Sentence, Proposition and Identity

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0. Introduction

The aim of this paper is to study the notions of sentence and proposition and the relation between them mainly from the point of view of the notion of identity that can be applied to them. In a first part we deal with this problem at the level of logical language, and in a second part at the level of natural language.

1. Logical language

1.1. Proposition, sentence and inscription

Quine has argued in favor of the rejection of propositions. His argument can be summarized by the following syllogism:

$$\frac{\text{No entity without identity} \qquad \text{There is no criterion of identity for propositions}}{\text{No propositions}}$$

Quine has been quite influential in the replacement of the expression *propositional logic* by the expression *sentential logic* to denote zero-order logic.

But do we have a criterion of identity for sentences? And what is a sentence? Some people prefer sentences to propositions, because sentences seem more concrete and definite objects. Nobody has “seen” a proposition, propositions have no smell and no color. But what about sentences? Some people confuse sentences with inscriptions. It was already noted long time ago by Lesniewski that a sentence is not a material object, but a set of equiform inscriptions. So sentences do not either have smell or color.

Is equiformity a satisfactory criterion of identity? In a previous paper (Béziau, 1998), we have discussed this question and pointed out the difficulty of *defining* equiformity. One thing is to have an operating criterion, another to be able to properly define a relation of identity. And one can have an operating criterion without knowing anything fundamental about identified objects.

Let us emphasize furthermore that anyway equiformity is a relation between inscriptions and not between sentences. At the end, what are the objects of zero-order logic: sentences or inscriptions? Should we be more Quinean that Quine and speak about *inscriptional logic*?

In fact we will argue that the objects of zero-order logic are neither sentences nor inscriptions, but something which is not erroneous to call propositions.

If one considers an inscription as a material object and a sentence as a set of equiform inscriptions, a sentence is already an abstract non material object. And in some sense such an abstract object is not less abstract than any set of inscriptions, in particular a set of non equiform inscriptions having the same meaning. Therefore if we define a proposition as a set of equisignificant inscriptions, we have an object which has a degree of abstraction comparable to the sentence's one. So defending the notion of proposition, we are not less materialist than the supporter of sentences.

Let us explain now why we don't think that logicians deal essentially with sentences. Consider the following formula of zero-order logic: $(p \to p)$. This formula in *Principia Mathematica* is written as follows: $p \supset p$. Using Lukasiewicz's notation, we can write this formula: *Cpp*. Each of the three above formulas can be viewed as a sentence, as a set of equiform inscriptions. But these three formulas are obviously not equiform, under any reasonable conception of equiformity. On the other hand most of the people think that they are the *same* formula.

Some would say that the difference between the three formulas is just a matter of *notation*. But what does this mean exactly? Is this difference of notation just like a *switching*, say like if we decide to use the symbol "!" instead of the symbol "$" for the dollar? Or is it more like a *translation*, a difference between two languages? Like the difference between "*p* implies *p*" and "*p* implique *p*"?

There is a natural way to solve the question of identity between these three formulas, it is simply to say that they represent the same logical proposition. But what is the logical proposition behind these three formulas? What is a logical proposition?

1.2. Free propositions

The set of zero-order formulas is constructed by recursion from a set of atomic formulas with some basic connectives. This forms a mathematical structure *F* known as an *absolutely free algebra*. There are various ways to define this notion. A simple definition is a categorical one: an absolutely free algebra is the initial object of the category of all algebras of similar type. (For more details about this notion see Béziau, 1999).

The way we deal with formulas in zero-order logic depends on this very structure. For example when we prove a theorem by induction on the complexity of the formula, the notions of *induction* and *complexity* are related to the concept of absolutely free algebra.

The above formulas are the same because they represent the same object of the absolutely free algebra. Shall we say that a logical proposition is simply an element of an absolutely free algebra? Let us rather call such an element a *free proposition*.

We can *represent* the same free proposition by objects which may have different properties. To understand this let us make a comparison with arithmetic. In fact, as it is known, arithmetic is also an absolutely free algebra (with only one generator, zero, and one function, successor). Elements of this structure, i.e. numbers, can be represented in different ways: e.g. using the Arabic system, or the Roman system. Important differences between these systems appear clearly at the level of manipulation of the numbers, in particular in the elaboration of algorithms for multiplication and addition. So the same number, e.g. the number seven, can be represented by different objects which have different properties. Does it mean that there are several numbers seven? Or that simply there are different ways of *writing* the same number?

What happens is that additional features can be added which facilitate the manipulation of numbers. These additional features are not considered as essential properties of numbers, but they are not either only "notational" features by opposition to "real" features. The same happens with formulas of zero-order logic.

One can wonder in which sense this question of "notation" is related to a material aspect of the objects under consideration. Nobody can seriously defend the position according to which the number seven is a set of equiform inscriptions for two reasons: the same number seven can be represented with two different notations which are not equiform (e.g. "7" and "VII"), moreover the basic properties of the number seven as an element of the absolutely free Peano algebra do not reduce to any material characterization.

However we can represent numbers by material objects and use the properties of these material objects to manipulate numbers. For example we can represent, let us say the first one hundred non zero numbers, by one hundred sticks of wood of different lengths, the number one being a stick of one centimeter, the number two a stick o two centimeters, etc. With these sticks one can perform addition: given two stick-numbers, to calculate their sum is to find the stick-number which has the same length as the two stick-numbers put together. In such a situation numbers are material objects and addition looks in this sense rather as a physical activity than a mental activity.

But we cannot say that numbers are really material objects, that they are sticks of wood: in this case the number seven could burn. The Oulipian writer R.Queneau showed the absurdity of a strict materialist view of numbers in a little essay entitled “Brief remarks on the aerodynamic properties of addition” (Queneau, 1950), in which the wind can influence the operation of addition.

One can use different representations of abstract objects like numbers and logical formulas, including representations with material features which will be used in a crucial way. But a specific representation must not be confused with the original object.

In fact Lukasiewicz, during a lecture in the twenties, represented formulas by geometrical colored figures, and used the properties of these figures to reason about formulas (cf. Tarski, 1936). Before him logicians already used diagrams, like Venn’s diagrams. Recently the study of logic via diagrams has taken a new start, people pointing out that this kind of representations are not necessarily only heuristical (see e.g. Barwise and Hammer, 1994).

At the end we must keep in mind that, in the case of numbers as well as for formulas of zero-order logic, first, all properties cannot be represented by material features, second, given a material representation, all aspects of it are not relevant.

When one turns to the question of identity, one can see that the notion of identity between abstract objects can be defined in a way not more problematic that the notion of identity between material objects. One can for example easily and rigorously define several notions of identity between free propositions which lead to several other notions of proposition.

1.3. Boolean propositions and other propositions

If we consider a relation of consequence ④ between formulas of zero-order logic, we can define a relation of logical equivalence between formulas, which is in fact a congruence relation. Factoring the structure by this congruence relation leads to a Boolean algebra.

One may prefer to call a proposition, an element of the Boolean algebra. This is the option of Halmos. He calls an element of the absolutely free algebra, a sentence, and an element of the Boolean algebra, a proposition:

> In analogy with calling an element of S* a sentence, an element of A* may be called a proposition. (The word "proposition" is not always defined this way. The intuitive reasons for using the definition are obvious, but, admittedly, they are not overwhelming). (Halmos, 1956, p.14)

What Halmos calls a sentence, we have called it a free proposition. What Halmos simply calls a proposition, we will call it a *Boolean proposition*.

The relation of logical equivalence is a congruence relation, the strongest one in fact, but there are other ones. For example one may want only to identify formulas with multiple negations, to reduce to two propositions: p and $\neg p$. Or to identify only repetitions of conjunction and disjunction, in order to identify a formula like $(p \wedge p)$ with $((p \wedge p) \wedge p)$. In fact generally logicians use conventions to write formulas which correspond to implicit congruence relations, they write for example $(p \wedge p \wedge p)$ identifying $((p \wedge p) \wedge p)$ with $(p \wedge (p \wedge p))$, using the congruence based on the associativity of conjunction.

Given the structure <*F*; ④>, there are a whole hierarchy of congruence relations, the lattice of congruences. In our view the elements of any factor structure can be called propositions, and also the elements of *F*, in particular due to the fact that <*F*;④> can be considered as the structure <*F*; ④> factored by the trivial congruence relation.

Let us have a look at two additional interesting related cases. Until now we have deal only with classical zero-order logic.

Let us consider da Costa's paraconsistent zero-order logic C1. In this logic, there are no non-trivial congruence relations, it is a simple structure (see Béziau, 1997). Therefore the only propositions are free propositions.

At this point it is important to emphasize that we are not making a real difference between free propositions considered as objects of the structure *F*, and free propositions considered as objects of the structure <*F*; ④>. Maybe one might want to call sentences objects of the structure *F* taken alone by opposition with objects of the full structure <*F*; ④> which only would be named propositions. This would be a bit artificial. It is true that the distinction between the structures *F* and <*F*; ④> is important but at the same time *F* makes sense only relatively to a more complex structure. In the same way it would not really make sense to consider Boolean propositions independently of a relation between them.

The traditional presentation of zero-order logic is somewhat confusing. The definition of the set of formulas is classified together with proof-theoretical systems as *syntax*, by opposition to *semantics* consisting of truth-tables. It is in fact better to replace this classification syntax/semantics by the classification morphology/proof-theory/model-theory, where the morphology is the theory dealing only with the definition of the set of formulas. For us, the relation ④ here is no specifically considered as being defined by a proof-theoretical system or by semantical means like truth-tables. It has to be considered as a relation in a structure just like the relation of order 🖮 in the structure <🕐; 🖮>.

The structure of the set of formulas of classical first-order logic is a also a kind of absolutely free algebra. Kleene defines a relation of congruence between first-order formulas (cf. Kleene, 1952), that we can call *Kleene congruence*, which consists in identifying formulas which differ only by the names of the variables: for example the formula $\forall x\varphi x$ is Kleene congruent to the formula $\forall y\varphi y$. This leads to what we can call *Kleene proposition*, or *Bourbachic proposition*, since Bourbaki adopts a devise in order to identify directly Kleene congruent formulas, i.e. he constructs directly the structure of Kleene propositions without going through the standard free propositions of first-order logic.

The first-order extension C1* of the paraconsistent logic C1 is also a simple structure. In particular two formulas differing only by the names of the variables are not logically equivalent. A reasonable way to identify two Kleene congruent formulas in this case is to define C1* directly with a set of Bourbachic propositions.

Bourbachic propositions are not often used because they are hard to write. In other words: it is difficult to find sentences to represent them, but it is not impossible. On the other hand they are very useful at the metatheoretical level. To consider Bourbachic propositions rather than the standard free propositions simplifies the (meta)proofs even in classical logic.

1. 4. Proposition, sentence and meaning

The relation between logical sentences and logical propositions is ambiguous because some features of propositions are related to inscriptions and some other not. The notion of free proposition seems related to the notion of inscription. In natural language sentences like:

John is not not not not happy

are not considered. Neither are they considered in the standard language of mathematics. In fact an absolutely free algebra is sometimes called a *words algebra*. In some sense a free proposition can be seen as an abstract representation of a sequence of material inscriptions.

On the other hand the structure of logical propositions is not abstracted from inscriptions. To define a first-order formula we must distinguish between connectives, quantifiers, variables, and it would be hard to say that we “abstract” these notions from inscriptions.

In the history of zero-order logic, the objects considered have varied and the final option motivated by formalism is only one possible solution. It is interesting to note that during a lot of time the objects of zero-order logic were of a quite floating nature. For example if one reads carefully Wittgenstein’s *Tractacus*, he will see that what Wittgenstein calls *Satz* is rather a Boolean proposition than a free proposition.[1] The exact distinction and correlation between free propositions and Boolean propositions became clear only with the work of Tarski and the introduction of the so-called Tarski-Lindenbaum algebra (cf. Tarski, 1935).

Anyway, we can say that logical propositions are abstract objects a some mathematical structure, be it a structure of type <*F*; ④>, or a Boolean algebra or even simply the structure *F*. These objects are represented by signs. These signs are sentences, sets of equiform inscriptions. In this sense a sentence is a name for a proposition, and a logical proposition is a meaning of a sentence. Therefore our terminology, logical *proposition*, corresponds to one of the standard definition of the notion of proposition, according to which a proposition is a meaning of a sentence.

Quine argues that it is against common sense to say that a sentence is a *name* of some *thing*:

> We do not, e.g., have occasion to observe that 'Boston is east of Chicago' and 'Chicago is west of Boston' are (or are not) two names for the same proposition; indeed , whereas we may have occasion to reflect that 'Boston' is the name of a city, we do not have occasion to regard 'Boston is east of Chicago' as a name of anything whatever. (Quine, 1934, p.58)

Maybe it is strange to say that a sentence is a *name*, but it is not at all against common sense to say that "Boston is east of Chicago" and "Chicago is west of Boston" *means* the same *thing*, or have the same *meaning*. If a sentence *means* something, we can say that it is a sign. When we say that a sentence is a name of a proposition, we just want to say that it is a sign (or a group of signs) which represents a proposition.

[1] According to 5.14, "if *p* follows from *q* and *q* from *p*, then they are one and the same proposition (Satz)". The definition of "follow" is given in 5.11: *p* follows from *q*, when *p* is a logical (semantical) consequence of *p*. Therefore 5.14 means that two propositions are the same when they are logically equivalent.

2. Natural language

Logical propositions are specific propositions. It is much more difficult to give an account of propositions in general. One reason is that it is more difficult to specify the structure where the propositions appear. A second reason is that the distinction between proposition and sentence is not so clear in natural language. The concept of proposition as discussed by philosophers is not indeed a concept of natural language, linguistics or grammar. In a basic English dictionary the notion of proposition as meaning of sentence is not ever mentioned. We find the two following meanings (the second meaning seems to be a special case of the first):

1. Anything presented for the purpose of discussion or evaluation.
2. A suggestion or proposal for sexual relations.

The concept of sentence used by linguists, not conceived via the duality sentence/proposition, does not have therefore the same meaning as in logic. Logicians have developed formal languages but have not yet developed a theory of natural language, even if their formal constructions may have influenced the study of natural language. Distinctions made by logicians like the very famous Fregean distinction between one sentence and the assertion of this sentence, expressed with the symbol ④, are not used in the standard analysis of natural language by linguists. Maybe it is because these distinctions do not really make sense in this context.

2.1. Translations between logic and natural language

In many books of logic exercises of translations between natural language and logical language are presented. However there are expressions of natural language that seem difficult to translate. Let us consider for example the following one:

Is it true that I dreamt that most of the monkeys were intelligent enough to get it done quickly, possibly sooner that I thought?

Questions are not treated in classical logic, but logics that deal with questions have been developed, they are the so-called erotetic logics. One may think that other aspects of this expression not grasped by classical logic can be grasped by other non-classical logics. So someone could say that it is not possible to translate this expression in first-order logic but that it is possible to translate it using a non-classical logic, for example a temporal modal non-monotonic erotetic polar turbo linear third-order logic with equality. Such a non-classical logic would be constructed by using advanced techniques that have been developed recently in order to combine logics.

It seems however that this is the wrong approach. The basic feature of a logical structure is the consequence relation. And it is clear that in natural language a proposition is not necessarily considered from the point of view of logical consequence.

In logic there is a close connection between the grammar of the proposition which is given by the absolutely free algebra *F* and the relation of consequence ④. The meaning of the so-called logical constants (connectives and quantifiers) is given by the whole structure <*F*;④>. The rest is left undetermined.

One could say that a logical proposition is a kind of logical skeleton of a proposition. So that logical propositions are not different propositions, but reflect only *one* aspect of *some* propositions of natural language. But what is a full proposition?

2.2. Grammatical rules and structures for propositions

In linguistics, a *sentence* is a compound of words, each word belonging to a certain category: noun, adjective, article, verb, etc. It is difficult to argue that for the linguist a sentence reduces to a set of equiform inscriptions. The standard definition of a sentence is ambiguous because it is a mix of grammatical and "material" features:

> A grammatically self-contained unit consisting of a word or a syntactically related group of words that expresses an assertion, a question, a command, a wish, or an exclamation that in writing usually begins with a capital letter and concludes with appropriate end punctuation, and that in speech is phonetically distinguished by various patters of stress, pitch and pauses. (Webster)

If one says that :

Drive likes apples

is not a grammatical correct sentence it is because the verb "drive" cannot be a subject of the verb "likes". The rules of grammar are rules for the constructions of *meaningful sentences*. But what is a meaningful sentence? A set of equiform inscriptions expressing a meaning? Or simply a proposition?

A possible way to understand what is the nature of meaningful sentences in natural language is to see how these are defined. First, a meaningful sentence must be grammatically correct. We already face a difficulty here, because as Suppes puts it: "There is nothing like an adequate grammar of any natural language" (Suppes, 1986b, p.53).

Linguists have tried to imitate logicians, they have tried to find sets of rules which generate meaningful sentences, they have developed generative grammars inspired by the construction of the set of formulas in logic.[2] But it is not obvious that we can treat natural language in the same way as logical language. In logic the rules which produce the absolutely free algebra of formulas make sense not by themselves but relatively to the consequence relation. The consequence relation is a relation between formulas (and

[2] According to Corcoran (1983, p.xx): "It is to be regretted that many linguists, philosophers, and mathematicians know so little of the history of the methodology of deductive science that they attribute the basic ideas of generative grammar to linguists working in the 1950s rather than to Tarski (and other logicians/methodologists) working in the early 1930s".

relations of congruence between formulas are defined using this relation). In natural language it is not clear which relations between meaningful sentences one should consider, or even if one should consider relations at all.

Grammatical correctness in natural language is a necessary condition, but not a sufficient one. For example, following the common sense and H.B.Curry, the sentence below is grammatically correct but seems meaningless:

Sincerity admires John.

According to Curry (1963, p.170), "Chomsky maintains that this is not a grammatical English sentence".

Logical positivists, following Wittgenstein, have argued that metaphysical sentences are meaningless. They have tried to argue that these sentences are meaningless *because* they are not syntactically correct (see in particular Carnap, 1932). But someone who wants to show that the above sentence or Hegel's metaphysical sentences are not syntactically correct, should provide grammatical rules according to which it is not possible to form such sentences.

This is not what logical positivists or logicians have done. Instead of this, they have created artificial languages in which most of the sentences of natural language cannot be translated. But these artificial languages do not solve the problem of meaning of the above sentence or sentences alike. Nothing forbids to consider that the above sentence is an atomic formula of zero-order logic.

Regarding Chomsky and other linguists, it doesn't seem that they have either produce such set of grammatical rules.

The question if it is possible to find such rules is still open. Anyway it seems that the meaning of a sentences is not only an "internal" property that one can get by constructing the sentence by rules from more elementary units, but has also to do with the relations between sentences (logical consequence and others).

2.3. Suppes's congruency theory of proposition

In a series of paper (Suppes, 1973, 1984, 1986a, 1986b), Patrick Suppes has developed a congruency theory of proposition. He summarizes it himself as follows:

> The main ingredients of the theory I propose are these: the concrete token utterance, which should in general be thought of as a sound-pressure wave uttered in a given context; an infinite hierarchy of congruence relations between utterances; and the creative definition of identity for propositions. Two propositions are identical just when the utterances from which they are (abstracted) are congruent. This means that the concept of identity of propositions is relative to a congruence relation, as in the standard condition of identity for congruence classes in algebra.
>
> The important point is that an utterance does not assert a single proposition but an infinite hierarchy of propositions. The choice of the proposition that is asserted depends upon the congruence relation under consideration of interest. Formally the creative definition of identity for propositions has the following form:
>
> (I) $[u]=[u']$ iff $u \cong u'$
>
> In (I), u and u' are two concrete utterances, and $[u]$ is the proposition expressed by u, or more explicitly, $[u]$ is the proposition expressed by u under the congruence relation $\cong$. (Suppes, 1986a, p.31)

Suppes's theory has several very interesting features. The first one is the substitution of the notion of congruence to the notion of identity, in the discussion about propositions. Without doubt this kind of distinction is the first step towards a subtler kind of philosophy. A serious discussion about or involving identity must deal with congruence (see Béziau, 2001). One fundamental consequence of Suppes's theory is the plurality of the theory: several notions of congruence of meaning lead to several notions of propositions. Another important feature is that this theory in some sense demystifies propositions by presenting them as abstract objects like any other ones. This point is illustrated by the following discussion between Barcan and Suppes:

> Mme Barcan-Marcus: I think it is misguided to pursue the question of “identity” conditions for propositions. Various weaker and stronger equivalences are appropriate and propositions are not the sort of “entities” which enter appropriately into the identity relation.
>
> M.Suppes: To the question of whether it is appropriate to ask for a criterion of identity for propositions my brief response is that the case seems as good as for other standard abstract objects. (Suppes, 1986a, p.299)

The basic idea of Suppes’s theory is in fact very general and can be applied to any kind of objects. One can say that any type of objects is the product of a hierarchy of congruence relations in such way that one can replace Quine’s slogan “no entity without identity” by the slogan “no entity without congruence”. The further step is to see how this works in a particular case, in the present discussion, how it works in the case of proposition.

First let us examine the distinction between sentence and proposition in the light of Suppes’s theory. Suppes doesn’t mention the notion of sentence but according to his theory, a spoken sentence, defined as a set of similar sound-pressure waves, is a particular case of proposition, since these waves have the same meaning. Are sentences propositions? In fact it seems difficult to characterize the notion of spoken sentence, because the “same” sound-pressure wave with a different intonation can mean something else. Suppes insists on the prosodic features which are sometimes neglected (see Suppes, 1986b).

The concept of spoken sentence is not clear because it is not clear from which kind of congruence relations we get sentences from sound-pressure waves. If one wants to define a sentence, as a conjunction of written inscriptions and sound-pressure waves the situation is even more difficult. Because one has to define a relation between these two kinds of entities, introducing features such as reading and pronunciation. Saussure has noticed the surprising fact that languages which are not written have more stability. This phenomenon is explained by the interaction between speech and writing which generates changes on both sides. A sentence of a language which is both written and spoken is an entity which has a weak stability through time, and therefore is even more difficult to characterize. Sentences turn to be relatively easy to define, only if we consider just a written language, as do the logicians.

It is important to note that if we consider the trivial congruence relation between sound-pressure waves as a meaning-congruence relation, then an utterance of a sound-pressure wave can itself be considered as a proposition. It can make sense if we consider that there are no two identical ways to pronounce the same sentence, and that these different pronunciations have different meanings. From this point of view it is not possible to make a serious distinction between sentences and propositions.

Suppes's theory puts everything at the same ontological level. Basic entities are given, let us say sound-pressure waves, and congruence relations between sound-pressure waves lead to propositions.

If we apply Suppes's theory to written languages rather than to spoken languages we have inscriptions and a proposition is any set of inscriptions which is a meaning-congruence class between inscriptions. From this point of view propositions are at the same level as sentences conceived as sets of equiform inscriptions.

The striking feature of Suppes's theory is the plurality of the notion of proposition. Whether such or such congruence relation between sound-pressure waves has to be taken as a meaning-congruence, and whether such or such objects which are the product of a congruence relation are rather propositions than sentences can be the subject of endless philosophical discussions. The difference between sentences and propositions is maybe rather continuous than discrete. The idea of Suppes is that there is not one right concept of proposition:

> After examining a number of congruence relations, I reiterate my opening claim that there is an infinite hierarchy. As we ascend this hierarchy there is no natural stopping place to select *the* concept of synonymy or sameness of meaning. Correspondingly, there is no natural point at which to abstract the proposition that *expresses* the meaning of an utterance. There is not one natural concept of meaning but many, with the choice to determined by context and purpose. (Suppes, 1986a, p.289)

According to Suppes's theory, propositions and sentences are abstract objects; they are abstracted from empirical data: sound-pressure waves or inscriptions.

But an important distinction has to be made: a congruence relation between objects like inscriptions can be based on features which are related to the material aspects of these objects, like equiformity, or on some more abstract features, like grammatical rules. The abstract features can be connected or not with material features. For example, an article is a short inscription, but a distinction between two grammatical categories like verb and adjective can hardly be defined on the basis of material features. And this distinction is certainly fundamental for the definition of a notion of meaning-congruence.

We have indeed a similar situation here to the situation of logic, where the distinction between a connective and a "proposition letter" is not founded on material features. It is

therefore difficult to claim that propositions in natural language, like in logical languages, are abstracted only from the empirical data of waves or inscriptions.

Mathematically speaking a congruence relation between some objects depends on the structure in which the objects are merged. Different congruence relations correspond to different kinds of structure, and therefore to different kinds of objects, if one admits that the nature of objects depends on the structure through which they are given.

The problem is to choose and properly define the structure and the related congruence relation, but as Suppes's points out, there is not a "right" meaning-congruence which leads to one concept of proposition, but several ones which lead to several concept of propositions.

To think that equiformity is the only right and well-defined congruence relation and that there are only sentences and no propositions seems as ridiculous as to say that there are only one well-defined congruence relation between people based on physical appearance (color of the skin, height, etc.). It is an easy solution but too simple.

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[3] The original version includes ten additional pages of discussion, which are not reprinted in (Suppes, 1991), the references of pages is to the original version.