are also a Platonist!) But notice the following. From my point of view, the correctness of Planets (Narrow Scope) (relative to a given w) does *not* require the existence of numbers. I am, after all, a noncommittalist. And I am no less a noncommittalist about the metalanguage in which Planets (Narrow Scope) is stated than I am about the object language of which 'the number of the planets is eight' is a part. So I take the presence of number-talk in Planets (Narrow Scope) to be no more committing than the presence of number-talk in 'the number of the planets is eight'."

"Our situation is therefore this."—Steve continues—"Each of us takes himself to be in a position to use Planets (Narrow Scope) to specify noncommittalist correctness conditions for assertions of 'the number of planets is eight', but we disagree about what is required for the specification to be successful. From your perspective, the specification presupposes Platonism; from my perspective it does not. Whether or not this disagreement is problematic depends on the task at hand. If I had the ambitious objective of converting you from committalism and Platonism to noncommittalism and nominalism, there would be a potential source of concern. For one might worry that you would only be in a position to understand my noncommittalism by presupposing Platonism and that you would only be prepared to give up your Platonism if I first convinced you of my noncommittalism. Fortunately, my objective is not as ambitious as that. All I set out to do was satisfy your original request: I set out to give you a way of understanding, to your own satisfaction, what noncommittalists take the correctness of arithmetical assertions to require of the world. And I take myself to have succeeded in this more modest task."

"Actually, I take myself to have achieved slightly more. Should you, for whatever reason, come to embrace noncommittalism, I will be able to give you a way of making explicit what you take the correctness of arithmetical assertions to require of the world. I will do so by offering you the same w-formulas I offer you now. But, because of your newly found noncommittalism, you will think these w-formulas can be used to specify correctness-conditions for arithmetical assertions without incurring commitment to numbers. So, like me, you will take yourself to be in a position to use the $\phi(w)$-technique without presupposing Platonism. Since you will have been presupposing noncommittalism to begin with, the exercise won't give you a new way of understanding arithmetical assertions. But it will shed light on the workings of arithmetical discourse in the same sort of way that semantic theorizing sheds light on the workings of quantification even though one uses quantification to carry out the theorizing."

There is a long pause. "Okay"—Kurt finally says—"I agree that you've satisfied my original request. I can now see what noncommittalists take the correctness-conditions of arithmetical assertions to consist in. But just because I understand noncommittalism it doesn't mean I accept it!"

"One final question."—Kurt adds—"Your implementation of the $\phi(w)$-technique seems to involve keeping arithmetical vocabulary outside the scope of '$[\ldots]_w$'. But now consider a sentence built up entirely from logical and arithmetical vocabulary: 'there are infinitely many primes', for instance. According to your proposal, the correctness-conditions of assertions of this sentence can be specified by using a w-formula in which all the action takes place outside the scope of '$[\ldots]_w$' ('there are infinitely many primes $\wedge\ [\forall x(x = x)]_w$', for instance). So won't it be a consequence of your view that *nothing* is required of the world in order for the correctness-conditions of assertions of a truth of pure arithmetic to be satisfied?" Steve's face lights up. "Exactly!"—he replies—"And it is this that allows me to account for the fact that the truths of pure arithmetic can be known *a priori*." We will see what Steve has in mind when we get to section 5.

2.5 Set Theory

What goes for arithmetic goes for set theory. The $\phi(w)$-technique can be used to assign noncommittalist truth-conditions to every sentence in a two-sorted version of the language of pure and applied set theory.[14]

3 Truth-Conditions

The purpose of this section is to explain what it takes for different w-formulas to have the same primary condition. This will immediately yield an explanation of what it takes for different w-formulas to have the same secondary condition, and for different w-formulas to deliver the same specification of truth-conditions via the $\phi(w)$-technique. For different w-formulas share their secondary condition just in case they share their primary condition, and they deliver the same specification of truth-conditions just in case they share their secondary condition.

[14]Here's is the relevant transformation:

$$\begin{aligned}
(\ulcorner P^n_i(x_{j_1},\ldots,x_{j_n})\urcorner)^* &= \ulcorner[\hat{P}^n_i(\dot{x}_{j_1},\ldots,\dot{x}_{j_n})]_w\urcorner \text{ (where } \ulcorner\hat{P}^n_i\urcorner \text{ translates } \ulcorner P^n_i\urcorner)\\
(\ulcorner x_i = x_j\urcorner)^* &= \ulcorner[\dot{x}_i = \dot{x}_j]_w\urcorner\\
(\ulcorner x_i \in \alpha_j\urcorner)^* &= \ulcorner x_i \in \alpha_j\urcorner\\
(\ulcorner \alpha_i \in \alpha_j\urcorner)^* &= \ulcorner \alpha_i \in \alpha_j\urcorner\\
(\ulcorner \phi\wedge\psi\urcorner)^* &= \ulcorner(\phi)^*\wedge(\psi)^*\urcorner\\
(\ulcorner \neg\phi\urcorner)^* &= \ulcorner\neg(\phi)^*\urcorner\\
(\ulcorner \exists x_i\,\phi\urcorner)^* &= \ulcorner\text{there is an object } x_i \text{ such that } ([\exists y(y=\dot{x}_i)]_w\wedge(\phi)^*)\urcorner\\
(\ulcorner \exists \alpha_i\,\phi\urcorner)^* &= \ulcorner\text{there is a set } \alpha_i \text{ such that } (\text{Good}_w(\alpha_i)\wedge(\phi)^*)\urcorner
\end{aligned}$$

where a set is good$_w$ just in case it occurs at some stage of the iterative hierarchy built up from objects x such that $[\exists y(y = \dot{x})]_w$.

3.1 Trivial Demands

It will be helpful to begin by explaining what it takes for the primary condition corresponding to a w-formula to be *trivial*. Informally, $\phi(w)$'s primary condition is trivial just in case the satisfaction of $\phi(w)$ by w does not depend on how the world is represented to be by w. Contrast the following examples:

(1) $[\text{Susan runs}]_w$

(2) $[\text{Susan runs}]_w \vee \neg[\text{Susan runs}]_w$

(3) Susan runs $\wedge$ $([\text{Susan runs}]_w \vee \neg[\text{Susan runs}]_w)$

The primary condition corresponding to (1) is non-trivial. For suppose w is a story. Then whether or not (1) is satisfied by w will depend on whether or not w is a story according to which Susan runs. In contrast, the primary condition corresponding to (2) is trivial. For (2) will be satisfied by w regardless of how the world is represented by w.

What about (3)? Which primary condition corresponds to (3) depends on whether Susan runs. If she does, then (3)'s primary condition is trivial. For (3) will be satisfied by w regardless of how w turns out to represent the world. But if Susan doesn't run, then (3)'s primary condition is non-trivial (in fact, it is unsatisfiable). For (3) will fail to be satisfied by w regardless of how the world is represented to be by w.

A formal characterization of the notion of triviality is provided in the appendix. It proceeds by letting w range over representations of a particular kind—*a-worlds*, as I call them—and taking a w-formula to have a trivial primary condition just in case it is satisfied by every a-world. For present purposes, however, we needn't worry about the details. Here I will supply an informal gloss of the notion of an a-world, and use it to elucidate the main consequences of the technical apparatus in the appendix.

Think of an a-world as a story which is complete, *de re* and intelligible. A story is *complete* (relative to a given language) if it includes any sentence in the language or its negation. A story is *de re* if every name used by the story is used to say of the name's *actual* bearer how it is according to the story,[15] and if every predicate used by the story is used to attribute the property *actually* expressed by the predicate to characters in the story. (Thus, a *de re* story that says 'Hesperus is covered with water' is a story according to which Venus itself is covered with H_2O.) Finally, a story is *intelligible* if one could make sense of it even if one knew everything of relevance there is to be known. To see what I have in mind here, it is useful to consider some examples. I can certainly imagine circumstances under which one might be inclined to conclude that Mark Twain turned out not to be Samuel Clemens—I can imagine, for example, discovering that *Huckleberry Finn* and other works written under the name 'Mark Twain' were authored by one of Clemens's literary rivals. And I could certainly make sense of a *de re* story describing such circumstances. But given that I know that Mark Twain is, in fact, Samuel Clemens, I am unable to make

[15] Thus, no empty names are allowed to figure in *de re* stories. For more on empty names, see my 'An Actualist's Guide to Quantifying-In'.

sense of a situation in which it is true of Mark Twain—the man himself—that he is not Samuel Clemens—the man himself. For I know that it would have to be a situation in which someone is not identical to himself. Since a *de re* story that says ‘Mark Twain is not Samuel Clemens’ would depict such situation, I am also unable to make sense of the story. Another example: I can certainly imagine circumstances under which one might be inclined to conclude that water does not contain hydrogen—I can imagine, for example, discovering that water electrolysis results in the release of oxygen and nitrogen rather than oxygen and hydrogen. And I could certainly make sense of a *de re* story describing such circumstances. But given that I know that part of what it is to be composed of water is to contain hydrogen, I am unable to make sense of a situation in which a portion of water—the very substance that results from binding oxygen and hydrogen atoms in a certain kind of way—contains no hydrogen. For I know that it would have to be a situation in which something containing hydrogen contains no hydrogen. Since a *de re* story that says ‘there is a portion of water containing no hydrogen’ would depict such a situation, I am also unable to make sense of the story.[16] (I hope your judgments about intelligibility coincide with mine. If they don’t, please keep in mind that the informal gloss of a-worlds can be safely ignored.)

The formal characterization of triviality in the appendix delivers the result that the primary condition corresponding to any instance of one of the following w-formula schemas is trivial:

1. Validity

   $[\psi]_w$

   (where ψ is logically true in a negative free logic[17])

2. Conjunction

   $[\psi \wedge \theta]_w \leftrightarrow ([\psi]_w \wedge [\theta]_w)$

3. Negation

   $[\neg\psi]_w \leftrightarrow \neg[\psi]_w$

4. Quantification

   $[\exists y(\phi(y))]_w \leftrightarrow \exists y([\exists z(z = \dot{y})]_w \wedge [\phi(\dot{y})]_w)$

5. Atomic Predication

   $[\mathrm{F}^n_j(v_1, \ldots, v_n)]_w \rightarrow ([\exists z(z = v_1)]_w \wedge \ldots \wedge [\exists z(z = v_n)]_w)$

   (where the v_i may occur dotted or undotted)

[16] I say more about *de re* stories, and about statements of the form ‘part of what it is to be F is to be G’, in ‘An Account of Possibility’.

[17] In a negative free logic, an atomic predication involving an empty name is false, and its negation is true.

6. Names

   $[\psi(c)]_w \leftrightarrow \exists x(x = c \wedge [\psi(x)]_w)$

   (where c is a nonempty individual constant)

7. Predicates

   $F(x) \ll_x G(x) \rightarrow ([F(x)]_w \rightarrow [G(x)]_w)$

   (where '$F(x) \ll_x G(x)$' is read 'part of what it is to be F is to be G')

   Remark 1: It is crucial that the validities in (1) be validities of a free logic. Otherwise one would get the unpleasant result that the primary condition corresponding to, say, '$[\exists x(\text{Socrates} = x)]_w$' is trivial, even though there are complete, *de re* and intelligible stories according to which Socrates doesn't exist.

   Remark 2: (3) may be thought of as entailing a *completeness* principle. In the presence of (1) and (2), it yields the result that the primary condition corresponding to '$[\phi]_w \vee [\neg\phi]_w$' is trivial for arbitrary ϕ.

   Remark 3: (1)–(3) entail the following *closure* principle for w-formulas: if $\ulcorner \phi \rightarrow \psi \urcorner$ is a truth of negative free logic, then the primary condition corresponding to $\ulcorner [\phi]_w \rightarrow [\psi]_w \urcorner$ is trivial.

   Remark 4: (4) is one of several places in the text where dotted variables appear. The dots are needed to deal with a technical issue that I have been ignoring so far (see footnotes 2 and 12). They would be unnecessary if, for instance, '[there are eight planets]$_w$' and 'there are eight xs such that [x is a planet]$_w$' were guaranteed to be equisatisfiable. But there is no such guarantee. Suppose w is a story according to which there are eight planets, none of which actually exists. Then, unless one believes in merely possible objects, one should say that '[there are eight planets]$_w$' is true but 'there are eight xs such that [x is a planet]$_w$' is not, since there is no (actual) individual of which it can be truly said that, according to w, it is a planet. Dotting variables is a technical trick for getting around the problem. A rigorous characterization of the dot-notation is supplied in the appendix, but the basic idea is that whereas '[x is a planet]$_w$' is satisfied by an individual x if, according to w, x is a planet, '[$\dot{x}$ is a planet]$_w$' is satisfied by an individual x if x represents a planet in w. I encourage you to have a look at the appendix. But readers who choose not to do so may simply ignore the dots throughout the remainder of the paper. Doing so carries little risk of serious misunderstanding.

   Remark 5: Since 'Hesperus = Phosphorus' is true, (6) guarantees that the primary condition corresponding to the following w-formula is trivial:

   $$[\text{Hesperus is cold}]_w \rightarrow [\text{Phosphorus is cold}]_w$$

This is as it should be. Consider a complete, *de re*, intelligible story that says 'Hesperus is cold'. Since the story is *de re*, it is a story according to which it is true of Hesperus itself that it is cold. But Hesperus is Phosphorus. So, by completeness and intelligibility, the story must also say 'Phosphorus is cold'.

Remark 6: Since part of what it is to be scarlet is to be red, (7) guarantees that the primary condition corresponding to the following w-formula is trivial:

$$[\text{there is a scarlet umbrella}]_w \rightarrow [\text{there is a red umbrella}]_w$$

This is as it should be. Consider a complete, *de re*, intelligible story that says 'there is a scarlet umbrella'. Since the story is *de re*, it describes an umbrella as being as scarlet things actually are. But part of what it is to be scarlet is to be red. So the story describes an umbrella as being as red things actually are. So, by completeness and intelligibility, the story must also say 'There is a red umbrella'.[18]

Remark 7: In conjunction with an auxiliary principle,[19] (1)–(7) guarantee that any w-formula is equisatisfiable with a w-formula in which only atomic formulas occur within the scope of '$[\ldots]_w$'.

3.2 On Expressing the Same Demand

Once one knows what it is for the primary condition corresponding to a w-formula to be trivial, it is straightforward to say what it takes for different w-formulas to share a primary condition: $\phi(w)$ and $\psi(w)$ share a primary condition just in case the primary condition corresponding to $\ulcorner \phi(w) \leftrightarrow \psi(w) \urcorner$ is trivial. (Informally, the idea is that two w-formulas share a primary condition just in case their equisatisfiability by a complete, *de re*, intelligible story does not depend on how the world is represented by the story.) It may be helpful to consider some examples. (As before, my informal glosses may be safely ignored.)

1. A humdrum case

   Different primary conditions correspond to '$[\text{John is happy}]_w$' and '$[\text{Susan runs}]_w$'. This is as it should be. Suppose one is told that w is a complete, *de re*, intelligible story according to which John is happy. Without further information about w there is no way of telling whether it is also a story according to which Susan runs.

2. Hesperus and Phosphorus

   It is a consequence of the fact that Hesperus is Phosphorus that the same primary condition corresponds to '$[\text{Hesperus is cold}]_w$' and ' $[\text{Phosphorus is cold}]_w$', and that

[18]For more on what it means to say that part of what it is to be F is to be G, see my 'An Account of Possibility'.

[19]The auxiliary principle is that '$[[\phi]_w]_{w'} \leftrightarrow [\phi]_w$' and '$[\exists w'(\phi)]_w \leftrightarrow \exists w'([\phi]_w)$' have trivial primary conditions.

the same primary condition corresponds to '[Hesperus = Phosphorus]$_w$', '[Hesperus = Hesperus]$_w$' and '[$\exists x(x = \text{Hesperus})$]$_w$'.

3. Triviality

   Any two w-formulas with trivial primary conditions have the same primary condition. This means, in particular, that any two formulas of the form $[\psi]_w$ (where ψ is a truth of negative free logic) share their primary condition. This is as it should be. Suppose one is told that w is a complete, *de re*, intelligible story. Assuming 'there are elephants' is part of the language, one needn't be told what the story is about in order to know that w is a story according to which there are elephants only if there are elephants.

   The point generalizes beyond simple logical truths. Suppose one is given a highly complex logical truth, ξ, and asked whether w satisfies the primary condition corresponding to $[\xi]_w$. If one is unable to see that ξ is a logical truth, one may find that one is unable to answer the question. But this will not be because one lacks information about w. The problem is rather that one is unable to determine what $[\xi]_w$'s primary condition is. If one comes to know that ξ is a logical truth by, say, carrying out a computation, one will thereby be in a position to know that $[\xi]_w$ has a trivial primary condition, and is therefore satisfied by w.

4. Metaphysical Necessity

   Nothing I have said so far entails that if ϕ is a necessary truth, then $[\phi]_w$'s primary condition is trivial.

   This is as it should be. There is no reason to doubt, for example, that one could tell a complete, *de re*, intelligible story according to which God doesn't exist. And this is so independently of whether one chooses to work with a conception of metaphysical necessity that makes room for the claim there are necessarily existing entities, and that God is among them.

5. Arithmetic

   It is a consequence of our definitions that the following w-formulas have different primary conditions:

   > Planets (Wide Scope)
   > [The number of the planets = 8]$_w$
   >
   > Planets (Narrow Scope)
   > The number of the xs such that [$\dot{x}$ is a planet]$_w$ = 8

   and that the latter has the same primary condition as '[$\exists!_8 x(\text{Planet}(x))$]$_w$'.

   This is as it should be. Suppose w is a complete, *de re*, intelligible story according to which there are eight planets. Then w satisfies the primary condition corresponding to PLANETS (NARROW SCOPE). But without further information about w there is no way of telling whether w is also a story according to which there are numbers. So there is no way of telling whether w also satisfies the primary condition corresponding to PLANETS (WIDE SCOPE). (Here I rely on the assumption that there are complete, *de re*, intelligible stories according to which there are no numbers. I take for granted that this assumption is plausible whether or not one one works with a conception of metaphysical necessity that makes room for the claim that numbers are necessarily existing entities.)

It is also a consequence of our definitions that when w-formulas are assigned to arithmetical sentences as in footnote 13, the w-formula corresponding to any truth of *pure* arithmetic has a trivial primary condition.

3.3 A Criterion of Ontological Commitment

To describe a sentence's ontological commitments is to describe from among the requirements that the world must meet in order for the sentence's truth-conditions to be satisfied those that concern ontology. To say, for example, that a sentence carries commitment to Fs is simply to say that the world is required to contain Fs in order for the sentence's truth-conditions to be satisfied.

This informal idea can be made precise by way of the following definition:[20]

> s carries commitment to Fs just in case $\ulcorner\phi(w) \rightarrow [\exists x(F(x))]_w\urcorner$ has a trivial primary condition;

where the truth-conditions of s consist of the requirement that the world meet the secondary condition corresponding to $\phi(w)$. Thus, 'Ebenezer is an elephant' carries commitment to elephants because

$$[\text{ELEPHANT}(\text{EBENEZER})]_w \rightarrow [\exists x\, \text{ELEPHANT}(x)]_w$$

has a trivial primary condition. And, on the assumption that part of what it is to be an elephant is to be a mammal, 'Ebenezer is an elephant' will also carry commitment to mammals, since

$$[\text{ELEPHANT}(\text{EBENEZER})]_w \rightarrow [\exists x\, \text{MAMMAL}(x)]_w$$

will turn out to have a trivial primary condition.[21] (A similar definition can be given for the case of assertion.)

[20] As formulated, the criterion applies to first-order commitments only. But it can easily be extended to encompass second-level commitments. One can say, for example, that s carries commitment to infinitely many objects just in case the primary condition corresponding to $\ulcorner\phi(w) \rightarrow [\exists X(\text{InfinitelyMany}(X))]_w\urcorner$ is trivial (or, equivalently, just in case the primary condition corresponding to $\ulcorner\phi(w) \rightarrow \exists\alpha(\forall x(x \in \alpha \rightarrow [\exists y(y = \dot{x})]_w) \wedge \text{Infinite}(\alpha))\urcorner$ is trivial). For more on second-level commitments, see (Rayo, 2002b) and (Rayo, 2007).

[21] A feature of the criterion of ontological commitment I propose is that it takes unsatisfiable w-formulas (w-formulas whose negations have trivial primary conditions) to be committed to Fs for any F. One way of avoiding this result is by restricting the criterion to satisfiable w-formulas, and letting the commitments carried by unsatisfiable w-formulas remain undefined. But I'm not sure this is fully satisfactory.

3.4 Thin and thick ontological commitments

(Hodes, 1990a) sets forth a distinction between 'thin' and 'thick' ontological commitments. The *thin* commitments of a piece of discourse are "what is said to be" by that piece of discourse; its *thick* commitments are "what there would have to be" in order for the piece of discourse to be true. In many cases thin and thick commitments coincide: 'There are elephants', for example, says that there are elephants, and in order for it to be true there would have to be elephants. But Hodes makes a case for the view that thin and thick commitments can come apart in arithmetical discourse: whereas 'there are numbers' says that there are numbers, it is not the case that there would have to be numbers in order for it to be true.

On my usage of the term 'commitment', a sentence's ontological commitments are its *thick* ontological commitments. Thus, what I mean when I say that the world is required to contain Fs in order for s's truth-conditions to be satisfied is what Hodes means when he says that there would have to be Fs in order for s to be true. To say that s carries *thin* commitment to Fs, as I understand Hodes's locution, is to say that s entails a sentence with the inferential behavior of 'there are Fs'. So, in my terminology, the claim that thin and thick commitments can come apart is the claim that one cannot generally go from the observation that a sentence entails something with the inferential behavior of 'there are Fs' to the conclusion that the world is required to contain Fs in order for the sentence's truth-conditions to be satisfied.[22]

The notion of truth-conditions I have tried to elucidate in this section can be seen as a generalization of Hodes's notion of thick ontological commitment. To describe a sentence's thick ontological commitments is to describe from among the requirements that the world must meet in order for s's truth-conditions to be satisfied those that concern ontology; to describe a sentence's truth-conditions, as I use the term, is to describe *all*

[22] Hodes's argument for this claim is based on the observation that expressions with identical syntactic and inferential roles may nonetheless perform radically different *semantic* jobs. He suggests, in particular, that the singular terms in 'Ebenezer is an elephant' and '0 is a number' do different semantic work. (Very roughly: whereas the job of the former is to denote an object, the job of the latter is to encode higher-level concepts, and can be modeled by way of a certain kind of supervaluational semantics.) So even though 'Ebenezer is an elephant' and '0 is a number' carry parallel *thin* commitments (since they entail 'there are elephants' and 'there are numbers', respectively), their semantic differences ensure that they differ in their *thick* commitments: whereas the truth of the former requires of the world that it contain elephants, the truth of the latter does not require of the world that it contain numbers.

My proposal is closely aligned with Hodes's. Like Hodes, I make full use of the idea that expressions with identical syntactic and inferential roles can perform different semantic jobs, and that such semantic differences can lead to differences in truth-conditions (and, in particular, differences in ontological commitment). So, like Hodes, I get the result that thin and thick ontological commitments can come apart. Where the projects diverge is when it comes to the particular assignments of truth-conditions under consideration, and the techniques that are used to specify these assignments. On Hodes's proposal, the world is required to contain infinitely many objects in order for the truth-conditions of 'there is no largest prime' to be satisfied; but according to noncommittalists of the sort I discuss, nothing whatsoever is required of the world for the correctness-conditions of the relevant assertion to be satisfied.

For a different implementation of the idea that expressions with identical syntactic and inferential roles can perform different semantic jobs, see (Hofweber, 2005a) and (Hofweber, 2005b).

such requirements.

4 A Noncommittalist Semantics

Earlier in the paper I suggested a mapping of w-formulas to arithmetical sentences, and claimed that it could be used to explain to the committalist what the noncommittalist takes the correctness-conditions of marketplace arithmetical assertions to be. I did *not* claim that the resulting assignment of correctness-conditions to arithmetical sentences—the Noncommittalist Assignment, as one might call it—should be seen as a characterization of the literal truth-conditions of arithmetical sentences. The purpose of this section is to assess the prospects of taking this additional step.

The difficulties are not of a technical nature. For someone who embraces the $\phi(w)$-technique, producing a semantics for the language of arithmetic that delivers the Noncommittalist Assignment is straightforward. (All one needs to do is rewrite the clauses in footnote 13 as clauses of a definition of satisfaction-relative-to-a-possible-world.) Our focus will be on the underlying philosophical terrain.

4.1 A Self-Defeating Proposal?

There is some temptation to think that any attempt to use the $\phi(w)$-technique to specify noncommittalist truth-conditions for arithmetical sentences would be self-defeating. Here is one way of spelling out the complaint:

> The w-formulas that would be needed to deliver the Noncommittalist Assignment are themselves up to their necks in number-talk. So they can only be used to specify the desired truth-conditions if there are numbers. It follows that the $\phi(w)$-technique is unavailable to the nominalist. It is certainly available to the Platonist, but the Platonist would have no use for it. So the proposal is pointless either way.

The trouble with this complaint is that it begs the question against the noncommittalist. For suppose the use of arithmetical vocabulary is indeed noncommitting. Then it is not true—contrary to what the complaint presupposes—that a w-formula containing number-talk can only be used to specify the desired truth-conditions for an arithmetical sentence if there are numbers. For the use of arithmetical vocabulary in the w-formula will be no more committing than the use of arithmetical vocabulary in a sentence of the object-language.

The view that arithmetical vocabulary is non-committing and that the $\phi(w)$-technique can be used to deliver an accurate specification of truth-conditions to arithmetical sentences is *stable*: it is a view that can be coherently adopted. At the same time, there are important limits to the sorts of arguments that are likely to be effective in an attempt to convince the unconvinced of embracing such a view. What the complaint brings out is that one will only be in a position to persuade a nominalist to adopt the view if she is prepared to take for granted that arithmetical vocabulary can be used in a noncommitting way.

Whether or not this is a problem depends on one's dialectical situation. Consider the following example. One starts out favoring a standard assignment of truth-conditions for the language of arithmetic. Accordingly, one believes that arithmetical sentences carry commitment to numbers. But one is also a fictionalist: one believes that marketplace mathematical assertions are made in a spirit of make believe, and do not commit their speaker to numbers. One then comes across the $\phi(w)$-technique. Because one is a fictionalist, one thinks the $\phi(w)$-technique can be used to specify the Noncommittalist Assignment without incurring in any offensive commitments. One is impressed by the Noncommittalist Assignment, and comes to believe that it is accurate as a characterization of the truth-conditions of arithmetical sentences. As a result, one comes to believe that one's fictionalism was unnecessary: what one has come to think of as the literal truth-conditions of arithmetical sentences is precisely what one had been taking to be the fictional-correctness conditions of the corresponding assertions all along. This is a dialectical situation in which the $\phi(w)$-technique might be used to convince the unconvinced that the Noncommittalist Assignment is accurate as a specification of truth-conditions for arithmetical sentences. (Notice, incidentally, that nowhere does the issue of whether or not one is a Platonist come up.)

This story assumes that one was a noncommittalist to begin with. But it may be possible to tell a similar story for someone like Kurt, who is able to understand what noncommittalists think is required of the world in order for the correctness-conditions of arithmetical assertions to be satisfied even though he himself is a committalist. Whether or not the new story will work depends on whether Kurt acquires the ability to understand arithmetical assertions as a noncommittalist would. In any event, there is no denying that when it comes to the project of convincing committalists to become noncommittalists, one would be in a far stronger position if one were able to supply noncommittalist paraphrases for arbitrary arithmetical sentences. The limits of such an approach were underscored in section 2.2.

It may well be the case that there is no way of specifying noncommittalist truth-conditions for arithmetical sentences without using something very much like arithmetic in one's metalanguage. But it would be hasty to conclude from this that the truth-conditions of arithmetical sentences cannot be illuminated by semantic theorizing. Consider the case of quantification. Russell taught us that quantified sentences are not correctly paraphrased as conjunctions or disjunctions, even if one avails oneself of infinitely long formulas and assumes that every individual has a name. It may well be the case that there is no way of specifying truth-conditions for sentences containing quantifiers without using something very much like quantification in one's metalanguage. But it would be a mistake to conclude from this that the truth-conditions of quantified sentences cannot be illuminated by semantic theorizing.

4.2 Ontological Commitment

If the Noncommittalist Assignment is an accurate characterization of the truth-conditions of arithmetical sentences, the language of arithmetic is not properly thought of as a first-order language. For although the syntax and inferential behavior of arithmetical sentences will be indistinguishable from that of first-order sentences, their semantics will be fundamentally different.

One could, if one wanted, introduce a characteristic notation as a grammatical reminder of this semantic difference. One option would be to place a mark on the quantifiers binding arithmetical variables; for instance, one could write '$\exists^* n$' in place of '$\exists n$'. But this choice of notation would be an unhappy one, for at least two reasons. Firstly, it could lead to a serious misunderstanding of the proposal by encouraging the thought that a semantic theory delivering the Noncommittalist Assignment makes use of a metaphysical picture that countenances different types of existence (existence *simpliciter vs.* existence 'in' the world, for instance). Just as singular and plural quantifiers should be understood as corresponding to different types of quantification rather than different types of existence, first-order quantifiers and the quantifiers in a language understood in accordance with the Noncommittalist Assignment should be understood as corresponding to different types of quantification rather than different types of existence. Secondly, the notation might mislead one into thinking that what is distinctive of a semantics delivering the Noncommittalist Assignment is simply its treatment of the quantifiers, when in fact it is in the treatment of atomic formulas that much of the action takes place (see footnote 13). A better choice of notation would follow the lead of second-order languages, and use a distinctive type of *variable* as an indicator of semantic differences. The easiest way of doing this is by taking advantage of the distinctive variables already in play: 'n' and 'm'. Hereafter I shall refer to middle-of-the-alphabet variables as *arithmetical variables*, and use them as an indicator that the predicates whose argument places they fill are to be understood in accordance with a semantic theory delivering the Noncommittalist Assignment. I shall refer to quantifiers binding arithmetical variables as *arithmetical quantifiers*.

There is a temptation to worry that by wheeling in arithmetical quantifiers one would be undoing all the hard work that Quine did for us in 'On What There Is'. This is a temptation that ought to be resisted. Quine's aim was to find a way of getting clear about the ontological commitments of our theories. He proposed a method for satisfying this aim for the special case in which our theories are stated in first-order languages: a first-order theory carries commitment to Fs just in case Fs must be admitted among the values of the variables in order for the theory to be true. Non-first-order theories were to be set aside for the purposes of ontological investigation, but only on the grounds that a rigorous method for assessing their ontological commitments was lacking. In our case, however, the method is not lacking. A suitable criterion of ontological commitment was supplied in section 3.3. It yields the same results as Quine's criterion when it comes to first-order sentences,[23] but

[23] This relies on the assumption that the 'must' in Quine's criterion can be properly cashed out in terms of a-worlds: 'Fs must be admitted among the values of the variables in order for the sentence to be true' is read 'every a-world in which the sentence is true has a domain with objects that are Fs according to the a-world'. (A characterization of a-worlds is supplied in the appendix.) If the assumption is not granted, there may be certain cases in which Quine's criterion and the section 3.3 criterion come apart. Just how this happens depends on how the 'must' in Quine's criterion is cashed out. Suppose, first, that Quine's 'must' is cashed out in terms of metaphysical necessity ('Fs must be admitted among the values of the

extends Quine's criterion by delivering results for any language whose truth-conditions are specifiable in terms of w-formulas, including a language containing arithmetical quantifiers.

The proposed criterion yields the result that arithmetical quantification carries no commitment to numbers. And this is so in spite of the fact that a semantic theory delivering the Noncommittalist Assignment takes arithmetical variables to range over numbers. Overexposure to Quinean doctrine might lead one to think that this result is unacceptable on the grounds that there must be a straightforward connection between the range of one's variables and the commitments carried by sentences with quantifiers binding such variables. But to think this is to put the cart before the horse. Whether or not a sentence carries commitment to Fs depends on whether the world is required to contain Fs in order for the truth-conditions of the sentence to be satisfied. If, as in the case of first-order languages, there happens to be a correlation between the sentences whose truth-conditions require the world to contain Fs and the sentences that can only be true when Fs fall within the range of their variables, then it will be acceptable to assess ontological commitments by looking at the range of the variables. But there is no good reason for thinking that variable ranges are generally correlated to ontological commitment.

In the Parable, George urged Charles to distinguish between the objects that form part of the semantic machinery that is used to specify truth-conditions for a sentence, and the objects that are required to exist in order for the truth-conditions thereby specified to be satisfied. I would like to commend George's advice in the present context.

4.3 The marketplace and the philosophy room

When noncommittalists make arithmetical assertions in the marketplace they take themselves to incur no commitments to numbers. But when they get together in the philosophy room to discuss ontology, they might regard an assertion of 'there are numbers' as carrying commitment to numbers.

There are different devices one might use to indicate whether one intends one's discourse to be understood committally or noncommittally. Suppose, for example, that one prefaces a theory with the claim that one will always regard 'there are Fs' as carrying commitment to Fs. Then one makes clear that one's quantifiers are to be regimented as full-fledged first-order quantifiers, rather than as the noncommittalist's arithmetical quantifiers. Alternatively, one can suggest committalist readings for one's assertions by making it contextually salient that one is concerned with matters of ontology. If, for example, one

variables in order for the sentence to be true' is read 'when evaluated with respect to an arbitrary possible world, the sentence is true just in case there are Fs amongst the values of the variables.' Then it is a consequence of Quine's criterion that 'there is a husband' is not committed to there being someone who has a husband, even though part of what it is to be a husband is for there to be someone one is a husband of. This problem is averted on the section 3.3 criterion. Now suppose that Quine's 'must' is cashed out in terms of logical consequence ('Fs must be admitted among the values of the variables in order for the sentence to be true' is read 'every model of the sentence has a domain with objects that are Fs according to the model'). Then a sentence like 'there are numbers' is committed to numbers but not to abstract objects. This result will be averted on the section 3.3 criterion on the assumption that part of what it is to be a number is to be an abstract object. For further discussion of Quine's criterion see (Rayo, 2007).

is interested in getting across the committalist reading of 'there are numbers', one might say 'arithmetical Platonism is true', or 'the furniture of the universe includes numbers', or 'the world contains numbers', or 'there *really* are numbers'. For although each of these locutions might be paraphrased as 'there are numbers', they all make it contextually salient that one is concerned with matters of ontology.[24] (Throughout the paper I have relied essentially on such contextual cues to get my meaning across; when I say, for instance, 'satisfaction of s's truth-conditions requires the existence numbers', I am relying on the fact that you will see that I am interested in matters of ontology, and read my use of arithmetical vocabulary committally.)

Another way of making one's commitments plain is by following Quine's advice, and regimenting one's claims in a language specifically designed to clarify questions of ontological commitment. The easiest way to do so is to use a language containing both arithmetical and first-order variables. (See footnote 11.) Accordingly, when a noncommittalist who is also a nominalist says 'it is true that there are infinitely many primes, but there aren't really any numbers' one might offer the following regimentation:[25]

$$\exists n(Num_m(n, Prime(m)) \wedge \neg Finite(n)) \wedge \neg\exists x(\textsc{Number}(x))$$

and when she says—opaquely—'the number of numbers is zero', one might offer:

$$\exists n(Num_x(n, \textsc{Number}(x)) \wedge 0(n)).$$

A two-sorted language of this kind treats sentences involving mixed identities as ill-formed ('$\exists n\exists x(n = x)$', for instance), but allows hybrid statements such as the following:

[24]Suppose that in stating her overall theory of the world Susan asserts that 'the number of the planets is 8' is true. She adds that in order for a sentence to be true its truth-conditions must be satisfied. Susan then goes on to specify the truth-conditions of 'the number of the planets is 8' by way of PLANETS (NARROW SCOPE), and uses this to justify the further claim that 'the number of the planets is 8' has the same truth-conditions as 'there are eight planets'. All the while she has been using numerical quantifiers. Is this enough to conclude that Susan's overall theory of the world carries commitment to numbers? Not unless one begs the question against the noncommittalist. For if noncommittalism is true, nothing Susan has said so far commits her to numbers. (Keep in mind that if noncommittalism is true, the use of number-talk in 'the number of the planets is 8' is no more committal than the use of number-talk in specifying truth-conditions for 'the number of the planets is 8'.)

In order to determine whether Susan intends her assertions to carry commitment to numbers, one must ask 'Do you take yourself to be committed to numbers?', or 'When you engaged in number-talk in the course of describing the truth-conditions of 'the number of the planets is 8', did you mean to suggest that numbers really exist?', thereby making clear that one is interested in questions of ontology. It would be better still if Susan was prepared to regiment her theory using a language specifically designed to clarify questions of ontological commitment.

[25]Here 'NUMBER' is a non-arithmetical predicate. This means that although '$\exists x(\textsc{Number}(x))$' and '$\exists n(n = n)$' might both be read 'there are numbers', they have very different truth-conditions. Whereas the world is required to contain numbers in order for the truth-conditions of '$\exists x(\textsc{Number}(x))$' to be satisfied (and is required to contain *no* numbers in order for the truth-conditions of '$\neg\exists x(\textsc{Number}(x))$' to be satisfied), nothing is required of the world in order for the truth-conditions of '$\exists n(n = n)$' to be satisfied.

$$\forall n \forall m(Num_x(n, \textsc{TerrMoon}(x)) \wedge Num_r(m, EvenPrime(r)) \rightarrow n = m)$$

(*Read:* The number of terrestrial moons equals the number of even primes)

$$\forall n \forall m(Num_r(n, Prime(r)) \wedge Num_x(m, \textsc{PrimeMinister}(x)) \rightarrow m < n)$$

(*Read:* There are more primes than prime ministers)

Enriching the language with second-order quantifiers is straightforward, as is developing a two-sorted set-theoertic language with non-committing set-theoretic vocabulary alongside the usual first-order vocabulary. But it is important to be clear about the limits of the proposal. Developing a language of regimentation for noncommittalist discourse calls for two separate tasks. First, one must get clear about what the correctness-conditions of the relevant pieces of discourse are supposed to be, and find a systematic way of specifying them. Second, one must characterize a language whose sentences have truth-conditions corresponding to the correctness-conditions specified. The former of these tasks is by far the most difficult. Insofar as the proposal in this paper has been able to get off the ground, it is because we have succeeded in developing a clear grasp of the correctness-conditions that the noncommittalist wishes to associate with elementary arithmetical discourse. But consider a sentence like 'I am thinking of a number between 1 and 10', or 'Smith likes the number 7'. It is not immediately clear what the noncommittalist thinks is required of the world in order for the correctness-conditions of assertions of such sentences to be satisfied. Unless one gets a handle on this issue, it is hard to develop a suitable language of regimentation.

If one wished to develop the proposal further, a good place to start would be the case of propositional attitude attributions with embedded arithmetical sentences (such as 'John believes that there are infinitely many primes'). In order to deal with such sentences, one would have to tackle the difficult question of how fine-grained to make the space of propositions that are to serve as the contents of propositional attitudes—whether, for example, to countenance distinct propositions with identical truth-conditions.[26] A similar problem emerges when it comes to accounting for propositional attitude attributions in which the embedded sentences are logical truths, and my suspicion is that the noncommittalist would wish to treat the two cases analogously. A proper discussion of the matter is beyond the scope of this essay.

4.4 Natural Language

In section 2 I suggested a way of specifying noncommittalist correctness-conditions for arithmetical discourse. In the present section I have argued that noncommittalism is a *stable* view, and considered a formalism that might be used to regiment assertions with noncommittalist correctness-conditions. But one thing I have *not* done is argue for noncommittalism. In particular, I have not argued that marketplace arithmetical assertions are ontologically innocent. This is not because I think noncommittalism is false. It is

[26]For discussion, see chapters 13 and 14 of (Stalnaker, 1999).

because I think ascertaining the correctness-conditions of natural-language assertions is a task of daunting complexity, which is far beyond the reach of this paper.

My remarks about commitment in natural language will therefore be confined to one small point, which is largely borrowed from (Yablo, 2001). The willingness of ordinary speakers to make arithmetical assertions appears to be independent of any beliefs they may have concerning matters of ontology. (Whether or not a speaker is prepared to assert 'the number of the planets is 8', for example, tends to depend on whether she believes that there are eight planets, but not on whether she has views about mathematical Platonism.) If this is right, then noncommittalist accounts of arithmetical discourse may be better placed than their committalist rivals to explain the connection between the correctness-conditions of arithmetical assertions, on the one hand, and the ways in which a speaker's arithmetical assertions depend on what she believes, on the other. To my mind, this supplies a piece of (defeasible) evidence for noncommittalism.

4.5 Anything goes?

One might worry that the methods described in this paper could be used to eliminate ontological commitment from any theory whatsoever, and that the proposal is therefore suspect. The best way of seeing that the worry would be unfounded is by noting that the proposal takes a sentence to be committed to Fs just in case the world is required to contain Fs in order for the sentence's truth-conditions to be satisfied. So there is no way of altering the ontological commitments of a sentence without also altering its truth-conditions. A semantic theory that delivers unexpectedly meager ontological commitments must also deliver unexpected truth-conditions. The case of arithmetic is special in that the truth-conditions delivered by the Noncommittalist Assignment are attractive—more attractive, I would say, than the truth-conditions delivered by a standard semantic theory. But in most cases a non-standard assignment of truth-conditions would be unwelcome.

An example might be helpful. Here are four different w-formulas that might be used to specify truth-conditions for 'Susan runs':

- $[\text{Susan runs}]_w$
- $\exists x(x = \text{Susan} \wedge [x \text{ runs}]_w)$
- $\exists x(x = \text{Susan} \wedge [\dot{x} \text{ runs}]_w)$
- $\text{Susan runs} \wedge [\forall x(x = x)]_w$

The first w-formula delivers the desired truth-conditions for 'Susan runs', but also yields the result that 'Susan runs' carries commitment to runners, and to Susan. The second w-formula has the same primary condition as the first, and therefore delivers identical truth-conditions and identical ontological commitments. The third w-formula has an unsatisfiable primary condition (this is because Susan is not an ordered-pair; see appendix). The fourth w-formula has a trivial primary condition if Susan runs and an unsatisfiable primary condition if she doesn't. So it either delivers the unwelcome result that 'Susan runs' has trivial truth-conditions or the unwelcome result that 'Susan runs' has impossible truth-conditions.

The lesson is that there are no shortcuts. If one wants non-standard ontological commitments for 'Susan runs' one must pay the price of non-standard truth-conditions. And in this case the nonstandard truth-conditions on offer are decidedly unattractive.[27] The proposal only really comes into its own when it comes to theories that are traditionally understood as involving reference to 'parasitic objects' (objects whose existence and all of whose properties are taken to supervene on the way the rest of the world is), and when one has a clear grasp of how the relevant supervenience would work.

5 The Prize

The purpose of this section is to argue that the Noncommittalist Assignment puts one in a position to provide an attractive explanation of how it is that arithmetical sentences might be rendered meaningful and of how it is that arithmetical truths might come to be known *a priori*. My proposal will be based on the idea that arithmetical sentences can be rendered meaningful by setting forth a suitable linguistic stipulation, and that one can come to know *a priori* that the stipulation is successful.

5.1 Stipulation

Let me begin by explaining why I think stipulational stories run into difficulties when one thinks of the quantifiers occurring in the standard arithmetical axioms as first-order quantifiers, rather than arithmetical quantifiers. Compare the following two cases:[28]

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[27]Suppose one is a mereological nihilist, and believes that only mereological atoms exist. One also believes that a person would have to be mereological complex, and therefore that there are no persons. One believes, however, that some atoms might be arranged 'personishly'—arranged just as the atomic parts of a person would be arranged—even if there are no persons. How about specifying truth-conditions for 'Susan runs' by way of the following?

$$[\exists X(\text{ArrangedSusanishly}(X) \wedge \text{ArrangedRunningly}(X))]_w$$

or, equivalently,

$$\exists X([\text{ArrangedSusanishly}(\dot{X}) \wedge \text{ArrangedRunningly}(\dot{X})]_w)$$

By doing so one would get the result that 'Susan runs' is committed to things arranged Susanishly and to things arranged runningly, but to neither Susan nor runners. The problem is that one would also get non-standard truth-conditions, since what the truth of 'Susan runs' will require of the world is that it be such that some things arranged Susanishly also be arranged runningly, whether or not it contains Susan, or runners. So the philosophical action comes from postulating devious truth-conditions, not from specifying these truth-conditions by way of w-formulas. (The non-standard truth-conditions could easily be specified with no appeal to w-formulas—by offering a second-order paraphrase, for instance.) For related discussion, see (Williams, ript).

[28]Some clarifications: (1) I take the standard arithmetical axioms to consist of Two-Sorted Fregean Arithmetic (see footnote 7). (2) For a linguistic stipulation to be *successful* is for it to have the effect of

Toy Stipulation

The new noun 'ilamus' can be rendered meaningful by stipulating that it is to be used as a syntactical abbreviation for 'individual with a large mustache'.

One can know *a priori* that such a stipulation will be successful. So one can know *a priori* that 'every ilamus has a large mustache' will be true as a result of the stipulation.

Arithmetical Stipulation

The standard arithmetical vocabulary can be rendered meaningful by stipulating that it is to be used in such a way that the standard arithmetical axioms turn out to be true.

One can know *a priori* that such a stipulation will be successful. So one can know *a priori* that the standard arithmetical axioms will be true as a result of the stipulation.

When arithmetical quantifiers are thought of as first-order quantifiers, there are two crucial sources of disanalogy between the toy stipulation and its arithmetical counterpart:

1. The toy stipulation can succeed regardless of how many mustached individuals there are and regardless of whether or not the mustaches in question are large. So there is no reason to doubt that one could know *a priori* that the stipulation succeeds, and therefore no reason to doubt that one could come to know *a priori* that 'every ilamus has a large mustache' is true as a result of the stipulation.

   In contrast, the arithmetical stipulation can only succeed if the world cooperates. More specifically, when the quantifiers are thought of as first-order quantifiers, rather than arithmetical quantifiers, the world is required to contain infinitely many individuals in order for the stipulation to succeed.[29] On the face of it, this means that

rendering the target vocabulary meaningful in such a way that the constraints imposed by the stipulation are satisfied. In particular, the stipulation that expressions $\xi_1, \ldots, \xi_n$ are to be used in such a way that sentences $\phi_1(\xi_1, \ldots, \xi_n), \ldots, \phi_m(\xi_1, \ldots, \xi_n)$ are true is successful just in case it has the effect of rendering $\xi_1, \ldots, \xi_n$ meaningful in such a way that $\phi_1(\xi_1, \ldots, \xi_n), \ldots, \phi_m(\xi_1, \ldots, \xi_n)$ all turn out to be true. (3) Stipulations don't work by magic: in order for a stipulation to have the effect of rendering a piece of vocabulary meaningful it must be taken on board by the relevant linguistic community in the right sort of way. This means that a stipulation that would have been successful if set forth within the right sort of linguistic community can nonetheless be unsuccessful if set forth within a linguistic community that fails to take it on board, or uses it to generate a practice that yields meanings whereby the sentences involved in the stipulation do not turn out to be true. Here and throughout I make the simplifying assumption that such deviant scenarios do not arise: a stipulation that would have been successful if set forth within the right sort of linguistic community will, in fact, be successful. The assumption could be avoided by qualifying claims of success in the main text with the presupposition that the relevant linguistic community is of the right sort, and qualifying ascriptions of *a priori* knowledge in the main text with the presupposition that the subject has the right sort of information about the workings of the linguistic community.

[29]In general, one can say that the requirement imposed on the world by the success of a stipulation to the effect that expressions $\xi_1, \ldots, \xi_n$ are to be used in such a way that sentences $\phi_1(\xi_1, \ldots, \xi_n), \ldots, \phi_m(\xi_1, \ldots, \xi_n)$ are true is the requirement imposed on the world by the truth of $\exists \alpha_1, \ldots, \alpha_n(\phi_1(\alpha_1, \ldots, \alpha_n) \wedge \ldots \wedge \phi_m(\alpha_1, \ldots, \alpha_n))$ (where the α_i are variables of the appropriate type).

one can only know that the stipulation is successful if one has an antecedent warrant for the claim that the world is infinite.[30] And the arithmetical stipulation supplies no reason for thinking that such a warrant could be acquired *a priori*. So it gives us no reason for thinking that one could come to know *a priori* that the standard arithmetical axioms are true as a result of the stiuplation.

2. The question of what truth-conditions are to be associated with an arbitrary sentence involving 'ilamus' is immediately settled by treating 'ilamus' as a syntactic abbreviation for 'individual with a large mustache'. The truth-conditions of '. . . ilamus . . . ' are simply the truth-conditions of '. . . individual with a large mustache . . . '.

   In contrast, it is not obvious that the question of what truth-conditions are to be associated with arbitrary arithmetical sentence is settled by taking the standard arithmetical axioms to be true. This is because the standard arithmetical axioms will be true no matter what the arithmetical terms refer to (provided the objects in question have the right structure, and provided the rest of one's arithmetical vocabulary is interpreted accordingly). But—against the background of a standard semantics—different choices of referents for the arithmetical terms result in different truth-conditions for arithmetical sentences. If, for example, '0' is taken to refer to the number zero, then the world is required to contain the number zero in order for '$\exists n(0 = n)$' to be true. But if '0' is taken to refer to the empty set, then what is required of the world is that it contain the empty set. (One way of addressing this problem is by giving up a standard semantics in favor of a supervaluationist semantics.[31])

In bringing out these disanalogies, I assumed that quantifiers are to be thought of as first-order quantifiers. But things are quite different if one thinks of them as arithmetical quantifiers instead (see section 4.2). With respect to the first disanalogy, the key observation is that when arithmetical quantifiers are in place *nothing* is required of the world in order for the standard arithmetical axioms to be true. So there is no reason to doubt that one could come to know *a priori* that the standard arithmetical axioms are true as a result of the stipulation. And, of course, once one has a story in place about how the axioms might be known *a priori*, it is straightforward to tell a story about how their deductive consequences might be known *a priori*, since logical deduction arguably preserves *a priori* knowledge.

With respect to the second disanalogy, the key observation is that even though the use of arithmetical quantifiers does not alter the fact that the standard arithmetical axioms can be true even if arithmetical terms are assigned non-standard referents, it does have the consequence that identical assignments of truth-conditions will result from any assignment of semantic values to the arithmetical terms that is consistent with the success of the arithmetical stipulation.[32]

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[30] Neo-Fregeans have questioned the need for an antecedent warrant. For discussion of this point, see (Rayo, 2003). For a defense of Neo-Fregeanism see (Wright, 1983) and (Hale and Wright, 2001).

[31] See (Hodes, 1990a), (Hodes, 1990b), (McGee, 1993) and (McGee, 1997).

[32] Actually, one needs a slight modification of Arithmetical Stiuplation:

The easiest way to see this is by noting that the following two w-formulas have the same primary condition:

> PLANETS (NARROW SCOPE)
>
> The number of the xs such that $[\dot{x}$ is a planet$]_w = 8$
>
> PLANETS (NARROW SCOPE – SET-THEORETIC VARIANT)
>
> The smallest ordinal in one-one correspondence with the set of xs such that $[\dot{x}$ is a planet$]_w =$ the ninth ordinal.

So one assigns the desired truth-conditions to 'the number of the planets is eight' even if the semantic values of arithmetical terms are taken to be sets rather than numbers.

5.2 Further Issues

I hope to have shown that by taking the quantifiers involved in arithmetical sentences to be arithmetical quantifiers one can eliminate some crucial disanalogies between the arithmetical stipulation and the toy stipulation. But it doesn't follow that the two stipulations are fully analogous. One difference is that whereas the toy stipulation proceeds by introducing a syntactic abbreviation, the arithmetical stipulation proceeds by insisting that certain sentences should turn out to be true (or that nothing be required of the world in order for their truth-conditions to be satisfied). So one shouldn't expect to gain a full understanding of the workings of the arithmetical stipulation by comparing it with its toy cousin.

There is a second difference that is far more pressing in the present context. Whereas it is immediately obvious that the toy stipulation can succeed, it is not immediately obvious that the arithmetical stipulation can succeed. That nothing is required of the world in order for the standard arithmetical axioms to be true is the sort of observation that calls for reflection. And unless it is plausible to suppose that such reflection is within our reach, the claim that one can know that the standard axioms are rendered true by the arithmetical stipulation may be undermined. To drive the point home, consider how things would play

---

> ARITHMETICAL STIPULATION (REVISED VERSION)
>
> The standard arithmetical vocabulary can be rendered meaningful by stipulating that it is to be used in such a way that nothing is required of the word in order for the truth-conditions of the standard arithmetical axioms to be satisfied.
>
> One can know *a priori* that such a stipulation will be successful. So one can know *a priori* that the standard arithmetical axioms will be true as a result of the stipulation.

The modification is needed to avoid an awkward technicality. When the Noncommittalist Assignment is in place, one gets the result that if the world contains only n objects, the arithmetical vocabulary might be rendered meaningful in such a way that the standard arithmetical axioms turn out to be true and 'the number of the planets $= n$' turns out to be such that it would have been true had the world contained $n + 1$ planets. So the success of the original arithmetical stipulation does not immediately guarantee that arithmetical sentences get the right truth-conditions, even if it guarantees that they get the right truth-values.

out if one tried to tell a story analogous to ARITHMETICAL STIPULATION for the set-theoretic system described in Quine's *New Foundations*:

> SET-THEORETIC STIPULATION (NF VERSION)
>
> The two place relation '∈' can be rendered meaningful by stipulating that it is to be used in such a way that nothing is required of the world in order for the truth-conditions of the axioms of Quine's NF to be satisfied.
>
> One can know *a priori* that such a stipulation will be successful. So one can know *a priori* that the axioms of NF will be true as a result of the stipulation.

It can only be the case that nothing is required of the world in order for the axioms of NF to be true if NF is consistent. But the consistency of NF remains an open question.[33] So it is by no means clear that knowledge that the stipulation can succeed is within our reach.

In general, acquiring a warrant for the consistency of a mathematical theory is a delicate matter. One can sometimes prove the consistency of one formal system in another. But it is a consequence of Gödel's Second Theorem that (when the systems in question are sufficiently strong) the system in which the proof is carried out cannot be a subsystem of the system the proof is about. There is therefore reason to expect that one's warrant for the consistency of a formal system will turn on more than just consistency proofs: it might turn, for example, on whether one has a good feel for the sorts of things that can be proved in the system, or on whether one has a good feel for the sorts of things that can be proved in formal systems within which one has been able to produce a consistency proof. It also means that one should expect one's warrant for the consistency of a formal system to be defeasible (whether or not it is *a priori*).

One's warrant for the claim that nothing is required of the world in order for the axioms of a theory of pure mathematics to be true is no better than one's warrant for the claim that the axioms in question are consistent. (Similarly, one's warrant for the claim that nothing is required of the world in order for the axioms of a theory of applied mathematics to be true is no better than one's warrant for the claim that the axioms in question are conservative over one's non-mathematical theories.[34]) So all the difficulties that are associated with establishing that consistency of a mathematical theory (or its conservativeness) will be inherited by stipulational stories of the sort I have been defending.

This is as it should be. One wouldn't want one's account of mathematical knowledge to yield the result that mathematical knowledge is easier to come by than the practice of mathematicians would suggest. But it does mean that the picture of mathematical knowledge that emerges from a stipulational story is a messy one. One's warrant for the success of a mathematical stipulation will typically be defeasible. It will sometimes turn on informal considerations, such as whether one has a good feel for the sorts of things that

[33] There is, on the other hand, a consistency proof for NFU (New Foundations with Urelements). See (Jensen, 1969).

[34] A set of axioms $\mathcal{A}$ is conservative over a theory $\mathcal{T}$ relative to mathematical vocabulary $\mathcal{V}$ not occurring in $\mathcal{T}$ just in case: if ψ is a logical consequence of $\mathcal{A} \cup \mathcal{T}$ and $\mathcal{V}$ does not occur in ψ, then ψ is a logical consequence of $\mathcal{T}$ alone.

can be proved on the basis of the theory in question. And it will sometimes depend on one's warrant for the success of further mathematical stipulations.

One final point. Whatever its plausibility as an explanation of how the standard arithmetical axioms *might* be rendered meaningful in such a way that their truth is knowable *a priori*, it should be clear that the arithmetical stipulation is not very plausible as an explanation of how the axioms were *actually* rendered meaningful or how it is that we *actually* acquire *a priori* knowledge of their truth. For it is not very plausible to suppose that anything like the arithmetical stipulation was actually carried out. I do not claim to have given an explanation of the latter kind here.

6 Closing Remarks

When philosophers argue about the ontological commitments carried by mathematical sentences it is easy to feel a Carnapian tug. It is natural to think that the use of a referential apparatus in mathematics is accidental to the linguistic framework we have chosen to work with as a matter of convenience, and that too much interest in matters of ontological commitment demonstrates a lack of understanding of this important fact.

I am in many ways sympathetic towards this line of thought. It seems to me that describing the world in terms of objects is largely a matter of convenience, and that if we found a new way of talking that allowed us to make suitably fine-grained distinctions between ways the world might be without postulating objects, that would do just as well.[35] But it is important to be clear that by adopting a different linguistic framework one won't solve any substantive philosophical problems. The interesting questions will all reemerge in the alternate framework, under a different description.

Suppose one is interested in the project of explaining how it is that the standard arithmetical axioms might come to be known. When a standard semantics is in place, non-trivial requirements must be satisfied by the world in order for the standard arithmetical axioms to be true. So one's account of arithmetical knowledge must somehow explain how it is that one is able to acquire a warrant for the belief that these requirements are satisfied. In order for the problem to go away, it is not sufficient to set forth an alternate semantics whereby the arithmetical axioms turn out to be ontologically innocent. For if the alternate truth-conditions are as non-trivial as their ontologically loaded counterparts, one will be left with the problem of explaining how it is that one is able to acquire a warrant for the belief that the requirements corresponding to the alternate truth-conditions are met. So the original problem has not been solved; it has simply been relocated. In its new guise, it no longer calls for a description in terms of ontological commitment. But it is no less of a problem.

On the proposal I have considered in this paper, nothing is required of the world in order for the standard arithmetical axioms to be true. Whether or not you agree with the proposal, I hope you agree that it does more than simply redescribe the problem of explaining how it is that the standard arithmetical axioms might come to be known.

[35] For a refreshing discussion of these issues, see (Burgess, 2005).

One final point. Throughout the paper I have relied on an unrefined understanding of Platonism, according to which the view that there are no numbers is *intelligible*, whether or not it is also taken to be false as a matter of metaphysical necessity. ("When God made the world, she began by creating spatiotemporal objects. She was tempted to go on to create abstract objects. But she changed her mind at the last minute. So numbers were never created.") There is, however, a subtler version of Platonism, according to which for the number of the planets to be 8 *just is* for there to be eight planets.[36] According to such a Platonist, the view that there are no numbers is not just false, but unintelligible. ("Suppose there are no numbers. For the number of Fs to be 0 *just is* for there to be no Fs. So the number 0 must exist after all!") Disagreement between a noncommittalist and a subtle Platonist is more elusive than one might think. Notice, in particular, that they both agree that all it takes for the truth-conditions of 'the number of the planets is 8' to be satisfied is that there be eight planets. One might reply that the views are nonetheless quite different. For whereas the subtle Platonist gets this result by relying on a standard semantics and setting forth a controversial identity statement—for the number of the planets to be 8 *just is* for there to be eight planets—the noncommittalist gets the result by relying on a non-standard semantics. But the extent to which there is a deep disagreement here, rather than a mere difference in bookkeeping, is not entirely clear to me.

Appendix

There are two different methods that might be used to characterize a possibility. The *direct* method proceeds by saying something about how things are according to the possibility. For example, one might characterize the possibility that Charles be a philosopher by saying that, according to that possibility, Charles is a philosopher. The *indirect* method proceeds by saying something about how the possibility might be *represented*. For example, a Lewisian about possible worlds might characterize the possibility that Charles be a philosopher by saying that it is represented by Lewisian worlds in which Charles's counterpart is a philosopher.[37] The purpose of this appendix is to describe a language in which possibilities can be characterized using either of these methods. (For a more detailed discussion, see my 'An Actualist's Guide to Quantifying In'.)

Although the ultimate aim is for the proposal to be suitable for modal actualists, it will be useful to start by seeing things from the perspective of a Lewisian. Compare the following two open formulas (where w is a Lewisian world):

$$[\text{Philosopher}(x)]_w \qquad\qquad [\text{Philosopher}(\dot{x})]_w$$

[36]Each of the following can be interpreted as defending a version of subtle Platonism: (Frege, 1884), (Parsons, 1983), (Wright, 1983) and (Stalnaker, 1996).

[37]Lewis argued that counterparts—and, accordingly, the possibilities represented by Lewisian worlds—are context-relative. Here I ignore such context relativity for ease of exposition.

The first formula is familiar, and corresponds to the direct method. It is satisfied by an individual z just in case w represents z as being a philosopher. Suppose, for example, that w is a Lewisian world in which Charles's counterpart is a philosopher. Then, even though w does not contain Charles himself, w represents Charles as being as a philosopher. And this is enough for '[Philosopher(x)]$_w$' to be satisfied by Charles himself.

The second formula is unfamiliar, and corresponds to the indirect method. It is satisfied by an individual z just in case z represents a philosopher in w. Since, according to the Lewisian, z represents a philosopher in w by being a part of w and being a philosopher, this means that '[Philosopher($\dot{x}$)]$_w$' is satisfied by z just in case z is a part of w and z is a philosopher. Thus, '[Philosopher($\dot{x}$)]$_w$' will not be satisfied by Charles himself when Charles does not inhabit w. But it will be satisfied by Charles's counterpart in w if Charles's counterpart in w is a philosopher.

How are dotted formulas to be understood by an actualist? The following is a rough outline of the proposal. (Full details are supplied below.)

> Just as the Lewisian uses Lewisian worlds to represent possibilities, we will use a-worlds (or actualist-worlds) to represent possibilities. An a-world is a pair $\langle D, I\rangle$, where D is a domain of (actually existing) objects and I is an interpretation function. Each object in D is either an ordered pair of the form $\langle x, \text{`actual'}\rangle$ or an ordered pair of the form $\langle x, \text{`nonactual'}\rangle$. (Pairs of the form $\langle x, \text{`actual'}\rangle$ will be used to represent their first components, and pairs of the form $\langle x, \text{`nonactual'}\rangle$ will be used to represent merely possible objects.[38]) I assigns extensions to atomic predicates, in the usual way. Lewisian worlds represent by analogy: a Lewisian world represents the possibility that Charles be a philosopher by containing a counterpart of Charles who is a philosopher. A-worlds, on the other hand, represent by ensuring the satisfaction of suitable formulas: an a-world represents the possibility that Charles be a philosopher by having the extension of 'Philosopher(...)' contain a representative of Charles (specifically: $\langle$Charles, 'actual'$\rangle$).

So what about the open formulas we had considered earlier?

$$[\text{Philosopher}(x)]_w \qquad\qquad [\text{Philosopher}(\dot{x})]_w$$

[38]What if there is a possibility wherein there are too many merely possible objects to be represented by ordered pairs of the form $\langle x, \text{`nonactual'}\rangle$? That there is no such possibility is guaranteed by the necessity the standard axioms of set-theory and the Urelement Set Axiom ('the non-sets form a set'). *Proof:* Let p be an arbitrary possibility. By the necessity of the Urelement Set Axiom, if p obtained there would be a set α of all non-sets. By the necessity of the standard axioms of set-theory, there would also be a set β of the same cardinality as α consisting solely of pure sets. But only pure sets that do exist could exist. So α actually exists. If f is an arbitrary one-one correspondence between α and β and x is an object that would exist if p obtained, let $x^f = f(x)$ for x a non-set and $x^f = \{y^f : y \in x\}$ for x a set. A merely possible object x of p can then be represented by $\langle x^f, \text{`nonactual'}\rangle$.

As before, we shall say that the first formula is satisfied by an individual z just in case w represents z as being a philosopher. But now w is taken to be an a-world, and representation is understood accordingly. Suppose, for example, that w is an a-world in which $\langle$Charles, 'actual'$\rangle$ is in the extension of 'Philosopher(...)'. Then, even though the domain of w does not contain Charles himself, w represents Charles as being as a philosopher. So '$[\text{Philosopher}(x)]_w$' is satisfied by Charles himself.

As before, we shall say that the second formula is satisfied by an individual z just in case z represents a philosopher in w. But now w is taken to be an a-world, and representation is understood accordingly. Since z represents a philosopher in an a-world w by being in the w-extension of 'Philosopher(...)', '$[\text{Philosopher}(\dot{x})]_w$' will not be satisfied by Charles himself (who, by virtue of not being an ordered pair, is guaranteed not to be in the domain of w). But it will be satisfied by Charles's representative in w ($\langle$Charles, 'actual'$\rangle$) if the representative is in the w-extension of 'Philosopher(...)'.

The following example will illustrate the proposal further.[39] Although I don't have a sister, I might have had one. And had I had a sister, I would have been in a position to consider two different possibilities concerning her: a possibility whereby she becomes a philosopher and a possibility whereby she becomes a cellist. These are possibilities I am not in a position to fully describe from the perspective of the actual world. I can, of course, describe a possibility whereby I have a sister who becomes a philosopher and a possibility whereby I have a sister who becomes a cellist. But this misses out on a crucial feature of the possibilities under consideration: namely, that they are meant to concern one and the same individual. Had the individual in question existed, I would have been in a position to fill in the missing information by saying that the possibilities both concern that individual. But the individual in question is the sister I would have had, and according to modal actualism there is no such individual.

How might a-worlds be used to represent the possibilities in question? Let w^p be an a-world with the following features:

- Both $\langle$Agustín, 'actual'$\rangle$ and $\langle$Socrates, 'nonactual'$\rangle$ are in the domain of w^p;
- $\langle$Socrates, 'nonactual'$\rangle$ is in the w^p-extension of 'Philosopher(...)';
- $\langle\langle$Agustín, 'actual'$\rangle$, $\langle$Socrates, 'nonactual'$\rangle\rangle$ is in the w^p-extension of 'Sister(...,...)'.

And let w^c be an a-world which is just like w^p except that $\langle$Socrates, 'nonactual'$\rangle$ is in the extension of 'Cellist(...)' rather than in the extension of 'Philosopher(...)'. It follows that each of the following is true:

$[\exists y(\text{Sister}(\text{Agustín}, y) \wedge \text{Philosopher}(y))]_{w^p}$
(according to w^p, Agustín has a sister who is a philosopher)

$[\exists y(\text{Sister}(\text{Agustín}, y) \wedge \text{Cellist}(y))]_{w^c}$
(according to w^c, Agustín has a sister who is a cellist)

[39] It is based on the illuminating discussion in Stalnaker's "On What there isn't (but might have been)".

So w^p and w^c represent, respectively, the possibility that I have a sister who is a philosopher, and the possibility that I have a sister who is a cellist. But the fact that the same individual—in this case, the arbitrarily chosen ⟨Socrates, 'nonactual'⟩—represents my sister in both w^p and w^c is used to capture the fact that the possibilities represented by w^p and w^c concern one and the same merely possible individual.

It is thanks of this cross-world identification that there can be an *a*-world w^* that makes the following true:

$$[\exists y(\text{Sister}(\text{Agustín}, y) \ \wedge \ [\text{Philosopher}(y)]_{w^p} \ \wedge [\text{Cellist}(y)]_{w^c})]_{w^*}$$

(according to w^*, I have a sister who is a philosopher according to w^p and a cellist according to w^c)

And, as one might expect, this sentence has the following as a consequence:

$$\exists w([\exists y(\text{Sister}(\text{Agustín}, y) \ \wedge \ \exists u \exists v([\text{Philosopher}(y)]_u \ \wedge [\text{Cellist}(y)]_v)]_w))$$

(I might have had a sister who might have been a philosopher and might have been a cellist)

An advantage of the dot-notation is that it allows one to express the cross-world identifications without bringing in w^*. The fact that w^p and w^c represent possibilities concerning one and the same individual, for example, can be captured by the following formula:

$$\exists y([\text{Philosopher}(\dot{y})]_{w^p} \ \wedge [\text{Cellist}(\dot{y})]_{w^c})$$

(there is an individual that represents a philosopher in w^p and a cellist in w^c).

The real payoff of the dot notation, however, is that it yields the truth of every instance of the following schema:

$$[\exists y(\phi(y))]_w \leftrightarrow \exists y([\exists z(z = \dot{y})]_w \wedge [\phi(\dot{y})]_w)$$

Without the dots, the schema is a version of the Barcan biconditional, which is highly controversial. But with the dots in place what the schema says is that there are ϕs according to w just in case there is something that represents an individual that exists and is a ϕ according to w. Such a schema is enormously useful. Among other things, it enables one to give actualist analogues of an intensional semantics for natural language in the style of (Lewis, 1970).

In the remainder of this appendix I give a formal semantics for a language $\mathfrak{L}^w$ containing $[\ldots]_w$ and the dot-notation, and characterize the notion of triviality. (The characterization of $\mathfrak{L}^w$ I supply here allows for empty names, and is therefore more general than required by the main text.) $\mathfrak{L}^w$ consists of the following symbols:

1. for $n > 0$, the n-place (non-modal) predicate letters: $\ulcorner \text{F}^n_1 \urcorner, \ulcorner \text{F}^n_2 \urcorner, \ldots$ (each with an intended interpretation);

2. for $n > 0$, the one-place modal predicate letters: $\ulcorner B_1 \urcorner, \ulcorner B_2 \urcorner, \ldots$ (each with an intended interpretation);

3. the identity symbol '=';

4. for $n > 0$, the individual non-empty constant-letter $\ulcorner c_n \urcorner$ (each with an intended referent);

5. for $n > 0$, the individual empty constant-letter $\ulcorner e_n \urcorner$;

6. the individual constant 'α'

7. the dot '·';

8. the monadic sentential operator '[...]';

9. the monadic sentential operator '¬';

10. the dyadic sentential operators '∧',

11. the quantifier-symbol '∃';

12. the modal variables: 'w', 'v' 'u' with or without numerical subscripts;

13. the non-modal variables: 'x', 'y', 'z' with or without numerical subscripts;

14. the auxiliaries '(' and ')'.

Undotted terms and formulas are defined as follows:

1. any modal variable is an undotted modal term;

2. 'α' is an undotted modal term;

3. any non-modal variable or individual constant-letter is an undotted non-modal term;

4. if $\tau_1, \ldots, \tau_n$ are undotted non-modal terms, then $\ulcorner F^n_i(\tau_1, \ldots, \tau_n) \urcorner$ is an undotted formula;

5. if τ_1 and τ_2 are either both undotted non-modal terms or both undotted modal terms, then $\ulcorner \tau_1 = \tau_2 \urcorner$ is an undotted formula;

6. if w is an undotted modal term, then $\ulcorner B_i(w) \urcorner$ is an undotted formula;

7. if ϕ is an undotted formula and w is an undotted modal term, then $\ulcorner [\phi]_w \urcorner$ is an undotted formula;

8. if v is an undotted (modal or non-modal) variable and ϕ is an undotted formula, then $\ulcorner \exists v(\phi) \urcorner$ is an undotted formula;

9. if ϕ and 'ψ' are undotted formulas, then $\ulcorner \neg\phi \urcorner$, $\ulcorner (\phi \wedge \psi) \urcorner$, $\ulcorner (\phi \vee \psi) \urcorner$ and $\ulcorner (\phi \supset \psi) \urcorner$ are undotted formulas;

10. nothing else is an undotted term or formula.

A *non-modal term* is either an undotted non-modal term or the result of dotting a non-modal variable; a *modal term* is an undotted modal term; a *formula* is the result of dotting any free or externally bounded occurrences of non-modal variables in an undotted formula.[40]

Next, we characterize the notion of an a-world and of a variable assignment. An a-world is a pair $\langle D, I \rangle$ with the following features:

1. D is a set of individuals in the range of the non-modal variables, each of which is either of the form $\langle x, \text{'actual'} \rangle$ or of the form $\langle x, \text{'nonactual'} \rangle$.[41]

2. I is a function assigning a subset of D to each 1-place predicate-letter, and a subset of D^n to each n-place predicate letter (for $n < 1$). In addition, if e is an empty name, I may or may not assign a referent to e (and if a referent is assigned, it may or may not be in D).

The *actualized* a-world $\langle D_\alpha, I_\alpha \rangle$ will be singled out for special attention. D_α is the set of pairs $\langle z, \text{'actual'} \rangle$ for z an individual in the range of the non-modal variables; and $I_\alpha(\text{'F}^n_j\text{'})$ is the set of sequences $\langle \langle z_1, \text{'actual'} \rangle, \ldots \langle z_n, \text{'actual'} \rangle \rangle$ such tha $z_1 \ldots z_n$ are in the range of the non-modal variables and satisfy F.

A variable assignment is a function σ with the following features:

1. σ assigns an a-world to each modal variable.

2. σ assigns an individual to each non-modal variable.

This puts us in a position to characterize notions of quasi-denotation and quasi-satisfaction. (Denotation and satisfaction proper will be characterized later). For v a non-modal variable, σ a variable assignment, ϕ a formula and w an a-world, we characterize the quasi-denotation function $\delta_{\sigma,w}(v)$ and the quasi-satisfaction predicate $Sat(\phi, \sigma)$. In addition, we characterize an auxiliary (a-world-relative) quasi-satisfaction predicate $Sat(\phi, \sigma, w)$. We proceed axiomatically, by way of the following clauses:

- If v is a (modal or non-modal) variable, $\delta_{\sigma,w}(v)$ is $\sigma(v)$;

[40] An occurrence of a non-modal variable in an undotted formula is *free* iff it is not bound by a quantifier (and does not occur as part of the quantifier phrase $\ulcorner \exists x_i \urcorner$); an occurrence of a non-modal variable in an undotted formula is *externally bounded* iff it is bound by a quantifier which is not within the scope of '[...]'.

[41] I assume that D is a *set* for the sake of simplicity. The assumption can be avoided by characterizing the notion of an a-world in second-order terms. This can be done by employing the technique in (Rayo and Uzquiano, 1999).

- If v is a non-modal variable, w is an a-world and $\sigma(v)$ is an ordered pair of the form $\langle z, \text{'actual'}\rangle$, then $\delta_{\sigma,w}(\ulcorner \dot{v} \urcorner)$ is the first member of $\sigma(v)$; otherwise $\delta_{\sigma,w}(\ulcorner \dot{v} \urcorner)$ is undefined;

- if c is a non-empty constant-letter and w is an a-world, $\delta_{\sigma,w}(c)$ is the intended referent of c.

- if e is an empty constant-letter and w is an a-world, $\delta_{\sigma,w}(e)$ is the w-referent of e if there is one, and is otherwise undefined;

- $\delta_{\sigma,w}(\text{'}\alpha\text{'})$ is $\langle D_\alpha, I_\alpha\rangle$;

- if τ_1 and τ_2 are terms (both of them modal or both of them non-modal) and neither of them is an empty constant-letter or lacks a $\delta_{\sigma,w}$-assignment, then $Sat(\ulcorner \tau_1 =, \tau_2)\urcorner, \sigma)$ if and only if $\delta_{\sigma,w}(\tau_1) = \delta_{\sigma,w}(\tau_2)$ for arbitrary w;

- if τ_1 and τ_2 are non-modal terms at least one of which is an empty constant-letter or lacks a $\delta_{\sigma,w}$-assignment, then not-$Sat(\ulcorner \tau_1 =, \tau_2)\urcorner, \sigma)$;

- if $\tau_1, \ldots \tau_n$ are non-modal terms none of which is an empty constant-letter or lacks a $\delta_{\sigma,w}$-assignment, then $Sat(\ulcorner \mathrm{F}^n_i(\tau_1, \ldots, \tau_n)\urcorner, \sigma)$ if and only if $F^n_i(\delta_{\sigma,w}(\tau_1), \ldots, \delta_{\sigma,w}(\tau_n))$, where w is arbitrary and $\ulcorner \mathrm{F}^n_i \urcorner$ is intended to express F^n_i-ness;

- if $\tau_1, \ldots \tau_n$ are non-modal terms at least one of which is an empty constant-letter or lacks a $\delta_{\sigma,w}$-assignment, then not-$Sat(\ulcorner \mathrm{F}^n_i(\tau_1, \ldots, \tau_n)\urcorner, \sigma)$;

- if v is a modal variable, $Sat(\ulcorner \mathrm{B}_i(v)\urcorner, \sigma)$ if and only if $B_i(\delta_{\sigma,w}(v))$ for arbitrary w, where B_i is intended to express B_i-ness;

- if v is a non-modal variable, $Sat(\ulcorner \exists v(\phi)\urcorner, \sigma)$ if and only if there is an individual z in the range of the non-modal variables such that $Sat(\phi, \sigma^{v/z})$, where $\sigma^{v/z}$ is just like σ except that it assigns z to v;

- if v is a modal variable, $Sat(\ulcorner \exists v(\phi)\urcorner, \sigma)$ if and only if there is an a-world z such that $Sat(\phi, \sigma^{v/z})$, where $\sigma^{v/z}$ is just like σ except that it assigns z to v;

- $Sat(\ulcorner \neg\phi \urcorner, \sigma)$ if and only if it is not the case that $Sat(\phi, \sigma)$;

- $Sat(\ulcorner \phi \wedge \psi \urcorner, \sigma)$ if and only if $Sat(\phi, \sigma)$ and $Sat(\psi, \sigma)$;

- $Sat(\ulcorner [\phi]_w \urcorner, \sigma)$ if and only if $Sat(\phi, \sigma', \sigma(w))$, where $\sigma'(x) = \sigma(x)$ for x a modal variable and $\sigma'(x) = \langle \sigma(x), \text{'actual'}\rangle$ for x a non-modal variable;

- if τ_1 and τ_2 are non-modal terms neither of which lacks a $\delta_{\sigma,w}$-assignment, then $Sat(\ulcorner \tau_1 =, \tau_2)\urcorner, \sigma, w)$ if and only if $\delta_{\sigma,w}(\tau_1)$ is in the domain of w and is identical to $\delta_{\sigma,w}(\tau_2)$;

- if τ_1 and τ_2 are non-modal terms at least one of which lacks a $\delta_{\sigma,w}$-assignment, then not-$Sat(\ulcorner \tau_1 =, \tau_2) \urcorner, \sigma, w)$;

- if τ_1 and τ_2 are modal terms, then $Sat(\ulcorner \tau_1 =, \tau_2) \urcorner, \sigma, w)$ if and only if $\delta_{\sigma,w}(\tau_1) = \delta_{\sigma,w}(\tau_2)$ for arbitrary w;

- if $\tau_1, \ldots \tau_n$ are non-modal terms none of which lacks a $\delta_{\sigma,w}$-assignment, then $Sat(\ulcorner \mathrm{F}_i^n(\tau_1, \ldots, \tau_n) \urcorner, \sigma, w)$ if and only if $\langle \delta_{\sigma,w}(\tau_1), \ldots, \delta_{\sigma,w}(\tau_n) \rangle$ is in the w-extension of $\ulcorner \mathrm{F}_i^n \urcorner$;

- if $\tau_1, \ldots \tau_n$ are non-modal terms at least one of which lacks a $\delta_{\sigma,w}$-assignment, then not-$Sat(\ulcorner \mathrm{F}_i^n(\tau_1, \ldots, \tau_n) \urcorner, \sigma, w)$;

- if v is a modal term, $Sat(\ulcorner \mathrm{B}_i(v) \urcorner, \sigma, w)$ if and only if $B_i(\delta_{\sigma,w}(v))$, where $\ulcorner \mathrm{B}_i \urcorner$ is intended to express B_i-ness;

- if v is a non-modal variable, $Sat(\ulcorner \exists v(\phi) \urcorner, \sigma, w)$ if and only if there is an individual z in the domain of w such that $Sat(\phi, \sigma^{v/z}, w)$, where $\sigma^{v/z}$ is just like σ except that it assigns z to v;

- if v is a modal variable, $Sat(\ulcorner \exists v(\phi) \urcorner, \sigma, w)$ if and only if there is an a-world z such that (a) any empty constant-letter which is assigned a referent by w is assigned the same referent by z, and (b) $Sat(\phi, \sigma^{v/z}, w)$, where $\sigma^{v/z}$ is just like σ except that it assigns z to v;

- $Sat(\ulcorner \neg\phi \urcorner, \sigma, w)$ if and only if it is not the case that $Sat(\phi, \sigma, w)$;

- $Sat(\ulcorner \phi \wedge \psi \urcorner, \sigma, w)$ if and only if $Sat(\phi, \sigma, w)$ and $Sat(\psi, \sigma, w)$;

- $Sat(\ulcorner [\phi]_u \urcorner, \sigma, w)$ if and only if $Sat(\phi, \sigma, \sigma(u))$.

Finally, we say that a formula ϕ is *quasi-true* if and only if $Sat(\phi, \sigma)$ for any variable assignment σ.

A notion of admissibility is needed to rule out a-worlds that fail to represent the world in intelligible ways. It is not the aim of this appendix to supply a reductive characterization of admissibility, but the following will do for the purposes of the main text: an a-world w is admissible if and only if for any predicates $\phi(x_1, \ldots, x_n)$ and $\psi(x_1, \ldots, x_n)$ with no modal variables and any variable assignment σ assigning w to v:

> if $\ulcorner \phi(x_1, \ldots, x_n) \ll_{x_1,\ldots,x_n} \psi(x_1, \ldots, x_n) \urcorner$ holds and $\ulcorner [\phi(x_1, \ldots, x_n)]_v \urcorner$ is quasi-satisfied by σ, then $\ulcorner [\psi(x_1, \ldots, x_n)]_v \urcorner$ is quasi-satisfied by σ;

where '$\phi(x_1, \ldots, x_n) \ll_{x_1,\ldots,x_n} \psi(x_1, \ldots, x_n)$' is read 'part of what it is for $x_1, \ldots, x_n$ to bear ϕ to each other is for them to bear ψ to each other'. (For a more refined notion of admissibility, see my 'An Account of Possibility'.)

Once a suitable notion of admissibility is on board, it is straightforward to characterize truth and satisfaction for $\mathfrak{L}^w$. Satisfaction is the special case of quasi-satisfaction in which attention is restricted to admissible a-worlds, and truth is the special case of quasi-truth in which attention is restricted to admissible a-worlds.

Finally, we characterize triviality. The primary condition corresponding to a formula $\phi(w)$ is trivial if and only if $\phi(w)$ is satisfied by every admissible a-world.

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